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Monetary Policy and the Predictability of Nominal Exchange Rates

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Abstract

This paper documents two facts about the behavior of floating exchange rates in countries where monetary policy follows a Taylor-type rule. First, the current real exchange rate is highly negatively correlated with future changes in the nominal exchange rate at horizons greater than two years. This negative correlation is stronger the longer is the horizon. Second, for most countries, the real exchange rate is virtually uncorrelated with future inflation rates both in the short and in the long run. We develop a class of models that can account for these and other key observations about real and nominal exchange rates.

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1 Introduction

This paper examines the behavior of floating exchange rates in countries where monetary policy follows a Taylor-type rule. To describe our findings, it is useful to define the real exchange rate ($REER$) as the price of the foreign consumption basket in units of the home consumption basket. Also define the nominal exchange rate (NER) as the price of the foreign currency in units of the home currency.

We document two facts about real and nominal exchange rates. First, the current $REER$ is highly negatively correlated with future changes in the NER at horizons greater than two years. This correlation is stronger the longer is the horizon. For most of the countries in our sample, the current $REER$ alone explains more than 50 percent of the variance of changes in nominal exchange rates at horizons greater than four years. Second, for most countries, the $REER$ is virtually uncorrelated with future inflation rates at all horizons. Taken together, these facts imply that the $REER$ adjusts in the medium and long-run overwhelmingly through changes in *nominal* exchange rates, not through differential inflation rates. When a country's consumption basket is relatively expensive, its NER eventually depreciates by enough to move the $REER$ back to its long-run level.

We redo our analysis for China which is on a quasi-fixed exchange rate regime versus the U.S. dollar, Hong Kong which has a fixed exchange rate versus the U.S. dollar, and the euro area countries which have fixed exchange rates with each other. In all these cases, the current $REER$ is highly negatively correlated with future relative inflation rates. In contrast to the flexible exchange rate countries, the $REER$ adjusts overwhelmingly through predictable inflation differentials.

We show that our first fact about the relationship between the current $REER$ and future changes in the NER emerges naturally in a wide class of models that have two features: home bias in consumption and a Taylor rule guiding monetary policy. This result holds regardless of whether or not we allow for nominal rigidities. We make these arguments using a sequence of models to develop intuition about the key mechanisms underlying our explanations of the facts. We then study a medium-size DSGE model to assess the quantitative plausibility of the proposed mechanisms. We argue that this model can account for the relationship between the current $REER$ and future changes in inflation and the NER .

A key question is whether the models we study are consistent with other features of the data that have been stressed in the open-economy literature. It is well known that, under flexible exchange rates, real and nominal exchange rates commove closely in the short run (Mussa (1986)). This property, along with the fact that real exchange rates ($REER$) are highly inertial (Rogoff (1996)), constitute bedrock observations which any plausible open-economy model must be consistent with. We show that our medium-size DSGE model with nominal rigidities is in fact consistent with these observations.

We begin our theoretical analysis with a simple flexible-price model where labor is the only factor in the production of intermediate goods. The intuition for why this simple model accounts for our empirical findings is as follows. Consider a persistent fall in domestic productivity or an increase in domestic government spending. Both shocks lead to a rise in the real cost of producing home goods that dissipates smoothly over time. Home bias means that domestically-produced goods have a high weight in the domestic consumer basket. So, after the shock, the price of the foreign consumption basket in units of the home consumption basket falls, i.e. the $REER$ falls. The Taylor rules followed by the central banks keep inflation relatively stable in the two countries. As a consequence, most of the adjustment in the $REER$ occurs through changes in the NER . In our model, the NER behaves in a way that is reminiscent of the overshooting phenomenon emphasized

by Dornbusch (1976). After a technology shock, the foreign currency depreciates on impact and then slowly appreciates to a level consistent with the return of the RER to its steady state value. The longer the horizon, the higher is the cumulative appreciation of the foreign currency. So in this simple model the current RER is highly negatively correlated with the value of the NER at future horizons and this correlation is stronger the longer is the horizon. These predictable movements in the NER can occur in equilibrium because they are offset by the interest rate differential, i.e. uncovered interest parity (UIP) holds.

Risk premia aside, UIP holds conditional on the realization of many types of shocks to the model economy. After the realization of one of these shocks, the nominal interest differential between two countries is equal to the expected change in the nominal exchange rate. But there is another class of shocks, namely shocks to the demand for bonds, for which UIP does not hold. So, when the variance of these shocks is sufficiently large, traditional tests of UIP applied to data from our model would reject that hypothesis.

An obvious shortcoming of the flexible-price model is that purchasing power parity (PPP) holds at every point in time. To remedy this shortcoming, we modify the model so that monopolist producers set the nominal prices of domestic and exported goods in local currency. They do so subject to Calvo-style pricing frictions. For simplicity, suppose for now that there is a complete set of domestic and international asset markets. Consider a persistent fall in domestic productivity or an increase in domestic government spending. Both shocks lead to a rise in domestic marginal cost. So, when they are able to, domestic firms increase their prices at home and abroad, and inflation rises. Because of home bias, domestic inflation rises by more than foreign inflation. The Taylor principle implies that the domestic real interest rate rises by more than the foreign real interest rate. So, domestic consumption falls by more than foreign consumption.

With complete asset markets, the RER is proportional to the ratio of foreign to domestic marginal utilities of consumption. So, the fall in the ratio of domestic to foreign consumption implies a fall in the RER . As in the flexible price model, the Taylor rule keeps inflation relatively low in both countries so that most of the adjustment in the RER is accounted for by movements in the NER . Again, the implied predictable movements in the NER can occur in equilibrium because they are offset by the interest rate differential, i.e. UIP holds.

While the intuition is less straightforward, our results are not substantively affected if we replace complete markets with incomplete markets or assume local currency pricing instead of producer currency pricing.

An important question is whether empirically plausible versions of our model can account for the new facts that we document. The key tension is as follows. We require that UIP holds for the key shocks that generate the correlation between the current RER and future $NERs$. But we also require that shocks to the demand for assets be sufficiently important so that traditional tests of UIP are rejected. In addition, we want the shocks in our model to be sufficiently persistent so that, for the reasons emphasized in Engel, Mark and West (2007), $RERs$ exhibit properties that are hard to distinguish from a random walk. Finally, to be plausible our model must be consistent with the bedrock observations associated with Mussa (1986) and Rogoff (1996). We study whether an open-economy medium-size DSGE version of our model is consistent with these observations. Amongst other features, the model allows for Calvo-style nominal wage and price frictions and habit formation in consumption of the type considered in the Christiano, Eichenbaum and Evans (2005). Our key finding is that the model can simultaneously account for our two empirical facts even though exchange rates behave like random walks at short horizons, unconditional UIP fails, nominal and real exchange commove closely, and the RER is inertial.

Our work is related to three important strands of literature. The first strand demonstrates the existence of long-run predictability in nominal exchange rates (e.g. Mark (1995) and Engel, Mark, and West (2007)). Rossi (2013) provides a thorough review of this literature. Our contribution here is to show the importance of the *RER* in predicting the *NER* at medium and long-run horizons.¹ The second strand of literature seeks to explain the persistence of real exchange rates. See, for example, Rogoff (1996), Kollmann (2001), Benigno (2004), Engel, Mark, and West (2007), and Steinsson (2008). Our contribution relative to that literature is to show that we can account for the relationship between the *RER* and future changes in inflation and the *NER* in a way that is consistent with the observed inertia in *RER*. The third strand of the literature emphasizes the importance of the monetary regime for the behavior of *RER*. See, for example Baxter and Stockman (1989), Engel, Mark, and West (2007), and Engel (2012). Our contribution relative to that literature is to document the critical role that Taylor-rule regimes play in determining the relative roles of inflation and the *NER* in the adjustment of the *RER* to its long-run levels.

Our paper is organized as follows. Section 2 contains our empirical results. Section 3 describes a sequence of models consistent with these results. We start with a model that has flexible prices, complete asset markets, and where labor is the only factor in the production of intermediate goods. We then replace complete markets with a version of incomplete markets where only one-period bonds can be traded. Next, we introduce Calvo-style frictions in price setting. In Section 4 we consider an estimated medium-scale DSGE model. Section 5 concludes.

2 Some empirical properties of nominal and real exchange rates

In this section we present our empirical results regarding nominal exchange rates, real exchange rates, and relative inflation rates. Our analysis is based on quarterly data for Australia, Canada, the euro area, Germany, Japan, New Zealand, Norway, Sweden, China, and Hong Kong. We use consumer price indexes for all items and average quarterly nominal exchange rates versus the U.S. dollar.²

2.1 Regression results

We begin by describing the results obtained for countries under flexible exchange rates and in which monetary policy is reasonably well characterized by a Taylor rule. We choose the sample period for each country using the following two criteria. First, the exchange rate must be floating. Second, following Clarida, Gali and Gertler (1998), we consider periods when monetary policies are reasonably characterized by Taylor

¹Authors like Engel and West (2004, 2005) Molodtsova and Papell (2009) have proposed using variables that might enter into a Taylor rule to improve out of sample forecasting. Such variables includes output gaps, inflation, and possibly real exchange rates. Our focus is not on out-of-sample forecasting.

²We use the H.10 exchange rate data published by the Federal Reserve, available at <http://www.federalreserve.gov/releases/H10/Hist/>, Federal Reserve Board, H.10 Foreign Exchange Rates. We compute quarterly averages of the daily data. For price indexes, we use the International Monetary Fund's International Financial Statistics database (Source: International Monetary Fund), with the exception of consumer prices for Germany, China, and the euro area. For those countries, we use OECD data, which we download from FRED (Federal Reserve Economic Data, Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org/>). The series names on FRED are CPHPTT01EZQ661N for the Euro Area, DEUCPIALLQINMEI for Germany, and CHNCPIALLQINMEI for China. When we use the OECD data for one of these countries country, we also use the OECD data for the U.S. in order to construct the real exchange rate. The FRED name for the U.S. consumer price index from the OECD is USACPIALLQINMEI. OECD (2017), "Main Economic Indicators - complete database", Main Economic Indicators (database).

rules. Our sample periods are as follows: Australia: 1973-2007, Canada: 1973-2007, Germany: 1979.Q2-1993, Japan: 1979.Q2-1994, New Zealand: 1989-2007, Norway: 1973-2007, Sweden: 1973-2007, Switzerland: 1973-2007, United Kingdom: 1992.Q4-2007.³ Unless indicated otherwise, a year means that the entire year's worth of data was used.

The RER is given by:

$$RER_t = \frac{NER_t P_t^*}{P_t}, \quad (1)$$

where NER_t is the nominal exchange rate, defined as U.S. dollars per unit of foreign currency. The variables P_t and P_t^* denote the domestic and foreign price levels, respectively.

Figures 1 through 10 show, for each country, scatter plots of the $\log(RER_t)$ against $\log(NER_{t+j}/NER_t)$ for different horizons, j . The maximal horizon (J) is country specific, equaling 5 or 10 years. Our rule for setting J to either 5 or 10 years is that we have at least one non-overlapping data point that exceeds that horizon. So, for example, for Canada $J = 10$ years, but for the U.K., $J = 5$ years. For countries where $J = 10$ years, we display the scatter plots at one, three, seven and ten year horizons. For countries where $J = 5$ years, we display the scatter plots at one, two, three and five year horizons.

Two features of these figures are worth noting. First, consistent with the notion that exchange rates behave like random walks at high frequencies, there is no obvious relationship between the $\log(RER_t)$ and $\log(NER_{t+j}/NER_t)$ at a one-year horizon. However, as the horizon expands, the correlation between $\log(RER_t)$ and $\log(NER_{t+j}/NER_t)$ rises. For the countries for which we have the most data, so that $J = 10$ years, the negative relationship is very pronounced at longer horizons.

We now discuss results obtained from running the following NER regression:

$$\log\left(\frac{NER_{t+j}}{NER_t}\right) = \beta_{0,j}^{NER} + \beta_{1,j}^{NER} \log(RER_t) + \epsilon_{t,t+j}, \quad (2)$$

for $j = 1, 2, \dots, J$ years. Panel A of Table 1 reports estimates and standard errors for the slope coefficient $\beta_{1,j}^{NER}$ obtained using data from flexible exchange rate countries.⁴ A number of features are worth noting. First, for every country and every horizon, the estimated value of $\beta_{1,j}^{NER}$ is negative. Second, for almost all countries, the estimated value of $\beta_{1,j}^{NER}$ is statistically significant at three-year horizons or longer. Third, in most cases the estimated value of $\beta_{1,j}^{NER}$ increases in absolute value with the horizon, j . Moreover, $\beta_{1,j}^{NER}$ is more precisely estimated for longer horizons.

Panel A of Table 2 reports the R^2 s from the fitted regressions. Consistent with the visual impression from the scatter plots, the R^2 s are relatively low at horizons of one year but rise with the horizon. Strikingly, for the longest horizons the R^2 exceeds 50 percent for all countries except for Japan (where it is 40 percent) and it is almost 88 percent for Canada.

Taken together, the results in Figures 1 – 10 and Table 1 strongly support the notion that, for flexible exchange rate countries where monetary policy is reasonably well characterized by a Taylor rule, the current RER is strongly correlated with changes in future nominal exchange rates, at horizons greater than roughly two years.

³We exclude France and Italy because the Clarida, Gali and Gertler (1998) dates would give us only 6 years of data for France and 8 years of data for Italy. These years include steep declines from very high initial inflation rates that are hard to reconcile with a stable Taylor-rule regime. Our data for the U.K. starts in 1992 to exclude the period in which the British pound was part of the Exchange Rate Mechanism of the European Monetary System.

⁴We compute standard errors for a generalized method of moments estimator of β_1 using a Newey-West estimator of the optimal weighting matrix with the number of lags equal to two quarters more than the forecasting horizon.

We now consider the relative-price regression:

$$\log\left(\frac{P_{t+j}^*/P_{t+j}}{P_t^*/P_t}\right) = \beta_{0,j}^\pi + \beta_{1,j}^\pi \log(RER_t) + \epsilon_{t,t+j}. \quad (3)$$

This regression quantifies how much of the adjustment in the RER occurs via changes in relative rates of inflation across countries. Panel A of Table 3 reports our estimates and standard errors for the slope coefficient $\beta_{1,j}^\pi$. In most cases, the coefficient is statistically insignificant and in some cases it is negative instead of positive. Panel A of Table 4 reports the R^2 s of the fitted regressions. Notice that the regression R^2 s are all much lower than the corresponding R^2 s from regression (2). As a whole, these results are consistent with the view that, for these countries, very little of the adjustment in the RER occurs via differential inflation rates.

We now redo our analysis for China, which is on a quasi-fixed exchange rate versus the U.S. dollar, and Hong Kong, which has a fixed exchange rate versus the U.S. dollar. The results are shown in Panel B of Table 3. The sample period is from 1985 to 2007 for Hong Kong and 1994 to 2007 for China. We also use data over the period 1999 to 2016 for France, Ireland, Italy, Portugal, and Spain where the RER and relative inflation rates are defined relative to Germany. The results for these countries are shown in Table 5. Two features of Panel B of Table 3 and Table 5 are worth noting. First, the estimated values of $\beta_{1,j}^\pi$ in equation (3) are statistically significant for every country at every horizon. Second, the estimated value of $\beta_{1,j}^\pi$ rises with the horizon, j . Panel B of Table 4 and Table 5 show that the regression R^2 s increase with the horizon. Interestingly, the 5 year R^2 s are very high, exceeding 79 percent for all euro area countries with a peak value of 93 percent for Portugal.

2.2 Power considerations

In the previous subsection we argued that for countries under flexible exchange rate regimes, changes in the NER at long horizons display a strong negative correlation with the current level of the RER . A potential problem with this claim is that it is based on the use of sample sizes that are short relative to the horizon of the regressions. A similar issue arises in the literature that uses regressions to argue that the equity premium is predictable at long-run horizons based on price-dividend ratios on equity return predictability. Authors like Stambaugh (1999) and Boudoukh, Richardson and Whitelaw (2006) argue that these regressions which are based on overlapping samples are no more informative than the corresponding short-horizon regressions. In their view the equity premium is plausibly a random walk and is not predictable based on price-dividend ratios. Cochrane (2008) suggests a series of diagnostics to evaluate these claims. In this subsection we report results based on those diagnostics to examine the statistical significance of our regressions findings.

Suppose that $\log(RER)$ has an AR(1) time series representation. Then the trivariate vector time series $X_{t+1} = \{\log(NER_{t+1}/NER_t), \log\left(\frac{P_{t+1}^*/P_{t+1}}{P_t^*/P_t}\right), \log(RER_{t+1})\}$ evolves according to

$$\begin{aligned} \log(NER_{t+1}/NER_t) &= \beta_{0,1}^{NER} + \beta_{1,1}^{NER} \log(RER_t) + \epsilon_{t,t+1}^{NER} \\ \log\left(\frac{P_{t+1}^*/P_{t+1}}{P_t^*/P_t}\right) &= \beta_{0,1}^\pi + \beta_{1,1}^\pi \log(RER_t) + \epsilon_{t,t+1}^\pi \\ \log(RER_{t+1}) &= a_{RER} + \rho_{RER} \log(RER_t) + \epsilon_{t,t+1}^{RER}. \end{aligned} \quad (4)$$

The definition of the RER implies a set of cross-equation restrictions on the coefficients of (4). Since

$$\log(NER_{t+1}/NER_t) = \log(RER_{t+1}) - \log\left(\frac{P_{t+1}^*/P_t^*}{P_{t+1}/P_t}\right) - \log(RER_t)$$

we have that

$$\beta_{1,1}^{NER} = -1 + \rho_{RER} - \beta_{1,1}^\pi$$

and

$$\epsilon_{t,t+1}^{NER} = \epsilon_{t,t+1}^{RER} - \epsilon_{t,t+1}^\pi.$$

Under the null hypothesis that $\log(NER)$ is a random walk we can re-write (4) as

$$X_{t+1} = \begin{bmatrix} \log(RER_{t+1}) \\ \log\left(\frac{P_{t+1}^*/P_t^*}{P_{t+1}/P_t}\right) \\ \log(NER_{t+1}/NER_t) \end{bmatrix} = \begin{bmatrix} \rho_{RER} \\ -1 + \rho_{RER} \\ 0 \end{bmatrix} \log(RER_t) + \begin{bmatrix} \epsilon_{t+1}^{RER} \\ \epsilon_{t+1}^\pi \\ \epsilon_{t+1}^{RER} - \epsilon_{t+1}^\pi \end{bmatrix}. \quad (5)$$

If $|\rho_{RER}| < 1$, and the $\log(NER)$ is a random walk, then, after a shock, relative inflation rates must move in such a way so as to eventually bring the RER back toward its unconditional mean. This observation explains why the coefficient on $\log(RER_t)$ in the second of (5) is equal to $-1 + \rho_{RER}$.

One way to test the random walk hypothesis using short-run regressions is as follows. First, estimate ρ_{RER} using data for the RER from a given country. Second, using that estimate of ρ_{RER} , back out a sequence for ϵ_{t+1}^π so that the second equation in (5) holds for all t . Third, using the fitted disturbances for ϵ_{t+1}^{RER} and ϵ_{t+1}^π , construct a large number of synthetic times series for X_{t+1} , each equal in length to the sample size of our actual time series. Fourth, estimate $\beta_{1,1}^{NER}$ and $\beta_{1,1}^\pi$ on each of the artificial time series by running regressions (2) and (3). Finally, examine how likely it is in the synthetic time to obtain values of $\beta_{1,1}^\pi$ as large as those that we obtain using the actual data.

Table 6 reports our results. With two exceptions the percentage of values of $\beta_{1,1}^\pi$ that are as large as those estimated using the actual data is extremely small. This pattern does not hold for Japan and the Euro area. In the latter case, we estimate a value of ρ_{RER} that is greater than one, so it is easy to generate positive values of $\beta_{1,1}^\pi$ using data generated from (5). For the case of Japan, we estimate a value of ρ_{RER} very close to 1, so it is relatively easy to generate positive values of $\beta_{1,1}^\pi$ using simulated time series.

Cochrane (2008) proposes a different test of the random walk hypothesis for equity returns. His procedure uses the long-horizon coefficients of a regression of equity returns on the past price-dividend ratios. We adopt his test to our setting. Recall that $\beta_{1,\infty}^{NER}$ denotes the regression coefficient of $\log(NER_{t+\infty}/NER_t)$ on $\log(RER_t)$. Assuming that the system evolves according to (4), we have that

$$\beta_{1,\infty}^{NER} \equiv \frac{\beta_{1,1}^{NER}}{1 - \rho_{RER}}.$$

Under the random walk hypothesis, $\beta_{1,\infty}^{NER} = 0$. Table 7 reports the point estimates of $\beta_{1,\infty}^{NER}$ implied by joint estimating $\beta_{1,1}^{NER}$ and ρ_{RER} using the first and third equations of (4). In addition we report the asymptotic standard errors for $\beta_{1,\infty}^{NER}$. With the exception of Japan, we easily reject the null hypothesis that $\beta_{1,\infty}^{NER}$ is equal to zero at conventional significance levels.⁵

⁵We do not report results for the Euro area because our point estimate of ρ_{RER} is greater than one and the Cochrane (2008) test

The equity return literature typically works with annual data. To assess the robustness of our results we redid the previous tests using annual data. These results are reported in Tables 6 and 7.⁶ The evidence against the random walk hypothesis is even stronger with the annual data, where we reject the random walk hypothesis for every country, including Japan and the Euro area.

Taken together the results in this subsection are strongly supportive of the view that at long horizons changes in the *NER* are strongly negatively correlated with the current *RER*. We conclude that, for countries on a flexible exchange rate regime and monetary policy well characterized by a stable Taylor rule, adjustments in the *RER*, occur slowly via predictable changes in the *NER*.

3 Benchmark models

In this section we use a sequence of simple models to explain the empirical findings documented above. We begin with a flexible price, two-country, complete-markets model, allowing for two different specifications of monetary policy. We then consider an incomplete-markets model, allowing for ‘spread shocks.’ These shocks imply that traditional tests applied to data from the model economy would reject UIP. We first assume that prices are flexible and then move on to a specification that allows for nominal rigidities.

3.1 Flexible-price, complete-markets model

Our model consists of two completely symmetric countries. We first describe the households’ problems and then discuss the firms’ problems.

3.1.1 Households

The domestic economy is populated by a representative household whose preferences are given by

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\log(C_{t+j}) - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi} + \mu \frac{(M_{t+j}/P_{t+j})^{1-\sigma_M}}{1-\sigma_M} \right]. \quad (6)$$

Here, C_t denotes consumption, L_t hours worked, M_t end-of-period nominal money balances, P_t the time- t aggregate price level, and E_t the expectations operator conditional on time- t information. In addition, $0 < \beta < 1$, $\sigma_M > 1$, and χ and μ are positive scalars.

Households can trade in a complete set of domestic and international contingent claims. The domestic household’s flow budget constraint is given by:

$$B_{H,t} + NER_t B_{F,t} + P_t C_t + M_t = R_{t-1} B_{H,t-1} + NER_t R_{t-1}^* B_{F,t-1} + W_t L_t + T_t + M_{t-1}. \quad (7)$$

Here, $B_{H,t}$ and $B_{F,t}$ are nominal balances of home and foreign bonds, NER_t is the nominal exchange rate, defined as in our empirical section to be the price of the foreign currency unit (units of home currency per unit of foreign currency), R_t is the nominal interest rate on the home bond and R_t^* is the nominal interest

requires a stationary *RER*.

⁶The annual data is constructed using every fourth observation of the quarterly data. This measure implies that if the $\log(RER)$ is an AR(1) at both the quarterly and annual data. In population the AR coefficient at the annual level is the quarterly AR coefficient to the fourth power. We find very little evidence against this hypothesis.

rate on the foreign bond, W_t is the wage rate, and T_t are lump-sum profits and taxes. For notational ease, we have suppressed the household's purchases and payoffs of contingent claims. With complete markets, the presence of one-period nominal bonds is redundant since these bonds can be synthesized using state-contingent claims.

The first-order conditions are:

$$\chi L_t^\phi C_t = \frac{W_t}{P_t}, \quad (8)$$

$$1 = \beta R_t E_t \frac{C_t}{C_{t+1} \pi_{t+1}}, \quad (9)$$

where, $\pi_t = P_t/P_{t-1}$, denotes the inflation rate.

$$\mu \left(\frac{M_t}{P_t} \right)^{-\sigma_M} = \left(\frac{R_t - 1}{R_t} \right) \frac{1}{C_t}. \quad (10)$$

Equation (10) characterizes money demand by domestic agents. Since households only derive utility from their country's money, domestic agents do not hold foreign money balances.

We use stars to denote the prices and quantities in the foreign country. The preferences of the foreign household are given by:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\log(C_{t+j}^*) - \frac{\chi}{1+\phi} (L_{t+j}^*)^{1+\phi} + \mu \frac{(M_{t+j}^*/P_{t+j}^*)^{1-\sigma_M}}{1-\sigma_M} \right]. \quad (11)$$

The foreign household's flow budget constraint is given by:

$$B_{F,t}^* + NER_t^{-1} B_{H,t}^* + P_t^* C_t^* + M_t^* = R_{t-1}^* B_{F,t-1} + NER_t^{-1} R_{t-1}^* B_{H,t-1} + W_t^* L_t^* + T_t^* + M_{t-1}^*. \quad (12)$$

The first-order conditions for the foreign household are:

$$\chi (L_t^*)^\phi C_t^* = \frac{W_t^*}{P_t^*}, \quad (13)$$

$$1 = \beta R_t^* E_t \frac{C_t^*}{C_{t+1}^* \pi_{t+1}^*}, \quad (14)$$

$$\mu \left(\frac{M_t^*}{P_t^*} \right)^{-\sigma_M} = \left(\frac{R_t^* - 1}{R_t^*} \right) \frac{1}{C_t^*}. \quad (15)$$

We define the real exchange rate, RER_t , as in our empirical section to be units of the home good per unit of the foreign good:

$$RER_t = \frac{NER_t P_t^*}{P_t}. \quad (16)$$

With this definition, an increase in RER_t corresponds to a lower real relative price of the home good, i.e. a real depreciation of the home good.

Complete markets and symmetry of initial conditions implies

$$\frac{C_t}{C_t^*} = RER_t. \quad (17)$$

Combining equations (14) and (17) we obtain:

$$1 = \beta R_t^* E_t \frac{C_t}{C_{t+1} \pi_{t+1}} \frac{NER_{t+1}}{NER_t}. \quad (18)$$

Similarly, combining equations (9) and (17) implies:

$$1 = \beta R_t E_t \frac{C_t^*}{C_{t+1}^* \pi_{t+1}^*} \frac{NER_t}{NER_{t+1}}. \quad (19)$$

3.1.2 Firms

The domestic final good, Y_t , is produced by combining domestic and foreign goods ($X_{H,t}$ and $X_{F,t}$, respectively) according to the technology

$$Y_t = \left[\omega^{1-\rho} (X_{H,t})^\rho + (1-\omega)^{1-\rho} (X_{F,t})^\rho \right]^{\frac{1}{\rho}}. \quad (20)$$

Here, $\omega > 0$ controls the importance of home bias in consumption. The parameter $\rho \leq 1$ controls the elasticity of substitution between home and foreign goods.

The foreign final good, Y_t^* , is produced according to:

$$Y_t^* = \left[\omega^{1-\rho} (X_{F,t}^*)^\rho + (1-\omega)^{1-\rho} (X_{H,t}^*)^\rho \right]^{\frac{1}{\rho}}. \quad (21)$$

The quantity $X_{H,t}$ denotes domestic goods used in domestic final production and produced according to the technology:

$$X_{H,t} = \left(\int_0^1 X_{H,t}(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \quad (22)$$

The quantity $X_{H,t}^*$ denotes domestic goods used in foreign final production and produced according to the technology:

$$X_{H,t}^* = \left(\int_0^1 X_{H,t}^*(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \quad (23)$$

Here, $X_{H,t}(j)$ and $X_{H,t}^*(j)$ are domestic intermediate goods produced by monopolist j using the linear technology:

$$X_{H,t}(j) + X_{H,t}^*(j) = A_t L_t(j). \quad (24)$$

The variable $L_t(j)$ denotes the quantity of labor employed by monopolist j and A_t denotes the state of time- t technology, which evolves so that

$$\log(A_t) = \rho_A \log(A_{t-1}) + \epsilon_{A,t}. \quad (25)$$

The parameter $\nu > 1$ controls the degree of substitutability between different intermediate inputs. The quantity $X_{F,t}$ denotes foreign goods used in domestic final production and produced according to the technology:

$$X_{F,t} = \left(\int_0^1 X_{F,t}(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \quad (26)$$

The quantity $X_{F,t}^*$ denotes foreign goods used in foreign final production and produced according to the

technology:

$$X_{F,t}^* = \left(\int_0^1 X_{F,t}^*(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \quad (27)$$

Here, $X_{F,t}(j)$ and $X_{F,t}^*(j)$ are foreign intermediate goods produced by monopolist j using the linear technology:

$$X_{F,t}(j) + X_{F,t}^*(j) = A_t^* L_t^*(j), \quad (28)$$

where $L_t^*(j)$ is the labor employed by monopolist j in the foreign country and A_t^* denotes the state of technology in the foreign country at time t , which evolves so that

$$\log(A_t^*) = \rho_A \log(A_{t-1}^*) + \epsilon_{A,t}^*. \quad (29)$$

In each period, monopolists in the home country choose $\tilde{P}_{H,t}(j)$ and $\tilde{P}_{H,t}^*(j)$ to maximize per-period profits, which are given by

$$\left(\tilde{P}_{H,t}(j) (1 + \tau_X) - W_t/A_t \right) X_{H,t}(j) + \left(NER_t \tilde{P}_{H,t}^*(j) (1 + \tau_X) - W_t/A_t \right) X_{H,t}^*(j), \quad (30)$$

subject to the demand curves of final good producers:

$$X_{H,t}(j) = \left(\frac{\tilde{P}_{H,t}(j)}{P_{H,t}} \right)^{-\nu} X_{H,t}, \quad (31)$$

and

$$X_{H,t}^*(j) = \left(\frac{\tilde{P}_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\nu} X_{H,t}^*. \quad (32)$$

Here, τ_X is a subsidy that corrects the steady state level of monopoly distortion.⁷ The aggregate price indexes for $X_{H,t}$ and $X_{H,t}^*$, denoted by $P_{H,t}$ and $P_{H,t}^*$, can be expressed as

$$P_{H,t} \equiv \left(\int_0^1 [\tilde{P}_{H,t}(j)]^{1-\nu} dj \right)^{\frac{1}{1-\nu}}, \quad (33)$$

and

$$P_{H,t}^* \equiv \left(\int_0^1 [\tilde{P}_{H,t}^*(j)]^{1-\nu} dj \right)^{\frac{1}{1-\nu}}. \quad (34)$$

Monopolists in the foreign country choose $\tilde{P}_{F,t}(j)$ and $\tilde{P}_{F,t}^*(j)$ to maximize profits

$$\left(\tilde{P}_{F,t}^*(j) (1 + \tau_X) - W_t^*/A_t^* \right) X_{F,t}^*(j) + \left(NER_t^{-1} \tilde{P}_{F,t}(j) (1 + \tau_X) - W_t^*/A_t^* \right) X_{F,t}(j). \quad (35)$$

subject to the demand curves of final good producers:

$$X_{F,t}(j) = \left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}} \right)^{-\nu} X_{F,t}, \quad (36)$$

⁷Impulse response functions from the model are little changed if we set $\tau_X = 0$.

and

$$X_{F,t}^*(j) = \left(\frac{\tilde{P}_{F,t}^*(j)}{P_{F,t}^*} \right)^{-\nu} X_{F,t}^* \quad (37)$$

Here, the aggregate price index for $X_{F,t}$ and $X_{F,t}^*$, denoted by $P_{F,t}$ and $P_{F,t}^*$, can be expressed as:

$$P_{F,t} \equiv \left(\int_0^1 [\tilde{P}_{F,t}(j)]^{1-\nu} dj \right)^{\frac{1}{1-\nu}}, \quad (38)$$

and

$$P_{F,t}^* \equiv \left(\int_0^1 [\tilde{P}_{F,t}^*(j)]^{1-\nu} dj \right)^{\frac{1}{1-\nu}}. \quad (39)$$

The first-order conditions for the monopolists imply:

$$\tilde{P}_{H,t}(j) = NER_t \tilde{P}_{H,t}^*(j) = \frac{W_t}{A_t}, \quad (40)$$

where $\tilde{P}_{H,t}(j)$ and $\tilde{P}_{H,t}^*(j)$ are prices that the home monopolist charges in the home and foreign markets, respectively. Similarly,

$$NER_t^{-1} \tilde{P}_{F,t}(j) = \tilde{P}_{F,t}^*(j) = \frac{W_t^*}{A_t^*}. \quad (41)$$

Here $\tilde{P}_{F,t}(j)$ and $\tilde{P}_{F,t}^*(j)$ are the prices that the foreign monopolist charges in the home and foreign markets, respectively. All monopolists charge a gross markup of one due to the subsidy that corrects the steady-state level of monopoly distortion. Equations (40) and (41) imply that PPP holds for both the home-produced and the foreign-produced intermediate goods.

3.1.3 Monetary policy, market clearing and the aggregate resource constraint

In our first specification of monetary policy, the domestic monetary authority sets the interest rate according to the following Taylor rule:

$$R_t = (R_{t-1})^\gamma \left(R \pi_t^{\theta_\pi} \right)^{1-\gamma} \exp(\epsilon_{R,t}). \quad (42)$$

We assume that the Taylor principle holds, so that $\theta_\pi > 1$. In addition, $r = \beta^{-1}$, and ϵ_t^R is an iid shock to monetary policy. To simplify, we assume that the inflation target is zero in both countries. The foreign monetary authority follows a similar rule so that:

$$R_t^* = (R_{t-1}^*)^\gamma \left(R (\pi_t^*)^{\theta_\pi} \right)^{1-\gamma} \exp(\epsilon_{R,t}^*). \quad (43)$$

We abstract from the output gap in the Taylor rule to make it easier to compare the flexible price version of the model (which has a zero output gap) with the sticky price version. In practice, the output-gap coefficient in estimated versions of the Taylor rule are quite small (see, e.g. Clarida, Gali and Gertler (1998)) and would have a negligible effect on our results.

In the Appendix we display our results for a Taylor rule in which the constant r is replaced by the natural rate of interest, i.e. the real interest rate in the economy replaces the intercept of the Taylor rule. We show that none of our key results are qualitatively affected by this change. The quantitative impact of switching to the natural rate version of the Taylor rule is similar to the impact of switching to the monetary growth rate rule we discuss below.

In our second specification of monetary policy, the domestic monetary authority sets the growth rate of nominal money balances to be:

$$\log\left(\frac{M_t}{M_{t-1}}\right) = x_t^M, \quad (44)$$

where

$$x_t^M = \rho_{X_M} x_{t-1}^M + \varepsilon_t^M. \quad (45)$$

Here, $\rho_{X_M} < 1$ and ε_t^M is an iid shock to monetary policy. For convenience, we have assumed that the unconditional mean growth rate of nominal money balances is zero. The foreign monetary authority follows a similar rule so that:

$$\log\left(\frac{M_t^*}{M_{t-1}^*}\right) = x_t^{M*}, \quad (46)$$

where

$$x_t^{M*} = \rho_{X_M} x_{t-1}^{M*} + \varepsilon_t^{M*}. \quad (47)$$

We assume that government purchases, G_t , evolve according to:

$$\log\left(\frac{G_t}{G}\right) = \rho_G \log\left(\frac{G_{t-1}}{G}\right) + \epsilon_t^G, \quad (48)$$

and, without loss of generality, that the government budget is balanced each period using lump-sum taxes. Here, ϵ_t^G is an iid shock to government purchases. The composition of government expenditures in terms of domestic and foreign intermediate goods ($X_{H,t}$ and $X_{F,t}$) is the same as the domestic household's final consumption good.

Similarly, government purchases in the foreign purchases, G_t^* , evolve according to:

$$\log\left(\frac{G_t^*}{G}\right) = \rho_G \log\left(\frac{G_{t-1}^*}{G}\right) + \epsilon_t^{G*}, \quad (49)$$

where ϵ_t^{G*} is an iid shock to government purchases and the government budget is balanced each period using lump-sum taxes. The composition of government expenditures in terms of domestic and foreign intermediate goods ($X_{F,t}^*$ and $X_{H,t}^*$) is the same as the foreign household's final consumption good. Since bonds are in zero net supply, bond-market clearing implies:

$$B_{H,t} + B_{H,t}^* = 0, \quad (50)$$

and

$$B_{F,t} + B_{F,t}^* = 0. \quad (51)$$

Labor-market clearing requires that:

$$L_t = \int_0^1 L_t(j) dj, \quad (52)$$

and

$$L_t^* = \int_0^1 L_t^*(j) dj. \quad (53)$$

Market clearing in the intermediate inputs market requires that

$$X_{H,t}(j) + X_{H,t}^*(j) = A_t L_t, \quad (54)$$

and

$$X_{F,t}(j) + X_{F,t}^*(j) = A_t^* L_t^*. \quad (55)$$

Finally, the aggregate resource constraints are given by

$$Y_t = C_t + G_t, \quad (56)$$

and

$$Y_t^* = C_t^* + G_t^*. \quad (57)$$

3.1.4 Impulse response functions

In the examples below we use the following parameter values. We assume a Frisch elasticity of labor supply equal to one ($\phi = 1$) and, as in Christiano, Eichenbaum and Evans (2005), set $\sigma_M = 10.62$. We set the value of β so that the steady state real interest rate is 3 percent. We follow Backus, Kehoe and Kydland (1992) and assume that the elasticity of substitution between domestic and foreign goods in the consumption aggregator is 1.5 ($\rho = 1/3$) and that the import share is 15 percent ($\omega = 0.85$), so that there is home bias in consumption. We assume that $\nu = 6$, which implies an average markup of 20 percent. This value falls well within the range considered by Altig, et al. (2011). We normalize the value of χ , which affects the marginal disutility of labor, and real balances, so that hours worked in the steady state equal one. We assume that monetary policy is given by the Taylor rules (180) and (183). We set θ_π to 1.5 so as to satisfy the Taylor principle. For ease of exposition, in this section we set $\gamma = 0$ so that the monetary authority does not do any interest rate smoothing. We choose 0.958 for the first-order serial correlation of the technology shock, which is very similar to standard values used in the literature (e.g. Hansen (1985)). We discuss how we chose this exact value later in the paper. In this section, we assume that the only shocks in the economy are shocks to the process for A_t and A_t^* .

Figure 13 displays the impulse response to a negative technology shock. Home bias in consumption has three implications. First, the *RER* falls since home goods are more costly to produce and the home consumption basket places a higher weight on these goods. Second, domestic consumption falls by more than foreign consumption because domestic agents consume more of the good whose relative cost of production has risen. Third, the households' Euler equations imply that the domestic real interest rate must rise by more than the foreign real interest rate. The Taylor rule and the Taylor principle imply that high real interest rates are associated with high nominal interest rates and high inflation rates. It follows that the domestic nominal interest rate and the domestic inflation rate rise by more than their foreign counterparts. This result is inconsistent with the naive intuition that differential inflation rates are the key mechanism by which the *RER* returns to its pre-shock level. The only way for the *RER* to revert to its steady state value is via a change in nominal exchange rates.

Since the Taylor rule keeps prices relatively stable, the fall in the *RER* on impact occurs via an appreciation of the home currency. To understand this result, note that the log-linearized equilibrium conditions imply that, in response to a technology shock, the behavior of the *RER* is given by:

$$\widehat{RER}_t = \kappa \hat{A}_t. \quad (58)$$

Here, κ is a positive constant that depends on the parameters of the model. This equation implies that the

RER inherits the AR(1) nature of the technology shock, so that:

$$E_t \widehat{RER}_{t+1} = \rho_A \widehat{RER}_t. \quad (59)$$

Combining the linearized home- and foreign-country intertemporal Euler equations (9) and (14), the relation between the two country's marginal utilities implied by complete markets (17), and the Taylor rules for the two countries (180) and (183) we obtain:

$$\hat{\pi}_t - \hat{\pi}_t^* = \frac{\rho_A - 1}{\theta_\pi - \rho_A} \widehat{RER}_t. \quad (60)$$

When the Taylor principle holds ($\theta_\pi > 1$), we have $\left| \frac{\rho_A - 1}{\theta_\pi - \rho_A} \right| < 1$. Recall that the RER is defined as $NER_t P_t^* / P_t$. Equation (60) implies that, on impact, the RER_t falls by more than P_t^* / P_t . It follows that NER_t must initially fall, i.e. the home currency *appreciates* on impact.

Recall that in response to the technology shock, both the real and the nominal interest rates rise more at home than abroad. The technology shock is persistent, so there is a persistent gap between the domestic and foreign nominal interest rates. Since UIP holds in the log-linear equilibrium, the domestic currency must depreciate over time to compensate for the nominal interest rate gap. So, the home currency appreciates on impact and then depreciates. This pattern is reminiscent of the overshooting phenomenon emphasized by Dornbusch (1976).

Domestic inflation is persistently higher than foreign inflation, so the domestic price level rises by more than the foreign price level. This result, along with PPP, implies that the home currency depreciates over time to an asymptotically lower value (the figure displays the price of the foreign currency which is rising to a higher value).

As the previous discussion makes clear, home bias plays a critical role in our analysis. Absent that bias, the consumption basket would be the same in both countries and the RER would be equal to one. Equation (60) implies that if the RER is constant so too is the relative inflation and the NER .

3.1.5 Implied regression coefficients

We now assess the model's ability to account for the basic regressions that motivate our analysis (equations (2) and (3)). In the Appendix we show that the probability limits of the regression coefficients, $\beta_{1,j}^{NER}$ and $\beta_{1,j}^\pi$, in our model drive only by shocks to A_t and A_t^* are given by:

$$\beta_{1,j}^{NER} = -\frac{1 - \rho_A^j}{1 - \rho_A / \theta_\pi}, \quad (61)$$

and

$$\beta_{1,j}^\pi = \frac{1 - \rho_A^j}{\theta_\pi / \rho_A - 1} \quad (62)$$

Equation (61) implies that $\beta_{1,j}^{NER}$ is negative for all j and increases in absolute value with j . The intuitions for these results is as follows. In the model, a low current value of the RER predicts a future appreciation of the foreign currency, so the slope of the regression is negative. The slope increases in absolute value with the horizon because the cumulative depreciation of the home currency increases over time.

Notice that the more aggressive is monetary policy (i.e. the larger is θ_π), the smaller is the absolute value

of $\beta_{1,j}^{NER}$. The intuition for this result is as follows. After a domestic technology shock, π_t is higher than π_t^* . Equation (59) implies that the *RER* must revert to its steady state level at a rate ρ_A . The higher is θ_π , the lower is π_t , and the less the domestic currency needs to depreciate to bring about the required adjustment in the *RER*. So, the absolute value of $\beta_{1,j}^{NER}$ is decreasing in θ_π . Equation (62) implies that $\beta_{1,j}^\pi$ is positive for all j and converges to $\rho_A / (\theta_\pi - \rho_A)$. Consistent with the previous intuition, the higher is θ_π , the lower is $\beta_{1,j}^\pi$ for all j .

The sum of the two slopes is given by:

$$\beta_{1,j}^{NER} + \beta_{1,j}^\pi = -(1 - \rho_A^j)$$

This sum converges to -1 as j goes to infinity. This property reflects the fact the *RER* must converge to its pre-shock steady state level either through changes in inflation or changes in the *NER*.

We illustrate these results using a version of our model driven only by technology shocks. Figure 14 displays the values of $\beta_{1,j}^{NER}$ and $\beta_{1,j}^\pi$. Notice that, consistent with our analytic expressions, $|\beta_{1,j}^\pi| < |\beta_{1,j}^{NER}|$ and the absolute value of each coefficient grows with horizon.

The ability of the model to rationalize the regression coefficients does not depend on technology shocks per se. For example, suppose that government purchases enter the utility function in a time-separable manner and that they follow an AR(1) with first-order serial correlation 0.95. Like a negative technology shocks, a positive shock to government purchases is associated with a negative wealth effect. Also a rise in government purchases leads to a rise in marginal cost. The basic reason is owing to their monopoly power, firms raise prices as total output rises.⁸ So the marginal revenue product rises leading to a rise in real wages. Figure 15 reports the response functions to a government spending shock. The results are very similar to the technology shock case.

The intuition underlying our results is as follows. Consider any shock which changes the *RER*, other than a shock for which UIP does not hold. Suppose that monetary policy is conducted so that inflation is relatively stable (e.g. a Taylor rule with a large value of θ_π). Then P_t^* and P_t are relatively stable. So, the only way for the *RER* to move is via changes changes in the nominal exchange rate. Since movements in the *RER* are predictable, so too are movements in the nominal exchange rate. For these predictable movements to be an equilibrium in which UIP holds, nominal interest rates must offset the expected movements in the *NER*.

As it turns out the implications of the model for the regressions involving relative inflation depends on various model details like the presence of nominal rigidities and which shocks are operative. Accordingly, we defer our discussion of those implications to the section on the medium size DSGE model.

3.1.6 Economy with money growth rule

Consistent with the intuition in Engel (2012), we now show that, when monetary policy follows a money growth rate rule (equation (181)), the flexible price model is much less successful at accounting for our regression result.

The impulse response functions to a technology shock are displayed in Figure 16. The following features are worth noting. First, prices in both countries move by much more than they did under the Taylor rule. So,

⁸The rise in government purchases is larger than the fall in consumption so total output rises.

the movements in the NER required to validate the given equilibrium path of the RER are much smaller than under a Taylor rule. Second, since the growth rate of money does not increase after the shock, the price level eventually reverts to its pre-shock steady state level. As a result, the nominal exchange rate also reverts to its steady state. Third, not all of the adjustment in the RER occurs via the price level, so there are still predictable movements in the NER . But these movements are much smaller than under a Taylor rule. This property is reflected model-implied regression slopes for our NER regression that are much smaller than under a Taylor rule (see Figure 17). The reason that movements in the NER are smaller than under a Taylor rule is that relative inflation rates help to move the RER back to steady state. Under a Taylor rule, prices move in the opposite direction.

3.2 Flexible-price, incomplete-markets model

In this subsection we assume that the only assets that can be traded internationally are one-period nominal bonds. We continue to assume that there are complete domestic asset markets. As in McCallum (1994), we allow for shocks that break UIP in log-linearized versions of the model. But rather than a shock directly to the UIP condition, we assume that households derive utility from domestic bond holdings and that this utility flow varies over time.

We modify the household's utility function to be:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\log(C_{t+j}) - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi} + \mu \frac{(M_{t+j}/P_{t+j})^{1-\sigma_M}}{1-\sigma_M} + \eta_t V \left(\frac{B_{H,t+j}}{P_{t+j}} \right) \right]. \quad (63)$$

The function V that governs the utility flow from the stock of domestic bonds is increasing, strictly concave, and has both positive and negative support.⁹ For convenience we assume that η_t is zero in steady state, meaning that the flow utility from bonds is also zero in steady state. In what follows, we refer to η_t as a spread shock.¹⁰ Outside of steady state, there may be shocks that put a premium on one bond or the other, arising from flights to safety or liquidity, for example. This type of spread shock is used in a closed-economy context by Smets and Wouters (2007), Christiano, Eichenbaum, and Trabandt (2014), Fisher (2015) and Gust, et al., (2016). Importantly, we assume that the home and foreign household are impacted by the same shocks to the utility flow from bond holdings. The foreign household's objective function is given by:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\log(C_{t+j}^*) - \frac{\chi}{1+\phi} (L_{t+j}^*)^{1+\phi} + \mu \frac{(M_{t+j}^*/P_{t+j}^*)^{1-\sigma_M}}{1-\sigma_M} + \eta_t V \left(\frac{B_{H,t+j}^*}{NER_t P_{t+j}^*} \right) \right]. \quad (64)$$

It is well known that with incomplete asset markets, the equilibrium process for the RER in models like ours has a unit root. To avoid this implication, authors like Schmitt-Grohe and Uribe (2003) assume that there is a small quadratic cost to holding bonds. We make a similar assumption in our model. The domestic

⁹It is straightforward to allow for a utility flow from holding foreign bonds of the form $\eta_t^* V \left(\frac{NER_t B_{F,t}}{P_t} \right)$. Abstracting from this term does not affect any of the results reported in this paper.

¹⁰In reality, the utility flow from bond holdings could well be positive because some agents in the economy must hold certain types of bonds for regulatory reasons.

household's budget constraint is given by

$$B_{H,t} + NER_t B_{F,t} + P_t C_t + M_t + \frac{\phi_B}{2} \left(\frac{NER_t B_{F,t}}{P_t} \right)^2 P_t = R_{t-1} B_{H,t-1} + NER_t R_{t-1}^* B_{F,t-1} + W_t L_t + T_t + M_{t-1}. \quad (65)$$

As in Erceg, et al. (2005), we assume that the quadratic cost of holding bonds applies to bonds from the other country. In steady state, $B_{F,t}$ is zero, and this term drops from the budget constraint. Symmetrically, the budget constraint of the foreign household is given by

$$B_{F,t}^* + NER_t^{-1} B_{H,t}^* + P_t^* C_t^* + M_t^* + \frac{\phi_B}{2} \left(\frac{NER_t^{-1} B_{H,t}^*}{P_t^*} \right)^2 P_t^* = R_{t-1}^* B_{F,t-1}^* + NER_t^{-1} R_{t-1} B_{H,t-1}^* + W_t^* L_t^* + T_t^* + M_{t-1}^*. \quad (66)$$

The first-order conditions of the households are unchanged, except that equation (9) is replaced by:

$$\frac{1}{C_t} = \eta_t V' \left(\frac{B_{H,t}}{P_t} \right) + \beta R_t E_t \frac{1}{C_{t+1} \pi_{t+1}}, \quad (67)$$

equation (19) is replaced by

$$\frac{1}{C_t^*} \left(1 + \phi_B \frac{B_{H,t}^*}{P_t RER_t} \right) = \eta_t V' \left(\frac{B_{H,t}}{NER_t P_t^*} \right) + \beta R_t E_t \frac{1}{C_{t+1}^* \pi_{t+1}^*} \frac{NER_t}{NER_{t+1}}, \quad (68)$$

equation (91) is replaced by

$$\frac{1}{C_t} \left(1 + \phi_B \frac{B_{F,t}}{P_t^* RER_t} \right) = \beta R_t^* E_t \frac{1}{C_{t+1} \pi_{t+1}} \frac{NER_{t+1}}{NER_t}, \quad (69)$$

and the money demand, equation (10), is replaced by

$$\mu \left(\frac{M_t}{P_t} \right)^{-\sigma_M} = \frac{\eta_t}{R_t} V' \left(\frac{B_{H,t}}{P_t} \right) + \left(\frac{R_t - 1}{R_t} \right) \Lambda_t. \quad (70)$$

In the absence of complete markets, equation (17) does not hold. So, the ratio of marginal utilities of consumption in the home and foreign country is not proportional to the real exchange rate.

All remaining elements of the model are the same as those of the flexible-price, complete-markets model. We confine our attention to the specification of monetary policy given by the Taylor rule specified in equation (180). In the Appendix, we solve for the steady state of the model and display the dynamic system of equations whose solution corresponds to the equilibrium for this economy.

Figure 18 displays the dynamic response of the economy to a positive iid spread shock in the home country (a positive shock to η_t). With flexible prices, only nominal variables are affected. The demand for domestic bonds rises at home and abroad so the domestic interest rate falls. The nominal interest rate declines by the same amount as the spread shock. The Taylor rule then implies that inflation also falls, although by less than the spread shock. Since P_t falls and P_t^* is unaffected, in order for PPP to hold NER_t has to decline. That is, the home currency appreciates.

3.2.1 Uncovered interest rate parity

In a log-linearized version of the model without shocks to the utility flow from real bond holdings, UIP holds. To show this result, log-linearize equations (67) and (69) to obtain

$$\hat{C}_t = CV'(0) \eta_t + \left[\hat{R}_t + E_t \left(-\hat{C}_{t+1} - \hat{\pi}_{t+1} \right) \right], \quad (71)$$

$$\hat{C}_t + \phi_B b_{F,t} = \hat{R}_t^* + E_t \left(-\hat{C}_{t+1} - \hat{\pi}_{t+1} + \Delta \hat{N}ER_{t+1} \right). \quad (72)$$

Here, the symbol ‘hat’ denotes log-deviation from the steady state, $\Delta \hat{N}ER_{t+1} = \log(NER_{t+1}/NER_t)$, and C is the steady-state level of consumption. It is convenient to normalize $V'(0)$ to be equal to $1/C$. Combining equation (71) and (72), and ignoring the small term in ϕ_B , we obtain

$$\hat{R}_t - \hat{R}_t^* = E_t \left(\Delta \hat{N}ER_{t+1} \right) - \eta_t. \quad (73)$$

This equation is identical to the reduced-form equation assumed by McCallum (1994).¹¹

Absent the spread shocks η_t , equation (73) corresponds to the classic UIP condition

$$\hat{R}_t - \hat{R}_t^* = E_t \left[\Delta \hat{N}ER_{t+1} \right]. \quad (74)$$

All the other shocks in our model induce movements in nominal interest rates and exchange rates that are consistent with equation (74). Conditional on these shocks occurring, UIP holds. However, UIP does not hold unconditionally in the presence of spread shocks and traditional tests would reject the hypothesis of UIP. For example, the classic Fama (1984) test involves running the regression

$$\Delta \hat{N}ER_{t+1} = \alpha_0 + \alpha_1 \left(\hat{R}_t - \hat{R}_t^* \right) + \varepsilon_t, \quad (75)$$

and testing the null hypothesis that $\alpha_0 = 0$ and $\alpha_1 = 1$. Our model implies that this null hypothesis should be rejected because of a negative covariance between the error term and the interest rate differential. To see this result, consider a positive iid shock to η_t . A rise in η_t is equivalent to a rise in ε_t . Since domestic bonds are in zero net supply, the yield on domestic bonds must fall leading to a decline in $\hat{R}_t - \hat{R}_t^*$. So, ε_t covaries negatively with $\hat{R}_t - \hat{R}_t^*$ which causes the probability limit of an ordinary least squares estimate of α_1 to be negative in an economy driven only by spread shocks.

3.3 Sticky-price, incomplete-markets model

In this section, we consider a version of the model with sticky prices. In what follows, we assume that monopolist producers set nominal prices in local currency units. The household’s problem is exactly the same as in the previous incomplete markets model. With the exception of spread shocks, the basic structure of this model is similar to Kollmann (2001).

The technology for producing final goods is still given by equation (20). Intermediate-good producing firms set prices according to a variant of the mechanism spelled out in Calvo (1983). In each period, a firm faces a constant probability, $1 - \xi$, of being able to re-optimize its nominal price. The ability to re-optimize

¹¹If we don’t ignore ϕ_B , equation (73) is replaced by $\hat{R}_t - \hat{R}_t^* = E_t \left[\Delta \hat{N}ER_{t+1} \right] - \eta_t - \phi_B b_{F,t}$.

prices is independent across firms and time. Domestic intermediate goods firms choose $\tilde{P}_{H,t}(i)$ and $\tilde{P}_{H,t}^*(i)$ to maximize the objective function:

$$E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \left\{ \begin{aligned} & \left(\frac{\tilde{P}_{H,t}(i)}{P_{t+j}} (1 + \tau_X) - MC_{t+j} \right) X_{H,t+j}(i) \\ & + \left(\frac{NER_{t+j} \tilde{P}_{H,t}^*(i)}{P_{t+j}} (1 + \tau_X) - MC_{t+j} \right) X_{H,t+j}^*(i), \end{aligned} \right\} \quad (76)$$

subject to the demand equations (31) and (32). Here, MC_{t+j} denotes the real marginal cost in period $t+j$ and $\beta^j \Lambda_{t+j}$ is the utility value of profits in period $t+j$ to the household in period t .

Foreign intermediate goods firms choose $\tilde{P}_{F,t}(i)$ and $\tilde{P}_{F,t}^*(i)$ to maximize the objective function:

$$E_t \sum_{j=0}^{\infty} \Lambda_{t+j}^* \left\{ \begin{aligned} & \left(\frac{\tilde{P}_{F,t}^*(i)}{P_{t+j}^*} (1 + \tau_X) - MC_{t+j}^* \right) X_{F,t+j}^*(i) \\ & + \left(\frac{NER_{t+j}^{-1} \tilde{P}_{F,t}(i)}{P_{t+j}^*} (1 + \tau_X) - MC_{t+j}^* \right) X_{F,t+j}(i), \end{aligned} \right\} \quad (77)$$

subject to equations (37) and (36).

In all other respects, the model is the same as the flexible-price, incomplete-markets model. The Appendix contains the equations that characterize the equilibrium of the model economy.

A technology shock Figure 19 displays the response of the economy to a negative technology shock in the home country. These effects are similar to those in the flexible-price model. The key difference is that in the sticky-price model the response of $\pi_{H,t}$, $\pi_{F,t}$, $\pi_{H,t}^*$, $\pi_{F,t}^*$ is attenuated relative to the flexible-price model. Interestingly, the effect of sticky prices on overall inflation is ambiguous. When prices are flexible, producers of the foreign good initially reduce the price they charge in the home market. This effect helps reduce the domestic rate of inflation in the flexible-price model. With sticky prices, this effect is attenuated relative to the flexible-price model. So depending on parameter values, domestic inflation can be higher or lower in the sticky price model than in the flexible price model.

Because the negative technology shock leads to a decline in RER_t followed by a persistent depreciation of the home currency, the model-implied values for $\beta_{1,j}^{NER}$ in the economy with only technology shocks, are negative and grow in absolute value with the horizon. As in the flexible price model, the basic intuition is that a negative technology shock drives down the real exchange rate. Over time the nominal exchange rate rises to its new steady state value. So, a low value of the contemporaneous real exchange rate is associated with increases in the exchange rate over time.

A monetary policy shock Figure 20 shows the effects of an iid contractionary monetary policy shock. We set the interest rate smoothing parameter, γ , to 0.75 so that the impact of this shock is easier to see in the figure. The monetary policy shock causes an increase in R_t . The resulting contraction leads to decrease in domestic consumption, wages, marginal cost, and inflation. The persistence of these effects arises from the interest rate smoothing parameter of the Taylor rule.

The fall in domestic marginal costs leads domestic producers to lower the price of exported goods, so that $\pi_{H,t}^*$ falls leading to a lower value of π_t^* . The foreign Taylor rule implies that R_t^* falls. Since the

Taylor principle holds, the foreign real interest rate falls, which generates a rise in foreign consumption. The RER returns to its initial steady state level after a few periods. The usual UIP logic implies that the interest rate differential must be offset by an expected depreciation of the home currency. This happens via an instantaneous appreciation of the home currency followed by a persistent depreciation.

Both the RER and the NER initially fall and then rise, which again produces negative values for $\beta_{1,j}^{NER}$ in our baseline regression, equation (2) for any economy with only monetary policy shocks. These model-implied values grow somewhat with the horizon and quickly reach their maximal value after about 1 year. As compared to the case when the economy is driven by technology shocks, the regression coefficients implied by monetary policy shocks are smaller. A shortcoming of the model when it is driven only by monetary policy shocks is that the adjustment in the RER occurs roughly equally through changes in the NER and relative inflation rates.

A spread shock Figure 21 displays the effect of an iid positive spread shock, η_t . In contrast with the flexible price case, a spread shock now has real effects. The shock increases the demand for the domestic bond, so the domestic interest rate falls to clear that market. In the home country, the Taylor rule implies that domestic inflation must fall. Since prices are sticky, inflation cannot fall as much as with flexible prices and the domestic nominal interest rate cannot fall enough to clear the domestic bond market. So the domestic currency appreciates to make domestic bonds more expensive, thereby reducing foreigners demand for domestic bonds.

According to Figure 21, the spread shock is larger than the difference between R_t and R_t^* . So, the modified UIP equation, equation (46), implies that $E_t \Delta NER_{t+1} < 0$, which corresponds to an expected appreciation of the home currency. This particular result depends on the degree of price stickiness. When prices are very sticky the nominal and the real exchange rate commove, so the domestic currency appreciates on impact and then slowly depreciates.

An interesting question is how the presence of spread shocks that overturn UIP affect standard analyses of optimal monetary policy in open economy environments like those reviewed in Corsetti, Dedola, and Leduc (2010).

4 Medium-scale DSGE, incomplete-markets model

In this section we investigate whether an empirically plausible version of our model can account for the new facts that we document. By empirically plausible we mean that the model is consistent with the persistence and volatility of real exchange rates, the failure of UIP and PPP, as well as the high correlation between real and nominal exchange rates. For simplicity we abstract from capital in this section. However, we redid our analysis for a version of the model that includes capital. It turns out that the results with capital are very similar to those reported above. See the Appendix for details.

4.1 Model structure

The basic structure of the model is the same as the sticky price model described above except that we allow for sticky nominal wages as in Erceg, Henderson and Levin (2000). Intermediate producers purchase a homogeneous labor input from a representative labor aggregator. The latter produces the homogeneous

labor input by combining differentiated labor inputs, $l_{j,t}$, $j \in (0, 1)$, using the technology

$$L_t = \left[\int_0^1 l_{j,t}^{\frac{\nu_L-1}{\nu_L}} dj \right]^{\frac{\nu_L}{\nu_L-1}}. \quad (78)$$

Labor contractors are perfectly competitive and take the nominal wage rate, W_t , which is the cost of hiring units of L_t , as given. They also take the wage rate, $W_{j,t}$, of the j^{th} labor type as given. Profit maximization on the part of contractors implies:

$$l_{j,t} = \left[\frac{W_{j,t}}{W_t} \right]^{-\nu_L} L_t. \quad (79)$$

Perfect competition and equation (78) imply:

$$W_t = \left[\int_0^1 W_{j,t}^{1-\nu_L} dj \right]^{\frac{1}{1-\nu_L}}. \quad (80)$$

There is a continuum of households of measure one, and each household has a continuum of members indexed $j \in (0, 1)$. Each member of the household belongs to a union that monopolistically supplies labor of type j . The union sets the wage $W_{j,t}$ subject to (79) and Calvo-style wage frictions. That is, the wage for j -type labor, $W_{j,t}$, is updated with probability $1 - \xi_w$. With probability ξ_w the wage rate is given by:

$$W_{j,t} = W_{j,t-1}.$$

The preferences of the j^{th} household are given by

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\log (C_{t+i} - h\bar{C}_{t+i-1}) - \frac{\chi}{1+\phi} \int_0^1 L_{j,t+i}^{1+\phi} dj + \mu \frac{(M_{t+i}/P_{t+i})^{1-\sigma_M}}{1-\sigma_M} + \eta_{t+i} V \left(\frac{B_{H,t+i}}{P_t} \right) \right]. \quad (81)$$

Here \bar{C}_t is aggregate consumption in time t . The household budget constraint becomes

$$B_{H,t} + NER_t B_{F,t} + P_t C_t + M_t + \frac{\phi_B}{2} \left(\frac{NER_t B_{F,t}}{P_t} \right)^2 P_t = R_{t-1} B_{H,t-1} + NER_t R_{t-1}^* B_{F,t-1} + \int_0^1 W_{j,t} L_{j,t} (1 + \tau_W) dj + T_t + M_{t-1} + Q_t. \quad (82)$$

where τ_W is a wage subsidy that corrects the steady state level of monopoly distortions. Here, $Q_{j,t}$ represents the net proceeds of an asset that provides insurance against the idiosyncratic uncertainty associated with the Calvo wage-setting friction. We have suppressed indexing variables by j that are the same across household member.¹²

The sequence of events in a period for a household is as follows. First, the technology shocks and spread shocks are realized. Second, the household makes its consumption and asset decisions, including securities whose payoffs are contingent upon whether it can re-optimize its wage decision. Third, wage rates are updated.

The changes introduced to the foreign economy are symmetric so that the preferences of the household

¹²With separable preferences, it is optimal to equalize consumption for each of its members.

are given by:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\log (C_{t+i}^* - h\bar{C}_{t+i-1}^*) - \frac{\chi}{1+\phi} \int_0^1 (L_{j,t+i}^*)^{1+\phi} dj + \mu \frac{(M_{t+i}^*/P_{t+i}^*)^{1-\sigma_M}}{1-\sigma_M} + \eta_{t+i} V \left(\frac{B_{H,t+i}}{P_t^*} \right) \right], \quad (83)$$

Here \bar{C}_t^* is aggregate consumption in the foreign country at time t . The budget constraint of the foreign household is given by:

$$B_{F,t}^* + NER_t^{-1} B_{H,t}^* + P_t^* C_t^* + M_t^* + \frac{\phi_B}{2} \left(\frac{NER_t^{-1} B_{H,t}^*}{P_t^*} \right)^2 P_t^* = R_{t-1}^* B_{F,t-1}^* + NER_t^{-1} R_{t-1} B_{H,t-1}^* + \int_0^1 W_{jt}^* L_{jt}^* (1 + \tau_W) dj + T_t^* + M_{t-1}^* + Q_t^*. \quad (84)$$

In Appendix we derive the set of equations whose solutions constitute an equilibrium for the model economy.

4.2 Parameter values

We divide the parameters into two categories: those that we calibrate and those that we estimate. We calibrate the parameters whose values are listed in Table 8.

We maintain the parameter values used in the previous sections and set the habit persistence parameter, h , the probability that firms can't adjust their price, ξ , and the probability that labor suppliers can't readjust their nominal wage, ξ_W to the point estimates reported in Christiano, Eichenbaum, and Evans (2005). We set the value of ν_L so as to imply a 5 percent steady state markup.

We now turn to ρ_η and σ_η which govern the AR(1) process for the spread shock. Equation (73) implies that if the one-quarter ahead nominal exchange rate behaves like a random walk, then

$$\hat{R}_t^* - \hat{R}_t = \eta_t. \quad (85)$$

So for any given country we can identify its spread relative to the U.S. with the corresponding interest rate differential. For each of the flexible exchange rate countries in Table 1 we estimate an AR(1) for the interest rate differential,

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_{\eta,t},$$

where $\varepsilon_{\eta,t}$ is an iid process and $E\varepsilon_{\eta,t}^2 = \sigma_{\varepsilon_\eta}^2$. We use money-market interest rate data from the IFS. For each country, we report our results in Table 9 using the same sample period as in Table 1.¹³ In terms of our model, there is no reason to focus on any one of these estimates since U.S. financial markets are integrated with all of these markets. In practice we set ρ_η to 0.85, which is well within the range of our point estimates. We chose the value of 0.85 because it is equal to the value of the persistence of the spread shock in Gust et.al. (2016), who estimate a closed-economy version of the new-Keynesian model.

We estimate the remaining parameters ρ_A , σ_A , and $\sigma_{\varepsilon_\eta}$ so that the model is consistent with the following moments of the data. We require that the first-order autocorrelation of HP-filtered model output and the standard deviation of the innovation to a fitted AR(1) and be the same as the analog objects in quarterly U.S.

¹³Because of limits on the available money-market interest rates in the IFS, in Table 1 the sample for Canada starts in 1975:Q1 and the sample for Sweden starts in 1975:Q4.

data over the sample 1973-2007.¹⁴ In this exercise we assume that the technology process is uncorrelated across countries. We also require that the model be consistent with the results of implementing the Fama regression defined by equation (75). In particular, we estimated that regression for each of the flexible exchange rate countries and corresponding sample period used to construct Table 1. Our results are reported in Table 10. In every case the coefficient α_1 is estimated very imprecisely so many target values would be very reasonable. In results reported below, we require that the probability limit for α_1 implied by our model be equal to 0.5. Table 11 reports our results, reported in the column labeled nominal rigidities. The value of $\sigma_{\varepsilon_\eta}$ is similar to the one estimated by Gust et. al.(2016). We also re-estimated these parameters for a flexible price and wage version of the model ($\xi = \xi_W = 0$), These results in table 11 in the column labeled no-nominal rigidities.

4.3 Empirical results

We now report and discuss the model’s implication for the key statistics that we emphasized in our empirical analysis. Panel C of Table 1 reports the models’ implications for the coefficients in regression (2).

A number of results are worth noting. First, the model with nominal rigidities does a good job of accounting for the estimated values of $\beta_{1,j}^{NER}$, including the fact that they rise in absolute value with the regression horizon. Second, the model without nominal rigidities also does reasonably well on this dimension of the data. But it overstates how quickly the absolute value of $\beta_{1,j}^{NER}$ rises with the horizon.

Panel C of Table 3 reports the model’s implications for the coefficients in the regression equation (3). Taking sampling uncertainty into account, the model with nominal rigidities does a very good job of accounting for the estimated values of $\beta_{1,j}^\pi$. The model without nominal rigidities does not do quite as well on this dimension of the data. Still, it does capture the fact that the estimated values of $\beta_{1,j}^\pi$ in regression (3) are much smaller than those in (2).

To understand this last result it is useful to consider the models’ impulse response functions. Figures 22 and 23 display the response functions of the model with nominal rigidities to a technology and spread shock, respectively. Figures 24 and 25 display the analog response functions for the model without rigidities. Consider the response of inflation in the model without rigidities to a technology shock. Notice that $\pi_{H,t}$ rises by roughly 1.5 percent after a negative technology shock. But $\pi_{F,t}$, the price of foreign goods in the domestic currency falls by roughly 0.75 percent after the shock. Domestic inflation is a weighted average of $\pi_{H,t}$ and $\pi_{F,t}$. So overall inflation doesn’t rise by as much as it would absent the offsetting behavior of $\pi_{F,t}$. This observation helps explain the ability of the model without nominal rigidities to generate relatively low estimated values of $\beta_{1,j}$ in regressions like (3) for low values of j . The model without nominal rigidities still has a quantitative problem because the offsetting effects on inflation are not present when there is a spread shock. Both $\pi_{H,t}$ and $\pi_{F,t}$ fall in response to a positive spread shock. All of the movements in inflation and its constituents are muted in the model with nominal rigidities.

In the introduction we noted three key facts which any plausible open-economy model ought to be consistent: real and nominal exchange rates commove closely in the short run (Mussa (1986)) and *RERs* are highly volatile and inertial (Rogoff (1996)). We conclude with a discussion of how our model fares with respect to these facts. Table 12 reports the standard deviations of ΔRER and ΔNER for the countries in our sample and our model. In addition, we report estimates for an AR(1) representation for the *RERs*. We

¹⁴We measure output using per-capita real GDP.

report the analog statistics for our model in the same table.

Four features of table Table (12) are worth noting. First, our data is consistent with the well know fact that real and nominal exchange are equally volatile (Mussa (1986), Rogoff (1996), and Burstein and Gopinath (2015)). More interestingly, both versions of our model (with and without nominal rigidities) are consistent with this fact. Second, even the model with nominal rigidities understates, for most countries, the volatility of ΔRER and ΔNER . The median estimates of these statistics across countries are 0.049 and 0.041, respectively. The analog values in the model with nominal rigidities and only shocks to technology and the spread shock are 0.023 for both statistics. This result owes, in part, to our including only three shocks (two technology shocks and a spread shock) in our model. Third, with the exception of Germany, the estimated AR(1) coefficients for the $RERs$ exceed 0.96 which is consistent with the results in Burstein and Gopinath (2015). Interestingly, taking sampling uncertainty into account, both versions of our model account for the estimated value of the AR(1) coefficient for countries with flexible exchange rates.

A different way to think about persistence of the RER , is to ask whether our model implies that, in small samples, an analyst would reject the hypothesis that the RER has a unit root. To this end we simulated 10,000 samples, each of length 120, from our model. For each sample we computed an augmented Dickey-Fuller test. We find that in only 41 percent of the samples could we reject, at the 5 percent significance level, the null hypothesis of a unit root. In the remaining 59 percent of the samples, the RER is sufficiently persistent (and the augmented Dickey-Fuller test is not sufficiently powerful) that we can't reject the null hypothesis that the RER has a unit root. Taken as a whole these results indicate that our model is broadly consistent with the properties of the data stressed by Mussa (1986) and Rogoff (1996).

Finally, according to Table (12) the model with model rigidities does very at accounting for the classic Mussa observations that real and nominal exchange rates are highly correlated. For every floating exchange rate country in our sample, the correlation is above 0.95. The correlation in our preferred model 0.96. Significantly, that correlation is only 0.65 in the model without nominal rigidities.

5 Conclusion

This paper documents that when exchange rates are floating and monetary policy is characterized by a Taylor rule, real exchange rates adjust overwhelmingly in the medium and long run through changes in *nominal* exchange rates. They do not adjust via cross-country differences in inflation rates. Two facts are the basis of this conclusion: for countries under a Taylor rule, changes in the NER at horizons of two years more more are highly correlated with the current value of the RER . But changes in the NER are uncorrelated with differential inflation rates across countries at all horizons that we consider.

In our theoretical analysis, we show that a wide variety of open-economy models are consistent with these facts: models with and without nominal rigidities as well complete and incomplete market models. But to account for our empirical findings, models must allow for home bias in consumption, monetary policy guided by a Taylor rule, and a conditional form of UIP.

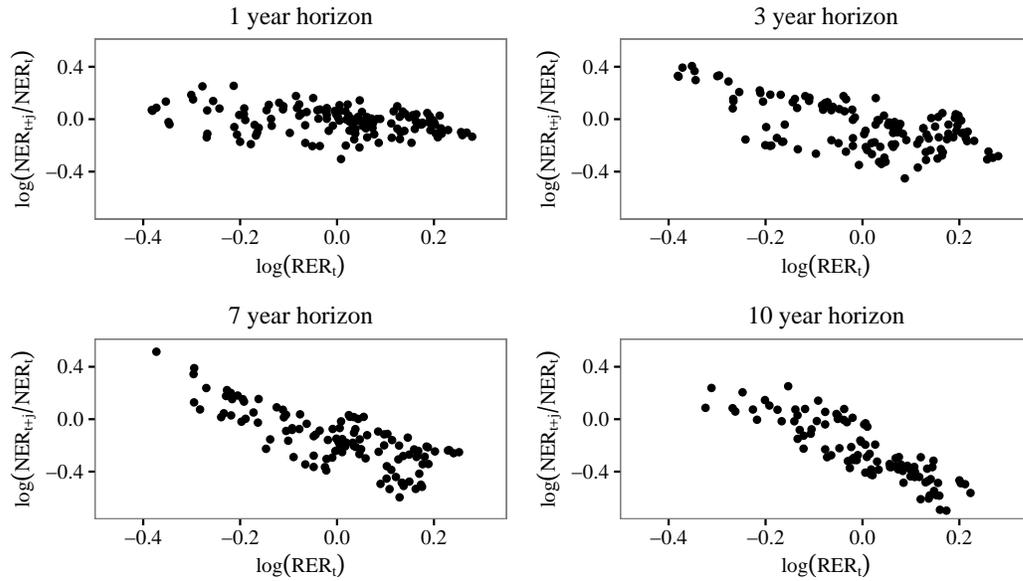
We assess the quantitative performance of a medium-scale DSGE model that embodies these elements. As it turns out, the version of the model that allows for sticky prices and wages does a very good job of accounting for our results. Significantly, the same model is consistent with other key observations about the volatility and persistence of real exchange rates, as well as the fact that standard tests of UIP reject that hypothesis.

References

- [1] Altig, David, Lawrence Christiano, Martin Eichenbaum, and Jesper Linde, “Firm-specific capital, nominal rigidities and the business cycle.” *Review of Economic Dynamics* 14.2 (2011): 225-247.
- [2] Backus, David K., Patrick J. Kehoe, and Finn E. Kydland. “International real business cycles.” *Journal of Political Economy* (1992): 745-775.
- [3] Baxter, Marianne, and Alan C. Stockman. “Business cycles and the exchange-rate regime: some international evidence.” *Journal of Monetary Economics* 23.3 (1989): 377-400.
- [4] Benigno, Gianluca. “Real exchange rate persistence and monetary policy rules.” *Journal of Monetary Economics* 51.3 (2004): 473-502.
- [5] Burstein, Ariel, and Gita Gopinath. “International Prices and Exchange Rates.” *Handbook of International Economics* 4 (2014): 391-451.
- [6] Calvo, Guillermo A. “Staggered prices in a utility-maximizing framework.” *Journal of Monetary Economics* 12.3 (1983): 383-398.
- [7] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. “Nominal rigidities and the dynamic effects of a shock to monetary policy.” *Journal of Political Economy* 113.1 (2005): 1-45.
- [8] Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. “Understanding the great recession.” *American Economic Journal: Macroeconomics* 7.1 (2015): 110-167.
- [9] Clarida, Richard, Jordi Gali, and Mark Gertler. “Monetary policy rules in practice: some international evidence.” *European Economic Review* 42.6 (1998): 1033-1067.
- [10] Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc “Optimal monetary policy in open economies” in Ben Friedman and Michael Woodford *Handbook of Monetary Economics, vol. III*, Elsevier, 2010.
- [11] Dornbusch, Rudiger. “Expectations and exchange rate dynamics.” *Journal of Political Economy* (1976): 1161-1176.
- [12] Engel, Charles, and Kenneth D. West. “Accounting for Exchange-Rate Variability in Present-Value Models When the Discount Factor Is Near 1.” *American Economic Review* 94.2 (2004): 119-125.
- [13] Engel, Charles and West, Kenneth, (2005), “Exchange Rates and Fundamentals,” *Journal of Political Economy*, 113, issue 3, p. 485-517.
- [14] Engel, Charles, Nelson C. Mark, and Kenneth D. West. “Exchange Rate Models Are Not As Bad As You Think.” *NBER Macroeconomics Annual 2007*, Volume 22. University of Chicago Press, 2008. 381-441.
- [15] Engel, Charles. “Real Exchange Rate Convergence: The Roles of Price Stickiness and Monetary Policy.” Manuscript (2012).
- [16] Engel, Charles, “Exchange Rates and Interest Parity,” Volume 4, 2014, Pages 453–522, *Handbook of International Economics*.
- [17] Erceg, Christopher J., Luca Guerrieri, and Christopher J. Gust. “SIGMA: a new open economy model for policy analysis.” Board of Governors of the Federal Reserve System Research Paper Series (2005).

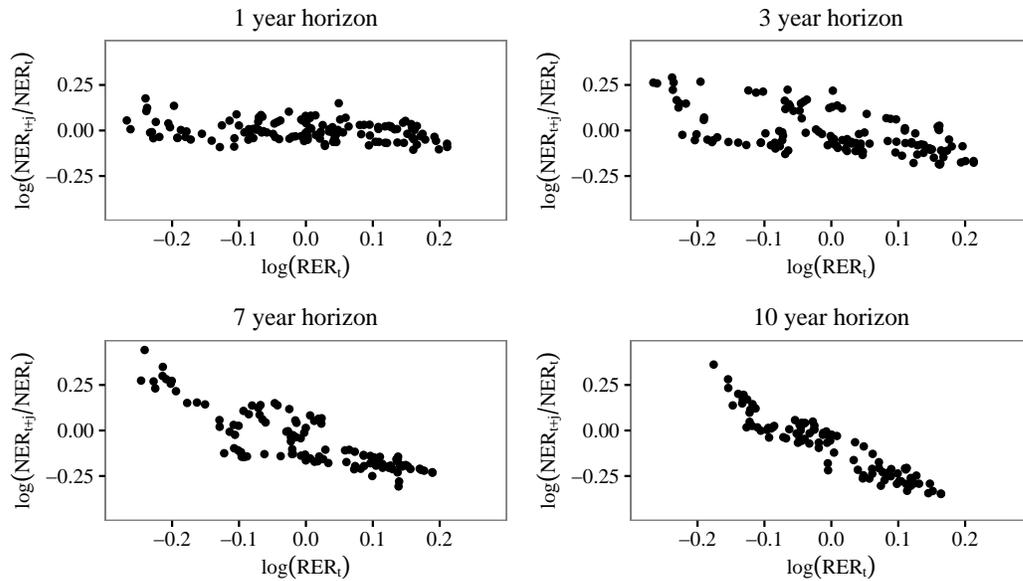
- [18] Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin. "Optimal monetary policy with staggered wage and price contracts." *Journal of Monetary Economics* 46.2 (2000): 281-313.
- [19] Fama, Eugene F. "Forward and spot exchange rates." *Journal of Monetary Economics* 14.3 (1984): 319-338.
- [20] Fisher, Jonas DM. "On the Structural Interpretation of the Smets-Wouters 'Risk Premium' Shock." *Journal of Money, Credit and Banking* 47.2-3 (2015): 511-516.
- [21] Gust, Christopher J., Edward Herbst, David López-Salido, and Matthew E. Smith. "The empirical implications of the interest-rate lower bound." Board of Governors of the Federal Reserve System Research Paper Series (2016).
- [22] Hansen, Gary D. "Indivisible labor and the business cycle." *Journal of Monetary Economics* 16.3 (1985): 309-327.
- [23] Kollmann, Robert. "The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: a quantitative investigation." *Journal of International Economics* 55.2 (2001): 243-262.
- [24] Mark, Nelson C. "Exchange rates and fundamentals: Evidence on long-horizon predictability." *American Economic Review* (1995): 201-218.
- [25] McCallum, Bennett T. "A reconsideration of the uncovered interest parity relationship." *Journal of Monetary Economics* 33.1 (1994): 105-132.
- [26] Molodtsova, Tanya, and David H. Papell. "Out-of-sample exchange rate predictability with Taylor rule fundamentals." *Journal of International Economics* 77.2 (2009): 167-180.
- [27] Mussa, Michael. "Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications." Carnegie-Rochester Conference series on public policy. Vol. 25. North-Holland, 1986.
- [28] Obstfeld, Maurice, and Kenneth Rogoff. "Exchange Rate Dynamics Redux." *Journal of Political Economy* 103.3 (1995): 624-60.
- [29] Obstfeld and Rogoff, "The six major puzzles in international macroeconomics: is there a common cause?" *NBER Macroeconomics Annual 2000*, Volume 15, 2001
- [30] Rogoff, Kenneth. "The purchasing power parity puzzle." *Journal of Economic Literature* 34.2 (1996): 647-668.
- [31] Rossi, Barbara. "Exchange rate predictability." *Journal of Economic Literature* 51.4 (2013): 1063-1119.
- [32] Schmitt-Grohé, Stephanie, and Martín Uribe. "Closing small open economy models." *Journal of international Economics* 61.1 (2003): 163-185.
- [33] Smets, Frank, and Rafael Wouters. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American Economic Review* 97.3 (2007): 586-606.
- [34] Steinsson, Jón. "The dynamic behavior of the real exchange rate in sticky price models." *American Economic Review* 98.1 (2008): 519-533.

Figure 1: Australia: NER and RER data



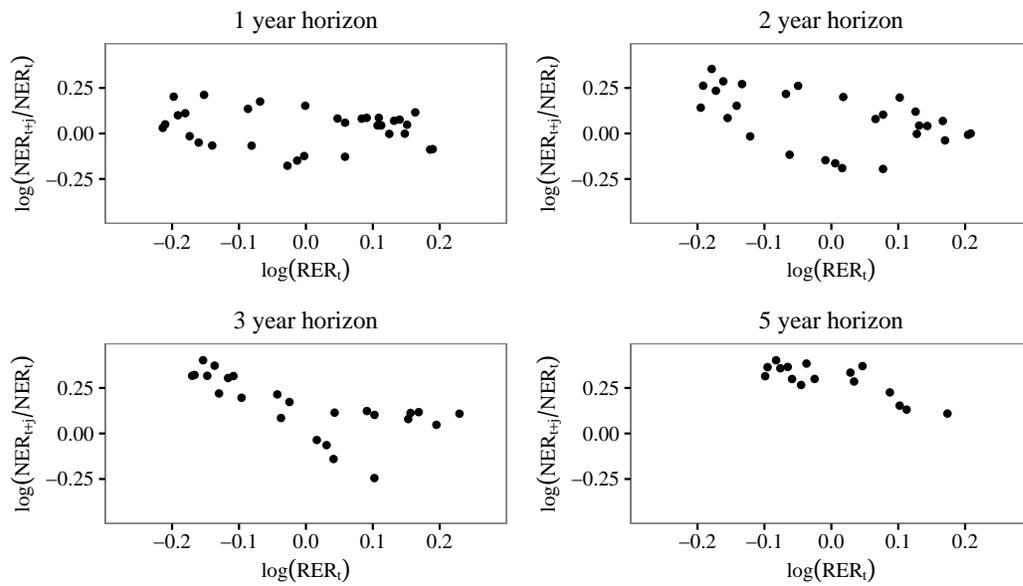
Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 2: Canada: NER and RER data



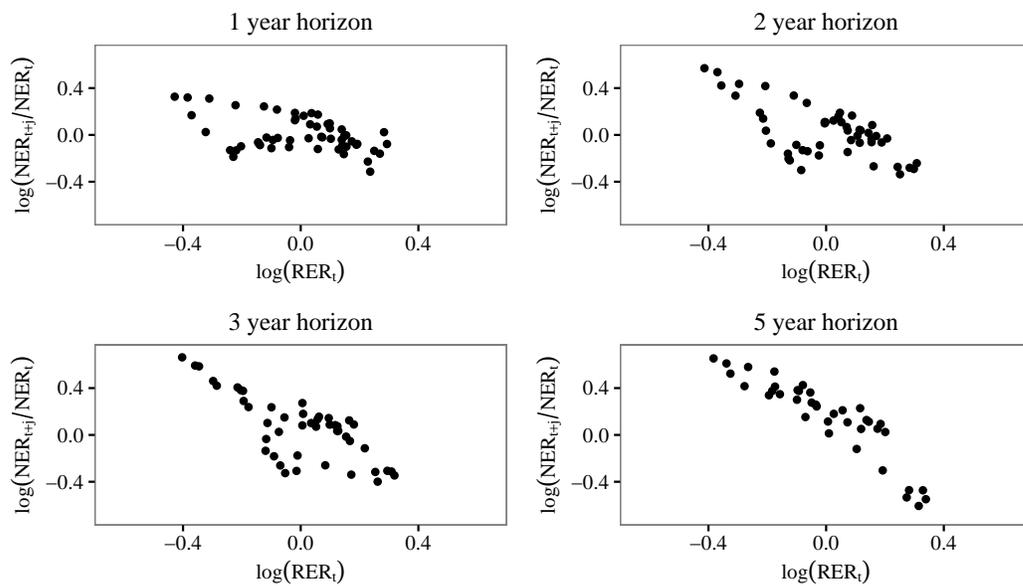
Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 3: Euro area: NER and RER data



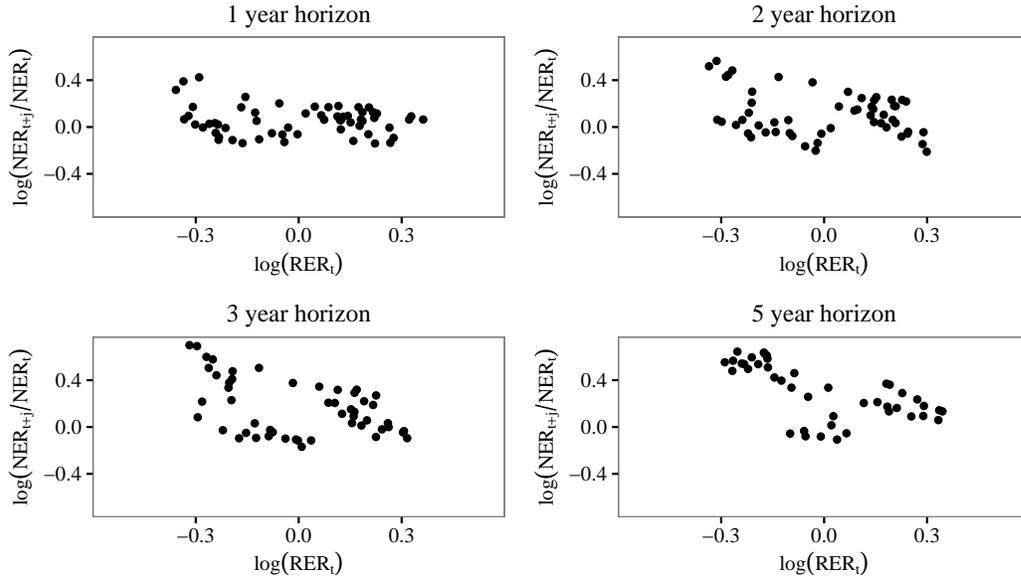
Source: OECD Main Economic Indicators, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 4: Germany: NER and RER data



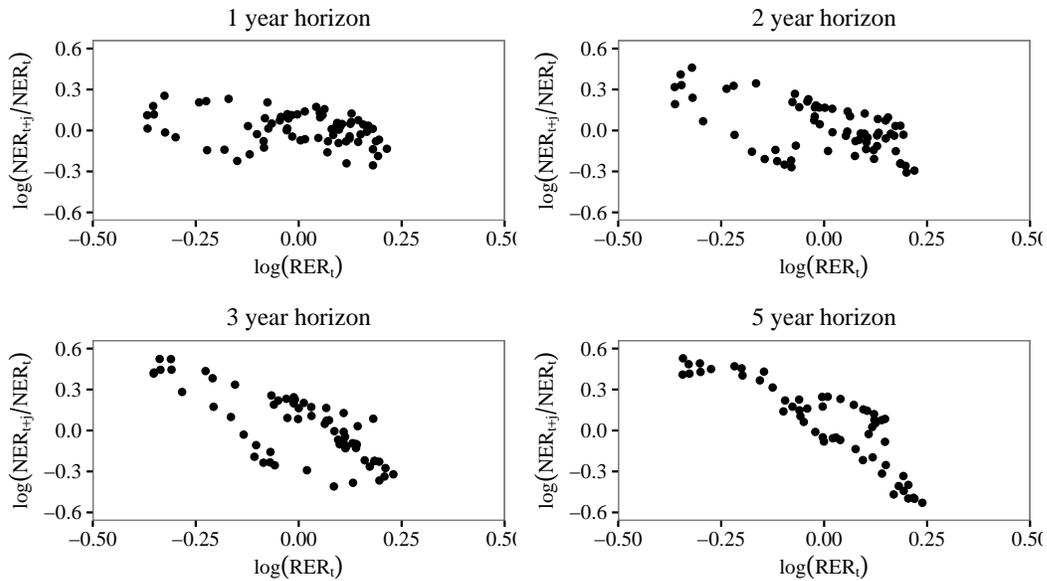
Source: OECD Main Economic Indicators, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 5: Japan: NER and RER data



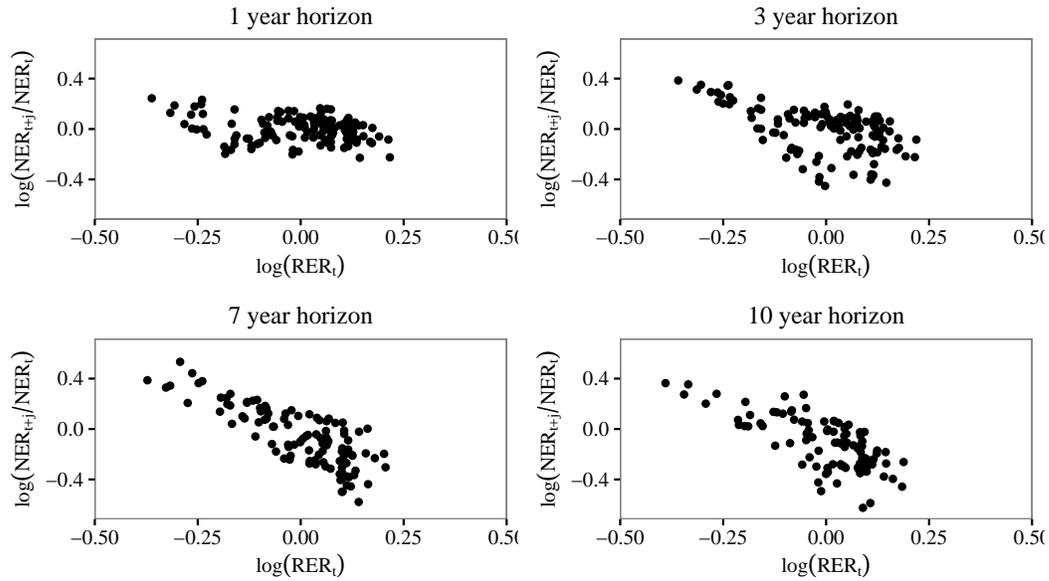
Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 6: New Zealand: NER and RER data



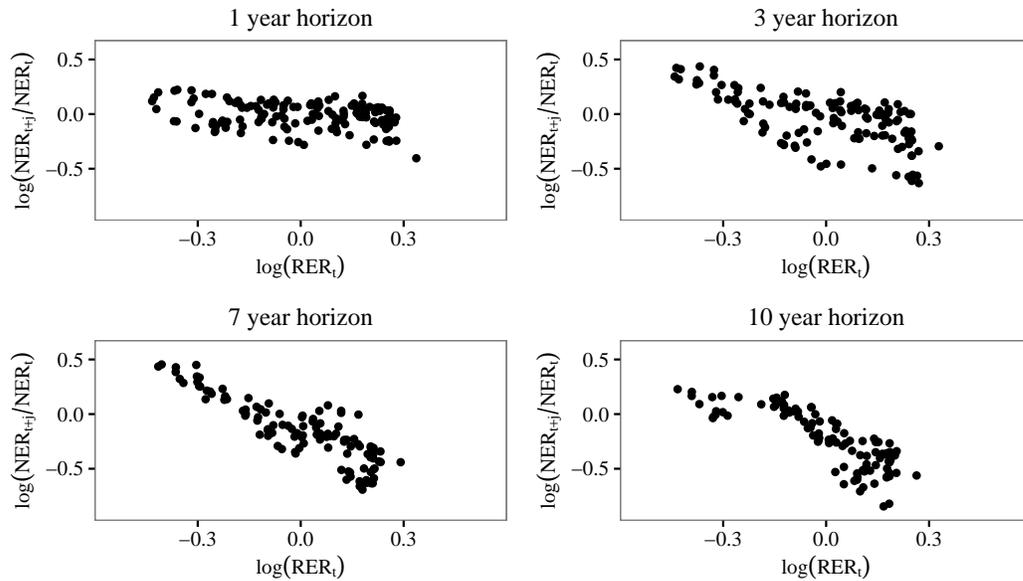
Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 7: Norway: NER and RER data



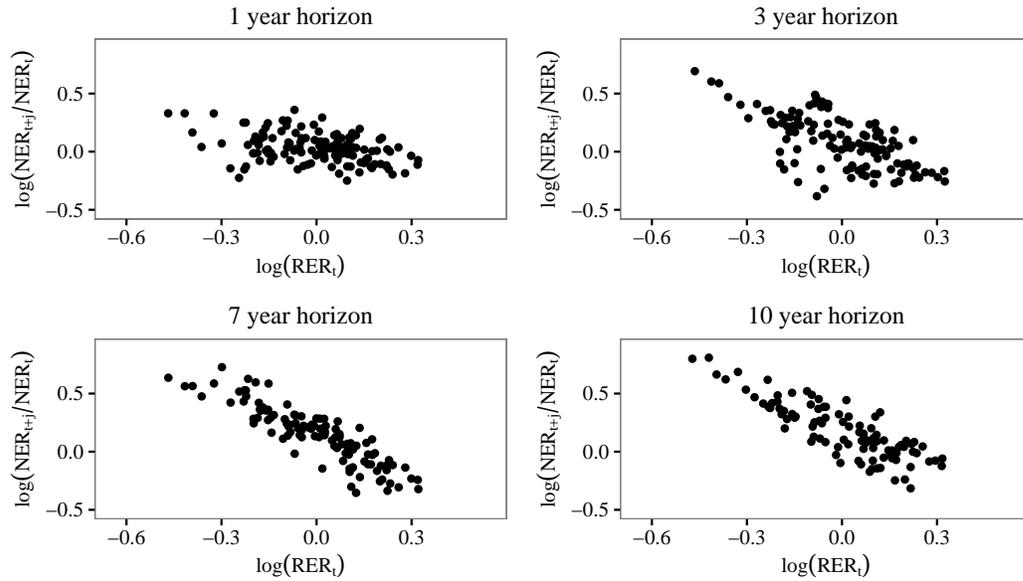
Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 8: Sweden: NER and RER data



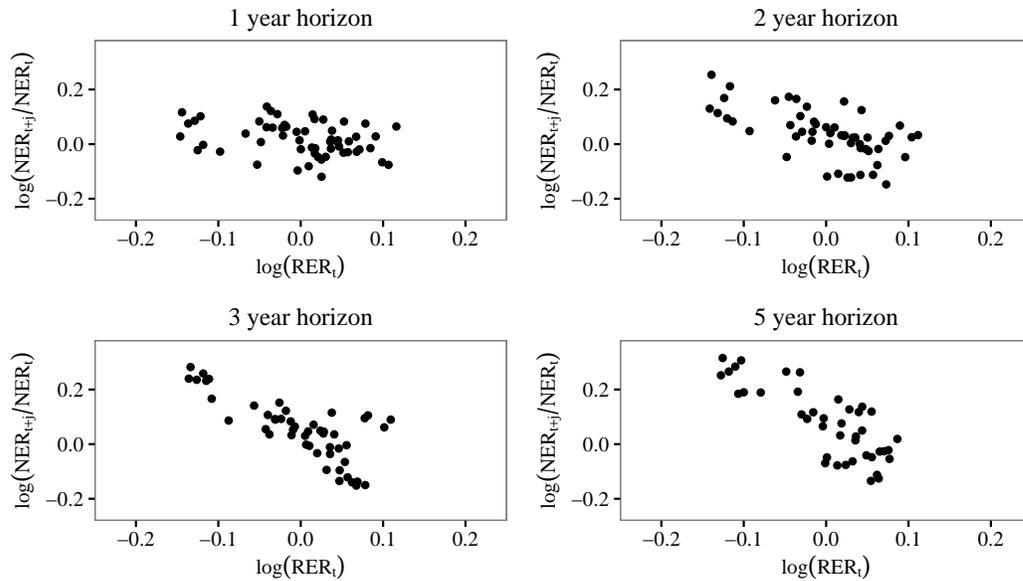
Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 9: Switzerland: NER and RER data



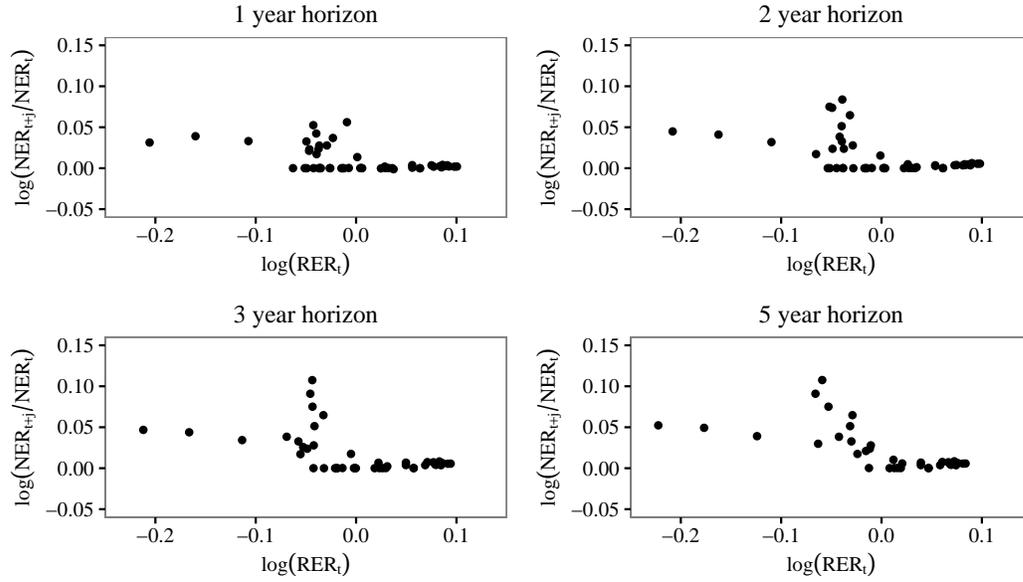
Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 10: United Kingdom: NER and RER data



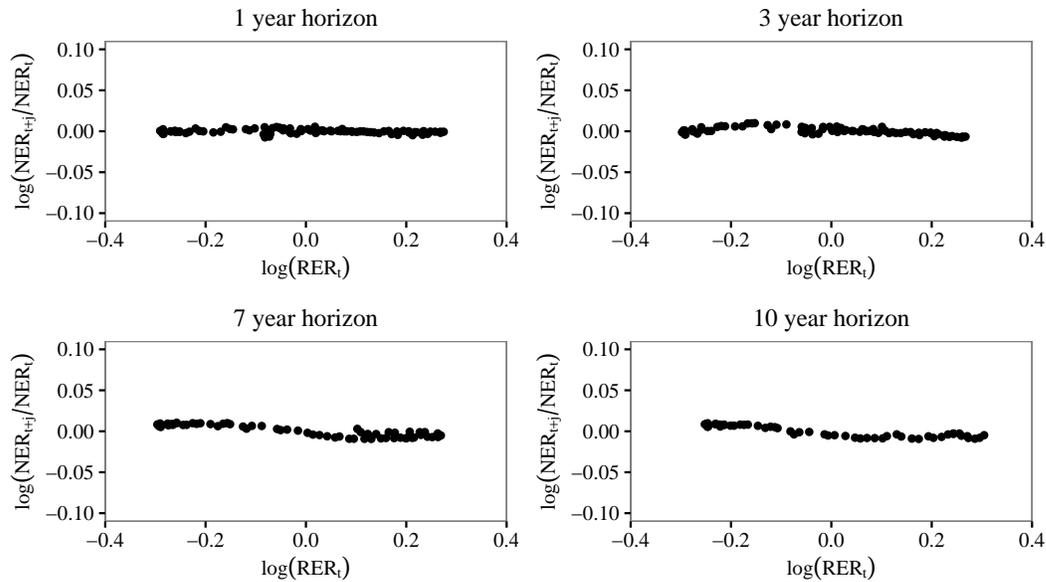
Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 11: China: NER and RER data



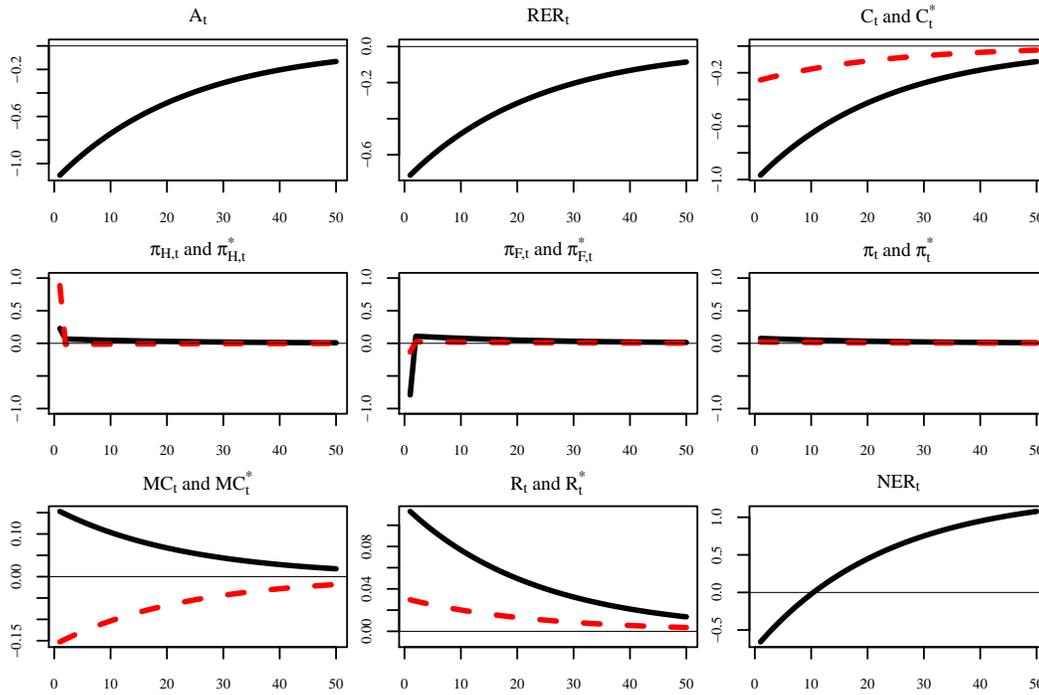
Source: OECD Main Economic Indicators, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 12: Hong Kong: NER and RER data



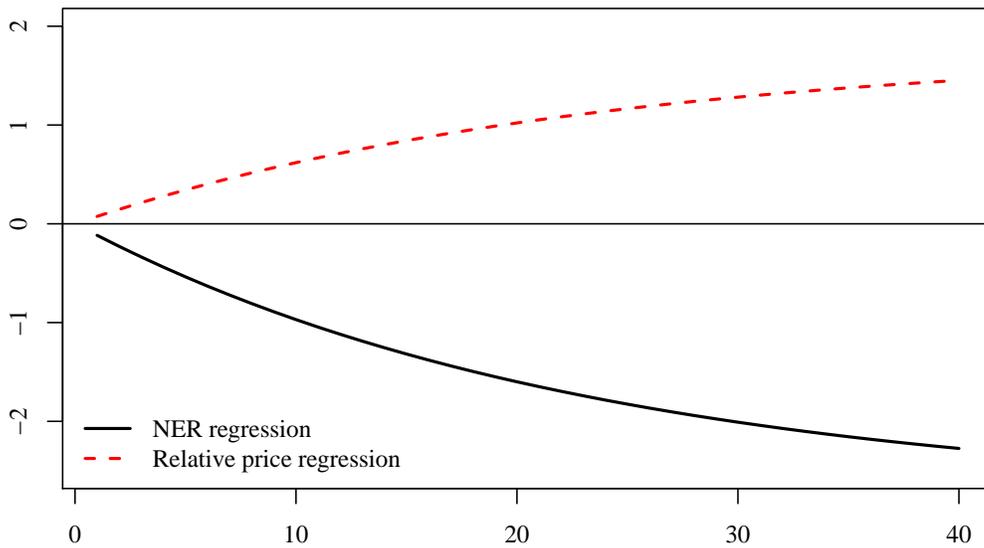
Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, and authors' calculations.

Figure 13: Response to technology shock under Taylor rule



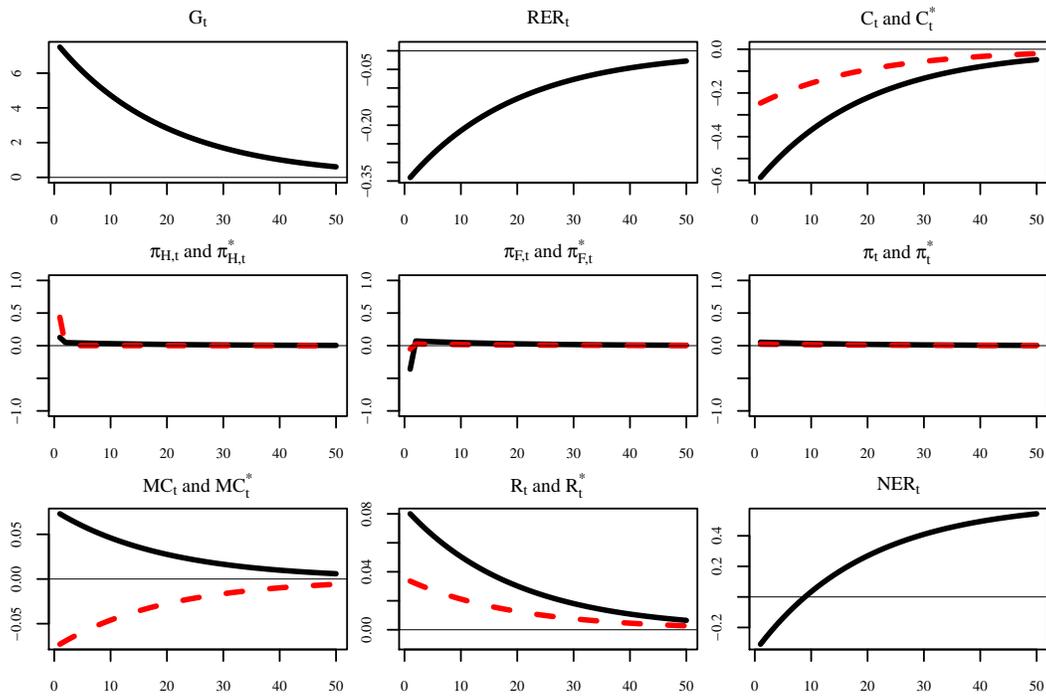
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 14: Implied values of $\beta_{1,j}^{NER}$ and $\beta_{1,j}^{\pi}$ from small-scale model



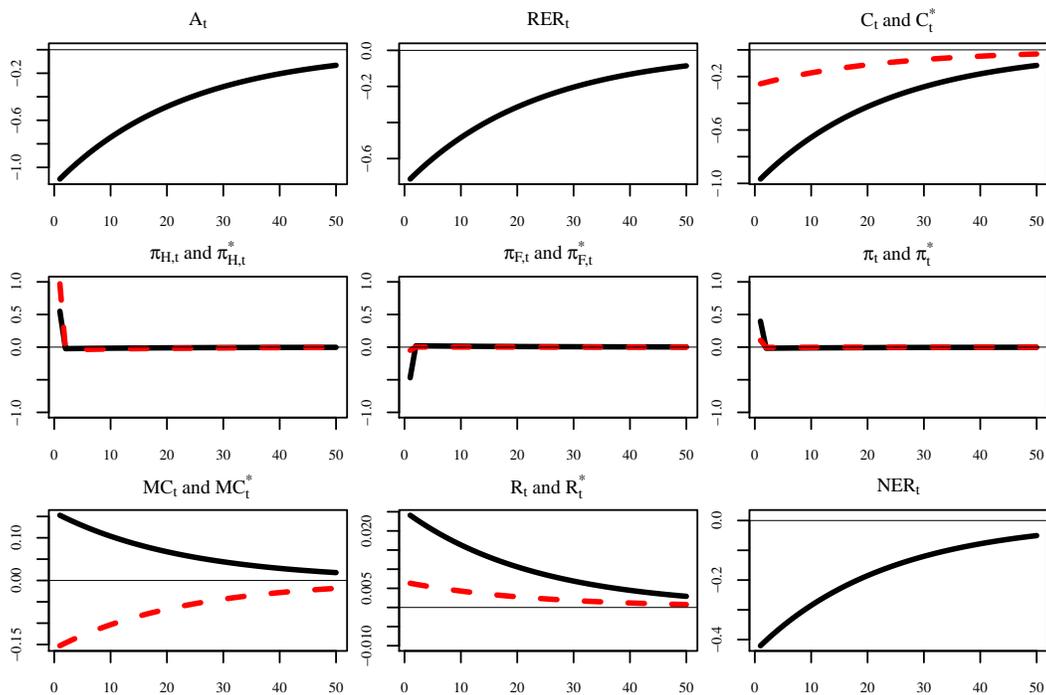
Note: The model-implied values come from our model with no nominal rigidities and only technology shocks.

Figure 15: Response to government spending shock under Taylor rule



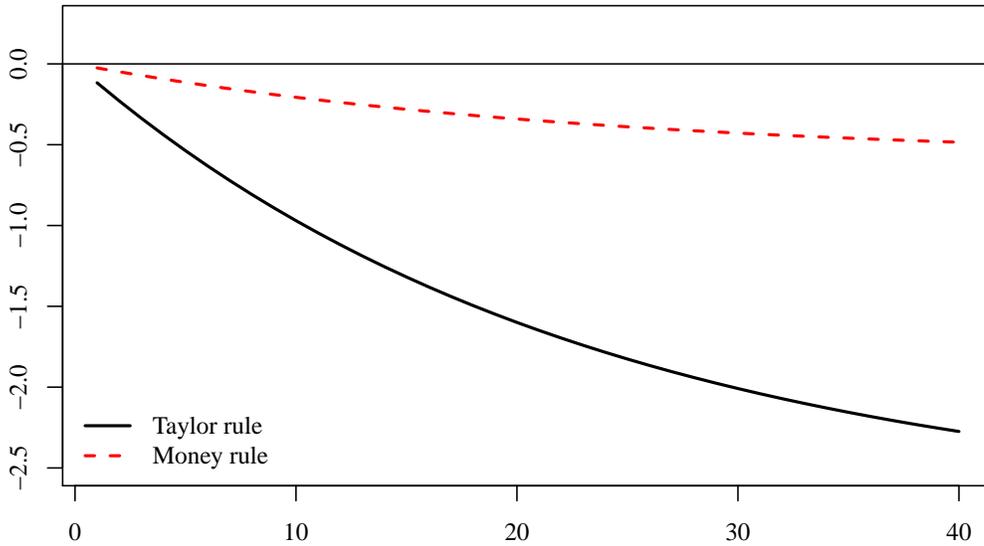
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 16: Response to technology shock under money-growth rule



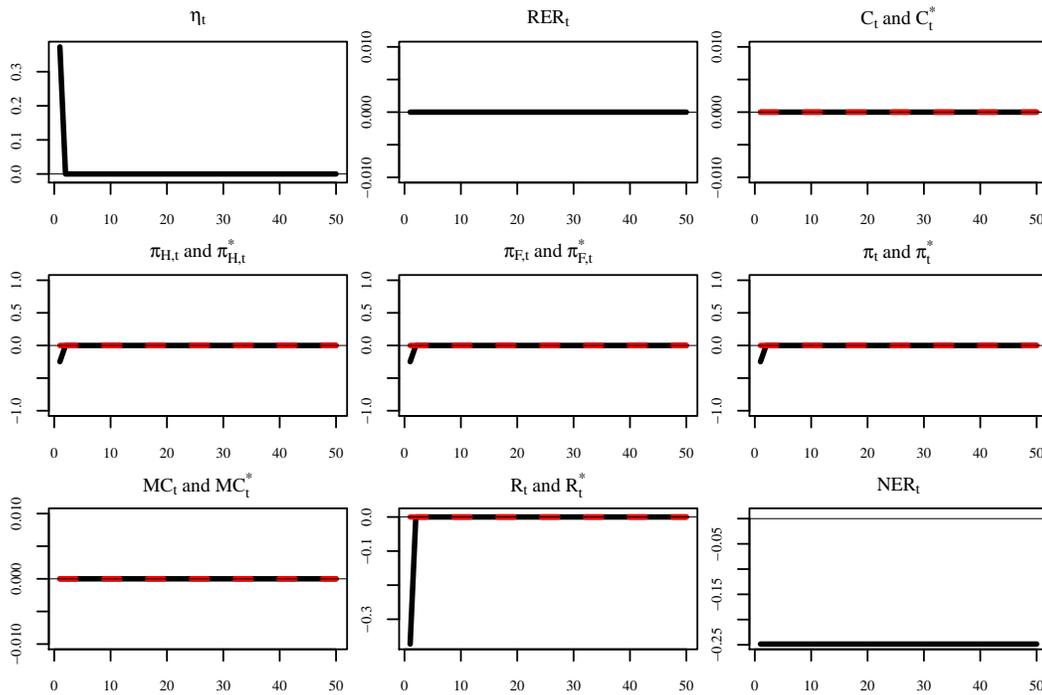
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 17: Implied values of $\beta_{1,j}^{NER}$ from small-scale model



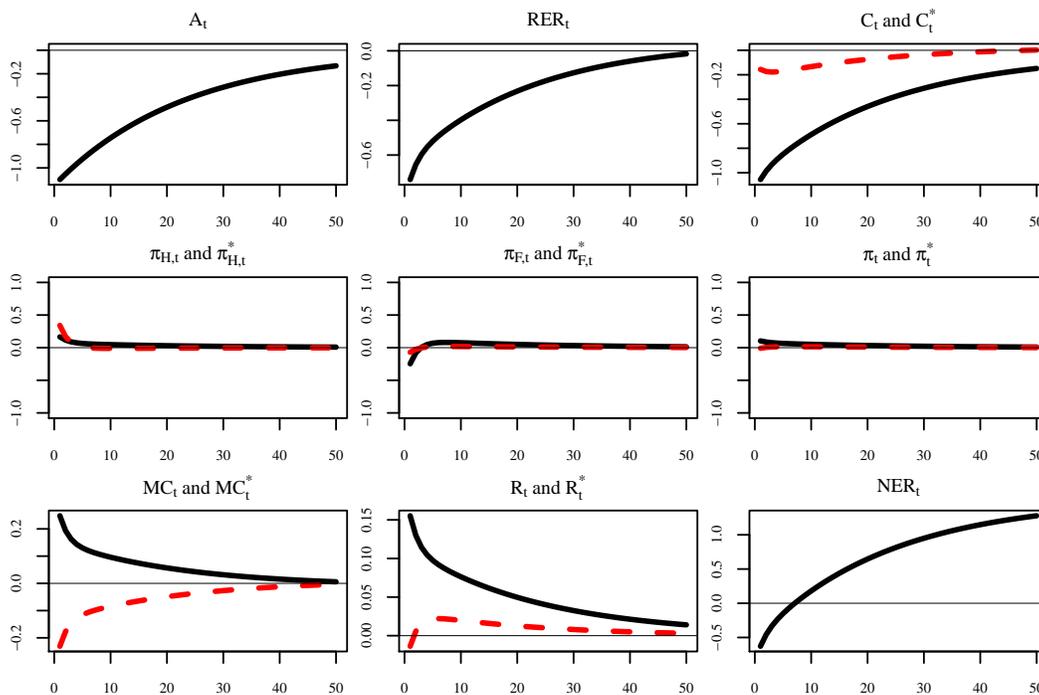
Note: The model-implied values come from our model with no nominal rigidities and only technology shocks.

Figure 18: Response to spread shock under Taylor rule with incomplete markets



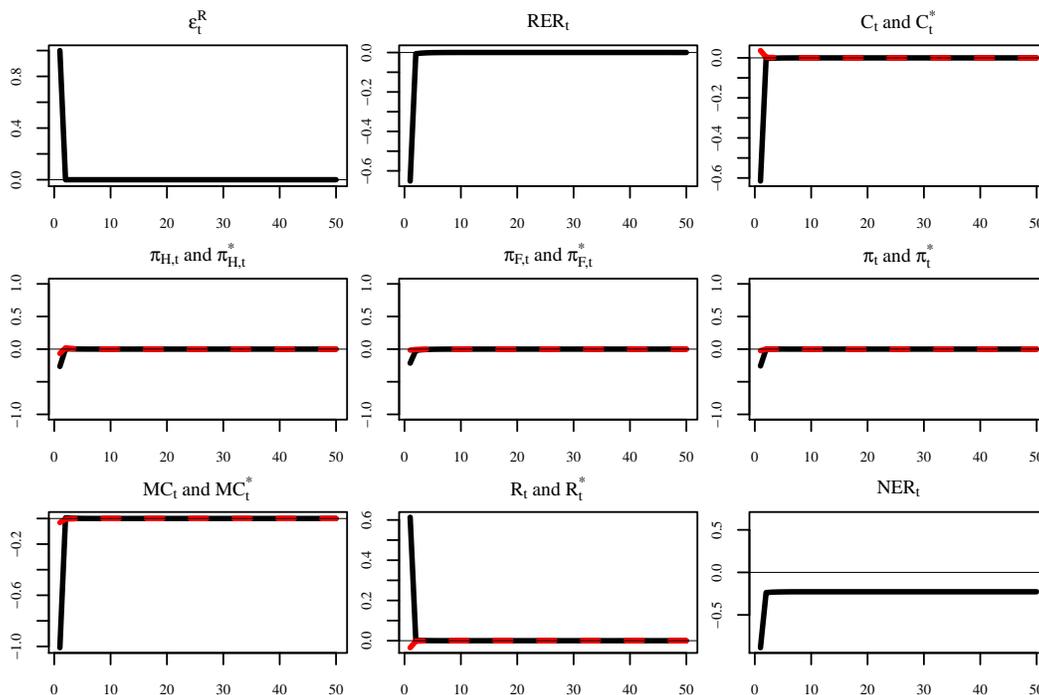
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 19: Response to technology shock under Taylor rule with incomplete markets and sticky prices



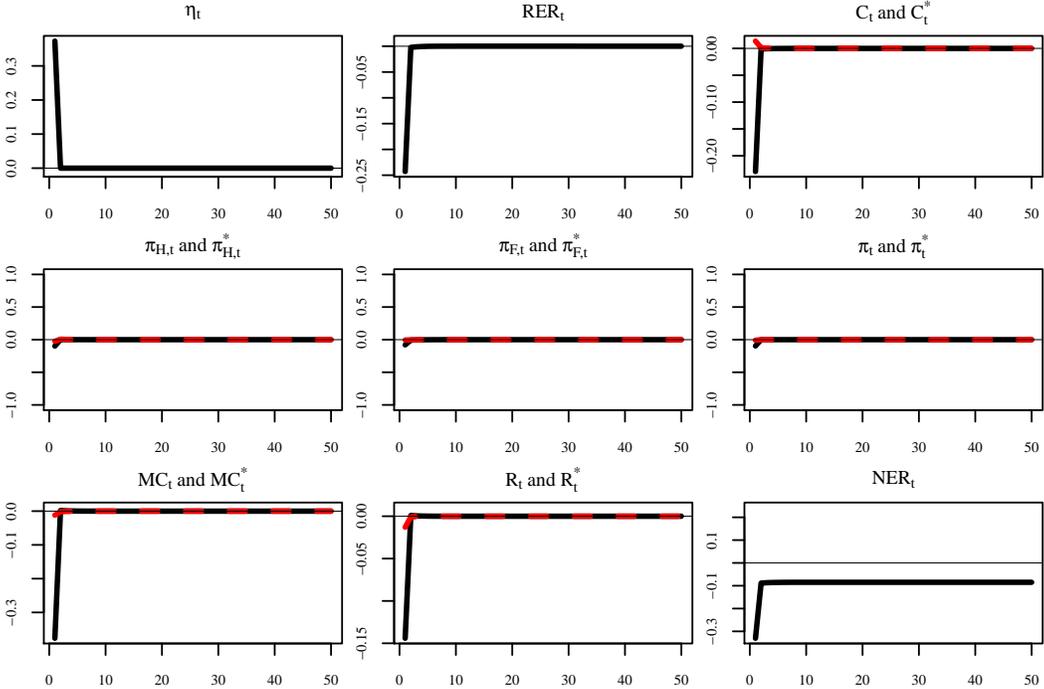
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 20: Response to monetary-policy shock under Taylor rule with incomplete markets and sticky prices



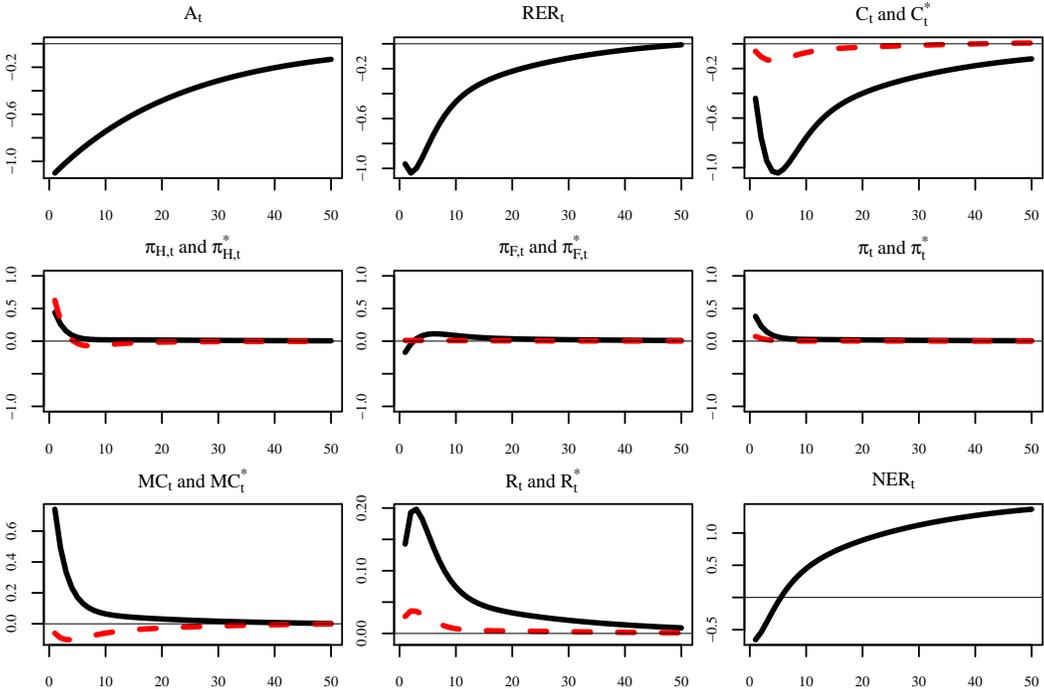
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 21: Response to spread shock under Taylor rule with incomplete markets and sticky prices



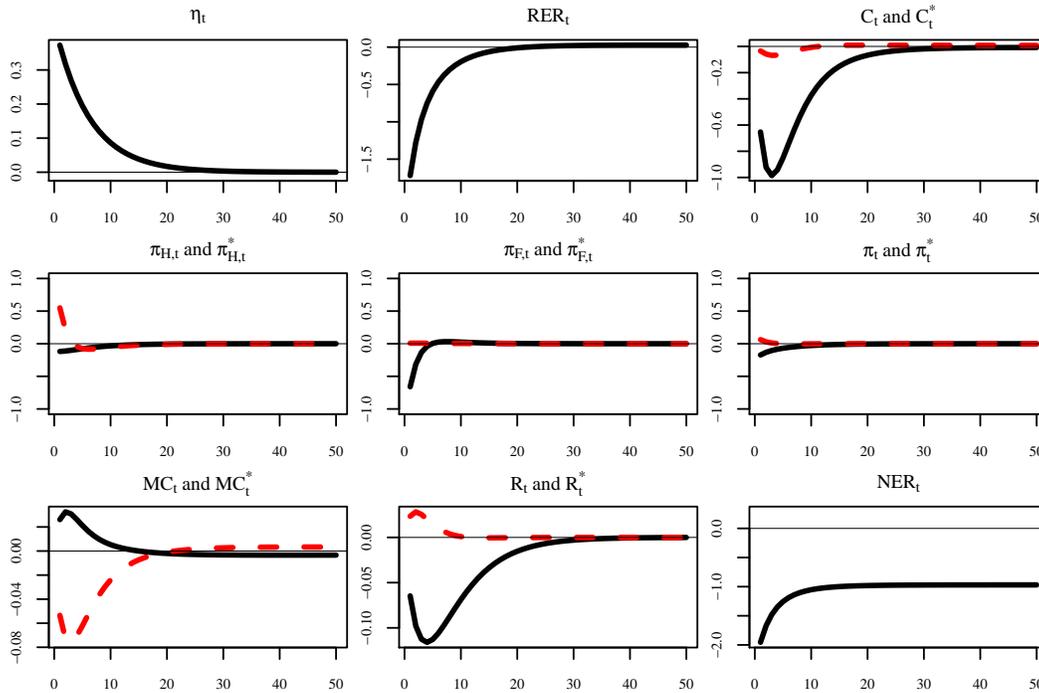
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 22: Response to technology shock under Taylor rule with incomplete markets and nominal rigidities



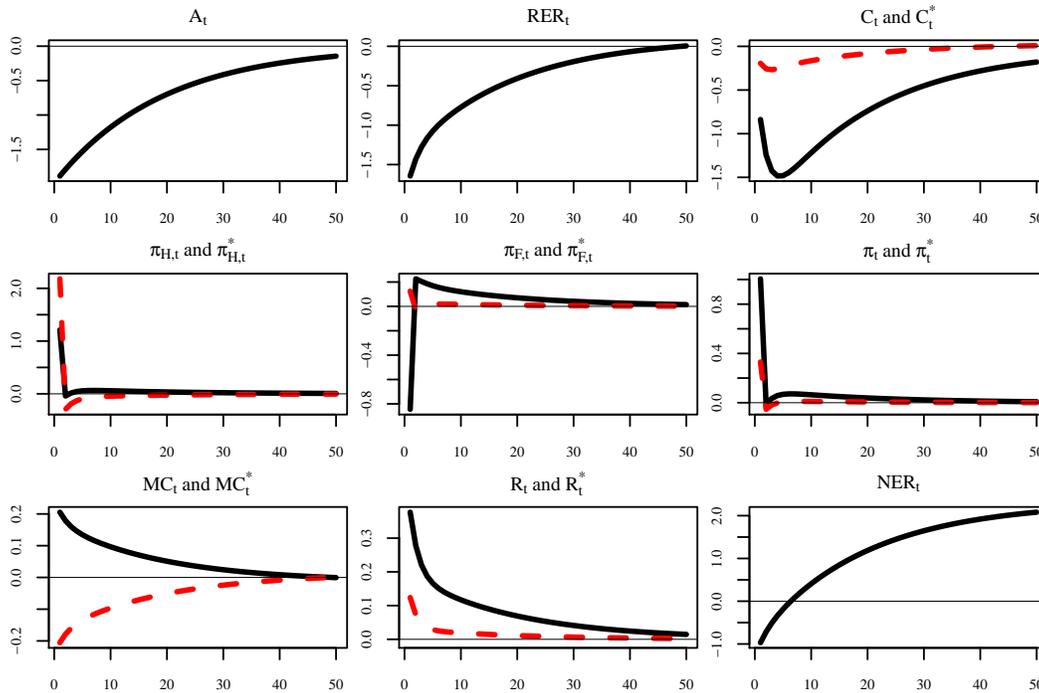
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 23: Response to spread shock under Taylor rule with incomplete markets and nominal rigidities



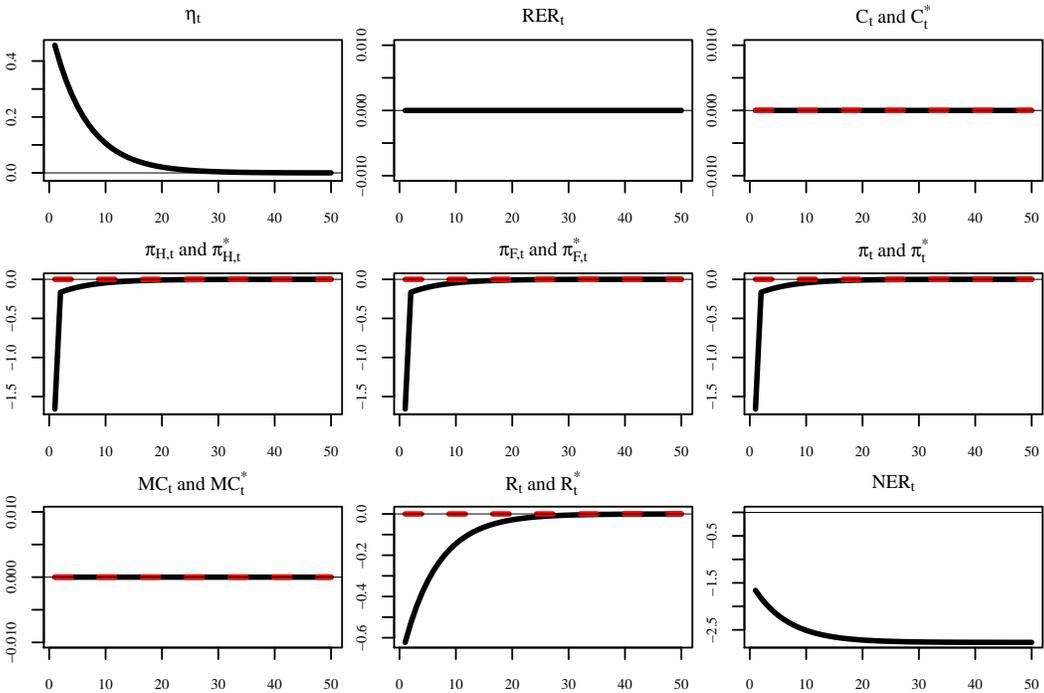
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 24: Response to technology shock under Taylor rule with incomplete markets and no nominal rigidities, medium-scale model



Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 25: Response to spread shock under Taylor rule with incomplete markets and no nominal rigidities, medium-scale model



Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Table 1: NER regression $\beta_{1,j}^{NER}$

	Horizon (in years)				
	1	3	5	7	10
A: Flexible					
Australia	-0.198 (0.095)	-0.704 (0.191)	-1.059 (0.211)	-1.128 (0.220)	-1.590 (0.135)
Canada	-0.122 (0.075)	-0.549 (0.184)	-0.944 (0.185)	-1.159 (0.142)	-1.662 (0.124)
Euro Area	-0.129 (0.169)	-0.858 (0.285)	-0.888 (0.126)	NA	NA
Germany	-0.368 (0.177)	-1.111 (0.172)	-1.551 (0.296)	NA	NA
Japan	-0.091 (0.147)	-0.555 (0.314)	-0.746 (0.204)	NA	NA
New Zealand	-0.230 (0.165)	-1.149 (0.125)	-1.566 (0.284)	NA	NA
Norway	-0.212 (0.120)	-0.764 (0.154)	-1.289 (0.250)	-1.467 (0.293)	-1.247 (0.052)
Sweden	-0.199 (0.095)	-0.746 (0.156)	-1.136 (0.187)	-1.365 (0.132)	-1.283 (0.213)
Switzerland	-0.305 (0.121)	-0.913 (0.141)	-1.373 (0.188)	-1.300 (0.125)	-1.134 (0.128)
United Kingdom	-0.294 (0.156)	-1.314 (0.341)	-1.644 (0.156)	NA	NA
B: Fixed					
China	-0.123 (0.035)	-0.208 (0.060)	-0.261 (0.096)	NA	NA
Hong Kong	-0.003 (0.002)	-0.014 (0.006)	-0.025 (0.006)	-0.031 (0.004)	-0.031 (0.004)
C: Model-implied					
Without NR	-0.414	-0.975	-1.341	-1.581	-1.797
With NR	-0.446	-0.855	-1.061	-1.199	-1.333

Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, OECD Main Economic Indicators, authors' calculations.

Table 2: NER regression R^2

	Horizon (in years)				
	1	3	5	7	10
A: Flexible					
Australia	0.103	0.388	0.586	0.600	0.755
Canada	0.078	0.349	0.590	0.687	0.878
Euro Area	0.029	0.455	0.668	NA	NA
Germany	0.215	0.563	0.826	NA	NA
Japan	0.024	0.214	0.401	NA	NA
New Zealand	0.099	0.559	0.752	NA	NA
Norway	0.075	0.293	0.552	0.647	0.514
Sweden	0.108	0.409	0.655	0.765	0.668
Switzerland	0.150	0.447	0.710	0.794	0.712
United Kingdom	0.105	0.583	0.647	NA	NA
B: Fixed					
China	0.260	0.291	0.445	NA	NA
Hong Kong	0.043	0.320	0.618	0.762	0.765

Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, OECD Main Economic Indicators, authors' calculations.

Table 3: Relative price regression $\beta_{1,j}^{\pi}$

	Horizon (in years)				
	1	3	5	7	10
A: Flexible					
Australia	0.011 (0.036)	0.046 (0.094)	0.098 (0.078)	0.198 (0.083)	0.484 (0.182)
Canada	0.014 (0.015)	0.033 (0.044)	0.040 (0.064)	0.075 (0.106)	0.257 (0.183)
Euro Area	-0.036 (0.006)	-0.079 (0.006)	0.028 (0.010)	NA	NA
Germany	-0.006 (0.033)	0.047 (0.050)	0.095 (0.058)	NA	NA
Japan	-0.003 (0.012)	0.009 (0.029)	0.040 (0.026)	NA	NA
New Zealand	-0.010 (0.012)	-0.066 (0.017)	-0.089 (0.012)	NA	NA
Norway	-0.066 (0.030)	-0.153 (0.112)	-0.112 (0.170)	-0.058 (0.194)	-0.061 (0.205)
Sweden	0.015 (0.022)	0.077 (0.055)	0.108 (0.096)	0.055 (0.187)	-0.022 (0.211)
Switzerland	-0.025 (0.023)	0.005 (0.056)	0.078 (0.091)	0.097 (0.163)	0.008 (0.175)
United Kingdom	-0.017 (0.013)	-0.031 (0.046)	-0.036 (0.036)	NA	NA
B: Fixed					
China	-0.427 (0.194)	-0.926 (0.203)	-1.052 (0.072)	NA	NA
Hong Kong	-0.093 (0.053)	-0.453 (0.141)	-0.928 (0.163)	-1.324 (0.143)	-1.629 (0.031)
C: Model-implied					
Without NR	0.184	0.476	0.670	0.797	0.912
With NR	0.074	0.176	0.269	0.340	0.413

Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, OECD Main Economic Indicators, authors' calculations.

Table 4: Relative price regression R^2

	Horizon (in years)				
	1	3	5	7	10
A: Flexible					
Australia	0.003	0.013	0.038	0.086	0.237
Canada	0.011	0.016	0.014	0.024	0.102
Euro Area	0.502	0.630	0.074	NA	NA
Germany	0.002	0.061	0.261	NA	NA
Japan	0.003	0.005	0.118	NA	NA
New Zealand	0.020	0.345	0.664	NA	NA
Norway	0.106	0.112	0.037	0.006	0.004
Sweden	0.013	0.062	0.064	0.008	0.001
Switzerland	0.033	0.000	0.023	0.025	0.000
United Kingdom	0.021	0.021	0.019	NA	NA
B: Fixed					
China	0.369	0.667	0.910	NA	NA
Hong Kong	0.126	0.374	0.660	0.878	0.990

Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, OECD Main Economic Indicators, authors' calculations.

Table 5: Euro area relative price regression

	Horizon (in years)		
	1	3	5
β_1			
France	-0.245 (0.126)	-1.029 (0.174)	-1.248 (0.158)
Italy	-0.158 (0.046)	-0.433 (0.072)	-0.555 (0.038)
Ireland	-0.302 (0.089)	-0.829 (0.086)	-1.089 (0.096)
Portugal	-0.223 (0.057)	-0.650 (0.063)	-0.819 (0.035)
Spain	-0.149 (0.031)	-0.411 (0.075)	-0.617 (0.063)
R^2			
France	0.151	0.642	0.795
Italy	0.386	0.695	0.798
Ireland	0.417	0.727	0.838
Portugal	0.475	0.849	0.933
Spain	0.483	0.747	0.880

Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, OECD Main Economic Indicators, authors' calculations.

Table 6: Results from testing random walk hypothesis using short-horizon regressions

	Quarterly Data	Annual Data
Australia	0.000	0.000
Canada	0.005	0.002
Euro Area	0.997	0.000
Germany	0.000	0.005
Japan	0.065	0.000
New Zealand	0.003	0.000
Norway	0.000	0.000
Sweden	0.000	0.000
Switzerland	0.000	0.000
United Kingdom	0.000	0.000

Note: Numbers reported are the probability of values of $\beta_{1,1}^{\pi}$ calculated from synthetic data being at least as large as in the data. We use 10,000 bootstrap samples. Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, OECD Main Economic Indicators, authors' calculations.

Table 7: Estimate of long-run regression statistic $\beta_{1,\infty}^{NER}$

	Quarterly Data			Annual Data		
	$\beta_{1,1}^{NER}$	ρ_{RER}	$\beta_{1,\infty}^{NER}$	$\beta_{1,4}^{NER}$	$(\rho_{RER})^4$	$\beta_{1,\infty}^{NER}$
Australia	-0.030 (0.021)	0.971 (0.021)	-1.023 (0.202)	-0.203 (0.105)	0.814 (0.096)	-1.089 (0.173)
Canada	-0.017 (0.015)	0.986 (0.015)	-1.234 (0.364)	-0.124 (0.070)	0.892 (0.073)	-1.143 (0.199)
Euro Area	0.010 (0.046)	1.005 (0.046)	NA	-0.257 (0.269)	0.710 (0.257)	-0.885 (0.151)
Germany	-0.052 (0.039)	0.938 (0.037)	-0.838 (0.146)	-0.305 (0.232)	0.663 (0.218)	-0.907 (0.143)
Japan	-0.005 (0.027)	0.995 (0.028)	-0.946 (0.720)	-0.120 (0.201)	0.873 (0.207)	-0.944 (0.104)
New Zealand	-0.020 (0.032)	0.979 (0.031)	-0.946 (0.229)	-0.248 (0.191)	0.745 (0.185)	-0.972 (0.066)
Norway	-0.037 (0.032)	0.948 (0.032)	-0.709 (0.203)	-0.257 (0.161)	0.685 (0.161)	-0.816 (0.120)
Sweden	-0.032 (0.021)	0.970 (0.020)	-1.083 (0.150)	-0.194 (0.106)	0.824 (0.104)	-1.098 (0.131)
Switzerland	-0.059 (0.031)	0.934 (0.029)	-0.897 (0.088)	-0.334 (0.150)	0.630 (0.142)	-0.928 (0.072)
United Kingdom	-0.027 (0.040)	0.968 (0.041)	-0.842 (0.336)	-0.295 (0.198)	0.676 (0.197)	-0.910 (0.078)

Note: Annual data are created as every fourth observation of the quarterly series. Values for $\beta_{1,1}^{NER}$ and ρ_{RER} are estimated separately from values for $\beta_{1,4}^{NER}$ and ρ_{RER}^4 . Standard errors for $\beta_{1,1}^{NER}$, ρ_{RER} , $\beta_{1,4}^{NER}$, ρ_{RER}^4 are GMM standard errors. Standard errors for $\beta_{1,\infty}^{NER}$ are computed by the delta method from the corresponding estimates and GMM standard errors.

Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, OECD Main Economic Indicators, authors' calculations.

Table 8: Calibrated Parameters

Parameter	Value	Model counterpart
σ_M	10.62	Elasticity of money demand
μ	1	Steady state money stock
β	$1.03^{-0.25}$	Steady state interest rate
h	0.65	Consumption persistence
σ	1	log utility
ϕ	1	Disutility of labor
γ	0.75	Policy rate smoothing
θ_π	1.5	Taylor principle
ν	6	Intermediate goods firm's markups
ρ_η	0.85	Persistence of interest rate differential
ρ	$\frac{1}{3}$	Substitutability of home and foreign goods
ξ	0.6	Frequency of price adjustment
ϕ_B	0.001	Cost of foreign bond holdings
ν_L	21	Differentiated wage markup
ξ_W	0.65	Frequency of wage adjustment
ω	0.90	Home bias in consumption

Table 9: Relative interest rate regressions

	ρ	σ
Australia	0.897 (0.040)	0.324 (0.023)
Canada	0.741 (0.093)	0.277 (0.020)
Euro Area	0.953 (0.033)	0.091 (0.003)
Germany	0.942 (0.040)	0.304 (0.033)
Japan	0.834 (0.098)	0.355 (0.040)
New Zealand	0.905 (0.044)	0.163 (0.009)
Norway	0.846 (0.082)	0.431 (0.029)
Sweden	0.757 (0.168)	0.603 (0.136)
Switzerland	0.944 (0.041)	0.309 (0.021)
United Kingdom	0.855 (0.059)	0.119 (0.003)

For each country listed, we estimate an AR(1) for the interest rate differential. We use money-market interest rate data from the IFS. Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, authors' calculations.

Table 10: Fama regression statistics

	α_0	α_1
Australia	0.005 (0.005)	-0.352 (0.419)
Canada	0.001 (0.003)	-0.387 (0.523)
Euro Area	-0.013 (0.006)	-5.011 (1.849)
Germany	-0.004 (0.009)	-0.630 (0.898)
Japan	-0.031 (0.010)	-2.982 (0.793)
New Zealand	0.013 (0.011)	-2.412 (1.459)
Norway	-0.001 (0.005)	-0.033 (0.657)
Sweden	0.001 (0.005)	0.586 (0.834)
Switzerland	-0.012 (0.007)	-0.583 (0.499)
United Kingdom	-0.004 (0.006)	-0.090 (1.632)

Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates,, authors' calculations.

Table 11: Estimated Parameters

Parameter	Value, No Nominal Rigidities	Values, Nominal Rigidities
ρ_A	0.949	0.958
$100 \times \sigma_A$	1.886	1.099
$100 \times \sigma_\eta$	0.457	0.373

Table 12: Empirical facts about exchange rates

	ρ_{RER}	$\sigma_{\Delta RER}$	$\sigma_{\Delta NER}$	$\text{cor}(\Delta RER, \Delta NER)$
Australia	0.971 (0.848,0.986)	0.040 (0.003)	0.040 (0.003)	0.968 (0.007)
Canada	0.986 (0.872,0.997)	0.022 (0.002)	0.022 (0.002)	0.969 (0.007)
Euro Area	1.005 (0.611,1.031)	0.039 (0.003)	0.039 (0.003)	0.994 (0.002)
Germany	0.936 (0.714,0.977)	0.055 (0.004)	0.055 (0.004)	0.991 (0.002)
Japan	0.995 (0.766,1.011)	0.053 (0.004)	0.051 (0.004)	0.991 (0.002)
New Zealand	0.979 (0.759,0.992)	0.040 (0.003)	0.040 (0.003)	0.990 (0.003)
Norway	0.948 (0.824,0.972)	0.043 (0.002)	0.042 (0.002)	0.975 (0.005)
Sweden	0.970 (0.849,0.986)	0.047 (0.004)	0.048 (0.004)	0.978 (0.004)
Switzerland	0.934 (0.828,0.963)	0.052 (0.003)	0.052 (0.003)	0.989 (0.002)
United Kingdom	0.968 (0.698,0.988)	0.027 (0.003)	0.025 (0.003)	0.978 (0.006)
China	0.857 (0.746,0.908)	0.020 (0.002)	0.005 (0.001)	0.543 (0.087)
Hong Kong	0.982 (0.938,0.999)	0.013 (0.001)	0.002 (0.000)	0.380 (0.079)
Nominal rigidities	0.890	0.023	0.023	0.957
Without nominal rigidities	0.928	0.024	0.023	0.649

Note: confidence intervals for ρ_{RER} are constructed from a parametric bootstrap for an AR(1) model of $\log(RER_t)$. We used 10,000 bootstrap draws and report the 0.025% and 0.975% quantiles of the bootstrap distribution of the statistic of interest. Standard errors for $\sigma_{\Delta RER}$ and $\sigma_{\Delta NER}$ are GMM standard errors. Source: International Monetary Fund, International Financial Statistics, Federal Reserve Board, H.10 Foreign Exchange Rates, OECD Main Economic Indicators, authors' calculations.

A Model

A.1 Household

The household problem for the representative household in the home country is

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{(C_{t+j} - h\bar{C}_{t+j-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi} \int_0^1 L_{t+j}(i)^{1+\phi} di + \mu \frac{\left(\frac{M_{t+j}}{P_{t+j}}\right)^{1-\sigma_M}}{1-\sigma_M} + \log(\eta_{t+j}) V\left(\frac{B_{H,t+j}}{P_{t+j}}\right) + \log(\eta_{t+j}^*) V\left(\frac{B_{F,t+j} NER_{t+j}}{P_{t+j}}\right) \right) \quad (86)$$

where C_t is consumption, \bar{C}_t is aggregate consumption, L_t is hours worked, $\frac{M_t}{P_t}$ are real money balances. The budget constraint is

$$B_{H,t} + NER_t B_{F,t} + P_t C_t + P_{I,t} I_t + M_t = R_{t-1} B_{H,t-1} + NER_t R_{t-1}^* B_{F,t-1} - \frac{\phi_B}{2} \left(\frac{NER_t B_{F,t}}{P_t}\right)^2 P_t + P_t R_{K,t} K_t + (1 + \tau_W) \int_0^1 W_t(i) L_t(i) di + T_t + M_{t-1} \quad (87)$$

where $B_{H,t}$ and $B_{F,t}$ are nominal balances of home and foreign bonds, NER_t is the nominal exchange rate quoted as the price of the foreign currency unit, P_t is the price of final goods in the home country, R_t is the nominal interest rate on the home bond and R_t^* is the nominal interest rate on the foreign bond, W_t is the wage rate, R_t^K is the rental rate on capital, K_t , I_t are investment goods and T_t are lump-sum profits and taxes. The capital accumulation equation is

$$K_{t+1} = I_t \left(1 - \frac{\phi_K}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) + (1 - \delta) K_t \quad (88)$$

The household-wide first-order conditions are

$$(C_t - hC_{t-1})^{-\sigma} = \Lambda_t \quad (89)$$

$$\Lambda_t = \log(\eta_t) V' \left(\frac{B_{H,t}}{P_t}\right) + \beta R_t E_t \frac{\Lambda_{t+1}}{\pi_{t+1}} \quad (90)$$

$$\Lambda_t + \phi_B \left(\frac{NER_t B_{F,t}}{P_t}\right) = \log(\eta_t^*) V' \left(\frac{B_{F,t} NER_t}{P_t}\right) + \beta R_t^* E_t \frac{\Lambda_{t+1}}{\pi_{t+1}} \frac{NER_{t+1}}{NER_t} \quad (91)$$

$$\mu \left(\frac{M_t}{P_t}\right)^{-\sigma_M} = \frac{\log(\eta_t)}{R_t} V' \left(\frac{B_{H,t}}{P_t}\right) + \left(\frac{R_t - 1}{R_t}\right) \Lambda_t \quad (92)$$

$$\frac{P_{I,t}}{P_t} \Lambda_t = Q_t \left[\left(1 - \frac{\phi_K}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) - \frac{I_t}{I_{t-1}} \phi_K \left(\frac{I_t}{I_{t-1}} - 1\right) \right] + \beta E_t Q_{t+1} \phi_K \left(\frac{I_{t+1}}{I_t} - 1\right) \frac{I_{t+1}^2}{I_t^2} \quad (93)$$

$$Q_t = \beta E_t [Q_{t+1} (1 - \delta) + \Lambda_{t+1} R_{K,t+1}] \quad (94)$$

There are similar first-order conditions for the foreign household. The household problem for the representative household in the foreign country is

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{(C_{t+j}^* - h\bar{C}_{t+j-1}^*)^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi} \int_0^1 L_{t+j}^*(i)^{1+\phi} di + \mu \frac{\left(\frac{M_{t+j}^*}{P_{t+j}^*}\right)^{1-\sigma_M}}{1-\sigma_M} + \log(\eta_{t+j}) V\left(\frac{B_{H,t+j}^*}{NER_{t+j} P_{t+j}^*}\right) + \log(\eta_{t+j}^*) V\left(\frac{B_{F,t+j}^*}{P_{t+j}^*}\right) \right) \quad (95)$$

The budget constraint is

$$B_{F,t}^* + NER_t^{-1} B_{H,t}^* + P_t^* C_t^* + P_{I,t}^* I_t^* + M_t^* = R_{t-1} B_{H,t-1}^* NER_t^{-1} + R_{t-1}^* B_{F,t-1}^* - \frac{\phi_B}{2} \left(\frac{B_{H,t}^*}{NER_t P_t^*}\right)^2 P_t + P_t^* R_{K,t}^* K_t^* + (1 + \tau_W) \int_0^1 W_t^*(i) L_t^*(i) di + T_t^* + M_{t-1}^* \quad (96)$$

The capital accumulation equation is

$$K_{t+1}^* = I_t^* \left(1 - \frac{\phi_K}{2} \left(\frac{I_t^*}{I_{t-1}^*} - 1 \right)^2 \right) + (1 - \delta) K_t^* \quad (97)$$

The household-wide first-order conditions are

$$(C_t^* - hC_{t-1}^*)^{-\sigma} = \Lambda_t^* \quad (98)$$

$$\Lambda_t^* + \phi_B \left(\frac{B_{H,t}^*}{NER_t P_t^*} \right) = \log(\eta_t) V' \left(\frac{B_{H,t}^*}{NER_t P_t^*} \right) + \beta R_t E_t \frac{\Lambda_{t+1}^*}{\pi_{t+1}^*} \frac{NER_t}{NER_{t+1}} \quad (99)$$

$$\Lambda_t^* = \log(\eta_t^*) V' \left(\frac{B_{F,t}^*}{P_t^*} \right) + \beta R_t^* E_t \frac{\Lambda_{t+1}^*}{\pi_{t+1}^*} \quad (100)$$

$$\mu \left(\frac{M_t^*}{P_t^*} \right)^{-\sigma_M} = \frac{\log(\eta_t^*)}{R_t^*} V' \left(\frac{B_{F,t}^*}{P_t^*} \right) + \left(\frac{R_t^* - 1}{R_t^*} \right) \Lambda_t^* \quad (101)$$

$$\frac{P_{I,t}^*}{P_t^*} \Lambda_t^* = Q_t^* \left[\left(1 - \frac{\phi_K}{2} \left(\frac{I_t^*}{I_{t-1}^*} - 1 \right)^2 \right) - \frac{I_t^*}{I_{t-1}^*} \phi_K \left(\frac{I_t^*}{I_{t-1}^*} - 1 \right) \right] + \beta E_t Q_{t+1} \phi_K \left(\frac{I_{t+1}^*}{I_t^*} - 1 \right) \left(\frac{I_{t+1}^*}{I_t^*} \right)^2 \quad (102)$$

$$Q_t^* = \beta E_t [Q_{t+1}^* (1 - \delta) + \Lambda_{t+1}^* R_{K,t+1}^*] \quad (103)$$

Note that we define

$$RER_t = \frac{NER_t P_t^*}{P_t} \quad (104)$$

A.2 The labor market

We assume that all of the household members consumer the same amount (perfect consumption insurance). Each household member of is a member of a union that supplies its type of labor, i . Labor is combined via

$$L_t = \left[\int_0^1 L_t(i)^{\frac{\nu_L - 1}{\nu_L}} di \right]^{\frac{\nu_L}{\nu_L - 1}}$$

to produce labor services, which go to the production sector. The aggregator that minimizes the cost of producing labor services is

$$W_t = \left[\int_0^1 W_t(i)^{1 - \nu_L} di \right]^{\frac{1}{1 - \nu_L}}$$

The demand for a given labor type is

$$L_t(i) = \left[\frac{W_t(i)}{W_t} \right]^{-\nu_L} L_t$$

Unions negotiate their wage with probability $1 - \xi_W$. When they do, they maximize household utility taking demand curves for their labor as given. The first-order condition with respect to the wage is

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\Lambda_{t+j} \frac{1}{P_{t+j}} (1 - \nu_L) \left[\frac{\tilde{W}_t}{W_{t+j}} \right]^{-\nu_L} (1 + \tau_W) + \chi \nu_L \left(\left[\frac{\tilde{W}_t}{W_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^\phi \left[\frac{\tilde{W}_t}{W_{t+j}} \right]^{-\nu_L} \frac{1}{\tilde{W}_t} \right] = 0$$

where \tilde{W}_t is the chosen wage by a union that can updates its wage. This is simplified to be

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \left[\Lambda_{t+j} \frac{1}{P_{t+j}} \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) \tilde{W}_t - \chi \left(\left[\frac{\tilde{W}_t}{W_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^\phi \right] = 0$$

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \left[\Lambda_{t+j} \frac{P_t^{1 + \nu_L \phi}}{P_{t+j}} \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) \tilde{w}_t^{1 + \nu_L \phi} - \chi \left(\left[\frac{1}{w_{t+j}} \frac{1}{P_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^\phi \right] = 0$$

$$E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \left[\Lambda_{t+j} \frac{P_t}{P_{t+j}} \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) \tilde{w}_t^{1+\nu_L \phi} - \chi \left(\left[\frac{1}{w_{t+j}} \frac{P_t}{P_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^{\phi} \right] = 0$$

where \tilde{w}_t is the real wage that is set by unions that optimize. Then, we can write

$$F_{W,t} \tilde{w}_t^{1+\nu_L \phi} = K_{W,t} \quad (105)$$

where

$$\begin{aligned} F_{W,t} &= E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \Lambda_{t+j} \frac{P_t}{P_{t+j}} \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) \\ &= L_t \Lambda_t \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) + E_t \sum_{j=1}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \Lambda_{t+j} \frac{P_t}{P_{t+j}} \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) \\ &= L_t \Lambda_t \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) + E_t \sum_{j=0}^{\infty} (\beta \xi_W)^{j+1} L_{t+1+j} \left[\frac{W_t}{W_{t+1+j}} \right]^{-\nu_L} \Lambda_{t+1+j} \frac{P_t}{P_{t+1+j}} \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) \\ &= L_t \Lambda_t \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) + \beta \xi_W E_t \left[\frac{W_t}{W_{t+1}} \right]^{-\nu_L} \frac{P_t}{P_{t+1}} E_{t+1} \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+1+j} \left[\frac{W_{t+1}}{W_{t+1+j}} \right]^{-\nu_L} \Lambda_{t+1+j} \frac{P_{t+1}}{P_{t+1+j}} \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) \\ &= L_t \Lambda_t \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) + \beta \xi_W E_t \left[\frac{w_t}{w_{t+1}} \pi_{t+1}^{-1} \right]^{-\nu_L} \pi_{t+1}^{-1} F_{W,t+1} \\ F_{W,t} &= L_t \Lambda_t \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) + \beta \xi_W E_t \pi_{t+1}^{-1} \left(\frac{w_{t+1}}{w_t} \pi_{t+1} \right)^{\nu_L} F_{W,t+1} \end{aligned} \quad (106)$$

and

$$\begin{aligned} K_{W,t} &= E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \chi \left(\left[\frac{1}{w_{t+j}} \frac{P_t}{P_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^{\phi} \\ &= \chi L_t^{1+\phi} w_t^{\phi \nu_L} + E_t \sum_{j=1}^{\infty} (\beta \xi_W)^j L_{t+j} \left[\frac{W_t}{W_{t+j}} \right]^{-\nu_L} \chi \left(\left[\frac{1}{w_{t+j}} \frac{P_t}{P_{t+j}} \right]^{-\nu_L} L_{t+j} \right)^{\phi} \\ &= \chi L_t^{1+\phi} w_t^{\phi \nu_L} + E_t \sum_{j=0}^{\infty} (\beta \xi_W)^{j+1} L_{t+1+j} \left[\frac{W_t}{W_{t+1+j}} \right]^{-\nu_L} \chi \left(\left[\frac{1}{w_{t+1+j}} \frac{P_t}{P_{t+1+j}} \right]^{-\nu_L} L_{t+1+j} \right)^{\phi} \\ &= \chi L_t^{1+\phi} w_t^{\phi \nu_L} + \beta \xi_W \left(\frac{W_t}{W_{t+1}} \right)^{-\nu_L} \left(\frac{P_{t+1}}{P_t} \right)^{\nu_L \phi} E_t \sum_{j=0}^{\infty} (\beta \xi_W)^j L_{t+1+j} \left[\frac{W_{t+1}}{W_{t+1+j}} \right]^{-\nu_L} \chi \left(\left[\frac{1}{w_{t+1+j}} \frac{P_{t+1}}{P_{t+1+j}} \right]^{-\nu_L} L_{t+1+j} \right)^{\phi} \\ &= \chi L_t^{1+\phi} w_t^{\phi \nu_L} + \beta \xi_W E_t \left(\frac{w_{t+1}}{w_t} \pi_{t+1} \right)^{\nu_L} (\pi_{t+1})^{\nu_L \phi} K_{W,t+1} \\ K_{W,t} &= \chi L_t^{1+\phi} w_t^{\phi \nu_L} + \beta \xi_W E_t \left(\frac{w_{t+1}}{w_t} \right)^{\nu_L} (\pi_{t+1})^{\nu_L (1+\phi)} K_{W,t+1} \end{aligned} \quad (107)$$

Then wages evolve so that

$$W_t = \left((1 - \xi_W) \tilde{W}_t^{1-\nu_L} + \xi_W W_{t-1}^{1-\nu_L} \right)^{\frac{1}{1-\nu_L}}$$

which yields

$$w_t = \left((1 - \xi_W) \tilde{w}_t^{1-\nu_L} + \xi_W \left(\frac{w_{t-1}}{\pi_t} \right)^{1-\nu_L} \right)^{\frac{1}{1-\nu_L}} \quad (108)$$

Note that in the case that $\xi_W = 0$, we have $\tilde{w}_t = w_t$ and

$$\Lambda_t \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) w_t = \chi L_t^{\phi}$$

so that if $\frac{\nu_L - 1}{\nu_L} (1 + \tau_W) = 1$ we have the usual intratemporal Euler equation.

In the foreign economy, we have

$$F_{W,t}^* (\bar{w}_t^*)^{1+\nu_L\phi} = K_{W,t}^* \quad (109)$$

$$F_{W,t}^* = L_t^* \Lambda_t^* \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) + \beta \xi_W E_t (\pi_{t+1}^*)^{-1} \left(\frac{w_{t+1}^*}{w_t^*} \pi_{t+1}^* \right)^{\nu_L} F_{W,t+1} \quad (110)$$

$$K_{W,t}^* = \chi (L_t^*)^{1+\phi} (w_t^*)^{\phi\nu_L} + \beta \xi_W E_t \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\nu_L} (\pi_{t+1}^*)^{\nu_L(1+\phi)} K_{W,t+1}^* \quad (111)$$

$$w_t^* = \left((1 - \xi_W) (\bar{w}_t^*)^{1-\nu_L} + \xi_W \left(\frac{w_{t-1}^*}{\pi_t^*} \right)^{1-\nu_L} \right)^{\frac{1}{1-\nu_L}}. \quad (112)$$

A.3 Goods aggregators

In each country, perfectly competitive firms aggregate country-specific intermedate inputs into $X_{H,t}$, $X_{F,t}$, $X_{H,t}^*$, and $X_{F,t}^*$. These intermediate inputs are used either for investment goods or final goods production, so

$$Y_{H,t} + I_{H,t} = X_{H,t} \quad (113)$$

$$Y_{F,t} + I_{F,t} = X_{F,t} \quad (114)$$

$$Y_{H,t}^* + I_{H,t}^* = X_{H,t}^* \quad (115)$$

$$Y_{F,t}^* + I_{F,t}^* = X_{F,t}^* \quad (116)$$

The values $X_{H,t}$ and $X_{F,t}$ are consumed in the home country are composites of goods purchased from monopolists so that

$$X_{H,t} = \left(\int_0^1 X_{H,t}(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}}$$

$$X_{F,t} = \left(\int_0^1 X_{F,t}(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}}$$

The profits of the goods aggregators are given by

$$P_{H,t} \left(\int_0^1 X_{H,t}(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} - \int_0^1 X_{H,t}(i) P_{H,t}(i) di$$

and

$$P_{F,t} \left(\int_0^1 X_{F,t}(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} - \int_0^1 X_{F,t}(i) P_{F,t}(i) di$$

First-order conditions are

$$P_{H,t} X_{H,t}^{\frac{1}{\nu}} X_{H,t}(i)^{-\frac{1}{\nu}} = P_{H,t}(i)$$

$$P_{F,t} X_{F,t}^{\frac{1}{\nu}} X_{F,t}(i)^{-\frac{1}{\nu}} = P_{F,t}(i)$$

Demand curves are then of the form

$$X_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\nu} X_{H,t} \quad (117)$$

$$X_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\nu} X_{F,t}. \quad (118)$$

The zero profit condition, along with these demand curves, implies

$$P_{H,t} \left(\int_0^1 \left(\left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\nu} X_{H,t} \right)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\nu} X_{H,t} P_{H,t}(i) di$$

$$P_{H,t}^{1+\nu} \left(\int_0^1 P_{H,t}(i)^{1-\nu} di \right)^{\frac{\nu}{\nu-1}} = P_{H,t}^\nu \int_0^1 P_{H,t}(i)^{1-\nu} di$$

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\nu} di \right)^{\frac{1}{1-\nu}} \quad (119)$$

Similarly,

$$P_{F,t} = \left(\int_0^1 P_{F,t}(i)^{1-\nu} di \right)^{\frac{1}{1-\nu}} \quad (120)$$

In country F, goods aggregators create $X_{H,t}^*$ and $X_{F,t}^*$ according to

$$X_{H,t}^* = \left(\int_0^1 X_{H,t}^*(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}},$$

$$X_{F,t}^* = \left(\int_0^1 X_{F,t}^*(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}}.$$

The profits of the goods aggregators are given by

$$P_{H,t}^* \left(\int_0^1 X_{H,t}^*(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} - \int_0^1 X_{H,t}^*(i) P_{H,t}^*(i) di$$

and

$$P_{F,t}^* \left(\int_0^1 X_{F,t}^*(i)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} - \int_0^1 X_{F,t}^*(i) P_{F,t}^*(i) di$$

Demand curves are then of the form

$$X_{H,t}^*(i) = \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{-\nu} X_{H,t}^* \quad (121)$$

$$X_{F,t}^*(i) = \left(\frac{P_{F,t}^*(i)}{P_{F,t}^*} \right)^{-\nu} X_{F,t}^*. \quad (122)$$

The zero profit conditions, along with these demand curves, imply

$$P_{H,t}^* = \left(\int_0^1 P_{H,t}^*(i)^{1-\nu} di \right)^{\frac{1}{1-\nu}} \quad (123)$$

and

$$P_{F,t}^* = \left(\int_0^1 P_{F,t}^*(i)^{1-\nu} di \right)^{\frac{1}{1-\nu}}. \quad (124)$$

A.4 Retailers

Final goods, Y_t , are created by combining goods from countries H and F ($X_{H,t}$ and $X_{F,t}$) using

$$Y_t = \left(\omega^{1-\rho} (Y_{H,t})^\rho + (1-\omega)^{1-\rho} (Y_{F,t})^\rho \right)^{\frac{1}{\rho}} \quad (125)$$

Profits are given by

$$P_t \left(\omega^{1-\rho} (Y_{H,t})^\rho + (1-\omega)^{1-\rho} (Y_{F,t})^\rho \right)^{\frac{1}{\rho}} - P_{H,t} Y_{H,t} - P_{F,t} Y_{F,t}. \quad (126)$$

where $P_{H,t}$ is the nominal price of $Y_{H,t}$, $P_{F,t}$ is the nominal price of $Y_{F,t}$. First-order conditions are

$$P_t \left(\omega^{1-\rho} (Y_{H,t})^\rho + (1-\omega)^{1-\rho} (Y_{F,t})^\rho \right)^{\frac{1-\rho}{\rho}} \omega^{1-\rho} (Y_{H,t})^{\rho-1} = P_{H,t}$$

$$P_t \left(\omega^{1-\rho} (Y_{H,t})^\rho + (1-\omega)^{1-\rho} (Y_{F,t})^\rho \right)^{\frac{1-\rho}{\rho}} (1-\omega)^{1-\rho} (Y_{F,t})^{\rho-1} = P_{F,t}$$

Use the definition of Y_t to get

$$\begin{aligned}\left(\frac{Y_{H,t}}{\omega Y_t}\right)^{\rho-1} &= \frac{P_{H,t}}{P_t} \\ \left(\frac{Y_{F,t}}{(1-\omega)Y_t}\right)^{\rho-1} &= \frac{P_{F,t}}{P_t}\end{aligned}$$

Demand curves are then

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} \omega Y_t \quad (127)$$

$$Y_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{\frac{1}{\rho-1}} (1-\omega) Y_t. \quad (128)$$

There is free entry for retailers, so profits are zero. Substituting demand curves into the profits expression yields

$$\begin{aligned}P_t Y_t &= P_{H,t} \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} \omega Y_t + P_{F,t} \left(\frac{P_{F,t}}{P_t}\right)^{\frac{1}{\rho-1}} (1-\omega) Y_t \\ P_t^{\frac{\rho}{\rho-1}} &= (P_{H,t})^{\frac{\rho}{\rho-1}} \omega + (P_{F,t})^{\frac{\rho}{\rho-1}} (1-\omega) \\ P_t &= \left(\omega P_{H,t}^{\frac{\rho}{\rho-1}} + (1-\omega) (P_{F,t})^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}\end{aligned} \quad (129)$$

Final goods in the foreign country, Y_t^* , are created by combining goods for countries H and F ($Y_{H,t}^*$ and $Y_{F,t}^*$) using

$$Y_t^* = \left(\omega^{1-\rho} (Y_{F,t}^*)^\rho + (1-\omega)^{1-\rho} (Y_{H,t}^*)^\rho\right)^{\frac{1}{\rho}} \quad (130)$$

Profits are given by

$$P_t^* \left(\omega^{1-\rho} (Y_{F,t}^*)^\rho + (1-\omega)^{1-\rho} (Y_{H,t}^*)^\rho\right)^{\frac{1}{\rho}} - P_{F,t}^* Y_{F,t}^* - P_{H,t}^* Y_{H,t}^* \quad (131)$$

where $P_{H,t}^*$ is the nominal price of $Y_{H,t}^*$, $P_{F,t}^*$ is the nominal price of $Y_{F,t}^*$. Demand curves are given by

$$Y_{H,t}^* = \left(\frac{P_{H,t}^*}{P_t^*}\right)^{\frac{1}{\rho-1}} (1-\omega) Y_t^* \quad (132)$$

$$Y_{F,t}^* = \left(\frac{P_{F,t}^*}{P_t^*}\right)^{\frac{1}{\rho-1}} \omega Y_t^* \quad (133)$$

The consumer price indexes are given by

$$P_t^* = \left(\omega (P_{F,t}^*)^{\frac{\rho}{\rho-1}} + (1-\omega) (P_{H,t}^*)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}. \quad (134)$$

A.5 Investment goods

Investment goods, I_t , are created by combining goods from countries H and F ($I_{H,t}$ and $I_{F,t}$) using

$$I_t = \left(\omega^{1-\rho} (I_{H,t})^\rho + (1-\omega)^{1-\rho} (I_{F,t})^\rho\right)^{\frac{1}{\rho}} \quad (135)$$

Profits are given by

$$P_{I,t} \left(\omega^{1-\rho} (I_{H,t})^\rho + (1-\omega)^{1-\rho} (I_{F,t})^\rho\right)^{\frac{1}{\rho}} - P_{H,t} I_{H,t} - P_{F,t} I_{F,t}. \quad (136)$$

where $P_{H,t}$ is the nominal price of $I_{H,t}$, $P_{F,t}$ is the nominal price of $I_{F,t}$. Note that we are imposing that the price of $I_{H,t}$ and $Y_{H,t}$ be the same because they are the same input. Similarly for $I_{F,t}$ and $Y_{F,t}$. First-order conditions are

$$P_{I,t} \left(\omega^{1-\rho} (I_{H,t})^\rho + (1-\omega)^{1-\rho} (I_{F,t})^\rho\right)^{\frac{1-\rho}{\rho}} \omega^{1-\rho} (I_{H,t})^{\rho-1} = P_{H,t}$$

$$P_{I,t} \left(\omega^{1-\rho} (I_{H,t})^\rho + (1-\omega)^{1-\rho} (I_{F,t})^\rho\right)^{\frac{1-\rho}{\rho}} (1-\omega)^{1-\rho} (I_{F,t})^{\rho-1} = P_{F,t}$$

Use the definition of I_t to get

$$\begin{aligned}\left(\frac{I_{H,t}}{\omega_I I_t}\right)^{\rho-1} &= \frac{P_{H,t}}{P_{I,t}} \\ \left(\frac{I_{F,t}}{(1-\omega_I) I_t}\right)^{\rho-1} &= \frac{P_{F,t}}{P_{I,t}}\end{aligned}$$

Demand curves are then

$$I_{H,t} = \left(\frac{P_{H,t}}{P_{I,t}}\right)^{\frac{1}{\rho-1}} \omega_I I_t \quad (137)$$

$$I_{F,t} = \left(\frac{P_{F,t}}{P_{I,t}}\right)^{\frac{1}{\rho-1}} (1-\omega_I) I_t. \quad (138)$$

There is free entry for retailers, so profits are zero. Substituting demand curves into the profits expression yields

$$\begin{aligned}P_{I,t} I_t &= P_{H,t} \left(\frac{P_{H,t}}{P_{I,t}}\right)^{\frac{1}{\rho-1}} \omega_I I_t + P_{F,t} \left(\frac{P_{F,t}}{P_{I,t}}\right)^{\frac{1}{\rho-1}} (1-\omega_I) I_t \\ P_{I,t}^{\frac{\rho}{\rho-1}} &= (P_{H,t})^{\frac{\rho}{\rho-1}} \omega_I + (P_{F,t})^{\frac{\rho}{\rho-1}} (1-\omega_I) \\ P_{I,t} &= \left(\omega_I P_{H,t}^{\frac{\rho}{\rho-1}} + (1-\omega_I) (P_{F,t})^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}\end{aligned} \quad (139)$$

Investment goods in the foreign country, I_t^* , are created by combining goods for countries H and F ($I_{H,t}^*$ and $I_{F,t}^*$) using

$$I_t^* = \left(\omega_I^{1-\rho} (I_{H,t}^*)^\rho + (1-\omega_I)^{1-\rho} (I_{F,t}^*)^\rho\right)^{\frac{1}{\rho}} \quad (140)$$

Profits are given by

$$P_{I,t}^* \left(\omega_I^{1-\rho} (I_{F,t}^*)^\rho + (1-\omega_I)^{1-\rho} (I_{H,t}^*)^\rho\right)^{\frac{1}{\rho}} - P_{F,t}^* I_{F,t}^* - P_{H,t}^* I_{H,t}^*. \quad (141)$$

where $P_{H,t}^*$ is the nominal price of $Y_{H,t}^*$, $P_{F,t}^*$ is the nominal price of $Y_{F,t}^*$. Again, we are imposing that the price of $Y_{H,t}^*$ and $I_{H,t}^*$ are the same. Similarly, the price of $Y_{F,t}^*$ and $I_{F,t}^*$ are the same. Demand curves are given by

$$I_{H,t}^* = \left(\frac{P_{H,t}^*}{P_{I,t}^*}\right)^{\frac{1}{\rho-1}} (1-\omega_I) I_t^* \quad (142)$$

$$I_{F,t}^* = \left(\frac{P_{F,t}^*}{P_{I,t}^*}\right)^{\frac{1}{\rho-1}} \omega_I I_t^* \quad (143)$$

The consumer price indexes are given by

$$P_{I,t}^* = \left(\omega_I (P_{F,t}^*)^{\frac{\rho}{\rho-1}} + (1-\omega_I) (P_{H,t}^*)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}. \quad (144)$$

A.6 Resource constraints and bond market clearing

Final good production is used for consumption and government purchases, so that

$$C_t + G_t = Y_t \quad (145)$$

and

$$C_t^* + G_t^* = Y_t^*. \quad (146)$$

We also have to have that

$$b_{H,t} + b_{H,t}^* = 0 \quad (147)$$

$$b_{F,t} + b_{F,t}^* = 0 \quad (148)$$

where $b_{H,t} \equiv B_{H,t}/P_t$, $b_{H,t}^* \equiv B_{H,t}^*/P_t$, $b_{F,t} \equiv B_{F,t}/P_t$, $b_{F,t}^* \equiv B_{F,t}^*/P_t$.

A.7 Monopolists Monopolists

We introduce price stickiness as a Calvo-style price-setting friction. Monopolists are only able to update their price with probability ξ in each period. If they are not able to update their price, it remains the same as the period before. If monopolist i in the country H can update its price, it chooses $\tilde{P}_{H,t}(i)$ and $\tilde{P}_{H,t}^*(i)$ to maximize

$$E_t \sum_{j=0}^{\infty} \Lambda_{t+j} \left\{ \left(\frac{\tilde{P}_{H,t}(i)}{P_{t+j}} (1 + \tau_X) - MC_{t+j} \right) \left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t+j}} \right)^{-\nu} X_{H,t+j} + \left(\frac{NER_{t+j} \tilde{P}_{H,t}^*(i)}{P_{t+j}} (1 + \tau_X) - MC_{t+j} \right) \left(\frac{\tilde{P}_{H,t}^*(i)}{P_{H,t+j}^*} \right)^{-\nu} X_{H,t+j}^* \right\}$$

The FOC with respect to $\tilde{P}_{H,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left[\frac{\tilde{P}_{H,t}}{P_t} \frac{P_t}{P_{t+j}} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} X_{H,t+j} - \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_{t+j} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} X_{H,t+j} \right] = 0$$

Then we have

$$F_{H,t} \tilde{p}_{H,t} = K_{H,t} \quad (149)$$

where we define $F_{H,t}$ and $K_{H,t}$ as recursive sums so that

$$F_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{P_t}{P_{t+j}} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} X_{H,t+j}$$

$$K_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_{t+j} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} X_{H,t+j}.$$

These can be written as

$$F_{H,t} = \Lambda_t X_{H,t} + \beta\xi E_t \pi_{t+1}^{-1} \pi_{H,t+1}^{\nu} F_{H,t+1} \quad (150)$$

$$K_{H,t} = \Lambda_t \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_t X_{H,t} + \beta\xi E_t \pi_{H,t+1}^{\nu} K_{H,t+1} \quad (151)$$

where

$$\pi_{H,t} \equiv P_{H,t} / P_{H,t-1} \quad (152)$$

The price index for home goods in the home market is given by

$$P_{H,t} = \left((1 - \xi) \tilde{P}_{H,t}^{1-\nu} + \xi P_{H,t-1}^{1-\nu} \right)^{\frac{1}{1-\nu}}$$

so that

$$p_{H,t} = \left((1 - \xi) \tilde{p}_{H,t}^{1-\nu} + \xi \frac{P_{H,t-1}^{1-\nu}}{\pi_t^{1-\nu}} \right)^{\frac{1}{1-\nu}} \quad (153)$$

The FOC with respect to $\tilde{P}_{H,t}^*(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left[\frac{NER_{t+j}}{NER_t} \frac{NER_t P_t^*}{P_t} \tilde{p}_{H,t}^* \frac{P_t}{P_{t+j}} \left(\frac{P_{H,t}^*}{P_{H,t+j}^*} \right)^{-\nu} X_{H,t+j}^* - \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_{t+j} \left(\frac{P_{H,t}^*}{P_{H,t+j}^*} \right)^{-\nu} X_{H,t+j}^* \right] = 0$$

Then we have

$$F_{H,t}^* RER_t \tilde{p}_{H,t}^* = K_{H,t}^* \quad (154)$$

where

$$F_{H,t}^* \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{NER_{t+j}}{NER_t} \frac{P_t}{P_{t+j}} \left(\frac{P_{H,t}^*}{P_{H,t+j}^*} \right)^{-\nu} X_{H,t+j}^*$$

$$K_{H,t}^* \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_{t+j} \left(\frac{P_{H,t}^*}{P_{H,t+j}^*} \right)^{-\nu} X_{H,t+j}^*.$$

These can be written as

$$F_{H,t}^* = \Lambda_t X_{H,t}^* + \beta \xi E_t \frac{NER_{t+1}}{NER_t} \pi_{t+1}^{-1} (\pi_{H,t+1}^*)^\nu F_{H,t+1}^* \quad (155)$$

$$K_{H,t}^* = \Lambda_t \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_t X_{H,t}^* + \beta \xi E_t (\pi_{H,t+1}^*)^\nu K_{H,t+1}^* \quad (156)$$

where

$$\pi_{H,t}^* \equiv P_{H,t}^* / P_{H,t-1}^* \quad (157)$$

. The price index for home goods in the foreign market is given by

$$P_{H,t}^* = \left((1 - \xi) (\tilde{P}_{H,t}^*)^{1-\nu} + \xi (P_{H,t-1}^*)^{1-\nu} \right)^{\frac{1}{1-\nu}}$$

so that

$$p_{H,t}^* = \left((1 - \xi) (\tilde{p}_{H,t}^*)^{1-\nu} + \xi \frac{(p_{H,t-1}^*)^{1-\nu}}{(\pi_t^*)^{1-\nu}} \right)^{\frac{1}{1-\nu}} \quad (158)$$

The foreign firms are symmetric. If monopolist i can update its price, it chooses $\tilde{P}_{F,t}^*(i)$ and $\tilde{P}_{F,t}$ to maximize

$$E_t \sum_{j=0}^{\infty} \Lambda_{t+j}^* \left\{ \left(\frac{\tilde{P}_{F,t}^*(i)}{P_{t+j}^*} (1 + \tau_X) - MC_{t+j}^* \right) \left(\frac{\tilde{P}_{F,t}^*(i)}{P_{t+j}^*} \right)^{-\nu} X_{F,t+j}^* + \left(\frac{\tilde{P}_{F,t}(i)}{NER_{t+j} P_{t+j}^*} (1 + \tau_X) - MC_{t+j}^* \right) \left(\frac{\tilde{P}_{F,t}(i)}{P_{F,t+j}^*} \right)^{-\nu} X_{F,t+j} \right\}$$

The FOC with respect to $\tilde{P}_{F,t}^*(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta \xi)^j \Lambda_{t+j}^* \left[\frac{\tilde{P}_{F,t}^*}{P_t^*} \frac{P_t}{P_{t+j}^*} \left(\frac{P_{F,t}^*}{P_{F,t+j}^*} \right)^{-\nu} X_{F,t+j}^* - \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_{t+j}^* \left(\frac{P_{F,t}^*}{P_{F,t+j}^*} \right)^{-\nu} X_{F,t+j}^* \right] = 0.$$

We write this as

$$F_{F,t}^* \tilde{P}_{F,t}^* = K_{F,t}^* \quad (159)$$

where

$$F_{F,t}^* = \Lambda_t^* X_{F,t}^* + \beta \xi E_t (\pi_{t+1}^*)^{-1} (\pi_{F,t+1}^*)^\nu F_{F,t+1}^* \quad (160)$$

and

$$K_{F,t}^* = \Lambda_t^* \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_t^* X_{F,t}^* + \beta \xi E_t (\pi_{F,t+1}^*)^\nu K_{F,t+1}^*. \quad (161)$$

where

$$\pi_{F,t}^* \equiv P_{F,t}^* / P_{F,t-1}^* \quad (162)$$

. The price index implies

$$p_{F,t}^* = \left((1 - \xi) (\tilde{p}_{F,t}^*)^{1-\nu} + \xi \frac{(p_{F,t-1}^*)^{1-\nu}}{(\pi_t^*)^{1-\nu}} \right)^{\frac{1}{1-\nu}} \quad (163)$$

The FOC with respect to $\tilde{P}_{F,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta \xi)^j \Lambda_{t+j}^* \left[\frac{NER_t}{NER_{t+j}} \frac{P_t}{NER_t P_t^*} \tilde{P}_{F,t}^* \frac{P_t}{P_{t+j}^*} \left(\frac{P_{F,t}^*}{P_{F,t+j}^*} \right)^{-\nu} X_{F,t+j}^* - \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_{t+j}^* \left(\frac{P_{F,t}^*}{P_{F,t+j}^*} \right)^{-\nu} X_{F,t+j} \right] = 0$$

We can write this as

$$F_{F,t} \frac{\tilde{P}_{F,t}^*}{RER_t} = K_{F,t} \quad (164)$$

where

$$F_{F,t} = \Lambda_t^* X_{F,t} + \beta \xi E_t \frac{NER_t}{NER_{t+1}} (\pi_{t+1}^*)^{-1} (\pi_{F,t+1}^*)^\nu F_{F,t+1} \quad (165)$$

and

$$K_{F,t} = \Lambda_t^* \frac{1}{1 + \tau_X} \frac{\nu}{\nu - 1} MC_t^* X_{F,t} + \beta \xi E_t (\pi_{F,t+1}^*)^\nu K_{F,t+1}. \quad (166)$$

where

$$\pi_{F,t} \equiv P_{F,t}/P_{F,t-1} \quad (167)$$

The price index implies that

$$p_{F,t} = \left((1-\xi) (\tilde{p}_{F,t})^{1-\nu} + \xi \frac{(p_{F,t-1})^{1-\nu}}{(\pi_t)^{1-\nu}} \right)^{\frac{1}{1-\nu}} \quad (168)$$

A.8 Marginal cost

Cost minimization implies

$$R_{K,t} = \alpha MC_t A_t (K_t)^{\alpha-1} (L_t)^{1-\alpha} \quad (169)$$

$$\frac{W_t}{P_t} = (1-\alpha) MC_t A_t K_t^\alpha (L_t)^{-\alpha} \quad (170)$$

which implies

$$\begin{aligned} (R_{K,t})^\alpha &= \alpha^\alpha MC_t^\alpha A_t^\alpha K_t^{(\alpha-1)\alpha} (L_t)^{(1-\alpha)\alpha} \\ \left(\frac{W_t}{P_t}\right)^{1-\alpha} &= (1-\alpha)^{1-\alpha} MC_t^{1-\alpha} A_t^{1-\alpha} K_t^{(1-\alpha)\alpha} (L_t)^{-\alpha(1-\alpha)} \end{aligned}$$

Meaning that

$$(R_{K,t})^\alpha \left(\frac{W_t}{P_t}\right)^{1-\alpha} = \alpha^\alpha MC_t^\alpha A_t^\alpha K_t^{(\alpha-1)\alpha} (L_t)^{(1-\alpha)\alpha} (1-\alpha)^{1-\alpha} MC_t^{1-\alpha} A_t^{1-\alpha} K_t^{(1-\alpha)\alpha} (L_t)^{-\alpha(1-\alpha)}$$

so that marginal cost is given by

$$MC_t = \frac{(R_{K,t})^\alpha \left(\frac{W_t}{P_t}\right)^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^\alpha A_t} \quad (171)$$

In the foreign economy

$$R_{K,t}^* = \alpha MC_t^* A_t^* (K_t^*)^{\alpha-1} (L_t^*)^{1-\alpha} \quad (172)$$

$$\frac{W_t^*}{P_t^*} = (1-\alpha) MC_t^* A_t^* (K_t^*)^\alpha (L_t^*)^{-\alpha} \quad (173)$$

A.9 Aggregation

Monopolists produce with technology so that

$$X_{H,t}(i) + X_{H,t}^*(i) = A_t K_t(i)^\alpha L_t(i)^{1-\alpha}$$

$$X_{F,t}(i) + X_{F,t}^*(i) = A_t^* K_t^*(i)^\alpha L_t^*(i)^{1-\alpha}$$

where A_t and A_t^* are stochastic processes and, in a slight abuse of notation, the (i) means the amount hired by a particular monopolist. Then aggregation is such that

$$\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} X_{H,t}(i) di + \int_0^1 \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*}\right)^{-\nu} X_{H,t}^*(i) di = A_t \int_0^1 K_t(i)^\alpha (L_t(i))^{1-\alpha} di$$

so that

$$d_{H,t} X_{H,t} + d_{H,t}^* X_{H,t}^* = A_t K_t^\alpha L_t^{1-\alpha} \quad (174)$$

where the last equation follows without the (i) 's because all firms choose the same capital-to-labor ratio and we have a measure 1 of firms. Similarly

$$d_{F,t} X_{F,t} + d_{F,t}^* X_{F,t}^* = A_t (K_t^*)^\alpha (L_t^*)^{1-\alpha} \quad (175)$$

Here the dispersion terms can be written as

$$d_{H,t} = (1-\xi) \tilde{p}_{H,t}^\nu (\tilde{p}_{H,t})^{-\nu} + \xi \pi_{H,t}^\nu d_{H,t-1} \quad (176)$$

$$d_{H,t}^* = (1 - \xi) (p_{H,t}^*)^\nu (\bar{p}_{H,t}^*)^{-\nu} + \xi (\pi_{H,t}^*)^\nu d_{H,t-1}^*, \quad (177)$$

$$d_{F,t}^* = (1 - \xi) (p_{F,t}^*)^\nu (\bar{p}_{F,t}^*)^{-\nu} + \xi (\pi_{F,t}^*)^\nu d_{F,t-1}^*, \quad (178)$$

$$d_{F,t} = (1 - \xi) (p_{F,t})^\nu (\bar{p}_{F,t})^{-\nu} + \xi (\pi_{F,t})^\nu d_{F,t-1}. \quad (179)$$

A.10 Government

The monetary authority follows a Taylor rule

$$R_t = (R_{t-1})^\gamma (R \pi_t^{\theta_\pi})^{1-\gamma} \exp(\epsilon_{R,t}) \quad \text{where } \theta_\pi > 1 \quad (180)$$

or alternatively follows a money growth rule

$$\log\left(\frac{M_t}{M_{t-1}}\right) = \log(x_{M,t}) \quad (181)$$

where

$$\log(x_{M,t}) = \rho_{x_M} \log(x_{M,t-1}) + \sigma_{x_M} \epsilon_{x_{M,t}} \quad (182)$$

The fiscal authority balances its budget with lump sum taxes. The foreign monetary authority follows a Taylor rule

$$R_t^* = (R_{t-1}^*)^\gamma (R (\pi_t^*)^{\theta_\pi})^{1-\gamma} \exp(\epsilon_{R,t}^*) \quad \text{where } \theta_\pi > 1 \quad (183)$$

or alternatively follows a money growth rule

$$\log\left(\frac{M_t^*}{M_{t-1}^*}\right) = \log(x_{M,t}^*) \quad (184)$$

where

$$\log(x_{M,t}^*) = \rho_{x_M} \log(x_{M,t-1}^*) + \sigma_{x_M} \epsilon_{x_{M,t}^*} \quad (185)$$

The fiscal authority balances its budget with lump sum taxes.

B Asset-market completeness

Assume that there is a complete set of Arrow securities. Let ω_t denote the state of the world in time t and $\omega^t = \{\omega_t, \omega_{t-1}, \dots\}$. The household in country H prices the Arrow securities that pay off one unit of the H currency in state ω_{t+1} so that

$$\frac{Q_t^{\omega_{t+1}}}{P_t} \Lambda_t = \beta \frac{\Lambda_{t+1}}{P_{t+1}} \Pr(\omega_{t+1} | \omega^t)$$

where $Q_t^{\omega_{t+1}}$ is the price at time t of the security that pays off at time $t+1$ in state ω_{t+1} . The household in country F prices the Arrow securities that pay off one unit of the H currency in state ω_{t+1} so that

$$\frac{Q_t^{\omega_{t+1}}}{NER_t P_t^*} \Lambda_t^* = \beta \frac{\Lambda_{t+1}^*}{NER_{t+1} P_{t+1}^*} \Pr(\omega_{t+1} | \omega^t).$$

Note that it is a global market, so the prices are the same. Then divide

$$\frac{NER_t P_t^*}{P_t} \frac{\Lambda_t}{\Lambda_t^*} = \frac{NER_{t+1} P_{t+1}^*}{P_{t+1}} \frac{\Lambda_{t+1}}{\Lambda_{t+1}^*}$$

$$RER_t \frac{\Lambda_t}{\Lambda_t^*} = RER_{t+1} \frac{\Lambda_{t+1}}{\Lambda_{t+1}^*}$$

Given that this must happen for every date and state,

$$RER_t \frac{\Lambda_t}{\Lambda_t^*} = \kappa \quad (186)$$

for all t .

C Equilibrium

An equilibrium required determining the following 37 endogenous objects: $C_t, \Lambda_t, L_t, w_t \equiv \frac{W_t}{P_t}, X_{H,t}, X_{F,t}, Y_t, Y_{H,t}, Y_{F,t}, R_t, MC_t, \pi_t, m_t \equiv \frac{M_t}{P_t}, b_{F,t}, b_{H,t}, K_t, I_t, I_{H,t}, I_{F,t}, Q_t, R_{K,t}, p_{F,t} \equiv \frac{P_{F,t}}{P_t}, p_{H,t} \equiv \frac{P_{H,t}}{P_t}, p_{I,t} \equiv \frac{P_{I,t}}{P_t}, \tilde{p}_{H,t}, F_{H,t}, K_{H,t}, d_{H,t}, \pi_{H,t}, \tilde{p}_{F,t}, F_{F,t}, K_{F,t}, d_{F,t}, \pi_{F,t}, \tilde{w}_t, F_{W,t}, K_{W,t}$, the 37 star versions, as well as $\Delta NER_t \equiv \frac{NER_t}{NER_t - 1}$ and $REER_t$. To determine these 76 variables, we use the following 73 equations: (88), (89), (90), (91), (92), (93), (94), (97), (98), (99), (100), (101), (102), (103), (104), (105), (106), (107), (108), (109), (110), (111), (112), (113), (114), (115), (116), (127), (128), (129), (132), (133), (134), (137), (138), (139), (142), (143), (144), (145), (146), (147), (148), (149), (150), (151), (152), (153), (154), (155), (156), (157), (158), (159), (160), (161), (162), (163), (164), (165), (166), (167), (168), (169), (170), (172), (173), (174), (175), (176), (177), (178), (179) along with either (180) and (183) or (181) and (184). Finally, we use the household budget constraint (87). The other budget constraint can be ignored because of Walras' law. If we want complete markets, we use (186) instead of (87). In addition, we replace equations (91) and (99) with the conditions that $b_{H,t} = b_{F,t} = 0$. If we want to exclude capital accumulation, we set $\alpha = 0$ and replace equations (88), (93), (94), and (169), by the conditions that $I_t = K_t = Q_t = R_{K,t} = 0$. Similarly, we replace equations (97), (102), (103), and (172) by the conditions that $I_t^* = K_t^* = Q_t^* = R_{K,t}^* = 0$. We assume that $\epsilon_{R,t}$ and $\epsilon_{R,t}^*$ are iid normal variables. Also, $A_t, A_t^*, x_{M,t}, x_{M,t}^*, G_t, \text{ and } G_t^*$ evolve so that

$$\log\left(\frac{x_t}{x}\right) = \rho_x \log\left(\frac{x_{t-1}}{x}\right) + \epsilon_{x,t}$$

where x is the steady state value of each variable. We normalize $A = A^* = 1, x_M = x_M^* = 1$, and $G = G^* = 0.2 \times Y$.

D Steady State

To determine steady state, we assume that target inflation in both countries is 1. So, $\pi = \pi^* = 1$. The intertemporal Euler equations determine $R = R^* = \beta^{-1}$. We normalized $L = L^* = 1$. From the definition of steady state, $\Delta NER = 1$. We define initial conditions so that $REER = 1$. Firm optimality and symmetry of the equilibrium, $p_H = p_H^* = p_F = p_F^* = 1$. As a result, $p_I = p_I^* = 1$ and $Q = \Lambda$. Marginal cost is given by

$$MC = \frac{\nu - 1}{\nu} (1 + \tau_X)$$

The rental rate of capital is

$$R_K = \frac{1 - \beta(1 - \delta)}{\beta}$$

So that

$$\frac{K}{X_H + X_H^*} = \frac{\beta \alpha MC}{1 - \beta(1 - \delta)}$$

Then

$$\frac{I}{X_H + X_H^*} = \delta \frac{K}{X_H + X_H^*}$$

Since

$$\begin{aligned} X_H + X_H^* &= K^\alpha \\ (X_H + X_H^*)^{1-\alpha} &= \left(\frac{K}{X_H + X_H^*} \right)^\alpha \end{aligned}$$

So that

$$X_H + X_H^* = \left(\frac{\beta \alpha \frac{\nu-1}{\nu} (1 + \tau_X)}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} = X_F + X_F^*$$

where the last equality follows by symmetry. With this, we also have K . Demand curves imply

$$Y_H = Y_F \frac{\omega}{1 - \omega}$$

and

$$Y_F^* = Y_H^* \frac{\omega}{1 - \omega}$$

Similarly,

$$I_H = I_F \frac{\omega_I}{1 - \omega_I}$$

and

$$I_F^* = I_H^* \frac{\omega_I}{1 - \omega_I}$$

Then

$$Y_H + I_H + Y_H^* + I_H^* = X_H + X_H^*$$

Symmetry and the demand curves imply

$$Y_H + I_H + Y_H \frac{1-\omega}{\omega} + I_H \frac{1-\omega_I}{\omega_I} = X_H + X_H^*$$

$$Y_H \frac{1}{\omega} + I_H \frac{1}{\omega_I} = X_H + X_H^*$$

$$Y + I = X_H + X_H^*$$

So

$$\frac{Y}{K} = \frac{X_H + X_H^*}{K} - \delta$$

which gives us Y . Then,

$$Y_F = (1-\omega)Y$$

$$Y_H = \omega Y$$

$$Y_F^* = \omega Y$$

$$Y_H^* = (1-\omega)Y$$

and

$$I_F = (1-\omega_I)I$$

$$I_H = \omega_I I$$

$$I_F^* = \omega_I I$$

$$I_H^* = (1-\omega_I)I$$

Given G and G^* this gives us C and C^* , which determine Λ and Λ^* . The values of m and m^* are determined by the money demand equations. The values of w and w^* are determined by

$$w = (1-\alpha)MCK^\alpha.$$

Finally, you get χ from

$$\chi = \Lambda \frac{\nu_L - 1}{\nu_L} (1 + \tau_W) w.$$

E Natural rate in Taylor rule

We define the natural rate of interest, r_t , to be the real rate of interest that would prevail in a flexible price and wage version of the model, conditional on the current state variables. We modify the Taylor rule so that

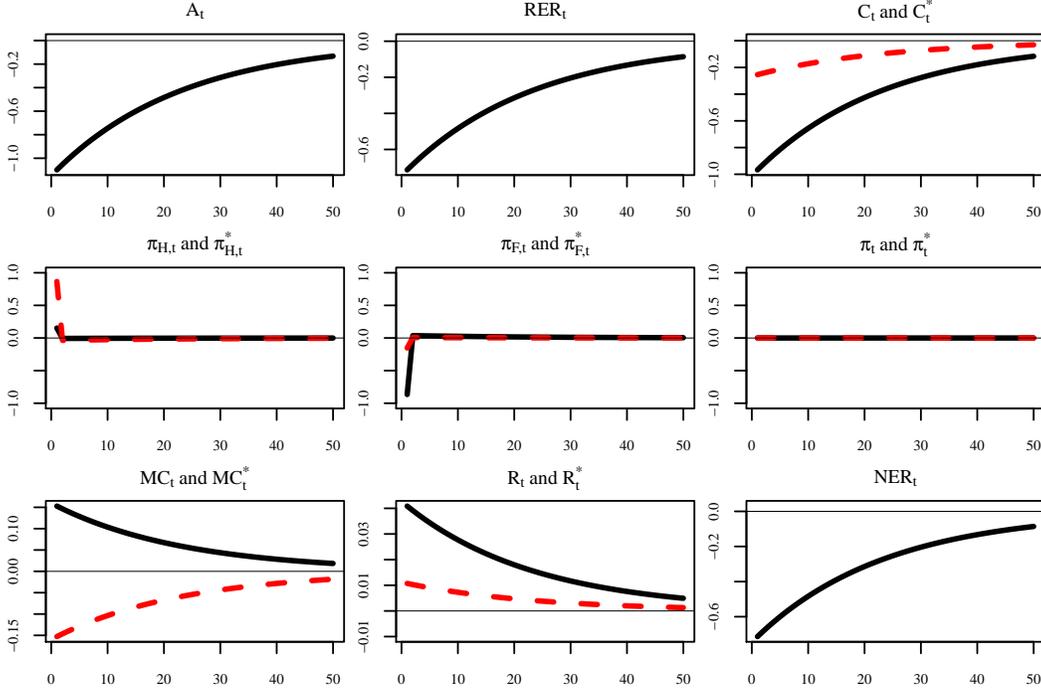
$$R_t = (R_{t-1})^\gamma (r_t \pi_t^{\theta_\pi})^{1-\gamma} \exp(\epsilon_{R,t}).$$

We make a similar adjustment to the foreign Taylor rule

$$R_t^* = (R_{t-1}^*)^\gamma (r_t^* (\pi_t^*)^{\theta_\pi})^{1-\gamma} \exp(\epsilon_{R,t}^*).$$

Assuming no capital, flexible prices and wages, complete markets, the Taylor rule is operative, no habit or interest rate smoothing, and our parameters from the text, the impulse response functions to a technology shock are shown below. Notice that, compared to the case without the natural rate in the Taylor rule shown in the main text, inflation moves by less, as does the nominal interest rate.

Figure 26: Response to technology shock under Taylor rule with the natural rate



Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

F Analytic expressions for regression coefficients

Assuming no capital, flexible prices and wages, complete markets, the Taylor rule is operative, no habit or interest rate smoothing, and that $\omega \neq 0.5$, the log-linear equations of the model simplify to the following system of equations:

$$-R\hat{E}R_t = \theta_\pi (\hat{\pi}_t - \hat{\pi}_t^*) + (\epsilon_{R,t} - \epsilon_{R,t}^*) - E_t R\hat{E}R_{t+1} - E_t (\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*). \quad (187)$$

$$R\hat{E}R_t = \left(\left[\frac{2\omega}{\rho-1} \frac{2(\omega-1)}{2\omega-1} + \frac{(2\omega-1)}{\sigma} \right] \phi + 1 - \frac{2(\omega-1)}{2\omega-1} \right)^{-1} (\phi+1) (\hat{A}_t - \hat{A}_t^*) \quad (188)$$

$$R\hat{E}R_t - R\hat{E}R_{t-1} = \Delta \hat{N}ER_t + \hat{\pi}_t^* - \hat{\pi}_t \quad (189)$$

These equations determine $R\hat{E}R_t$, $(\hat{\pi}_t - \hat{\pi}_t^*)$, and $\Delta \hat{N}ER_t$. Note that (188) implies that $R\hat{E}R_t$ is an AR(1), just as the technology processes. Then, assuming $\lim_{j \rightarrow \infty} E_t (\hat{\pi}_{t+j} - \hat{\pi}_{t+j}^*) = 0$, (187) implies

$$\hat{\pi}_t - \hat{\pi}_t^* = \frac{\rho_A - 1}{\theta_\pi - \rho_A} R\hat{E}R_t - \frac{1}{\theta_\pi} (\epsilon_{R,t} - \epsilon_{R,t}^*) \quad (190)$$

Using (189), we can write

$$R\hat{E}R_{t+j} - R\hat{E}R_t = N\hat{E}R_{t+j} - N\hat{E}R_t + \sum_{k=1}^j (\hat{\pi}_{t+k}^* - \hat{\pi}_{t+k})$$

Take expectations, and ignoring monetary policy shocks, this implies

$$\left[\frac{\rho_A^{j+1} - \rho_A}{\theta_\pi - \rho_A} - (1 - \rho_A^j) \right] R\hat{E}R_t = N\hat{E}R_{t+j} - N\hat{E}R_t \quad (191)$$

Define

$$\beta_{1,j}^{NER} \equiv \left[\frac{\rho_A^{j+1} - \rho_A}{\theta_\pi - \rho_A} - (1 - \rho_A^j) \right] \quad (192)$$

which can be simplified to the expression in the text. This corresponds to our NER regression. Note that $\beta_{1,j}^{NER} < 0$, $\beta_{1,j+1}^{NER} < \beta_{1,j}^{NER}$, and

$$\lim_{j \rightarrow \infty} \beta_{1,j}^{NER} = \left[-\frac{\rho_A}{\theta_\pi - \rho_A} - 1 \right].$$

Now let's think about the relative-price regression. Note that

$$\sum_{k=1}^j E_t (\hat{\pi}_{t+k} - \hat{\pi}_{t+k}^*) = \frac{\rho_A - 1}{\theta_\pi - \rho_A} \sum_{k=1}^j E_t R \hat{E} R_{t+k}$$

meaning

$$\log \left(\frac{P_{t+j}^*/P_t^*}{P_{t+j}/P_t} \right) = \frac{\rho_A - \rho_A^{j+1}}{\theta_\pi - \rho_A} R \hat{E} R_t \quad (193)$$

Define

$$\beta_{1,j}^\pi \equiv \frac{\rho_A - \rho_A^{j+1}}{\theta_\pi - \rho_A} \quad (194)$$

which can be rearranged into the expression in the text. This corresponds to our relative-price regression. Note that $\beta_{1,j}^\pi > 0$, $\beta_{1,j+1}^\pi > \beta_{1,j}^\pi$, and

$$\lim_{j \rightarrow \infty} \beta_{1,j}^\pi \equiv \frac{\rho_A}{\theta_\pi - \rho_A}.$$

Now it is apparent that

$$\lim_{j \rightarrow \infty} (\beta_{1,j}^{NER} + \beta_{1,j}^\pi) = -1.$$

G The nominal exchange rate after a technology shock

Consider the same version of the model as in the previous section. One interesting feature of the impulse response functions to a technology shock is that the exchange rate has an initial appreciation (depreciation) followed by persistent depreciation (appreciation). The algebra in the previous section can explain why. According to (188), a technology shock moves the real exchange rate directly. As a result, the relative inflation rates are given by (190), setting $\epsilon_{R,t} = \epsilon_{R,t}^* = 0$. Noting that

$$\left| \frac{\rho_A - 1}{\theta_\pi - \rho_A} \right| < 1$$

we can conclude that relative prices do not change by as much as the real exchange rate. As such, the nominal exchange rate must move in the same direction as the real exchange rate. Thereafter, (190) implies

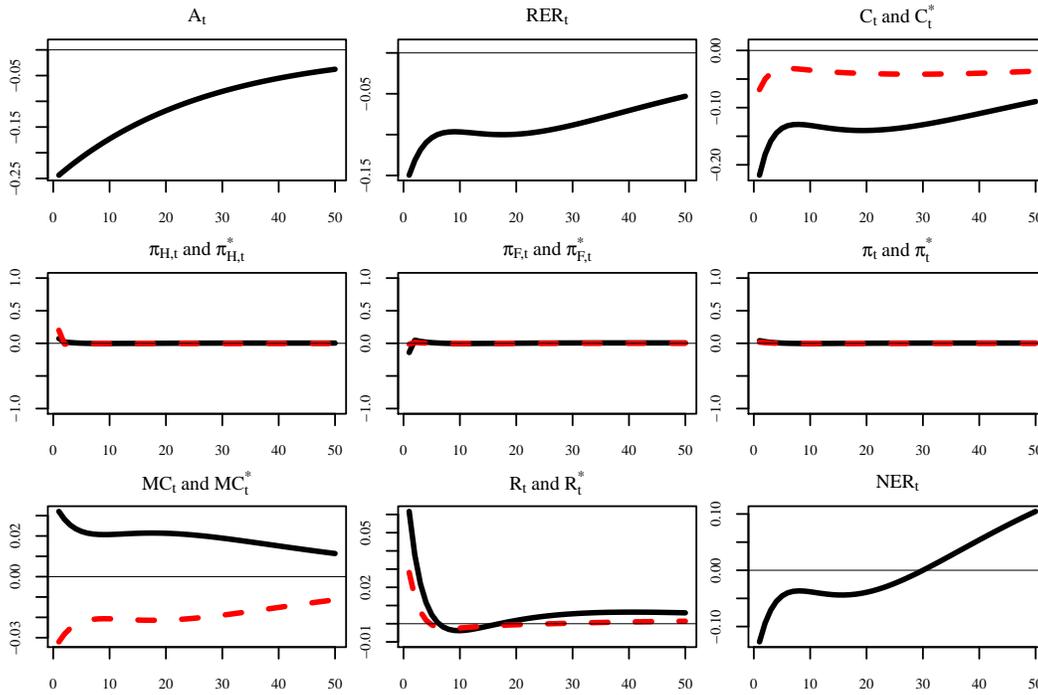
$$\text{sgn}(\hat{\pi}_t - \hat{\pi}_t^*) = \text{sgn}(R \hat{E} R_t)$$

which means prices are moving in the opposite direction as would be required to bring the *RER* back to steady state. As such, the nominal exchange rate has to move in the opposite direction as it did in the period of the shock so as to bring the *RER* back to steady state. Notably, the movement in the nominal exchange rate is larger than the movement in the real exchange rate.

H Figures using model with capital

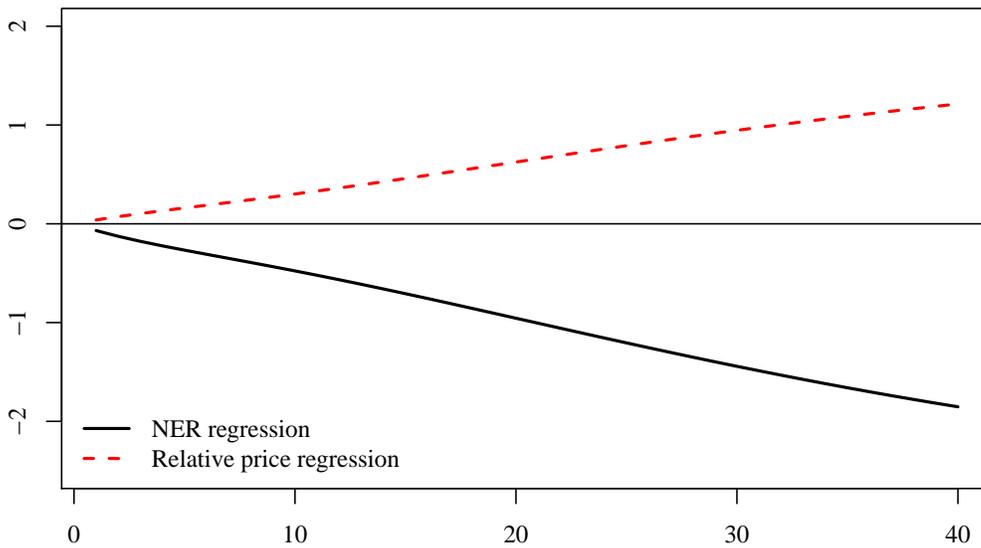
We re-did our analysis using the version of the model with capital. We assumed that $\delta = 0.025$, $\phi_K = 1.58$, and $\alpha = 0.36$. These are the parameters used in CEE (2005) for the version of their model without variable capital utilization. All other parameters are the same as in the text except that we estimate $\sigma_A = 0.00243$, $\rho_A = 0.96263$, and $\sigma\eta = 0.00352$ as in the text but so that we hit a coefficient in the Fama regression of zero. The graphs corresponding to those in the text for the model without capital follow.

Figure 27: Response to technology shock under Taylor rule



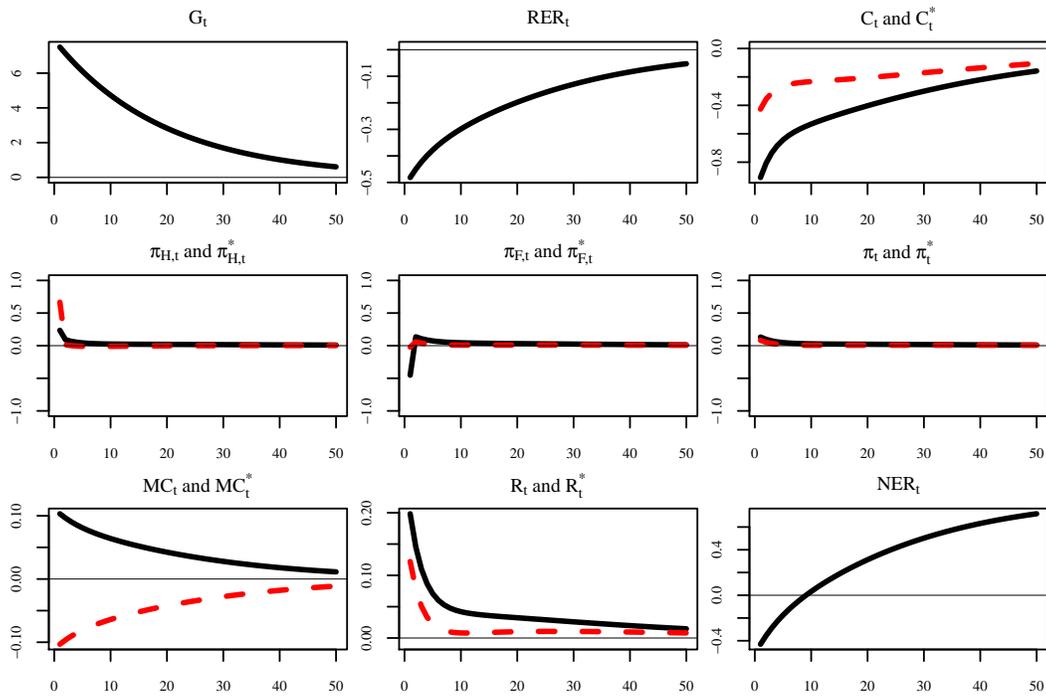
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 28: Implied values of $\beta_{1,j}^{NER}$ and $\beta_{1,j}^{\pi}$ from small-scale model



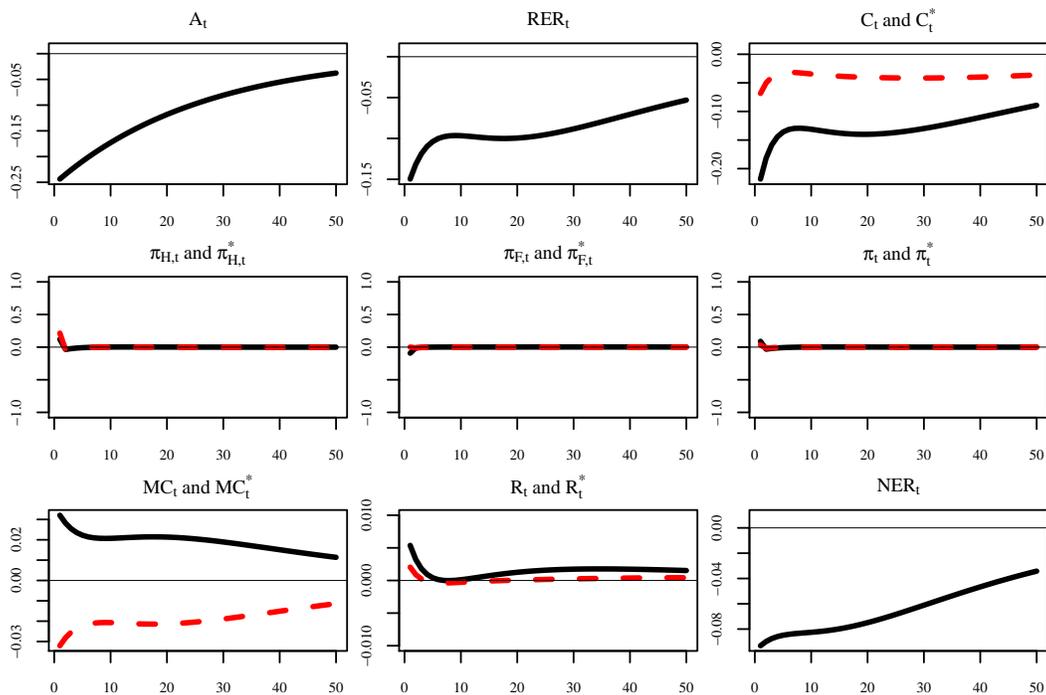
Note: The model-implied values come from our model with no nominal rigidities and only technology shocks.

Figure 29: Response to government spending shock under Taylor rule



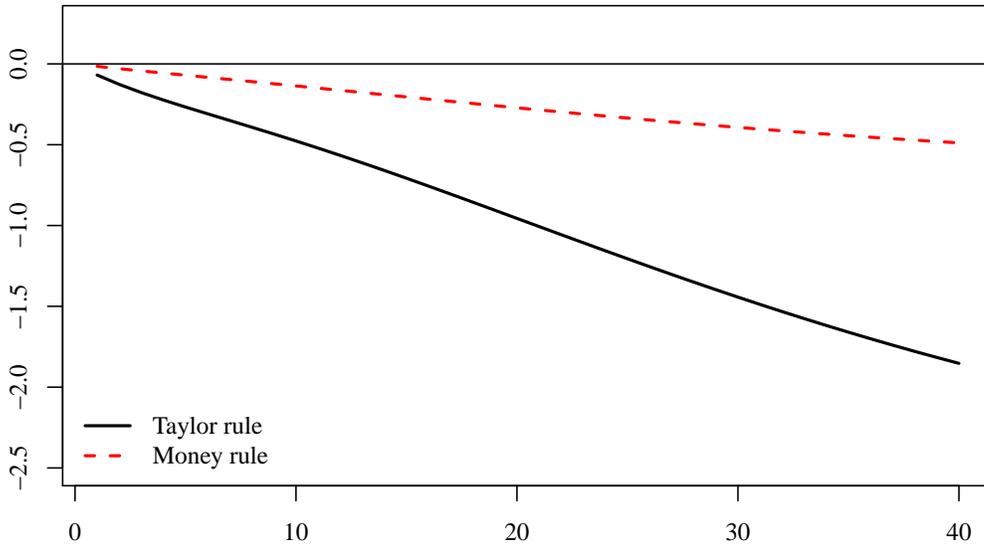
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 30: Response to technology shock under money-growth rule



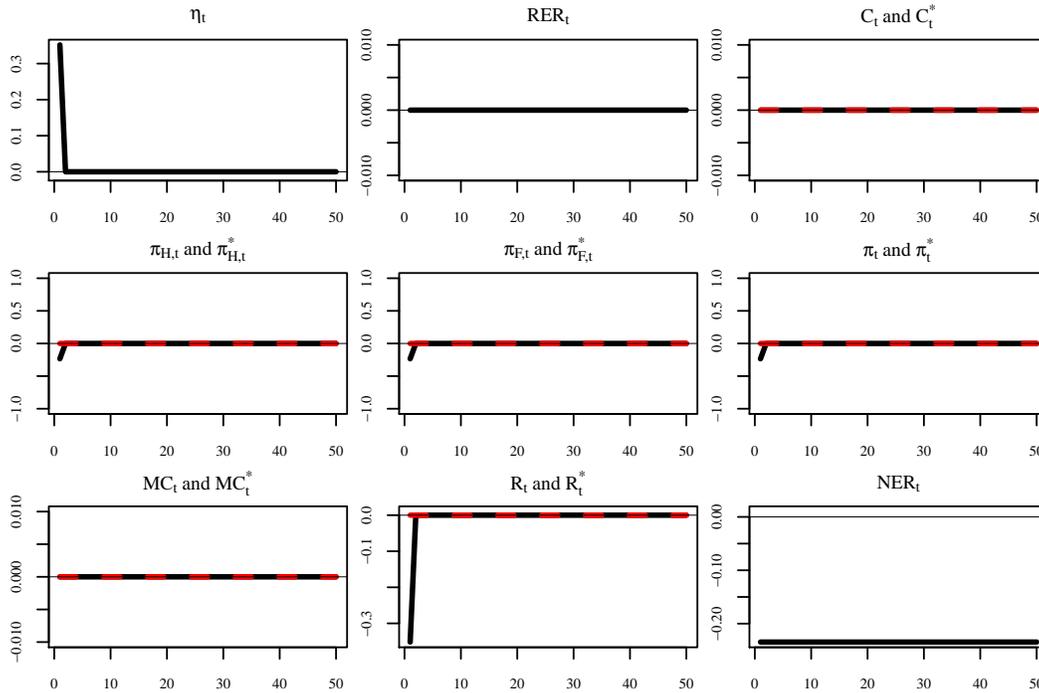
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 31: Implied values of $\beta_{1,j}^{NER}$ from small-scale model



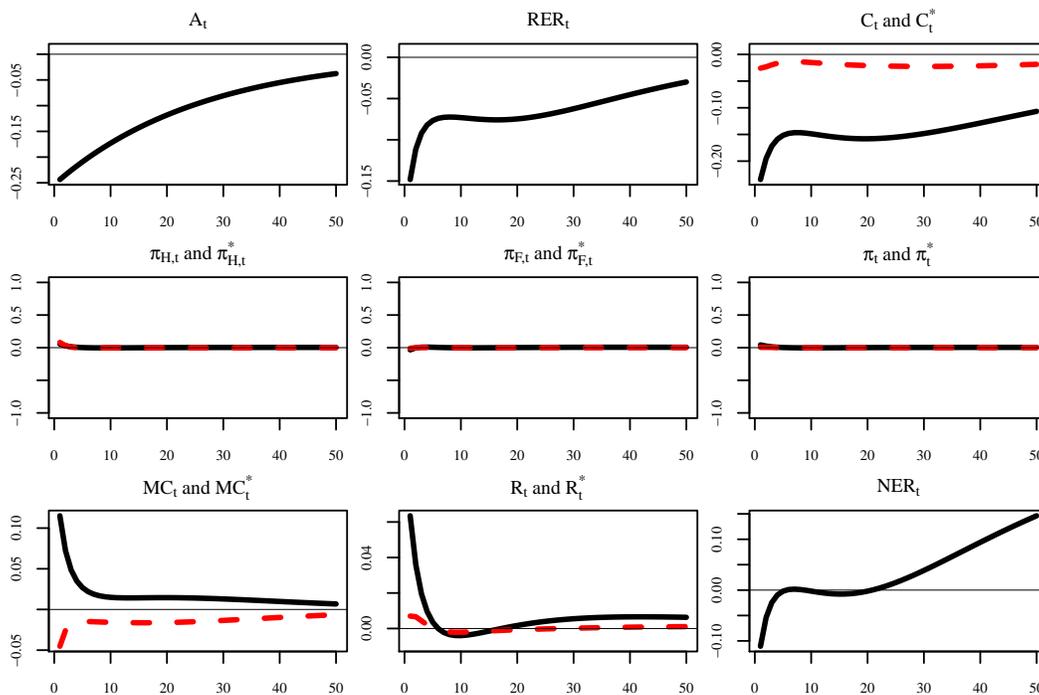
Note: The model-implied values come from our model with no nominal rigidities and only technology shocks.

Figure 32: Response to spread shock under Taylor rule with incomplete markets



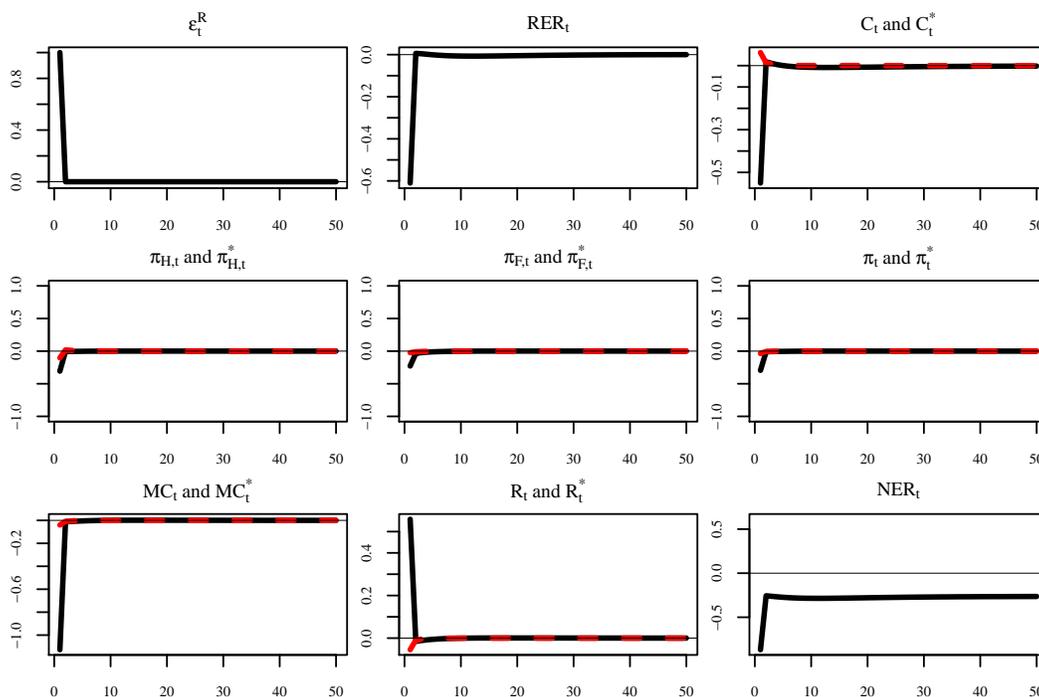
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 33: Response to technology shock under Taylor rule with incomplete markets and sticky prices



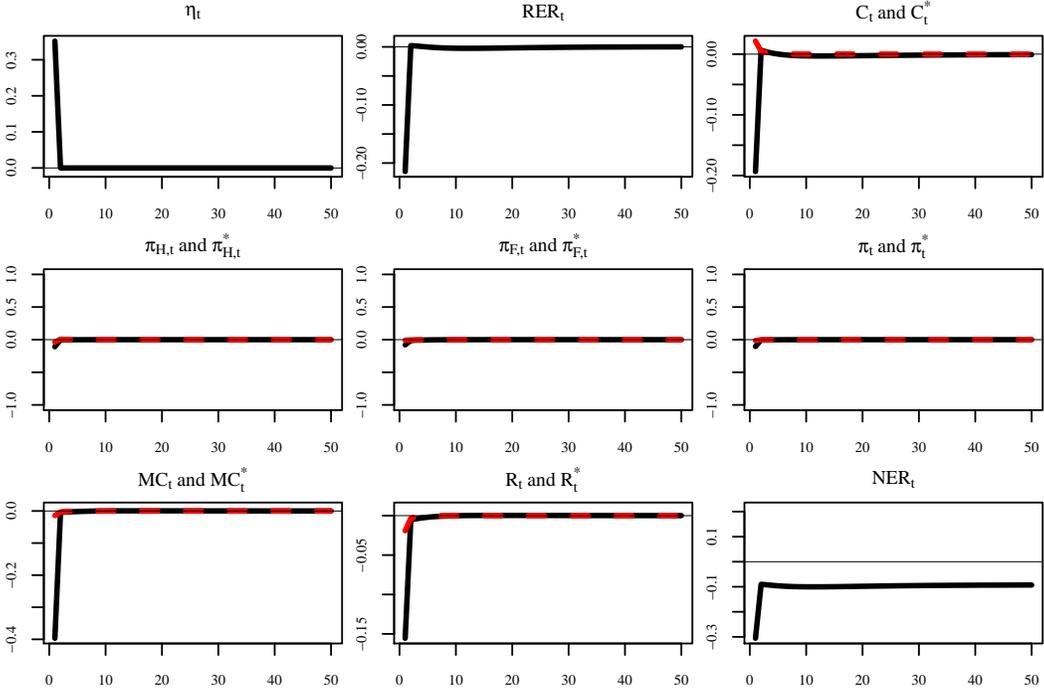
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 34: Response to monetary-policy shock under Taylor rule with incomplete markets and sticky prices



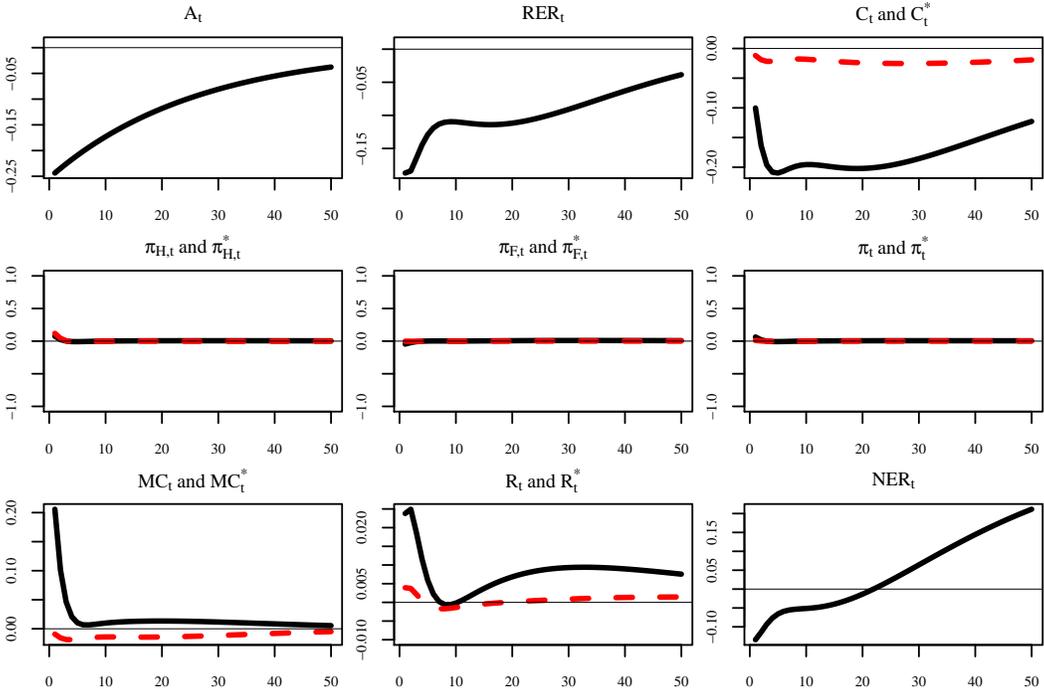
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 35: Response to spread shock under Taylor rule with incomplete markets and sticky prices



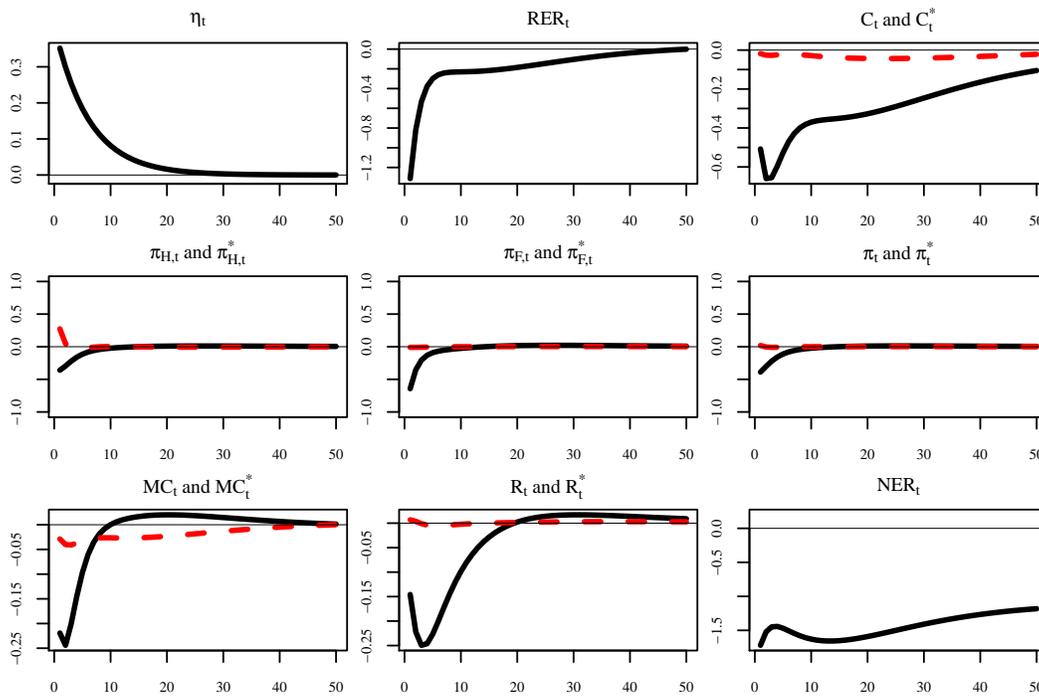
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 36: Response to technology shock under Taylor rule with incomplete markets and nominal rigidities



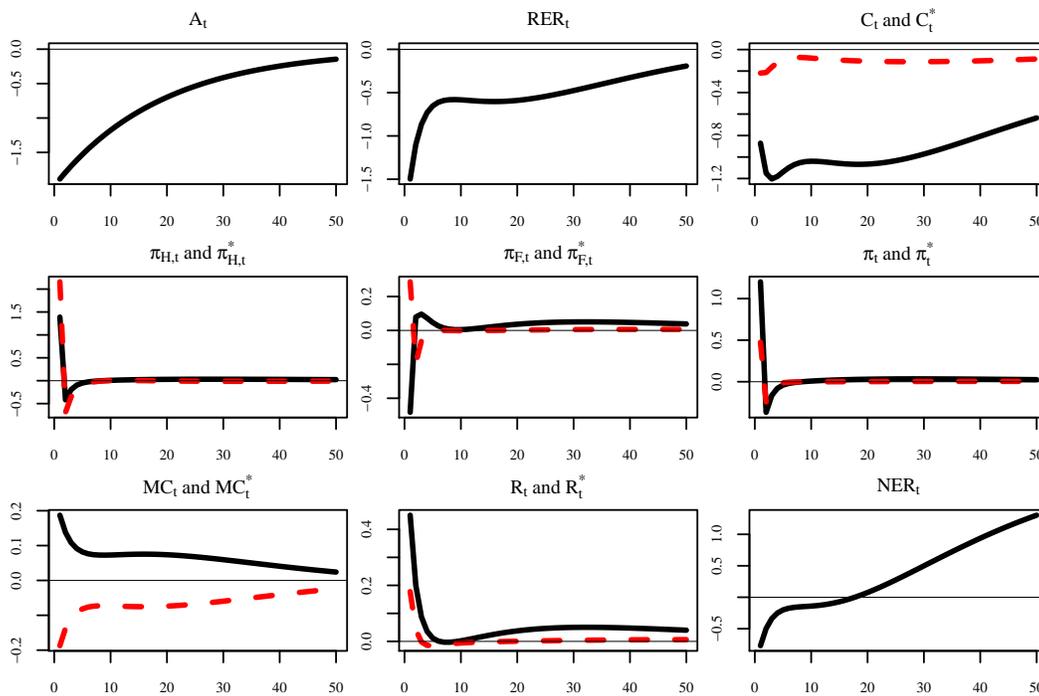
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 37: Response to spread shock under Taylor rule with incomplete markets and nominal rigidities



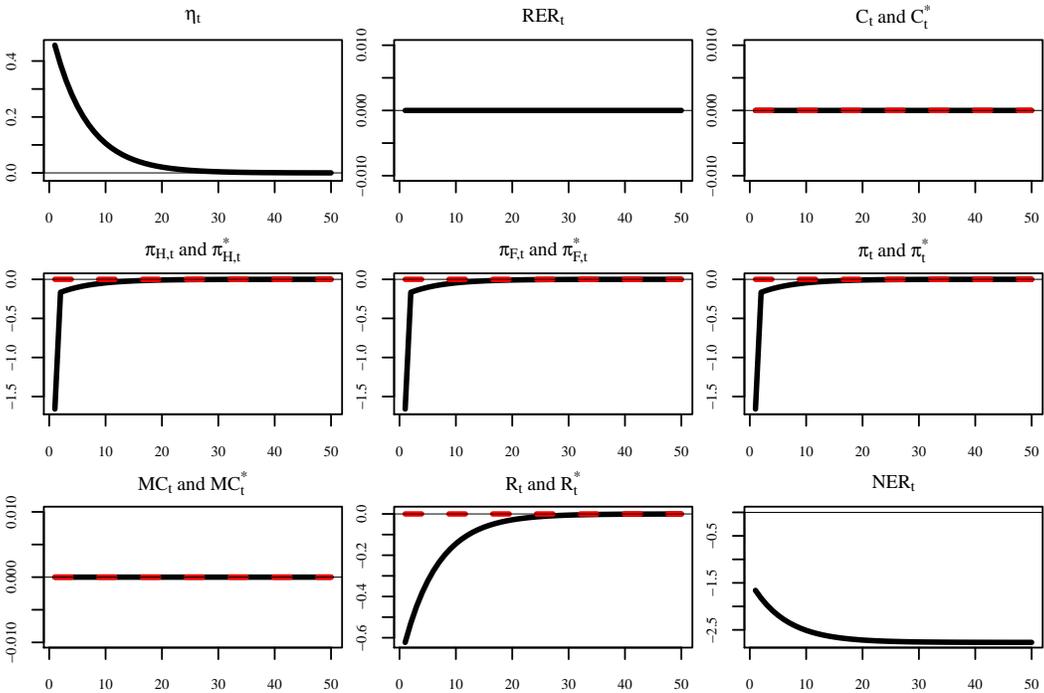
Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 38: Response to technology shock under Taylor rule with incomplete markets and no nominal rigidities, medium-scale model



Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.

Figure 39: Response to spread shock under Taylor rule with incomplete markets and no nominal rigidities, medium-scale model



Note: The vertical axis is expressed in percent. Inflation and interest rates are in annualized percent. The horizontal axis shows quarters after the shock. Red-dashed lines indicate the variables with a *.