Growth, Trade, and Inequality*

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Abstract

We introduce firm and worker heterogeneity into a model of innovation-driven endogenous growth. Individuals who differ in ability sort into either a research activity or a manufacturing sector. Research projects generate new varieties of a differentiated product. Projects differ in quality and the resulting technologies differ in productivity. In both sectors, there is a complementarity between firm quality and worker ability. We study the co-determination of growth and income inequality in both the closed and open economy, as well as the spillover effects of policy in one country to outcomes in others.

Keywords: endogenous growth, innovation, income distribution, income inequality, trade and growth

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1 Introduction

The relationship between growth and income distribution has been much studied. Researchers have identified several channels through which inequality might affect growth, such as if rich and poor households differ in their propensity to save (Kaldor, 1955-56), if poor households face credit constraints that limit their ability to invest in human capital (Galor and Zeira, 1993), or if greater inequality generates more redistribution and thus a different incentive structure via the political process (Alesina and Rodrik, 1994; Persson and Tabellini, 1994). Growth might affect distribution if the activities that drive growth make more intensive use of skilled labor than do other activities in the economy (Grossman and Helpman, 1991).

In this paper, we propose a novel mechanism that links distribution to growth, one that has not previously been considered in the literature. In an environment with heterogeneous workers and heterogeneous firms, markets provide incentives for certain types of workers to *sort* to certain activities and for the workers in a sector to *match* with certain types of firms. The fundamental forces that drive growth also determine the composition of worker types in each activity and thereby influence the matching of workers to firms. In this way of thinking, growth does not cause inequality, nor does inequality influence growth, but rather the two outcomes are jointly determined. We examine several potential determinants of growth and inequality, such as the productivity of an economy's manufacturing operations, its capacity for innovation, and its policies to promote R&D. Since we know from previous work that the form and extent of international integration can have important influences on growth, we also investigate how the mechanism of sorting and matching of heterogeneous workers operates in an open economy.

We introduce our mechanism in a simple and stylized setting—although we believe that it would operate similarly in a wide variety of growth models with heterogeneous workers and heterogeneous firms. We imagine that the economy undertakes two distinct activities that we refer to abstractly as *idea creation* and *idea using*. Our mechanism rests on two key assumptions. First, among a group of workers with heterogeneous abilities, greater ability confers a comparative advantage in creating ideas relative to using ideas. This implies rather directly that the more able types will sort into the idea-creating activity. Second, when research or production takes place, there exists a complementarity between the quality of an idea and the ability of the workers that implement the idea. As a consequence, there is positive assortative matching between heterogeneous firms and heterogeneous workers in both sectors of the economy. The forces that affect the sizes of the two sectors also affect the composition of workers in each sector and thereby affect the matching of workers with firms.

In our model, as in Romer (1990), the accumulation of knowledge serves as the engine of growth. Knowledge is treated as a by-product of purposive innovation undertaken to develop new products. Our treatment of trade, international knowledge diffusion, and growth extends the simplest, one-sector model from Grossman and Helpman (1991).[1] The advantage of the framework we develop

[1]In Grossman and Helpman (1991), we devote several chapters to models with two or more industrial sectors in order to address the impact of intersectoral resource allocation on growth and relative factor prices. By considering

here is that it focuses on the new mechanism and allows us to consider the entire distribution of earnings that emanates from a given distribution of worker abilities and firm productivity levels, and not just, say, the skill premium (i.e., the relative wage of "skilled" versus "unskilled" workers), which has been the focus of much of the existing theoretical literature.

In the next section, we develop our model in the context of a closed economy. A country has a fixed endowment of research equipment and a fixed supply of labor with an exogenous distribution of abilities. The economy assembles a single consumption good from differentiated intermediate inputs. Blueprints for the intermediate goods result from R&D services that are purchased by firms. The manufacturing firms, which engage in monopolistic competition, have access to different technologies and can hire workers of any ability. A firm's total output is the sum of what is produced by its various employees. The productivity of any employee depends on his ability and on the firm's technology. Ability and technology are complementary, so that, in equilibrium, the firms that have access to the better technologies hire the more able workers.

Innovation drives growth. Entrepreneurs rent research equipment to pursue their research ideas. Once an entrepreneur has established a research lab, she learns the quality of her project. The lab produces "R&D services" at a rate that depends on the quality of its project, the ability of the researchers that it hires, and the stock of knowledge capital available in the economy. Knowledge accumulates with research experience and is non-proprietary, as in Romer (1990). R&D services can be converted into designs for new varieties of the differentiated product. Each design comes with a random draw of a production technology, so that some manufacturing firms ultimately operate sophisticated technologies and others simpler technologies. There is free entry in both sectors of the economy. Expected returns are zero, although the lucky research entrepreneurs (those that draw above average research ideas) and the lucky manufacturers (those that draw above average production technologies) earn positive profits, while the others do not fully cover their fixed costs.

In equilibrium, all individuals with ability above some endogenous cutoff level sort into the research sector. They are hired there by the heterogeneous labs according to their ability. Similarly, for those that enter the manufacturing sector, there is endogenous matching between firms and employees. The complementarity between ability and technology delivers positive assortative matching in both sectors. These competitive forces of sorting and matching dictate the economy's wage distribution. We focus the analysis on the resulting inequality of wages.

After developing the model, we show how the long-run growth rate and wage distribution are codetermined in a long-run equilibrium. More specifically, we derive a pair of equations that jointly determine the steady-state growth rate in the number of varieties and the cutoff ability level that divides manufacturing workers from inventors. Once we know the growth rate of intermediate varieties, we can calculate the growth rate of final output and the growth rate of wages. Once we know the cutoff ability level, we can calculate the entire distribution of relative wages.

In Section 3, we compare growth rates and wage inequality across countries that differ in their

here a model with one industrial sector, we neglect this important, additional channel for trade to influence growth and income distribution.

technological parameters and policy choices. In this section, we focus on isolated countries that do not trade and do not capture any knowledge spillovers from abroad. We show that Hicks-neutral differences in labor productivity in manufacturing that apply across the full range of ability levels do not generate long-run differences in growth rates or wage inequality, although they do imply differences in income and consumption levels. In contrast, differences in "innovation capacity" generate differences in growth and inequality. Innovation capacity is represented by a sufficient statistic that reflects the size of a country's labor force, its endowment of research capital, its ability to convert research experience into knowledge capital, and its inventors' productivity in generating new ideas. A country with greater innovation capacity grows faster in autarky but experiences greater wage inequality. Subsidies to R&D financed by taxes on wage and capital income also contribute to faster growth but greater inequality.

Section 4 addresses the impacts of globalization. We consider a world economy with an arbitrary number of countries that trade the differentiated intermediate goods as well as the homogeneous final good. We follow Grossman and Helpman (1991) by assuming that international integration might also facilitate the international sharing of knowledge capital. In our baseline specification, we allow for an arbitrary pattern of complete or partial (but positive) international spillovers. In particular, the knowledge stock in each country is a weighted sum of accumulated innovation experience in all countries including itself, with an arbitrary matrix of strictly positive weighting parameters. We study a balanced-growth equilibrium in which the number of varieties of intermediate goods grows at the same constant rate in all countries. Even allowing for a wide range of differences in technologies and policies, we find that the long-run growth rate is higher in every country in the trading equilibrium than in autarky, but so too is the resulting wage inequality. We reach the same conclusion regardless of whether financial capital is internationally mobile, so that countries can engage in intertemporal trade, or is completely immobile, so that a country's trade must be balanced at every moment in time.

To better understand what is driving these results, we also study cases in which knowledge spillovers do not occur. When only goods are traded, the opening of trade has no effect on the long-run innovation rate or on wage inequality in any country. However, trade does accelerate wage and consumption growth in all countries except the one that grows the fastest in autarky. The other countries can take advantage of the rapid innovation in the fastest growing country by importing the new varieties of intermediate goods that it develops and produces. When goods and assets are both traded, then innovation slows in all countries except the fastest innovator. Yet these other countries all experience a boost in growth of wages and consumption. Moreover, wage inequality narrows in all of them. We conclude that the adverse distributional consequences of international integration are driven by knowledge flows and not by trade on international markets *per se*.

In Section 5, we study further the long-run trading equilibrium with partial or complete international knowledge spillovers. There, we are interested in how wage inequality compares in countries that differ in their productivities and policies and how parameter and policy changes in one country affect growth and inequality in others. Countries that differ in size, in research productivity, in manufacturing productivity and in their ability to create and absorb knowledge spillovers will converge not only in their growth rates, but also in their wage inequality in the long run. However, differences in government inducements to R&D generate enduring differences in the shape of their wage distributions; a country with a greater R&D subsidy will devote a larger fraction of its labor force to the research activity and will experience greater wage inequality as a result. Parameter and policy changes that accelerate growth and promote inequality in one country will have qualitatively similar effects on growth and inequality in all of its trade partners by the mechanism of sorting and matching that we describe.

In this paper, we do not conduct any empirical tests for the operation of our mechanism, nor do we attempt to quantify its significance. In general, attempts to substantiate the operation of mechanisms linking inequality to growth have been hampered by inadequate data and methodological pitfalls. Kuznets (1955, 1963), for example, famously advanced the hypothesis that income inequality first rises then falls over the course of economic development. While the "Kuznets curve"—an inverted-U shaped relationship between inequality and stage of development—has been established for the small set of countries that Kuznets considered, subsequent studies using broader data sets cast doubt on the ubiquity of this relationship.[2] More generally, empirical assessment of the links between distribution and growth has proven elusive due to the fact that a country's growth rate and its income inequality are jointly determined and there are few if any exogenous variables to serve as instruments for identifying causal relationships.[3] It might be possible to calibrate a growth model to get a sense of the relative quantitative significance of various mechanisms that link distribution with growth, but the model that we have presented here is too simple for calibration purposes. We have chosen the simple (and familiar) specification in order to present starkly the mechanism that we have in mind, and leave quantification of the mechanism for future research.

2 The Basic Model

We develop a model of economic growth featuring heterogeneous workers, heterogeneous firms, and heterogeneous research opportunities. In the model, endogenous innovation drives growth. Workers that differ in ability engage either in creating ideas or using ideas. In keeping with the literature, we refer to the creation of ideas as "R&D" and the implementation of ideas as "manufacturing," although we prefer not to interpret these terms too narrowly. Research firms ("labs") generate both research services and general knowledge as joint outputs, using labor, laboratory equipment ("equipment") and knowledge as inputs. To simplify our analysis, we take the stock of equipment as fixed. Research services are proprietary and are sold by the labs to manufacturing firms that convert them into blueprints for differentiated intermediate inputs. Knowledge, in contrast, is non-rival and a freely-available public input. When a manufacturing firm produces an intermediate input, it operates a randomly-chosen technology that is an identifying characteristic of the firm.

[2] See Helpman (2004, ch.4) for a survey of this evidence.

[3] A similar problem has plagued attempts to assess the relationship between trade and growth (see Helpman, 2004, ch.6).

Similarly, labs differ in their realized research productivity. There is free entry into both activities before the uncertainty is resolved. In the equilibrium, the heterogeneous workers sort into one of the two sectors and firms and labs with different productivities hire different types of workers. The economy converges to a long-run equilibrium with a constant growth rate of final output and a fixed and continuous distribution of wage income.

We describe here the economic environment for a closed economy and defer the introduction of international trade, international knowledge spillovers and international capital mobility until Section 4 below.

2.1 Demand and Supply for Consumption Goods

The economy is populated by a mass N of individuals indexed by ability level, a. The cumulative distribution of abilities is given by $H(a)$, which is twice continuously differentiable and has a positive density $H'(a) > 0$ on the bounded support, $[a_{\min}, a_{\max}]$.

Each individual maximizes a logarithmic utility function,

$$u_t = \int_t^{\infty} e^{-\rho(\tau - t)} \log c_\tau d\tau, \tag{1}$$

where c_τ is consumption at time τ and ρ is the common, subjective discount rate. The consumption good serves as numeraire; its price at every moment is normalized to one. It follows from the individual's intertemporal optimization problem that

$$\frac{\dot{c}_t}{c_t} = \iota_t - \rho, \tag{2}$$

where ι_t is the interest rate at time t in terms of consumption goods. Inasmuch as a varies across individuals, so does income and consumption.

Consumption goods are assembled from an evolving set Ω_t of differentiated intermediate inputs. Dropping the time subscript for notational convenience, the production function for these goods at a moment when the set of available inputs is Ω is given by

$$X = \left[\int_{\omega \in \Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{3}$$

where $x(\omega)$ is the input of variety ω. The elasticity of substitution between intermediate inputs is constant and equal to σ.

The market for consumption goods is competitive. It follows that the equilibrium price of these goods reflects the minimum unit cost of producing them. Since X is the numeraire, we have

$$\left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = 1, \tag{4}$$

where $p(\omega)$ is the price of intermediate input ω.

2.2 Supply, Demand, Pricing, and Profits of Intermediate Goods

Once a firm has converted research services into the blueprint for an intermediate good, it produces that good indefinitely using labor as the sole input to production. Each firm that manufactures an intermediate good is distinguished by its technology, φ. A firm with a higher φ is more productive, no matter what type(s) of workers it hires. Consider a firm that produces variety ω using technology φ and that hires a set L_ω of workers types with densities $\ell_\omega(a)$. In such circumstances, the firm's output is

$$x(\omega) = \int_{a \in L_\omega} \psi(\varphi, a)\, \ell_\omega(a)\, da, \tag{5}$$

where $\psi(\varphi, a)$ is the productivity of workers of type a when applying technology φ. Notice that labor productivity (given φ) is independent of ω.

We suppose that more productive technologies are also more complex and that more able workers have a comparative advantage in operating the more complex technologies. In other words, we posit a complementarity between the type of technology φ and the type of worker a in determining labor productivity. Formally, we adopt

Assumption 1 The productivity function $\psi(\varphi, a)$ is twice continuously differentiable, strictly increasing, and strictly log supermodular.

Assumption 1 implies $\psi_{\varphi a} > 0$ for all φ and a.

As is known from Costinot (2009), Eeckhout and Kircher (2016), Sampson (2014) and others, the strict log supermodularity of $\psi(\cdot)$ implies that, for any upward-sloping wage schedule $w(a)$, each manufacturing firm hires the particular type of labor that is most appropriate given its technology φ, and there is *positive assortative matching* (PAM*)* between firm types and worker types. We denote by $m(\varphi)$ the ability of workers employed by firms that operate a technology φ; PAM is reflected in the fact that $m'(\varphi) > 0$.

Shephard's lemma gives the demand for any variety ω as a function of the prices of all available intermediate goods, namely

$$x(\omega) = X \left[\int_{\upsilon \in \Omega} p(\upsilon)^{1-\sigma}\, d\upsilon \right]^{\frac{\sigma}{1-\sigma}} p(\omega)^{-\sigma}.$$

In view of (4), demand for variety ω can be expressed as

$$x(\omega) = Xp(\omega)^{-\sigma} \quad \text{for all } \omega \in \Omega. \tag{6}$$

Each firm takes aggregate output of final goods X as given and so it perceives a constant elasticity of demand, $-\sigma$. As is usual in such settings, the profit-maximizing firm applies a fixed percentage markup to its unit cost.

Considering the optimal hiring decision, a firm that operates a technology φ has productivity $\psi[\varphi, m(\varphi)]$ and pays a wage $w[m(\varphi)]$. Hence, the firm faces a minimal unit cost of

$w\left[m\left(\varphi\right)\right]/\psi\left[\varphi,m\left(\varphi\right)\right]$. The firm's profit-maximizing price is given by[4]

$$p\left(\varphi\right)=\left(\frac{\sigma}{\sigma-1}\right)\frac{w\left[m\left(\varphi\right)\right]}{\psi\left[\varphi,m\left(\varphi\right)\right]}. \tag{7}$$

This yields an operating profit of

$$\pi\left(\varphi\right)=\sigma^{-\sigma}\left(\sigma-1\right)^{\left(\sigma-1\right)}X\left\{\frac{w\left[m\left(\varphi\right)\right]}{\psi\left[\varphi,m\left(\varphi\right)\right]}\right\}^{1-\sigma}. \tag{8}$$

2.3 Inventing New Varieties

Any research entrepreneur can establish a lab to pursue a research project. When an entrepreneur contemplates a new project, she does not know its quality, q. At this stage, she perceives q as being drawn from some cumulative distribution function for project types, $G_R\left(q\right)$, with $G_R'\left(q\right)>0$ on a bounded support $\left[q_{\min},q_{\max}\right]$. Each project requires f units of equipment. Once an entrepreneur has rented the requisite equipment to undertake her project, she discovers the project's quality. She then hires some number $\ell_R\left(a\right)$ of workers of some ability level a to carry out the research, paying the equilibrium wage, $w\left(a\right)$.

All projects generate R&D services. The volume of services that results from a project depends upon its quality, the number of researchers engaged in the project, their ability, and the state of knowledge at the time. We follow Romer (1990) in assuming that knowledge accumulates as a by-product of research experience. The knowledge stock at time t is $\theta_K M_t$, where M_t is the mass of varieties that have been developed before time t and θ_K is a parameter that reflects how effectively the economy converts cumulative research experience into applicable knowledge. The output of a research project of quality q that employs $\ell_R\left(a\right)$ workers with ability in the interval $\left[a,a+da\right]$ when the state of knowledge is $\theta_K M$ is given by $\theta_K M\psi_R\left(q,a\right)\ell_R\left(a\right)^{\gamma}da$, where $\psi_R\left(q,a\right)$ captures a complementarity between project quality and worker ability in determining innovation productivity. In particular, we adopt the following assumption, analogous to Assumption 1.

Assumption 2 Research productivity $\psi_R\left(q,a\right)$ is twice continuously differentiable, strictly increasing, and strictly log supermodular.

In equilibrium, the workers with type $m_R\left(q\right)$ work on projects of quality q. Assumption 2 ensures PAM in the research sector, so that $m_R'\left(q\right)>0$.

Let Q be the economy's fixed endowment of laboratory equipment and define $R\equiv Q/f$. Then R gives the measure of active research projects at any point in time. This fixed quantity does not pin down the innovation rate in the economy, however, because the scale and productivity of the research labs are determined endogenously in equilibrium.

Manufacturing firms buy R&D services from research labs at the price p_R. One unit of R&D services generates a design for a differentiated intermediate good along with an independent draw

[4] We henceforth index intermediate goods by the technology with which they are produced (φ) rather than their variety name (ω), since all varieties are symmetric except for their different technologies.

from a cumulative technology distribution, $G(\varphi)$, as in Melitz (2003). The technology parameter φ determines the complexity and productivity of the technology, as described in Section 2.2 above.

2.4 Free Entry

There is free entry into both research and manufacturing. A research entrepreneur must pay rf to rent the equipment needed to carry out a project, where r is the equilibrium rental rate. The investment yields an expected return of $E\pi_R$, where

$$E\pi_R = \int_{q_{\min}}^{q_{\max}} \pi_R(q)\, dG_R(q)$$

and

$$\pi_R(q) = \max_{a,\ell_R} \left[p_R \theta_K M \psi_R(q,a)\, \ell_R^{\gamma} - w(a)\, \ell_R \right]$$

is the maximal profit for a lab that implements a project with quality q. Since a lab that undertakes a project of quality q hires researcher workers with ability $m_R(q)$, we have[5]

$$\pi_R(q) = (1-\gamma)\, \gamma^{\frac{\gamma}{1-\gamma}} \left\{ p_R \theta_K M \psi_R[q, m_R(q)]\, w[m_R(q)]^{-\gamma} \right\}^{\frac{1}{1-\gamma}}. \tag{9}$$

Free-entry into R&D implies

$$rf = (1-\gamma)\, \gamma^{\frac{\gamma}{1-\gamma}} \left(p_R \theta_K M \right)^{\frac{1}{1-\gamma}} \int_{q_{\min}}^{q_{\max}} \psi_R[q, m_R(q)]^{\frac{1}{1-\gamma}}\, w[m_R(q)]^{-\frac{\gamma}{1-\gamma}}\, dG_R(q), \tag{10}$$

which determines r.

Similarly, a manufacturing firm pays p_{Rt} to purchase the R&D services needed to introduce a variety of intermediate good at time t. If it draws a manufacturing technology φ, it will earn a stream of profits $\pi_\tau(\varphi)$ for all $\tau \geq t$. We have derived the expression for operating profits and recorded it (with time index suppressed) in (8). On a balanced-growth path, wages of all types of workers grow at the common rate g_w and final output grows at a constant rate g_X. Final output serves only consumption, so, by (2), $g_X = \iota - \rho$. Operating profits also grow at a constant rate g_π, independent of φ, and, by (8), $g_\pi = g_X - (\sigma - 1)\, g_w$. Finally, (4) and (7) imply that, in a steady state, $(\sigma-1)\, g_w = g_M$. Combining these long-run relationships, the expected discounted profits for a new manufacturing firm at time t can be written as

$$\int_t^{\infty} e^{-\iota(\tau - t)} \int_{\varphi_{\min}}^{\varphi_{\max}} \pi_\tau(\varphi)\, dG(\varphi)\, d\tau = \frac{\int_{\varphi_{\min}}^{\varphi_{\max}} \pi_t(\varphi)\, dG(\varphi)}{\rho + g_M}.$$

[5] We derive the maximal research profit for a lab with a project of quality q by choosing $\ell_R(q,a)$ according to the first-order condition,

$$\ell_R(q,a) = \left[\frac{\gamma p_R \theta_K M \psi_R(q,a)}{w(a)} \right]^{\frac{1}{1-\gamma}},$$

and substituting this expression for optimal employment into the expression for operating profits.

Equating the cost of R&D services to the expected discounted value of a new product, and again dropping the time subscript, we have

$$p_R = \frac{\int_{\varphi_{\min}}^{\varphi_{\max}} \pi(\varphi)\, dG(\varphi)}{\rho + g_M}. \tag{11}$$

2.5 Sorting, Matching, and Labor-Market Equilibrium

Individuals choose employment in either research or manufacturing. In so doing, they compare the wages they can earn (given their ability) in the alternative occupations. Let $w_M(a)$ be the wage paid to employees in the manufacturing sector and let $w_R(a)$ be the wage paid to those entering research. To identify the equilibrium sorting pattern, we make use of two lemmas that characterize the wage schedules in the two sectors. First, we have

Lemma 1 *Consider any closed interval of workers $[a', a'']$ that is employed in the manufacturing sector in equilibrium. In the interior of this interval, wages must satisfy*

$$\frac{w_M'(a)}{w_M(a)} = \frac{\psi_a\left[m^{-1}(a), a\right]}{\psi\left[m^{-1}(a), a\right]} \text{ for all } a \in \left(a', a''\right), \tag{12}$$

where $m^{-1}(\cdot)$ is the inverse of $m(\cdot)$.

The lemma reflects the requirement that, in equilibrium, a manufacturing firm with productivity φ must prefer to hire the worker with ability $m(\varphi)$ than any other worker. The lemma follows from the first-order condition for the profit-maximizing choice of $a = m(\varphi)$; it says that, the shape of the wage schedule mirrors the rise in productivity as a function of ability, with productivity evaluated at the equilibrium match. In the event, no firm will have any incentive to upgrade or downgrade its labor force.

The second lemma applies to the research sector, and has a similar logic.

Lemma 2 *Consider any closed interval of workers $[a', a'']$ that is employed in the R&D sector in equilibrium. In the interior of this interval, the wage schedule must satisfy*

$$\frac{w_R'(a)}{w_R(a)} = \frac{\psi_{Ra}\left[m_R^{-1}(a), a\right]}{\gamma\psi_R\left[m_R^{-1}(a), a\right]} \text{ for all } a \in \left(a', a''\right). \tag{13}$$

This lemma expresses a preference on the part of each lab for the type of researcher that it hires in equilibrium compared to all alternatives. It comes from the first-order condition for maximizing research profits in (9). The shape of the wage schedule in R&D is slightly different from that in manufacturing, because the R&D sector has diminishing returns to employment in a given lab with its fixed research capital, whereas the manufacturing sector exhibits a constant marginal product of labor. The entrepreneur's choice of researcher type reflects not only the direct effect of ability on the productivity shifter, but also the fact that different types imply different employment levels and therefore different diminishing returns; see Grossman et al. (2017) for further discussion of this point in a related setting.

We assume that high-ability workers enjoy a comparative advantage in R&D; in particular, we make

Assumption 3 $\frac{\psi_{Ra}(q,a)}{\gamma\psi_R(q,a)} > \frac{\psi_a(\varphi,a)}{\psi(\varphi a)}$ for all q, φ, and a.

Assumption 3, together with Lemmas 1 and 2, dictate the equilibrium sorting pattern. They ensure that there exists a cutoff ability level a_R such that all workers with ability above a_R are employed in the research sector and all workers with ability below a_R are employed in manufacturing.[6] In a steady-state equilibrium with positive growth, $a_R < a_{\max}$. In any case, the equilibrium wage schedule, $w(a)$ satisfies

$$w(a) = \begin{cases} w_M(a) & \text{for } a \leq a_R \\ w_R(a) & \text{for } a \geq a_R \end{cases}, \tag{14}$$

with $w_M(a_R) = w_R(a_R)$.

We next derive a pair of differential equations that characterize the matching functions in the two sectors. In the manufacturing sector, the wages paid to all workers with ability less than or equal to some $a = m(\varphi)$ matches what the firms with technology indexes less than or equal to φ are willing to pay, considering their labor demands. This equation of labor supply and labor demand implies

$$MX\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\int_{\varphi_{\min}}^{\varphi}\left\{\frac{w_M[m(\phi)]}{\psi[\phi, m(\phi)]}\right\}^{1-\sigma} dG(\phi) = N\int_{a_{\min}}^{m(\varphi)} w_M(a)\, dH(a) \ . \tag{15}$$

Differentiating this equation with respect to φ yields

$$m'(\varphi) = \frac{MX}{N}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{w_M[m(\varphi)]^{-\sigma}}{\psi[\varphi, m(\varphi)]^{1-\sigma}}\frac{G'(\varphi)}{H'[m(\varphi)]} \quad \text{for all } \varphi \in [\varphi_{\min}, \varphi_{\max}]\,. \tag{16}$$

Following Grossman et al. (2017), we show in Appendix A2.5 that this equation, together with the wage equation (12) and the boundary conditions,

$$m(\varphi_{\min}) = a_{\min},\ m(\varphi_{\max}) = a_R, \tag{17}$$

uniquely determine the matching function $m(\varphi)$ and the wage function $w_M(a)$ for workers in manufacturing, for a given cutoff value a_R.

The demand for R&D workers by all labs with projects qualities between some q and $q_{\max}$ is

$$R\int_{q}^{q_{\max}}\left\{\frac{\gamma p_R\theta_K M\psi_R[z, m_R(z)]}{w_R[m_R(z)]}\right\}^{\frac{1}{1-\gamma}} dG_R(z)$$

[6] The wage schedule must be everywhere continuous, or else those paying the discretely higher wage will prefer to downgrade slightly. The two lemmas ensure that wages rise faster in the research sector just to the right of any cutoff point, and they rise slower in manufacturing just to the left of any cutoff point. It follows that there can be at most one such cutoff point.

and the wage paid to a worker by a lab with a project of quality z is $w_R[m_R(z)]$. Wage payments equal wage earnings. Therefore, labor-market clearing for this set of workers requires

$$R\int_{q}^{q_{\max}} w_R[m_R(z)]\left\{\frac{\gamma p_R\theta_K M\psi_R[z,m_R(z)]}{w_R[m_R(z)]}\right\}^{\frac{1}{1-\gamma}} dG_R(z) = N\int_{m_R(q)}^{a_{\max}} w_R(a)\,dH(a)\ . \tag{18}$$

Differentiating this equation with respect to q yields a differential equation for the matching function in the research sector,

$$m_R'(q) = \frac{R}{N}\left\{\frac{\gamma p_R\theta_K M\psi_R[q,m_R(q)]}{w_R[m_R(q)]}\right\}^{\frac{1}{1-\gamma}}\frac{G_R'(q)}{H'[m_R(q)]}\ , \tag{19}$$

with boundary conditions

$$m_R(q_{\min}) = a_R,\quad m_R(q_{\max}) = a_{\max}. \tag{20}$$

The differential equation (19) together with (13) and the boundary conditions (20) uniquely determine the matching function $m_R(q)$ and the wage function $w_R(a)$ for a given cutoff a_R. The proof is similar to the proof of uniqueness for the matching and wage functions in the manufacturing sector.

The solution to these differential equations give us matching functions for the two sectors that are parameterized by the cutoff point, a_R, which enters through the boundary conditions (17) and (20). To emphasize this dependence on a_R, we write the solutions as $m(\varphi;a_R)$ and $m_R(q;a_R)$. Note that the matching functions do not depend directly on N, R, X, θ_K, p_R or M. As shown in Grossman et al. (2017), the wage *ratios* in manufacturing—that is, the ratio of wages paid to any pair of workers employed in that sector—are also uniquely determined by a_R, independently of N, X or M. Similarly, the relative wages of R&D workers are uniquely determined by a_R, independently of N, R, θ_K, p_R or M. We define relative wage functions $\lambda(a;a_R)$ and $\lambda_R(a;a_R)$ that describe inequality among workers in each sector as

$$\left.\begin{array}{ll}\lambda(a;a_R) = \frac{w_M(a)}{w_M(a_{\min})} & \text{for } a\in[a_{\min},a_R]\\ \lambda_R(a;a_R) = \frac{w_R(a)}{w_R(a_R)} & \text{for } a\in[a_R,a_{\max}]\end{array}\right\}\ . \tag{21}$$

We note that the *levels* of the wages—for example, of $w_M(a_{\min})$ and $w_R(a_R)$—do depend on parameters and variables like N, X, R, θ_K, p_R, and M that determine the momentary equilibrium.

2.6 The Steady-State Equilibrium

In this section, we derive a pair of equations that jointly determine the growth rate in the number of varieties and the cutoff ability level that separates researchers from production workers in a steady-state equilibrium. The first curve can be understood as a kind of resource constraint; the more workers that sort to R&D in equilibrium, the more new varieties are invented. The second relationship combines the free-entry condition for manufacturing with the labor-market-clearing condition for that sector. Once we have the steady-state values of g_M and a_R, we can calculate the

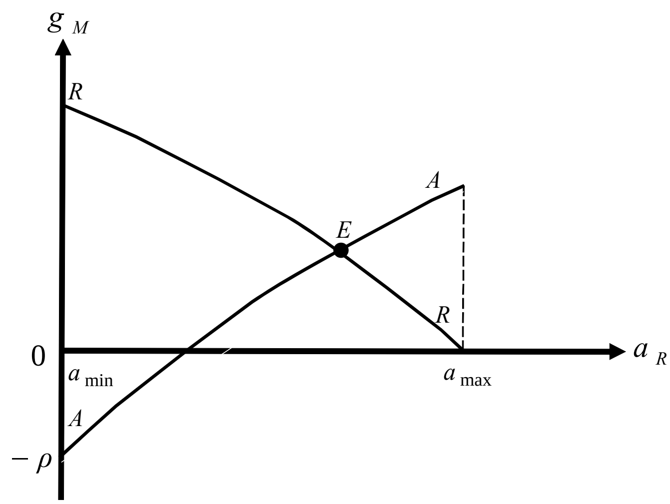

Figure 1: Equilibrum growth rate and ability cutoff

other variables of interest, such as the growth rates of output and consumption and the distribution of income.

The growth in varieties reflects the aggregate output of R&D services by the research sector. In steady state,

$$g_M = \theta_K R \int_{q_{\min}}^{q_{\max}} \psi_R \left[q, m_R(q)\right] \ell_R \left[q, m_R(q)\right]^{\gamma} dG_R(q) \ ,$$

where $\ell_R \left[q, m_R(q)\right]$ is steady-state employment by projects of quality q. In the appendix, we derive what we call the RR curve by substituting the labor-market-clearing condition for the research sector (18) into the expression for g_M. The RR curve is given by

$$g_M = \theta_K N^{\gamma} R^{1-\gamma} \Phi(a_R) \int_{a_R}^{a_{\max}} \lambda_R(a; a_R)\, dH(a)\,, \tag{22}$$

where

$$\Phi(a_R) \equiv \left\{ \frac{\int_{q_{\min}}^{q_{\max}} \psi_R \left[q, m_R(q; a_R)\right]^{\frac{1}{1-\gamma}} \lambda_R \left[m_R(q; a_R); a_R\right]^{-\frac{\gamma}{1-\gamma}} dG_R(q)}{\int_{a_R}^{a_{\max}} \lambda_R(a; a_R)\, dH(a)} \right\}^{1-\gamma} .$$

Notice that the right-hand side of (22) depends only on the cutoff value a_R and on exogenous parameters, inasmuch as the cutoff fully determines matching in the research sector and relative wages there. In the appendix, we show that the RR curve slopes downward, as depicted in Figure 1, despite the fact that $\Phi'(a_R) > 0$. The RR curve is a resource constraint, indicating that faster growth in the number of varieties requires that more resources be devoted to R&D and hence a lower cutoff ability level for the marginal research worker. Given the cutoff a_R, (22) indicates that the growth rate will be higher the more productive is experience in generating knowledge capital, the larger is the population of workers, and the larger is the stock of laboratory equipment, which allows that more research projects can be undertaken.

Next we substitute the expression for profits of an intermediate good producer in (8) into the free-entry condition (11) and combine the result with the labor-market clearing condition for manufacturing (15), evaluated with $\tilde{\varphi} = \varphi_{\max}$. The result can be written as

$$\rho + g_M = \frac{1}{\sigma - 1} \frac{N}{p_R M} \int_{a_{\min}}^{a_R} w(a)\, dH(a) \ .$$

Again we can use (18), the labor-market-clearing condition for the research sector, together with the definition of the relative wages $\lambda(a; a_R)$ and $\lambda_R(a; a_R)$ to eliminate $p_R M$, so that we can write a second steady-state relationship involving only g_M and a_R. This is the AA curve depicted in Figure 1, and it is given by

$$\rho + g_M = \frac{\gamma}{\sigma - 1} \theta_K N^{\gamma} R^{1-\gamma} \Phi(a_R) \frac{\int_{a_{\min}}^{a_R} \lambda(a; a_R)\, dH(a)}{\lambda(a_R; a_R)} \ . \tag{23}$$

In the appendix, we prove that the AA curve must slope upward, as drawn.

The figure shows a unique balanced-growth equilibrium at point E.[7] Once we know the steady-state cutoff level of ability a_R, we can compute the long-run distribution of relative wages using the wage structures dictated by Lemmas 1 and 2. From the long-run rate of growth in the number of intermediate goods, g_M, we can calculate the long-run growth rates for consumption and wages. We have noted already that $g_w = g_M / (\sigma - 1)$. Equality between savings and investment requires

$$p_R \dot{M} = N \int_{a_{\min}}^{a_{\max}} w(a; a_R)\, dH(a) + rQ + M \int_{\varphi_{\min}}^{\varphi_{\max}} \pi(\varphi)\, dG(\varphi) - C \ , \tag{24}$$

where C is aggregate consumption and, therefore the right-side is the difference between aggregate income (the sum of wages, rents and profits) and aggregate consumption. Equation (33) in the appendix implies that $p_R \dot{M} = g_M p_R M = \frac{N}{\gamma} \int_{a_R}^{a_{\max}} w_R(a)\, dH(a)$, so aggregate investment grows in the long run at the same rate as wages, g_w. On the right-side, aggregate wage income grows at rate g_w, while the free-entry condition for R&D (10) implies that rental income grows at this same rate. Also, we have noted from (8) that $g_\pi = g_X - (\sigma - 1) g_w$ while the labor-market clearing condition (15) implies $\sigma g_w = g_M + g_X$. Together, this gives $g_\pi + g_M = g_w$; i.e., aggregate profits also grow at the rate g_w. it follows from (24) that aggregate consumption must grow in the long run at the same rate as wages; $g_C = g_w$. We conclude, therefore, that $g_w = g_C = g_X = g_M / (\sigma - 1)$.

3 Growth and Inequality in Autarky Equilibrium

In this section, we compare growth rates and wage inequality in a pair of closed economies. We consider countries i and j that are basically similar but differ in some technological or policy parameters. We focus on balanced-growth equilibria, such as those described in Section 2. In the

[7] If the AA curve falls below the horizontal axis for all $a_R \leq a_{\max}$, then no workers are employed in the research sector in the steady state. In such circumstances, growth rates of varieties, final output, consumption and wages are all zero.

following sections, we will perform similar cross-country comparisons for a set of open economies and examine how the opening of trade affects growth and wage inequality with and without international knowledge spillovers and with and without international borrowing and lending.

3.1 Productivity in Manufacturing

We begin by supposing that the countries differ only in their productivity in manufacturing, as captured by a Hicks-neutral technology parameter θ_{Mc}. In country c, a unit of labor of type a applied in a firm with technology φ can produce $\psi_c(\varphi, a) = \theta_{Mc}\psi(\varphi, a)$ units of a differentiated intermediate good. For the time being, the other characteristics of the countries are the same, including their sizes, their distributions of ability, their distributions of firm productivity, their discount rates and the efficiency of their knowledge accumulation.

In these circumstances, the matching function $m(\varphi; a_R)$ in the manufacturing sector is common to both countries; i.e., a difference between θ_{Mi} and θ_{Mj} does not affect matching in the manufacturing sector for a given a_R.[8] Therefore, the relative-wage function $\lambda(a; a_R)$ also will be the same in both countries if they have the same cutoff point. But then the solution to (22) and (23) is the same for any values of θ_{Mi} and θ_{Mj}. In other words, countries that differ only in the (Hicks-neutral) productivity of their manufacturing sectors share the same long-run growth rate and the same marginal worker in manufacturing. It follows that relative wages for any pair of ability levels are also the same. We summarize in

Proposition 1 *Suppose that countries i and j differ only in manufacturing labor productivity $\psi_c(\cdot)$ and that these differences are Hicks-neutral; i.e., $\psi_c(\cdot) = \theta_{Mc}\psi(\cdot)$ for $c = i, j$. Then in autarky, both countries grow at the same rate in a balanced-growth equilibrium and both share the same structure of relative wages and the same degree of wage inequality.*

3.2 Capacity to Innovate

In our model, a country's capacity for innovation is described by four parameters: population size, which determines the potential scale of the research sector; the productivity of research workers; the efficiency with which research experience is converted into knowledge; and the endowment of laboratory equipment or, equivalently, the measure of research projects that can be undertaken simultaneously. In this section, we compare autarky growth rates and wage distributions in countries that differ in labor force, N_c, in efficiency of knowledge accumulation, θ_{Kc}, in research capital Q_c and thus in the measure of active research projects, $R_c \equiv Q_c/f$, and in the productivity of research workers, as captured by a Hicks-neutral shift parameter θ_{Rc}, where $\psi_{Rc}(q, a) = \theta_{Rc}\psi_R(q, a)$.

[8] To see this, differentiate the labor-market clearing condition (16) with respect to φ, to derive the second-order differential equation,

$$\frac{m''(\varphi)}{m'(\varphi)} = (\sigma - 1)\frac{\psi_\varphi[\varphi, m(\varphi)]}{\psi[\varphi, m(\varphi)]} - \sigma\frac{\psi_a[\varphi, m(\varphi)]}{\psi[\varphi, m(\varphi)]} + \frac{G''(\varphi)}{G'(\varphi)} - \frac{H''[m(\varphi)]\, m'(\varphi)}{H'[m(\varphi)]} .$$

The productivity parameter θ_{Mc} appears in the numerator and the denominator of ψ_φ/ψ and of ψ_a/ψ, and so it does not affect matching for a given a_R.

The RR curve in Figure 1 is defined by equation (22). In this equation, the right-hand side is proportional to $\theta_{Kc}N_c^{\gamma}R_c^{1-\gamma}\theta_{Rc}$, for a given a_R. The same expression also appears in equation (23) for the AA curve. We observe that $\theta_{Kc}N_c^{\gamma}R_c^{1-\gamma}\theta_{Rc}$ is a sufficient statistic for the innovation capacity in country c; variation in this term explains cross-country variation in (autarky) long-run growth rates and wage distributions, all else the same.[9]

Consider two countries i and j that differ only in their innovation capacities, such that $\theta_{Ki}N_i^{\gamma}R_i^{1-\gamma}\theta_{Ri} > \theta_{Kj}N_j^{\gamma}R_j^{1-\gamma}\theta_{Rj}$. Under these circumstances, the AA and RR curves for country i lie above those for country j. But relative to the equilibrium cutoff point a_{Rj} in country j, the AA curve in country i passes above the RR curve in that country.[10] It follows that the equilibrium point for country i lies above and to the left of that for country j; i.e., country i devotes more resources to R&D and it grows at a faster rate in the long run.

To compare wage inequality in the two countries, we first need to compare the matching of workers with firms and research projects that takes place in each. In Figure 2, the left panel depicts matching of firms and workers in the manufacturing sector. The solid curve represents the matching function $m_j(a) \equiv m(a; a_{Rj})$ in country j. The firms with the simplest technologies, namely, those with indexes $\varphi_{\min}$, hire the least-able workers, namely, those with indexes $a_{\min}$. The firms with the most sophisticated technologies, namely, those with indexes $\varphi_{\max}$, hire the most-able workers employed in the manufacturing sector, namely, those with indexes a_{Rj}. There is positive assortative matching in the sector and thus the matching function slopes upward. Now compare the matching function for country i, represented by the broken curve. Recall that $a_{Ri} < a_{Rj}$. In this country, too, the firms with technology $\varphi_{\min}$ hire the workers with ability $a_{\min}$. And the firms with technology $\varphi_{\max}$ hire the best workers in that country's manufacturing sector, who have index, a_{Ri}. Since we show in Appendix A2.5 that a pair of solutions to (12) and (16) that apply for different boundary conditions can intersect at most once, and since the curves for the two countries intersect at their common lower boundary, they cannot intersect elsewhere. It follows that the broken curve lies everywhere above the solid curve, except at the leftmost endpoint. This implies that a worker in country i with some ability level $a < a_{Ri} < a_{Rj}$ matches with a more productive firm than does his counterpart with similar ability in country j.

The right panel of Figure 2 depicts the matching between researchers and research projects in the two countries. In both countries, the best projects, namely, those with indexes $q_{\max}$, hire the most-able researchers, namely, those with indexes $a_{\max}$. The solid curve again represents matching in country j. Here, entrepreneurs that find themselves with the least productive research projects hire the researchers with ability a_{Rj}, who are the least able among those employed in the R&D

[9] The reader may have noticed that the relative-wage function for R&D, $\lambda_R(a; a_R)$, appears under an integral in both equations, and the relative wage function for manufacturing, $\lambda(a; a_R)$, appears under an integral in (23). However, none of the four parameters under consideration affects the solution for the matching function in research or in manufacturing, given the cutoff ability a_R that appears in the boundary conditions. Given that the matching functions are not affected by these parameters except through a_R, the same is true of the relative-wage functions.

[10] An increase in $\theta_K N^{\gamma} R^{1-\gamma}\theta_R$ of some proportion shifts every point on the RR curve vertically upward by that same proportionate amount, but it shfits the AA curve up more than in proportion. Therefore, the new AA curve must pass above the new RR curve at the initial equilibrium value of a_R, and the new steady-state equilibrium must fall to the left and above point E in Figure 1.

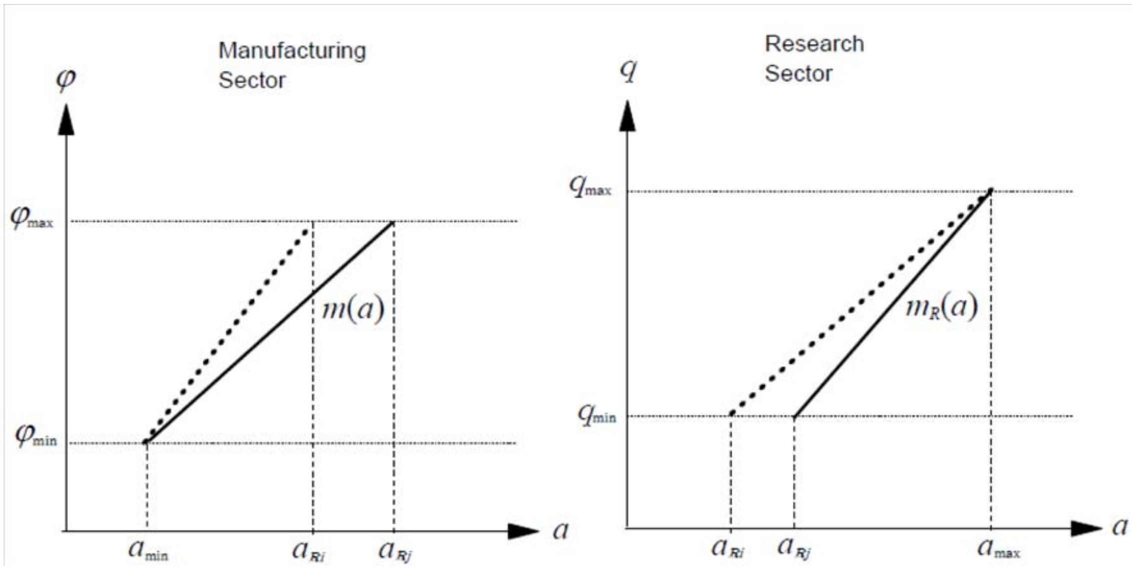

Figure 2: Matching in manufacturing and research

sector. The broken curve represents the matching in country i, where the least-able researchers have ability $a_{Ri} < a_{Rj}$. By a similar argument as before, the solid and dashed curves cannot intersect except at their common extreme point. It follows that a researcher in country i with some ability $a > a_{Rj} > a_{Ri}$ pursues a higher quality research project than his counterpart in country j with the same ability.

The different matching in the two countries translates into differences in wage inequality. Consider first inequality in the manufacturing sectors. We have seen in Figure 2 that manufacturing workers of any ability level in country i are paired with firms that have access to better technologies than the firms that hire their similarly-talented counterparts in country j. The better technology pairings boost the productivity of workers in i relative to those in j at all ability levels. But the complementarity between technology and ability implies that the productivity gain is relatively greatest for those who have more ability. This translates into a relative wage advantage for the more able of a pair of manufacturing workers in the country with the greater capacity for innovation. We have[11]

Lemma 3 *Suppose* $a_{\min} < a_{Ri} < a_{Rj} < a_{\max}$. *Then*

$$\frac{\lambda(a''; a_{Ri})}{\lambda(a''; a_{Rj})} > \frac{\lambda(a'; a_{Ri})}{\lambda(a'; a_{Rj})} \text{ for all } a'' > a' \text{ and } a', a'' \in [a_{\min}, a_{Ri}].$$

[11] Given the ability cutoff a_R and the matching function $m(\varphi; a_R)$ the wage equation for manufacturing implies

$$\ln \lambda(a; a_R) = \int_{a_{\min}}^{a} \frac{\psi_a\left[m^{-1}(v; a_R), v\right]}{\psi\left[m^{-1}(v; a_R), v\right]} dv \text{ for } a \in [a_{\min}, a_R] .$$

By Assumption 1, a deterioration in the match for the worker with ability v reduces the expression under the integral. It therefore reduces the relative wage of the worker with greater ability among any pair of workers employed in the manufacturing sector.

Now consider inequality in the research sector. Research workers also achieve better matches in country i than in country j, as illustrated in the right panel Figure 2. The relative research productivity of the more able in any pair of researchers is greater in country i than in country j, due to the complementarity between project quality and worker ability that we posited in Assumption 2. Akin to Lemma 3, we have

Lemma 4 *Suppose* $a_{\min} < a_{Ri} < a_{Rj} < a_{\max}$. *Then*

$$\frac{\lambda_R(a''; a_{Ri})}{\lambda_R(a''; a_{Rj})} > \frac{\lambda_R(a'; a_{Ri})}{\lambda_R(a'; a_{Rj})} \text{ for all } a'' > a' \text{ and } a', a'' \in [a_{Rj}, a_{\max}].$$

Finally, consider an individual who has an ability level $a'' \in [a_{Ri}, a_{Rj}]$. Such a worker sorts to the research sector in country i, but to the manufacturing sector in country j. If a'' were to work in manufacturing in country i, he would already earn a relatively higher wage in that country compared to some $a' \in [a_{\min}, a_{Ri}]$, thanks to the better technologies that all manufacturing workers access there. The fact that this individual instead chooses employment in the research sector implies that the wage offer there is even better than what he could earn in manufacturing. It follows that a'' earns relatively more compared to a' in country i than in country j. By the same token, if we compare the relative wages of $a'' \in [a_{Ri}, a_{Rj}]$ to $a''' \in [a_{Ri}, a_{\max}]$ in the two countries, a''' would earn relatively more in i than in j even if a'' were to work in the research sector in country j. The fact that this worker prefers to work in manufacturing in country j only strengthens the relative advantage for these lower-ability workers from residing in the country with the relatively smaller research sector.

Putting all the pieces together, we can compare the relative wages paid to any pair of workers of similar ability levels in the two countries. We have established

Proposition 2 *Suppose countries i and j differ only in their capacities for innovation, with* $\theta_{Ki} N_i^{\gamma} R_i^{1-\gamma} \theta_{Ri} > \theta_{Kj} N_j^{\gamma} R_j^{1-\gamma} \theta_{Rj}$. *In autarky, country i grows faster than country j in a balanced-growth equilibrium and it has greater inequality throughout its wage distribution. That is,* $g_{Mi} > g_{Mj}$, *and for any pair of workers* $a', a'' \in [a_{\min}, a_{\max}]$ *such that* $a'' > a'$,

$$\frac{w_i(a'')}{w_i(a')} > \frac{w_j(a'')}{w_j(a')},$$

where $w_c(a)$ is the equilibrium wage schedule in country c.

The proposition implies that, when countries differ only in their capacity for innovation, fast growth and wage inequality go hand in hand. A greater innovation capacity generates a relatively larger research sector and therefore a lower cutoff ability level for the marginal worker who is indifferent between employment in the two sectors. The fact that $a_{Ri} < a_{Rj}$ means that manufacturing workers access better production technologies in country i than in country j and that research workers work on better projects there. In both cases, the better matches favor the relatively more able among any pair of ability levels, due to the complementarity between ability and technology on the one hand, and between ability and project quality on the other. Finally, the fact that ability confers a comparative advantage in R&D reinforces the tendency for the more able (and better paid) workers to earn relatively higher wages in the country that conducts more research.

3.3 Support for R&D

Next we examine the role that research policy plays in shaping growth and inequality, focusing specifically on cross-country differences in R&D subsidies. We consider symmetric countries i and j that differ only in their subsidy rates, s_i and s_j. The subsidy applies to the purchase of R&D services by manufacturing firms, so that the private cost of a product design and its associated technology draw becomes $(1 - s_c)\, p_{Rc}$ in country c. The subsidy is financed by a proportional tax on wages or on research capital.

With a subsidy in place, the equation for the AA curve in Figure 1 is replaced by

$$(1 - s_c)\,(\rho + g_{Mc}) = \frac{\gamma}{\sigma - 1}\theta_K N^{\gamma} R^{1-\gamma}\Phi\,(a_{Rc})\,\frac{\int_{a_{\min}}^{a_{Rc}} \lambda\,(a; a_{Rc})\, dH\,(a)}{\lambda\,(a_{Rc}; a_{Rc})}\ .$$

Since the relationship between the resources invested in R&D and the growth rate is not affected by the subsidy, neither is the RR curve that depicts this relationship..

It follows immediately that, if $s_i > s_j$, the AA curve for country i lies above and to the left of that for country j. Not surprisingly, the subsidy draws labor into the research sector and, thereby, stimulates growth. The link to the income distribution should also be clear. With $a_{Ri} < a_{Rj}$, the technology matches are better for manufacturing workers of a given ability in country i than in country j, and the project matches are better for the researchers there as well. The larger size of the research sector in country i also contributes to its greater inequality, because ability is more amply rewarded in R&D than in manufacturing. Together, these forces generate a more unequal distribution of wages in both sectors of country i compared to country j, and in the economy as a whole.

Proposition 3 *Suppose that countries i and j differ only in their R&D subsidies and that $s_i > s_j$. Then, in autarky, country i grows faster than country j in a balanced-growth equilibrium and it has more inequality throughout its wage distribution. That is, $g_{Mi} > g_{Mj}$, and for any pair of workers $a', a'' \in [a_{\min}, a_{\max}]$ such that $a'' > a'$,*

$$\frac{w_i\,(a'')}{w_i\,(a')} > \frac{w_j\,(a'')}{w_j\,(a')}\ .$$

In Section 5.3, we will revisit the effects of R&D subsidies for an open economy and will address the spillover effects of such subsidies on growth and inequality in a country's trading partners. We will see that, with partial or complete international knowledge spillovers, R&D subsidies increase wage inequality not only in the economy that applies them, but also around the globe.

4 The Effects of Trade on Growth and Inequality

In this section, we introduce international trade among a set of countries that differ in size, in research productivity, in manufacturing technologies, and in their capacity to create and absorb international knowledge spillovers. We focus here on how international integration affects growth and income inequality in the various countries. Our baseline case, discussed in Section 4.1, allows for partial or complete international spillovers of knowledge capital between all pairs of country. In Section 4.2, we examine the long-run effects of goods trade on growth and inequality when neither knowledge capital nor financial capital flows between countries, as well as in a world without knowledge spillovers but with free mobility of financial capital.

Our trading environment has C countries indexed by $c = 1, \ldots, C$. In country c, there are N_c workers with a distribution of abilities, $H(a)$. A worker with ability a who applies a technology φ in country c can produce $\theta_{Mc}\psi(\varphi, a)$ units of any intermediate good, where $\psi(\varphi, a)$ again has the complementarity properties described by Assumption 1. We assume that manufacturing firms in all countries draw production technologies from a common distribution $G(\varphi)$.

All varieties of intermediate goods can be freely traded at zero cost.[12] Therefore, the cost of producing final goods is the same in all countries and since these goods are competitively priced, so too are the prices of these goods (irrespective of whether final goods are tradable or not). Once again, we can choose the price of a final good (anywhere) as numeraire, and then we have

$$\left\{ \sum_{c=1}^{C} \left[\int_{\omega \in \Omega_c} p_c(\omega)^{1-\sigma} d\omega \right] \right\}^{\frac{1}{1-\sigma}} = 1 \ , \tag{25}$$

where $p_c(\omega)$ is the price of variety ω of an intermediate good produced in country c and Ω_c is the set of intermediate goods produced there. We denote by X_c the output of final goods in country c and by $\bar{X} = \sum_c X_c$ the aggregate world output of final goods.

In the research sector in country c, a team of researchers of size ℓ_R and with ability a who work on a project of quality q produces $\theta_{Rc}\psi_R(q, a) K_c \ell_R^\gamma$ units of research services, where θ_{Rc} reflects the overall research productivity in country c and K_c is the national stock of knowledge capital. Assumption 2 again describes a complementarity between the researchers' abilities and quality of the project. An entrepreneur in country c must hire f units of local equipment at the rental rate r_c in order to operate a research lab. This enables her to draw a research project from the common distribution of project qualities, $G_R(q)$. Once the project quality is known, the lab hires local researchers to produce the R&D services. R&D services are not internationally tradable, so the price p_{Rc} of these services may vary across countries.

The national knowledge stock in country c reflects the country's cumulative experience in R&D,

[12] In our working paper, Grossman and Helpman (2014), we allow for both *ad valorem* tariffs and iceberg trading costs. We show that, in a world with partial or complete knowledge spillovers, the long-run effects of opening trade on a country's growth rate and wage inequality are qualitatively the same for any level of physical or policy-generated trade costs. Moreover, changes in the trade costs do not affect the long-run growth rate or relative wages in any country. We assume away all trading frictions here in order to simplify the exposition.

its ability to learn from that experience, and the extent of any knowledge spillovers from abroad. We assume that

$$K_c = \sum_{j=1}^{C} \theta_{Kjc} M_j, \tag{26}$$

where θ_{Kjc} is a parameter that measures the extent to which cumulative research experience in country j contributes to inventors' productivity in country c. Note that θ_{Kcc} captures the effectiveness with which country c converts its own research experience into usable knowledge; this parameter is the same as what we denoted by θ_K in Section 2.3 above. Any positive spillovers between country j and country c imply $\theta_{Kjc} > 0$. The special case of complete international spillovers into country c can be represented by setting $\theta_{Kjc} = \theta_{Kc}$ for all j. An absence of international knowledge spillovers corresponds to $\theta_{Kjc} = 0$ for all $j \neq c$.

Besides goods trade and any knowledge flows, international integration might enable cross-border borrowing and lending. With perfect capital mobility, interest rates are equalized worldwide, i.e., $\iota_c = \iota$ for all c. With no capital mobility, trade in goods must be balanced in each country at every moment in time. Then, R&D investment must be financed by local savings, or

$$p_{Rc}\dot{M}_c = N_c \int_{a_{\min}}^{a_{\max}} w_c\left(a; a_R\right) dH\left(a\right) + r_c Q_c + M_c \int_{\varphi_{\min}}^{\varphi_{\max}} \pi_c\left(\varphi\right) dG\left(\varphi\right) - C_c \ , \tag{27}$$

where C_c is aggregate consumption in country c.

4.1 Partial or Complete International Knowledge Spillovers

The available evidence points to the existence of significant but incomplete international R&D spillovers. Coe et al. (2009), for example, find that a country's researchers benefit differentially from domestic and foreign R&D experience and that the capacity to absorb domestic and foreign knowledge depends on a country's institutions and in particular on its regime for protection of intellectual property rights and the quality of its tertiary education.[13] To capture this reality, our baseline case posits $\theta_{Kjc} > 0$ for all j and c, so that every country reaps some spillover benefits from research that takes place anywhere in the world. Our qualitative findings do not depend on whether international knowledge spillovers are partial or complete, so we simply assume that $\theta_{Kjc} \leq \theta_{Kcc}$ for all $j \neq c$.

We begin with the case of balanced trade (no capital mobility) and describe a balance-growth path along which each country's share of the total number of intermediate goods remains strictly positive and constant; i.e., there is convergence in national rates of innovation, and therefore $g_{Mc} = g_M$ for all c.[14] The output of final goods, X, in the closed-economy expression for the profits of

[13] For a review of additional evdience, see the survey by Helpman (2004, ch.5).

[14] Such an equilibrium always exists. We show in Section 5.2 that in the equilibrium described below, the solution to the share of country c in the number of inermediates available in the world economy, $\mu_c \equiv M_c / \sum_j M_j$, satisfies

$$\zeta\mu_c = \sum_{j=1}^{C} \gamma_{jc}\mu_j \text{ for } c = 1, 2, ..., C,$$

a typical intermediate good (8) and in the labor-market clearing condition (15), is replaced in the open economy by aggregate world output, $\bar{X} = \sum_j X_j$.[15] Since this variable enters multiplicatively on the left-hand side of (15), the form of the matching function in the manufacturing sector, as described by the differential equation (16), remains the same for the open economy as for the closed economy.

We can solve for the growth rate of varieties in country c and the cutoff point for labor allocation a_{Rc} using two equations analogous to (22) and (23). In place of the former, we have

$$g_{Mc} = \kappa_c \theta_{Rc} N_c^{\gamma} R_c^{1-\gamma} \Phi\left(a_{Rc}\right) \int_{a_{Rc}}^{a_{\max}} \lambda_R\left(a; a_{Rc}\right) dH\left(a\right), \tag{28}$$

where $\kappa_c \equiv K_c/M_c$ is the ratio of the knowledge stock in country c to the country's own cumulative experience in research and

$$\Phi\left(a_{Rc}\right) \equiv \left\{ \frac{\int_{q_{\min}}^{q_{\max}} \psi_{Rc}\left[q, m_{Rc}\left(q; a_{Rc}\right)\right]^{\frac{1}{1-\gamma}} \lambda_R\left[m_{Rc}\left(q; a_{Rc}\right); a_{Rc}\right]^{-\frac{\gamma}{1-\gamma}} dG_R\left(q\right)}{\int_{a_{Rc}}^{a_{\max}} \lambda_R\left(a; a_{Rc}\right) dH\left(a\right)} \right\}^{1-\gamma},$$

as before (except that now we add a country-specific index, c). In place of the latter, we have[16]

$$\rho + g_{Mc} = \frac{\gamma}{\sigma - 1} \kappa_c \theta_{Rc} N_c^{\gamma} R_c^{1-\gamma} \Phi\left(a_{Rc}\right) \frac{\int_{a_{\min}}^{a_{Rc}} \lambda\left(a; a_{Rc}\right) dH\left(a\right)}{\lambda\left(a_{Rc}; a_{Rc}\right)}. \tag{29}$$

Notice the similarity between (28) and (29) and the equations that jointly determine the steady-state equilibrium in the closed economy; the new equations incorporate the parameter θ_{Rc} that represents Hicks-neutral differences in researcher productivity and they include κ_c in place of θ_K (or what we now denote by θ_{Kcc}). Similar arguments as before imply that the RR curve for the open economy slopes downward and the AA curve slopes upward. Using (28) and (29), we can solve for the long-run values of g_{Mc} and a_{Rc} as a function of κ_c. Then, we can use a_{Rc} and the

where $\zeta_c \equiv \kappa_c N_c^{\gamma} R_c^{1-\gamma} \theta_{Rc}$ takes a common value across all countries, i.e., $\zeta_c = \zeta$ for all c, and $\gamma_{jc} \equiv \theta_{Kjc} N_c^{\gamma} R_c^{1-\gamma} \theta_{Rc}$. It follows that ζ is a characteristic root of the matrix $\mathbf{\Gamma} = \{\gamma_{jc}\}$, with the associated characteristic vector $\boldsymbol{\mu} = \{\mu_c\}$. Moreover, by the assumption that $\theta_{Kjc} > 0$ for all j and c, all elements of $\mathbf{\Gamma}$ are strictly positive. Then the Perron-Frobenius Theorem implies that all elements of $\boldsymbol{\mu}$ are positive only if ζ is the *largest* characteristic root of $\mathbf{\Gamma}$. It follows that the balanced growth path with $\mu_c > 0$ for all c is uniquely determined by the matrix $\mathbf{\Gamma}$.

[15] We show in our working paper, Grossman and Helpman (2016) that, in the presence of trade costs, the output X is replaced instead by the *market access* $\bar{X}_c$ facing a typical producer of intermediates in country c, where

$$\bar{X}_c = \sum_j \tau_{jc}^{1-\sigma} p_{Mj}^{\sigma} X_j$$

and p_{Mj} is the price index of intermediate goods in country j. This variable, as defined by Redding and Venables (2004), scales the aggregate demand facing an intermediate good producer in country c (given its price), considering the production of final goods in each market, the cost of overcoming the trade barrier specific to the market, and the competition the firm faces from other intermediate goods sold in that market (as reflected in the price index for intermediate goods). The following arguments about the effects of trade on growth and inequality remain the same when we use $\bar{X}_c$ in place of $\bar{X}$.

[16] See Section A4.1 in the appendix for details.

differential equations for wages in each sector to solve for the distribution of relative wages in country c. Separately, we can use a set of trade balance conditions and labor-market clearing conditions to solve for the relative prices of final goods and the wage *levels* in each country.

A key observation is that $\kappa_c > \theta_{Kcc}$ for all c. That is, in an integrated world with international knowledge spillovers, researchers anywhere can draw not only on their own country's accumulated research experience when inventing new products, but also to some extent on the research experience that has accumulated outside their borders. No matter what the extent of international knowledge spillovers, so long as they are positive, a research team in any country can be more productive in the open economy than in autarky. This greater productivity translates a given labor input into greater innovation by (28) and it reduces the cost of R&D that is embedded in the zero-profit condition in (29).

Now we are ready to compare (28) and (29) to their analogs that describe the closed-economy equilibrium. Note that the bigger κ_c appears in place of the smaller θ_{Kcc} (i.e., θ_K) in each equation. Thus, the RR curve for the open economy lies proportionately above that for the closed economy, whereas the AA curve for the open economy lies more than proportionately above that for the closed economy. The two curves that determine the open-economy equilibrium in country c cross above and to the left of the intersection depicted in Figure 1. Thus, in a trade equilibrium, every country devotes more labor to research than in autarky and invents new varieties at a faster rate.

What about consumption growth and wage inequality? Concerning the former, the trade-balance condition (27) implies that consumption in country c grows in the long run at the same rate as wages do, by arguments analogous to those used in Section 2.6; aggregate investment grows at rate g_{wc} as do all components of aggregate income, so aggregate consumption must grow at this rate in order that savings match investment.[17] Using the labor-market clearing condition analogous to (15) and the convergence in innovation rates such that $g_{Mc} = g_M$ for all c, we have $\sigma g_{wc} = g_M + g_{\bar{X}}$ for all c and thus $g_{wc} = g_w$ for all c. Then, from the pricing equation analogous to (7) and the competitive pricing of final goods (25), we have $g_{wc} = g_{Cc} = g_M/(\sigma - 1)$; wages and consumption in every country grow in proportion to the aggregate rate of innovation, just as in the closed economy. The opening of trade accelerates the latter and therefore it accelerates wage and consumption growth in every country. Meanwhile, the expansion of the research sector (fall in a_{Rc}) exacerbates wage inequality, both as a reflection of the re-matching that takes place in both sectors (i.e., workers match with better firms and projects) and of the reallocation of labor to R&D, where ability is more amply rewarded. Meanwhile, the acceleration of innovation generates faster growth of wages and final output. We have established

Proposition 4 *Suppose that goods are freely tradable and each country's trade is balanced at every moment in time. Countries may differ in their research productivities, their manufacturing productivities, their capacities to generate and absorb international knowledge spillovers, and in their labor supplies. In a balanced-growth equilibrium, consumption and wages in every country grow faster*

[17] See Section A4.1 of the appendix for more details of this argument.

with trade than in autarky and every country has a more unequal wage distribution with trade than in autarky.

Now suppose that financial capital is perfectly mobile. At every point in time, capital flows from countries with (incipient) low interest rates to countries with (incipient) high interest rates, until interest rates are equalized worldwide. Note, however, that interest rates anyway are equalized along a balanced growth path in an equilibrium without capital flows; from the optimal consumption path, $\iota_c = g_{Cc} + \rho$, and we have just seen that $g_{Cc} = g_M / (\sigma - 1)$ for all c in the absence of capital mobility. It follows that the long-run rates of wage and consumption growth and the long-run wage distribution are the same in a trade equilibrium whether or not capital is internationally mobile. We summarize in

Proposition 5 *Suppose that intermediate goods are freely tradable and capital is perfectly mobile. Countries may differ in their research productivities, their manufacturing productivities, their capacities to generate and absorb international knowledge spillovers, and in their labor supplies. In a balanced-growth equilibrium, consumption and wages in every country grow faster with trade than in autarky and every country has a more unequal wage distribution with trade than in autarky.*

In our working paper, Grossman and Helpman (2016), we show that the equilibrium rate of innovation and wage inequality do not depend on the level of *ad valorem* tariffs or on the size of iceberg trading costs. Taking this argument to the limit, it follows that long-run innovation rates and measures of wage inequality would converge to the same levels in a global equilibrium with international knowledge spillovers even if intermediate and final goods were not traded at all.

4.2 No International Knowledge Spillovers

In Section 4.1, we studied global integration that combines goods trade and partial or complete international knowledge spillovers. In order to better understand the distinct role played by each of these components of globalization, we compare now the autarky equilibrium to one with free trade in goods but no knowledge spillovers. Here, each country has a national stock of knowledge that reflects only its own experience in R&D. As before, capital immobility might require balanced goods trade at every moment in time or integration may allow imbalanced trade subject to an intertemporal budget constraint. We consider each possibility in turn.

With no knowledge spillovers, the knowledge stock in country c is proportional to the country's own experience in R&D, so the relationship between the inputs into research and the production of R&D services is the same as in autarky. We can follow the same steps as in Sections 2.4 and 2.6 to derive the RR curve that relates the steady-state growth rate of intermediate inputs in country c to the ability a_{Rc} of its marginal worker in the R&D sector. This curve, represented by equation (22), is the same for each country c as in autarky.[18] Moreover, when trade must be balanced for

[18] With our new notatoin, θ_{Kcc} replaces θ_K in this equation, as well as in that for the AA curve.

each country at every moment in time, the AA curve also is the same as autarky.[19] It follows that the steady-state growth rate of intermediates g_{Mc} and the steady-state marginal worker, a_{Rc} are the same as in autarky. Goods trade in the absence of knowledge spillovers and capital flows has no affect on the long-run resource allocation and therefore no affect on any relative wages.

How does trade affect the growth of output and consumption in this case? In the appendix we show that the labor-market clearing condition in manufacturing implies

$$g_{Mc} + g_{\bar{X}} = \sigma g_{wc}. \tag{30}$$

We also show that the growth of aggregate output is a weighted average of the growth rates of wages in every country, where the weights vary with the share of country j in the total number of varieties of intermediate goods in the world economy. In the long run, the weight approaches one for the country j that has the fastest rate of innovation in the global economy and approaches zero for all others. Therefore, in the steady state,

$$g_{\bar{X}} = \frac{\tilde{g}_M}{\sigma - 1} \ ,$$

where $\tilde{g}_M \equiv \max_j\{g_{Mj}\}$. It follows from (30) that wages grow faster with trade than in autarky in every country except the one with the fastest rate of autarky growth. Moreover, the trade-balance condition (27) ensures that output and consumption grow in every country at the same rate as wages do, in the long run. Therefore, we have

Proposition 6 *Suppose that there are no international knowledge spillovers, that intermediate goods are freely tradable, and that each country's trade is balanced at every moment in time. Countries may differ in their research productivities, their manufacturing productivities, their capacities to absorb local knowledge spillovers, and in their labor supplies. In the long run, consumption, output and wages grow faster with trade than in autarky in every country except that with the fastest rate of autarky growth. The innovation rate and wage inequality are the same in every country as in autarky.*

Now, we allow for international capital mobility, while maintaining the assumption that there are no international knowledge spillovers. The resulting economy is like the one studied by Feenstra (1996) and Grossman and Helpman (1991, ch. 9.3), except that we have introduced firm and worker heterogeneity. We find effects of trade on growth that are similar to the ones he described, but with additional implications for wage inequality.

First, note that the resource constraint that determines the relationship between the marginal worker a_{Rc} in R&D and the growth rate of intermediates is the same as in autarky; the RR curve again is given by (22). As for the AA curve, we show in the appendix that the right-hand is the same as in (23), but the left-hand side is replaced by $(\tilde{g}_w - g_{wc}) + g_{Mc} + \rho$, where $\tilde{g}_w \equiv \max_j\{g_{wj}\}$. In the country with the fastest growth of wages in the trade equilibrium, the left-hand side is the

[19] See Section A4.2 of the appendix for the proof of this statement.

same as in autarky, which means that its marginal worker and its innovation rate are the same as in autarky. For all other countries, the AA curve with trade and capital mobility lies below that for autarky, which means that these countries devote less labor to R&D than in autarky ($a_{Rc} > a_{Rc}^{autarky}$) and they invent new intermediates at a slower rate ($g_{Mc} < g_{Mc}^{autarky}$). The fact that $a_{Rc} > a_{Rc}^{autarky}$ means that wage inequality narrows in each of these countries upon the opening of goods and asset trade.

Also, comparing the AA curve for the trade equilibrium and autarky, and noting that $a_{Rc} > a_{Rc}^{autarky}$, it follows that $(\tilde{g}_w - g_{wc}) + g_{Mc} > g_{Mc}^{autarky}$. But, (30) continues to describe the relationship between the growth of wages in country c, the growth of intermediates there, and the growth of aggregate output. We conclude from this that $g_{wc} > g_{wc}^{autarky}$ for all c except the fastest-growing country, which experiences the same long-run growth of wages as in autarky.[20] Inasmuch as final output grows in the long run at the same rate as wages, each of these countries also experiences faster output growth than in autarky. Finally, consumption grows at the same rate in every country by (2), since capital flows equalize interest rates. That rate is equal to the growth rate of output in the fastest growing country and thus faster than the growth of consumption in autarky in all countries besides that one. In short, the opportunity to import intermediates from countries that innovate rapidly allows all other countries to share a high rate of consumption, wage and output growth even as they devote fewer resources themselves to R&D and so realize a more equal distribution of wages. We summarize these findings in

Proposition 7 *Suppose that there are no international knowledge spillovers, that goods are freely tradable, and that interest rates are equalized worldwide by perfect capital mobility. Countries may differ in their research productivities, their manufacturing productivities, their capacities to absorb local knowledge spillovers, and in their labor supplies. In the long run, consumption, output and wages grow faster with trade than in autarky in every country except that with the fastest rate of autarky growth. The rate of innovation and wage inequality decline in every country except that with the fastest rate of autarky growth.*

We can also say how the addition of capital mobility affects long-run growth and wage inequality in an open economy with no knowledge spillovers. Starting from a steady-state equilibrium with free trade in goods but no capital mobility, the opening of global asset markets speeds the growth of consumption in all countries other than that which enjoys the fastest growth, while slowing their growth of wages and output. Meanwhile, the capital flows narrow the wage distribution in all countries besides the fastest growing one.[21]

[20] First note that aggregate output grows in the long run at the growth rate of output in the fastest growing country, which in turn is equal to the growth rate of wages in that country; i.e., $g_{\bar{X}} = \tilde{g}_w$. Therefore, we have $(\tilde{g}_{\bar{X}} - g_{wc}) + g_{Mc} > (\tilde{g}_w - g_{wc}) + g_{Mc} > g_{Mc}^{autarky}$. But (30) implies $g_{\bar{X}} = \sigma g_{wc} - g_{Mc}$, so $(\sigma - 1)\, g_{wc} > (\tilde{g}_w - g_{wc}) + g_{Mc} > g_{Mc}^{autarky} = (\sigma - 1)\, g_{wc}^{autarky}$, or $g_{wc} > g_{wc}^{autarky}$.

[21] The fact that innovation is the same as in autarky when there is no capital mobility but slower than in autarky with capital mobility implies that opening capital markets shifts labor out of R&D in all of these countries. The rematching of workers and firms generates a tighter distribution of wages. The decline in the innovation rate implies a decline in the growth of wages and output, per (30). Meanwhile, the equalization of world interest rates allows consumption in all countries to grow at the faster rate experience by the fastest-growing country.

5 Cross-Country Comparisons and Comparative Statics

In the last section, we examined how the opening of trade affects the long-run growth of wages, output and consumption and long-run wage inequality in the various countries of a multi-nation world. We studied the effects of trade on growth and inequality with and without international spillovers of knowledge and with and without international capital mobility. In this section, we explore how growth rates and wage inequality compare across countries in a trading equilibrium, as well as how parameter or policy changes in one country affect growth and inequality in its trading partners. We focus only on our baseline case, with partial or complete knowledge spillovers, because this case seems the most empirically relevant. As we noted in Section 4.1, product market integration equalizes long-run interest rates when there are knowledge spillovers, even if financial capital is completely immobile. Therefore, the same analysis applies when comparing steady states no matter whether financial claims are internationally tradable or not.

5.1 Differences in Manufacturing Productivity

Suppose now that countries differ only in their manufacturing productivities, as parameterized by θ_{Mc}. For the moment, assume they are equal in size ($N_c = N$ for all c), equal in research productivity ($\theta_{Rc} = \theta_R$ for all c), and benefit symmetrically from complete international knowledge spillovers ($\theta_{Kjc} = \theta_K$ for all j and c). In these circumstances, a balanced-growth path with $g_{Mc} = g_M$ requires $\kappa_c = \kappa$ and $a_{Rc} = a_R$ for all c, per equations (28) and (29). It follows that not only do the long-run growth rates converge internationally, but so too do the sizes and compositions of the research sectors. Then, matching between technologies and production workers in manufacturing and between research projects and researchers in R&D is the same in all countries. Consequently the structure of relative wages is the same in all countries. The differences in manufacturing productivity and import tariff rates generate cross-country heterogeneity only in wage *levels*. We summarize in

Proposition 8 *Suppose that goods are tradable and countries differ only in manufacturing productivities. Then all countries grow at the same rate in a balanced-growth equilibrium and all have the same wage inequality in the long run.*

It is also clear that, in these circumstances, the long-run value of κ is independent of any θ_{Mc}, in which case (28) and (29) imply that changes in manufacturing productivities do not affect the long-run growth rate or relative wages in any country.[22] Moreover, κ_c would be independent of θ_{Mc} (albeit not necessarily common across countries) if countries were of different sizes, had had different research productivities, or had different capacities to generate or absorb international R&D spillovers. The parameters θ_{Mc} do, of course, affect income levels and consumer welfare.[23]

[22] With $\theta_{Kjc} = \theta_K$ for all j and c, (26) yields $K_c = \theta_K \sum_{j=1}^{C} M_j$ for all c, and thus $\kappa_c = \theta_K \left(\sum_{j=1}^{C} M_j\right)/M_c$ for all c. Then (29) and the fact established above that $a_{Rc} = a_R$ for all c imply that $\kappa_c = \kappa = \theta_K C$. Clearly, κ is independent of any θ_{Mc}.

[23] In our working paper, Grossman and Helpman (2016), where we allow for trade frictions either in the form of iceberg trading costs or ad valorem tariffs, we find convergenece in growth rates in wage inequality even with

5.2 Differences in Innovation Capacity and in Ability to Create and Absorb Knowledge Spillovers

Now suppose that countries differ in size (N_c), in their research productivity (θ_{Rc}), and in their numbers of active research projects ($R_c = K_{Rc}/f$). Moreover, there may be differences in their abilities to absorb R&D spillovers from abroad and in their abilities to convert research experience (their own and foreign) into usable knowledge that facilitates subsequent innovation. Such differences are reflected in the matrix $\Theta_K = \{\theta_{Kjc}\}$ of spillover parameters that determines knowledge capital in country c, according to (26). Finally, as in Section 5.1, they may operate with different manufacturing productivities, θ_{Mc}. In all of these cases, (28) and (29) imply

$$\frac{g_{Mc}}{\rho + g_{Mc}} = \frac{\sigma - 1}{\gamma} \frac{\int_{a_{Rc}}^{a_{\max}} \frac{\lambda_R(a;a_{Rc})}{\lambda_R(a_{Rc};a_{Rc})} dH(a)}{\int_{a_{\min}}^{a_{Rc}} \frac{\lambda(a;a_{Rc})}{\lambda(a_{Rc};a_{Rc})} dH(a)} \quad \text{for all } c, \tag{31}$$

with the right-hand side a decreasing function of a_{Rc} (as we show in Appendix A5.3)..

It is clear from (31) that, since all countries converge on the same long-run growth rate of varieties, they must also have the same ability cutoff level $a_{Rc} = a_R$. Then, all share a common long-run relative wage profile. It is interesting to note that international integration generates a convergence in income inequality around the globe, whereas differences in innovation capacity give rise to different degrees of inequality in autarky.

Although relative wages are the same in all countries, wage levels are not equalized internationally. We show in Appendix A5.2, for example, that if knowledge spillovers are complete ($\theta_{Kjc} = \theta_{Kc}$ for all c), the relative wages of workers of any common ability level in countries i and j hinges on a comparison of innovation capacities per capita in these countries; i.e., on $\theta_{Ki}\theta_{Ri}(R_i/N_i)^{1-\gamma}$ versus $\theta_{Kj}\theta_{Rj}(R_j/N_j)^{1-\gamma}$. The greater is a country's ability to convert cumulative experience in R&D into usable knowledge, θ_{Ki}, or the greater is the productivity of its workers in R&D, θ_{Ri}, or the larger is its endowment of research capital relative to its labor force, R_j/N_j, the greater is its wage level.

Next observe that with $a_{Rc} = a_R$ for all c, (28) implies that $\zeta_c \equiv \kappa_c N_c^{\gamma} R_c^{1-\gamma}\theta_{Rc}$ takes a common value across all countries, i.e., $\zeta_c = \zeta$ for all c. Substituting ζ into (26), we have

$$\zeta\mu_c = \sum_{j=1}^{C} \gamma_{jc}\mu_j \ ,$$

where $\mu_c \equiv M_c/\sum_j M_j$ is the share of country c in the total number of varieties of intermediate goods in the world economy and $\gamma_{jc} \equiv \theta_{Kjc}N_c^{\gamma}R_c^{1-\gamma}\theta_{Rc}$ is a measure of innovation capacity in a setting in which knowledge spillovers are not complete . We recognize ζ as being a characteristic root of the matrix $\mathbf{\Gamma} = \{\gamma_{jc}\}$, with associated characteristic vector $\boldsymbol{\mu} = \{\mu_c\}$. Moreover, by the assumption that $\theta_{Kjc} > 0$ for all j and c, all elements of $\mathbf{\Gamma}$ are strictly positive. Then the Perron-

differences in trade frictions, and that changes in the size of any trade barrier do not affect growth or inequality in any country in the long run.

Frobenius Theorem implies that all elements of $\boldsymbol{\mu}$ can be positive (as they must be) only if ζ is the *largest* characteristic root of $\mathbf{\Gamma}$. Finally, the envelope theorem implies that ζ must be increasing in every element γ_{jc} of $\mathbf{\Gamma}$.[24]

We have thus established that an increase in any spillover parameter θ_{Kjc}, in any country size N_c, in any R&D productivity parameter θ_{Rc}, or in any country's measure of projects R_c, shifts upward the RR curve and the AA curve for every country, and the former by more (at the initial a_R) than the latter. The result is an increase in the common rate of long-run growth and an increase in wage inequality in every country.[25]

We record our findings in

Proposition 9 *Suppose that goods are freely tradable. Then all countries grow at the same rate in a balanced-growth equilibrium and all have the same wage inequality in the long run. An increase in any spillover parameter θ_{Kc}, in any country size N_c, in any R&D productivity parameter θ_{Rc}, or in any country's measure of research projects R_c leads to faster growth and greater wage inequality in every country.*

5.3 Differences in R&D Subsidies

Now we reintroduce R&D subsidies. As in Section 3.3 the subsidy applies to the purchase of R&D services by manufacturing firms, so that the private cost of a product design and its associated technology draw becomes $(1 - s_c)\, p_{Rc}$ in country c. The subsidies are financed by a proportional tax on wages or on research equipment.

Suppose that international knowledge spillovers are complete and that countries are similar in all ways except in their R&D subsidies and in the proportional wage taxes used to finance these subsidies.[26] When $N_c = N$, $R_c = R$, and $\theta_{Rc} = \theta_R$ for all c and when long-run growth rates converge to g_M, (28) and (29) imply

$$(1 - s_c)\, \frac{\rho + g_M}{g_M} = \frac{\gamma}{\sigma - 1} \frac{1}{\lambda\,(a_{Rc}; a_{Rc})} \frac{\int_{a_{\min}}^{a_{Rc}} \lambda\,(a; a_{Rc})\, dH\,(a)}{\int_{a_{Rc}}^{a_{\max}} \lambda_R\,(a; a_{Rc})\, dH\,(a)} \; .$$

We show in the appendix that the right-hand side of this equation is increasing in a_{Rc}. Therefore, if $s_i > s_j$, $a_{Ri} < a_{Rj}$; i.e., the country with the larger R&D subsidy devotes more of its labor force

[24] Multiplying the characteristic equation by μ_c and summing over all c yields

$$\zeta = \frac{\sum_{c=1}^{C} \sum_{j=1}^{C} \gamma_{jc} \mu_j \mu_c}{\sum_{c=1}^{C} (\mu_c)^2} \; .$$

The largest characteristic root is found by maximizing the right hand side with respect to $\{\mu_c\}$. By the envelope theorem, the largest ζ is an increasing function of every γ_{jc}.

[25] Again, the same results apply with (possibly heterogeneous) trade frictions of any sizes; see Grossman and Helpman (2014).

[26] It is relatively easy to verify that the implications of differences in research support would be the same as we describe here, even if we allowed for cross-country differences in innovation capacity. However, we assume that these features are common in order to simplify the exposition.

to research activities. This does not generate faster long-run growth in i than in j, but it does spell a more unequal long-run wage distribution there.

Although wage profiles do not converge in the presence of (differential) R&D subsidies, such policies do affect growth and inequality throughout the world. To examine these spillover effects of innovation policy, we treat (28) and (29) as a system of $C+1$ equations that determines the C cutoff ability levels and the common growth rate, g_M. We prove in Appendix A5.3 that an increase in an arbitrary subsidy rate s_i leads to an expansion of the research sectors in all countries. In other words, $da_{Rj}/ds_i < 0$ for all $i, j \in \{1, \ldots, C\}$. It follows that an increase in a single subsidy rate contributes not only to faster innovation throughout the world economy, but also to a spreading of the long-run wage distribution everywhere. We summarize in

Proposition 10 *Suppose that goods are tradable, that international knowledge spillovers are complete, and that countries differ only in their R&D subsidy rates. Comparing any two countries, the long-run wage distribution is more unequal in the one with the greater subsidy rate. An increase in any subsidy rate raises the common long-run growth rate and generates a spread in the distribution of wages in every country.*

The main lessons from this section are twofold. First, national conditions that create differential incentives for research versus manufacturing generate long-run differences in wage distributions, whereas conditions that affect a country's ability to contribute to or draw on the world's stock of knowledge capital lead to a convergence in wage distributions but with cross-country differences in wage levels. Second, technological conditions or government policies that cause an expansion of the research sector in one country typically have spillover effects abroad. In particular, when the incentives for R&D rise somewhere, the induced expansion in knowledge capital generates a positive growth spillover for other countries but also a tendency for wage inequality to rise worldwide.

6 Concluding Remarks

In this paper, we have focused on one mechanism that links income distribution to long-run growth. The mechanism operates via sorting and matching in the labor market. We posit that the most able individuals in any economy specialize in creating ideas and that innovation is the engine of growth. Among those that conduct research, the most able are relatively more proficient at performing the most promising research projects. Among those that use ideas rather than create them, the most able are relatively more proficient at using the most sophisticated technologies. In each case, the complementarity between worker ability and firm productivity dictates positive assortative matching. In the long run, the size of what we call the research sector determines not only the pace of innovation, but also the composition of the two sectors and the matches that take place.

Our model highlights an important mechanism in a simple economic environment. We have abstracted from diversity in manufacturing industries, from team production activities that involve multiple individuals in both research and manufacturing, from capital inputs that may be complementary to certain worker or inventor types, and from a host of market frictions that can impede job placement and financing for innovation. In this setting, faster growth often goes hand in hand with greater wage inequality. In response to events that encourage faster growth, the research sector expands by drawing the most able workers from the manufacturing sector, who then become the least able researchers. The expansion of the research sector at the extensive margin generates a re-matching between researchers and research projects that brings the relatively greatest benefit to those with greatest ability. Meanwhile, the contraction of the manufacturing sector generates re-matching between production workers and technologies that also favors relatively most those in this sector with greatest ability. The complementarity between ability and technologies implies an increase in wage inequality. This effect is strengthened by the fact that those with most ability have comparative advantage in the activity that underlies growth.

By allowing for international trade and international knowledge spillovers, we introduced links between inequality measures in different countries. We find that within-country income inequality is exacerbated by the knowledge sharing, because the knowledge spillovers make innovation more productive and so create incentives for expansion of the idea-generating portion of economies worldwide. As the research sector expands in every country so too does the relative pay for the most able individuals (who engage in innovation) as well as for the more able individuals among those that sort to each sector. The more able researchers benefit relatively more from the improved matching with research projects while the most able workers in manufacturing benefit relatively more from the improved matching with technologies.

To better understand how international integration affects growth and inequality, we also study economies without international knowledge spillovers in which R&D productivity in every country reflects only prior local experience. With trade in intermediate goods but no international borrowing or lending, the long-run allocation of resources is the same in every country as in autarky, and so too are the innovation rate and all relative wages. However, all but the country with the greatest capacity for innovation enjoy faster growth in output, income, and wages with trade than without, thanks to the productivity gains that come from importing foreign varieties. When we allow for capital mobility in a world with goods trade but no knowledge spillovers, competition with innovators in the faster-growing country displaces investment in R&D in the others. This movement of resources from R&D to manufacturing generates a more equal distribution of wages in all countries except the one with the fastest innovation rate. Meanwhile, long-run growth of consumption accelerates in all of these countries thanks to the equalization of world interest rates.

Our treatment of the open economy also allows us to study the links between conditions and policies in one country and growth and distributional outcomes in its trade partners. For example, in the presence of partial or complete international knowledge spillovers, we find that an R&D subsidy in one country accelerates growth in all countries and increases within-country income inequality throughout the globe. While previous work on endogenous growth emphasized cross-country dependence in growth rates (e.g., Grossman and Helpman 1991), our model also features cross-country dependence in wage inequality. Moreover, while long-run growth rates converge, cross-country differences in wage distributions can persist even along a balanced-growth path.

Numerous possible extensions of our model come to mind. Additional elements of interdependence would arise if production functions involved multiple factors of production (or teams of individuals) and if sectors differed in their relative factor intensities. We also suspect that investment in ideas has more dimensions of uncertainty than just the productivity of the resulting technology, and that the prospects for success in innovation and the range of reachable technologies depend on the abilities of the individuals who generate the new ideas. Imperfect information about worker characteristics and frictions in labor markets undoubtedly impede the smooth, assortative matching that features in our model. Similarly, asymmetric information about research ideas and financing constraints impede investment in innovation and bias technological outcomes. All of these extensions would be interesting.

We view our contribution in this paper not as a final word on the link between growth and inequality, but as an exploration of a core mechanism that will play a role in richer economic environments. The empirical importance of this mechanism remains to be settled, although at this stage it is not obvious how to do so in light of the limited availability of historical data and the endogeneity of the variables of interest. Yet we are convinced that a better understanding of the relationship between growth and inequality can be obtained by studying economies in which both are endogenously determined.

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Appendix

A2.5 Uniqueness and Single Crossing of the Matching Function

In Section 2.5 we stated that the solution to the pair of differential equations (12) and (16) that satisfies the boundary conditions (17) is unique, and later that the matching functions of two solutions to (12) and (16) that apply for different boundary conditions can intersect at most once. Here, we prove these statements by adapting Lemma 2 in the appendix of Grossman et al. (2015) to the present circumstances.

We begin with the latter claim. As in Grossman et al. (2017), let $[m_{\varkappa}(\varphi), w_{\varkappa}(a)]$ and $[m_{\varrho}(\varphi), w_{\varrho}(a)]$ be solutions to the differential equations (12) and (16), each for different boundary conditions,

$$m(\varphi_{\min}) = a_{z,\min} \text{ and } m(\varphi_{\max}) = a_{z,\max}\ ,\ z = \varkappa, \varrho. \tag{32}$$

Let the solutions intersect for some $\varphi = \varphi_0$ and $a = a_0$. Without loss of generality, suppose that $m'_{\varrho}(\varphi_0) > m'_{\varkappa}(\varphi_0)$. We will now show that $m_{\varrho}(\varphi) > m_{\varkappa}(\varphi)$ for all $\varphi > \varphi_0$ and $m_{\varrho}(\varphi) < m_{\varkappa}(\varphi)$ for all $\varphi < \varphi_0$ in the overlapping set of (φ, a).

To see this, suppose to the contrary there exists a $\varphi_1 > \varphi_0$ such that $m_{\varrho}(\varphi_1) \leq m_{\varkappa}(\varphi_1)$. Then differentiability of $m_z(\cdot)$, $z = \varkappa, \varrho$, implies that there exists a φ_2 with $\varphi_2 > \varphi_0$ such that $m_{\varrho}(\varphi_2) = m_{\varkappa}(\varphi_2)$, $m_{\varrho}(\varphi) > m_{\varkappa}(\varphi)$ for all $\varphi \in (\varphi_0, \varphi_2)$ and $m'_{\varrho}(\varphi_2) < m'_{\varkappa}(\varphi_2)$. This also implies that $m_{\varrho}^{-1}(a) < m_{\varkappa}^{-1}(a)$ for all $a \in (m_{\varrho}(\varphi_0), m_{\varrho}(\varphi_2))$, where $m_z^{-1}(\cdot)$ is the inverse of $m_z(\cdot)$. But then (16) implies that $w_{\varrho}[m_{\varrho}(\varphi_0)] < w_{\varkappa}[m_{\varrho}(\varphi_0)]$ and $w_{\varrho}[m_{\varrho}(\varphi_2)] > w_{\varkappa}[m_{\varrho}(\varphi_2)]$, and therefore

$$\ln w_{\varkappa}[m_{\varrho}(\varphi_2)] - \ln w_{\varkappa}[m_{\varrho}(\varphi_0)] < \ln w_{\varrho}[m_{\varrho}(\varphi_2)] - \ln w_{\varrho}[m_{\varrho}(\varphi_0)].$$

On the other hand, (12) implies that

$$\ln w_z[m_{\varrho}(\varphi_2)] - \ln w_z[m_{\varrho}(\varphi_0)] = \int_{m_{\varrho}(\varphi_0)}^{m_{\varrho}(\varphi_2)} \frac{\psi_a\left[m_z^{-1}(a), a\right]}{\psi\left[m_z^{-1}(a), a\right]} da, \quad z = \varkappa, \varrho.$$

Together with the previous inequality, this gives

$$\int_{m_{\varrho}(\varphi_0)}^{m_{\varrho}(\varphi_2)} \frac{\psi_a\left[m_{\varkappa}^{-1}(a), a\right]}{\psi\left[m_{\varkappa}^{-1}(a), a\right]} da < \int_{m_{\varrho}(\varphi_0)}^{m_{\varrho}(\varphi_2)} \frac{\psi_a\left[m_{\varrho}^{-1}(a), a\right]}{\psi\left[m_{\varrho}^{-1}(a), a\right]} da.$$

Note, however, that the strict log supermodularity of $\psi(\cdot)$ and $m_{\varrho}^{-1}(a) < m_{\varkappa}^{-1}(a)$ for all $a \in (m_{\varrho}(\varphi_0), m_{\varrho}(\varphi_2))$ imply the reverse inequality, which establishes a contradiction. It follows that $m_{\varrho}(\varphi) > m_{\varkappa}(\varphi)$ for all $\varphi > \varphi_0$. A similar argument shows that $m_{\varrho}(\varphi) < m_{\varkappa}(\varphi)$ for all $\varphi < \varphi_0$.

The fact that the matching functions for different boundary conditions can cross at most once immediately implies the uniqueness of the solution to (12) and (16) for a given set of boundary conditions, $m(\varphi_{\min}) = a_{\min}$ and $m(\varphi_{\max}) = a_R$. If there were two different solutions for these boundary conditions, the resulting matching functions would have to intersect at least twice, which is not possible.

A2.6 The RR Curve and the AA Curve

We derive now the equation for the RR curve and establish that it is downward sloping. In steady state,

$$g_M = \theta_K R \int_{q_{\min}}^{q_{\max}} \psi_R \left[q, m_R(q)\right] \ell_R \left[q, m_R(q)\right]^{\gamma} dG_R(q),$$

where $\ell_R\left[q, m_R(q)\right]$ is employment for a project of quality q. From footnote 7 we have

$$\ell_R\left[q, m_R(q)\right] = \left[\frac{\gamma p_R M \psi_R\left[q, m_R(q)\right]}{w_R(a)}\right]^{\frac{1}{1-\gamma}},$$

and therefore

$$g_M = \theta_K^{\frac{1}{1-\gamma}} \left(\gamma p_R M\right)^{\frac{\gamma}{1-\gamma}} R \int_{q_{\min}}^{q_{\max}} \psi_R\left[q, m_R(q)\right]^{\frac{1}{1-\gamma}} w_R\left[m_R(q)\right]^{-\frac{\gamma}{1-\gamma}} dG_R(q).$$

Next, substituting (18) with $\tilde{q} = q_{\min}$ into this equation yields

$$g_M = \frac{N}{\gamma p_R M} \int_{a_R}^{a_{\max}} w_R(a)\, dH(a). \tag{33}$$

This is a version of the RR curve.

From (16) and (21), we obtain:

$$\begin{aligned} p_R M &= \frac{1}{\gamma\theta_K} \left\{ \frac{N \int_{a_R}^{a_{\max}} w_R(a)\, dH(a)}{R \int_{q_{\min}}^{q_{\max}} \psi_R\left[z, m_R(z)\right]^{\frac{1}{1-\gamma}} w_R\left[m_R(z)\right]^{-\frac{\gamma}{1-\gamma}} dG_R(z)} \right\}^{1-\gamma} \\ &= \frac{w(a_R; a_R)}{\gamma\theta_K} \left\{ \frac{N \int_{a_R}^{a_{\max}} \lambda_R(a; a_R)\, dH(a)}{R \int_{q_{\min}}^{q_{\max}} \psi_R\left[q, m_R(q)\right]^{\frac{1}{1-\gamma}} \lambda_R\left[m_R(q; a_R); a_R\right]^{-\frac{\gamma}{1-\gamma}} dG_R(q)} \right\}^{1-\gamma}, \end{aligned}$$

and therefore

$$p_R M = \frac{w(a_R; a_R)}{\gamma\theta_K} \left(\frac{N}{R}\right)^{1-\gamma} \frac{1}{\Phi(a_R)}, \tag{34}$$

where

$$\Phi(a_R) \equiv \left\{ \frac{\int_{q_{\min}}^{q_{\max}} \psi_R\left[q, m_R(q; a_R)\right]^{\frac{1}{1-\gamma}} \lambda_R\left[m_R(q; a_R); a_R\right]^{-\frac{\gamma}{1-\gamma}} dG_R(q)}{\int_{a_R}^{a_{\max}} \lambda_R(a; a_R)\, dH(a)} \right\}^{1-\gamma}. \tag{35}$$

Substituting this expression into (33) yields the modified RR curve,

$$g_M = \theta_K N^{\gamma} R^{1-\gamma} \Phi(a_R) \int_{a_R}^{a_{\max}} \lambda_R(a; a_R)\, dH(a). \tag{36}$$

We now prove

Lemma 5 *The function $\Phi(a_R)$ is increasing while the product $\Phi(a_R)\int_{a_R}^{a_{\max}}\lambda_R(a;a_R)\,dH(a)$ is decreasing in a_R. Therefore the RR curve slopes downward.*

First, note that, in view of (13),

$$\log\lambda_R(a;a_R)=\int_{a_R}^{a}\frac{\psi_{Ra}\left[m_R^{-1}(z;a_R),z\right]}{\gamma\psi_R\left[m_R^{-1}(z;a_R),z\right]}dz \text{ for } a>a_R$$

and therefore

$$-\frac{\lambda_{Ra_R}(a;a_R)}{\lambda_R(a;a_R)}=\frac{\psi_{Ra}(q_{\min},a_R)}{\gamma\psi_R(q_{\min},a_R)}-\int_{a_R}^{a}\frac{\partial}{\partial a_R}\left\{\frac{\psi_{Ra}\left[m_R^{-1}(z;a_R),z\right]}{\gamma\psi_R\left[m_R^{-1}(z;a_R),z\right]}\right\}dz.$$

The derivative under the integral on the right-hand side of this equation is negative, because an increase in a_R worsens each worker's match (see Figure 2), i.e., $m_R^{-1}(z;a_R)$ is declining in a_R and $\psi_{Ra}(q,z)/\psi_R(q,z)$ is increasing in q due to Assumption 2. Together with equation (13) and Assumption 3, this implies:

$$-\frac{\lambda_{Ra_R}(a;a_R)}{\lambda_R(a;a_R)}>\frac{\psi_{Ra}(q_{\min},a_R)}{\gamma\psi_R(q_{\min},a_R)}>\frac{\psi_a(\varphi,a_R)}{\psi(\varphi,a_R)}>0 \text{ for all } \varphi \text{ and all } a>a_R.$$

From (12) we obtain:

$$\log\lambda(a;a_R)=\int_{a_{\min}}^{a}\frac{\psi_a\left[m^{-1}(z;a_R),z\right]}{\psi\left[m^{-1}(z;a_R),z\right]}dz \text{ for } a<a_R$$

and therefore

$$\frac{\lambda_a(a;a_R)}{\lambda(a;a_R)}=\frac{\psi_a\left[m^{-1}(a;a_R),a\right]}{\psi\left[m^{-1}(a;a_R),a\right]}>0 \text{ for all } a<a_R.$$

Thus, we have

Lemma 6

$$-\frac{\lambda_{Ra_R}(a;a_R)}{\lambda_R(a;a_R)}>\frac{\lambda_a(a_R;a_R)}{\lambda(a_R;a_R)}=\frac{\psi_a(\varphi_{\max},a_R)}{\psi(\varphi_{\max},a_R)} \quad \textit{for all } a>a_R.$$

Next, consider the definition of $\Phi(a_R)$ in (35); it can be expressed as

$$\begin{aligned}\log\Phi(a_R) &= (1-\gamma)\log\left\{\int_{q_{\min}}^{q_{\max}}\psi_R\left[q,m_R(q;a_R)\right]^{\frac{1}{1-\gamma}}\lambda_R\left[m_R(q;a_R);a_R\right]^{-\frac{\gamma}{1-\gamma}}dG_R(q)\right\}\\ &\quad-(1-\gamma)\log\left\{\int_{a_R}^{a_{\max}}\lambda_R(a;a_R)\,dH(a)\right\}.\end{aligned}$$

Differentiating this equation yields

$$\begin{aligned}\frac{\Phi'(a_R)}{\Phi(a_R)} &= -\gamma\int_{q_{\min}}^{q_{\max}}\omega_G(q,a_R)\frac{\lambda_{Ra_R}\left[m_R(q;a_R);a_R\right]}{\lambda_R\left[m_R(q;a_R);a_R\right]}dq-(1-\gamma)\int_{a_R}^{a_{\max}}\omega_H(a,a_R)\frac{\lambda_{Ra_R}(a;a_R)}{\lambda_R(a;a_R)}da\\ &\quad+\frac{(1-\gamma)\lambda_R(a_R;a_R)H'(a_R)}{\int_{a_R}^{a_{\max}}\lambda_R(a;a_R)\,dH(a)},\end{aligned} \tag{37}$$

where

$$\omega_G(q, a_R) = \frac{\psi_R[q, m_R(q; a_R)]^{\frac{1}{1-\gamma}} \lambda_R[m_R(q; a_R); a_R]^{-\frac{\gamma}{1-\gamma}} G'_R(q)}{\int_{q_{\min}}^{q_{\max}} \psi_R[q, m_R(q; a_R)]^{\frac{1}{1-\gamma}} \lambda_R[m_R(q; a_R); a_R]^{-\frac{\gamma}{1-\gamma}} dG_R(q)}$$

and

$$\omega_H(a, a_R) = \frac{\lambda_R(a; a_R) H'(a)}{\int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)}$$

are weights that satisfy

$$\int_{q_{\min}}^{q_{\max}} \omega_G(q, a_R) dq = \int_{a_R}^{a_{\max}} \omega_H(a, a_R) da = 1.$$

Lemma 6 implies

$$\begin{aligned} -\frac{\lambda_{Ra_R}[m_R(q; a_R); a_R]}{\lambda_R[m_R(q; a_R); a_R]} &> \frac{\lambda_a(a_R; a_R)}{\lambda(a_R; a_R)} \quad \text{for all } q, \\ -\frac{\lambda_{Ra_R}(a; a_R)}{\lambda_R(a; a_R)} &> \frac{\lambda_a(a_R; a_R)}{\lambda(a_R; a_R)} \quad \text{for all } a > a_R, \end{aligned}$$

and since the last term in (37) is positive, we have

Lemma 7

$$\frac{\Phi'(a_R)}{\Phi(a_R)} > \frac{\lambda_a(a_R; a_R)}{\lambda(a_R; a_R)} > 0,$$

The lemma establishes that $\Phi(a_R)$ is an increasing function.

Although, as shown above, $\Phi(a_R)$ is an increasing function and $\int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)$ is a decreasing function of a_R, we now show that their product is decreasing in a_R, and therefore the RR curve slopes downward.[27] To see this, first note that $da = m'_R(q) dq$ together with (19) implies, via a change of variables, that

$$\int_{q_{\min}}^{q_{\max}} \omega_G(q, a_R) \frac{\lambda_{Ra_R}[m_R(q; a_R); a_R]}{\lambda_R[m_R(q; a_R); a_R]} dq = \int_{a_R}^{a_{\max}} \omega_H(a, a_R) \frac{\lambda_{Ra_R}(a; a_R)}{\lambda_R(a; a_R)} da,$$

and therefore (37) can be rewritten as:

$$\frac{\Phi'(a_R)}{\Phi(a_R)} = -\int_{a_R}^{a_{\max}} \omega_H(a, a_R) \frac{\lambda_{Ra_R}(a; a_R)}{\lambda_R(a; a_R)} da + \frac{(1-\gamma)\lambda_R(a_R; a_R) H'(a_R)}{\int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)}.$$

Using this expression we obtain:

$$\frac{d}{da_R} \log\left[\Phi(a_R) \int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)\right] = -\frac{\gamma \lambda_R(a_R; a_R) H'(a_R)}{\int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)} < 0.$$

That is, we have

Lemma 8 $\Phi(a_R) \int_{a_R}^{a_{\max}} \lambda_R(a; a_R) dH(a)$ *is declining in* a_R*.*

[27] We are indebted to Elisa Rubbo for this argument.

Next, we derive the equation for the AA curve and establish that the curve is upward sloping. Equations (8) and (11) yield

$$p_R = \sigma^{-\sigma} (\sigma - 1)^{(\sigma-1)} X \frac{\int_{\varphi_{\min}}^{\varphi_{\max}} \left\{ \frac{w_M[m(\varphi)]}{\psi[\varphi, m(\varphi)]} \right\}^{1-\sigma} dG(\varphi)}{\rho + g_M}. \tag{38}$$

while (15) yields

$$MX \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} \int_{\varphi_{\min}}^{\varphi_{\max}} \left\{ \frac{w_M [m(\varphi)]}{\psi[\varphi, m(\varphi)]} \right\}^{1-\sigma} dG(\varphi) = N \int_{a_{\min}}^{a_R} w_M(a) \, dH(a) \ . \tag{39}$$

Therefore:

$$\rho + g_M = \frac{1}{\sigma - 1} \frac{N}{p_R M} \int_{a_{\min}}^{a_R} w_M(a) \, dH(a)$$

This is a version of the AA curve. Using (21) and (34), this can be expressed as:

$$\rho + g_M = \frac{\gamma}{\sigma - 1} \theta_K N^{\gamma} R^{1-\gamma} \Phi(a_R) \int_{a_{\min}}^{a_R} \frac{\lambda(a; a_R)}{\lambda(a_R; a_R)} dH(a),$$

which is the AA curve in the text (see (23)). We now show that $\Phi(a_R) \int_{a_{\min}}^{b} \frac{\lambda(a;b)}{\lambda(a_R;b)} dH(a)$ is an increasing function of both a_R and b, for $b \to a_R$, and therefore the AA curve slopes upwards.

From (12) we obtain

$$\log \left[\frac{\lambda(a;b)}{\lambda(a_R;b)} \right] = - \int_{a}^{a_R} \frac{\psi_a \left[m^{-1}(z;b), z \right]}{\psi \left[m^{-1}(z;b), z \right]} dz \ \text{ for } a < a_R.$$

Due to Assumption 2 the right-hand side of this equation is rising in b, because an increase in b reduces the quality of matches for manufacturing workers (see Figure 2), i.e., $m^{-1}(z;b)$ is declining in b. Therefore $\Phi(a_R) \int_{a_{\min}}^{b} \frac{\lambda(a;b)}{\lambda(a_R;b)} dH(a)$ is rising in b. In addition, Lemma 7 implies that $\Phi(a_R) \int_{a_{\min}}^{b} \frac{\lambda(a;b)}{\lambda(a_R;b)} dH(a)$ is rising in a_R for $b \to a_R$, which establishes that the AA curve slopes upward.

A4.1 International Knowledge Spillovers

In this section, we derive equilibrium relationships that hold in open economies with international knowledge spillovers. From equation (25), which equates the minimum unit cost of production of final output in country c to $p_X = 1$, we obtain:

$$x_c(\omega) = p_c(\omega)^{-\sigma} \bar{X}, \tag{40}$$

where $\bar{X} = \sum_j X_j$. The profit maximizing price of a firm with productivity φ in country c is therefore

$$p_c(\varphi) = \left(\frac{\sigma}{\sigma - 1} \right) \frac{w_c[m_c(\varphi)]}{\theta_{Mc} \psi[\varphi, m_c(\varphi)]}. \tag{41}$$

This price generates an operating profit of

$$\pi_c(\varphi) = \sigma^{-\sigma}(\sigma-1)^{(\sigma-1)}\bar{X}\left\{\frac{w_c[m_c(\varphi)]}{\theta_{Mc}\psi[\varphi, m_c(\varphi)]}\right\}^{1-\sigma}, \tag{42}$$

where $m_c(\varphi)$ is the matching function in manufacturing in country c.

Following the analysis of the innovation sector, with international knowledge spillovers the profit function of a project of quality q in country c is (where K_c replaces $\theta_K M$ in (9)):

$$\pi_{Rc}(q) = (1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}\left\{p_{Rc}K_c\theta_{Rc}\psi_R[q, m_{Rc}(q)]\,w_c[m_{Rc}(q)]^{-\gamma}\right\}^{\frac{1}{1-\gamma}}.$$

Free-entry by entrepreneurs implies

$$r_c f = (1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}(p_{Rc}K_c\theta_{Rc})^{\frac{1}{1-\gamma}}\int_{q_{\min}}^{q_{\max}}\psi_R[q, m_{Rc}(q)]^{\frac{1}{1-\gamma}}w_c[m_{Rc}(q)]^{-\frac{\gamma}{1-\gamma}}dG_R(q), \tag{43}$$

which determines r_c.

A manufacturing firm pays $p_{Rc,t}$ at time t to purchase the R&D services needed to introduce a variety of the intermediate good at time t. If it draws a manufacturing technology φ, it will earn a stream of profits $\pi_{c\tau}(\varphi)$ for all $\tau \geq t$. We have derived the expression for operating profits and recorded it in (42). Assuming free entry of manufacturing firms, this profit function implies:

$$p_{Rc,t} = \int_t^{\infty} e^{-\int_t^{\tau}\iota_{c,b}db}\int_{\varphi_{\min}}^{\varphi_{\max}}\pi_{c,\tau}(\varphi)\,dG(\varphi)\,d\tau,$$

which yields the familiar no arbitrage condition:

$$\frac{\dot{p}_{Rc,t}}{p_{Rc,t}} = -\frac{\int_{\varphi_{\min}}^{\varphi_{\max}}\pi_{c,t}(\varphi)\,dG(\varphi)\,d\tau}{p_{Rc,t}} + \iota_{c,t}.$$

Using the profit function (42) and the consumption growth equation (2), and dropping the time index t, we obtain

$$\frac{\dot{p}_{Rc}}{p_{Rc}} = -\frac{\sigma^{-\sigma}(\sigma-1)^{(\sigma-1)}\bar{X}}{p_{Rc}}\int_{\varphi_{\min}}^{\varphi_{\max}}\left\{\frac{w_c[m_c(\varphi)]}{\theta_{Mc}\psi[\varphi, m_c(\varphi)]}\right\}^{1-\sigma}dG(\varphi)\,d\tau + \frac{\dot{C}_c}{C_c} + \rho, \tag{44}$$

where $C_j = N_j c_j$ is aggregate consumption in country j.

The differential equations for wages in manufacturing and R&D, (12) and (13), do not change. The labor market clearing condition (15) implies

$$M_c\bar{X}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\int_{\varphi_{\min}}^{\varphi}\left\{\frac{w_c[m_c(\phi)]}{\theta_c\psi[\phi, m_c(\phi)]}\right\}^{1-\sigma}dG(\phi) = N_c\int_{a_{\min}}^{m(\varphi)}w_{Mc}(a)\,dH(a)\;, \tag{45}$$

which, when differentiated, yields

$$m_c'(\varphi) = \frac{M_c \bar{X}}{N_c} \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \frac{w_{Mc}\left[m_c(\varphi)\right]^{-\sigma}}{\left\{\theta_{Mc}\psi\left[\varphi, m_c(\varphi)\right]\right\}^{1-\sigma}} \frac{G'(\varphi)}{H'\left[m_c(\varphi)\right]} \quad \text{for all } \varphi \in \left[\varphi_{\min}, \varphi_{\max}\right].$$

This differential equation together with the differential equation for manufacturing wages and the boundary conditions uniquely determine the matching function in manufacturing, which depends on a_{Rc} but not on $M_c\bar{X}/N_c$ nor θ_{Mc}. In other words, $m_c(\varphi) = m(\varphi; a_{Rc})$. Similarly, (18) becomes

$$R_c \int_q^{q_{\max}} w_{Rc}\left[m_{Rc}(z)\right] \left\{\frac{\gamma p_{Rc} K_c \theta_{Rc} \psi_R\left[z, m_{Rc}(z)\right]}{w_{Rc}\left[m_{Rc}(z)\right]}\right\}^{\frac{1}{1-\gamma}} dG_R(z) = N_c \int_{m_{Rc}(q)}^{a_{\max}} w_{Rc}(a)\, dH(a) \quad , \tag{46}$$

and differentiating yields

$$m_{Rc}'(q) = \frac{R_c}{N_c} \left\{\frac{\gamma p_{Rc} K_c \theta_{Rc} \psi_R\left[q, m_{Rc}(q)\right]}{w_{Rc}\left[m_{Rc}(q)\right]}\right\}^{\frac{1}{1-\gamma}} \frac{G_R'(q)}{H'\left[m_{Rc}(q)\right]} \text{ for all } q \in \left[q_{\min}, q_{\max}\right].$$

This differential equation together with the differential equation of wages in the innovation sector and the boundary conditions uniquely determine the matching function in the innovation sector, which depends on a_{Rc} but not on R_c/N_c nor $p_{Rc}K_c\theta_{Rc}$. That is, $m_{Rc}(q) = m_R(q; a_{Rc})$.

Dynamics

The growth in varieties reflects the aggregate output of the research sector and therefore

$$\begin{aligned} \dot{M}_c &= K_c R_c \theta_{Rc} \int_{q_{\min}}^{q_{\max}} \psi_R\left[q, m_R(q; a_{Rc})\right] \ell_R\left[q, m_R(q; a_{Rc})\right]^{\gamma} dG_R(q) \\ &= \kappa_c M_c R_c \theta_{Rc} \int_{q_{\min}}^{q_{\max}} \psi_R\left[q, m_R(q; a_{Rc})\right] \ell_R\left[q, m_R(q; a_{Rc})\right]^{\gamma} dG_R(q), \end{aligned}$$

where $\kappa_c = K_c/M_c$. However,

$$\ell_R\left[q, m_R(q; a_{Rc})\right] = \left[\frac{\gamma p_{Rc} K_c \theta_{Rc} \psi_R\left[q, m_R(q; a_{Rc})\right]}{w_{Rc}(a; a_{Rc})}\right]^{\frac{1}{1-\gamma}},$$

where $w_{Rc}(a; a_{Rc})$ is the wage function in the innovation sector in country c, and therefore

$$g_{Mc} = \kappa_c (\theta_{Rc})^{\frac{1}{1-\gamma}} (\gamma p_{Rc} K_c)^{\frac{\gamma}{1-\gamma}} R_c \int_{q_{\min}}^{q_{\max}} \psi_R\left[q, m_R(q; a_{Rc})\right]^{\frac{1}{1-\gamma}} w_{Rc}\left[m_R(q; a_{Rc}); a_{Rc}\right]^{-\frac{\gamma}{1-\gamma}} dG_R(q).$$

Next, substituting (46) with $q = q_{\min}$ into this equation yields

$$g_{Mc} = \frac{\kappa_c N_c}{\gamma p_{Rc} K_c} \int_{a_{Rc}}^{a_{\max}} w_{Rc}(a; a_{Rc})\, dH(a). \tag{47}$$

From (46) and (21), we obtain

$$
\begin{aligned}
p_{Rc}K_c &= \frac{1}{\gamma\theta_{Rc}}\left\{\frac{N_c\int_{a_{Rc}}^{a_{\max}} w_{Rc}(a)\,dH(a)}{R_c\int_{q_{\min}}^{q_{\max}}\psi_R\left[q,m_R(q;a_{Rc})\right]^{\frac{1}{1-\gamma}} w_{Rc}\left[m_R(q;a_{Rc});a_{Rc}\right]^{-\frac{\gamma}{1-\gamma}}dG_R(q)}\right\}^{1-\gamma}\\
&= \frac{w_c(a_{Rc};a_{Rc})}{\gamma\theta_{Rc}}\left\{\frac{N_c\int_{a_{Rc}}^{a_{\max}}\lambda_R(a;a_{Rc})\,dH(a)}{R_c\int_{q_{\min}}^{q_{\max}}\psi_R\left[q,m_R(q;a_{Rc})\right]^{\frac{1}{1-\gamma}}\lambda_R\left[m_R(q;a_{Rc});a_{Rc}\right]^{-\frac{\gamma}{1-\gamma}}dG_R(q)}\right\}^{1-\gamma},
\end{aligned}
$$

and therefore

$$
p_{Rc}K_c = \frac{w_c(a_{Rc};a_{Rc})}{\gamma\theta_{Rc}}\left(\frac{N_c}{R_c}\right)^{1-\gamma}\frac{1}{\Phi(a_{Rc,t})}, \tag{48}
$$

where

$$
\Phi(a_{Rc}) \equiv \left\{\frac{\int_{q_{\min}}^{q_{\max}}\psi_R\left[q,m_R(q;a_{Rc})\right]^{\frac{1}{1-\gamma}}\lambda_R\left[m_R(q;a_{Rc});a_{Rc}\right]^{-\frac{\gamma}{1-\gamma}}dG_R(q)}{\int_{a_{Rc}}^{a_{\max}}\lambda_R(a;a_{Rc})\,dH(a)}\right\}^{1-\gamma}.
$$

Note that the function $\Phi(a_{Rc})$ is common to all countries and that it is the same function we defined for the closed economy. Now, however, it must be evaluated at the cutoff a_{Rc} in country c, which trades with the other countries. Substituting this expression into (47) yields

$$
g_{Mc} = \kappa_c\theta_{Rc}N_c^{\gamma}R_c^{1-\gamma}\Phi(a_{Rc})\int_{a_{Rc}}^{a_{\max}}\lambda_R(a;a_{Rc})\,dH(a). \tag{49}
$$

Equation (28), which must hold at every moment in time, traces out a temporary trade-off between a_{Rc} and g_{Mc}. While $\Phi(a_{Rc})$ is an increasing function, the right-hand side of (49) is decreasing in a_{Rc} (see Section A2.6 above). It follows that a_{Rc} uniquely determines g_{Mc} through (49), and since K_c is a state variable, it also uniquely determines $p_{Rc}/w_c(a_{Rc};a_{Rc})$ given K_c via (48).

The labor market clearing condition in manufacturing (45) implies

$$
\begin{aligned}
\frac{M_c\bar{X}}{w_{Mc}(a_{\min};a_{Rc})^{\sigma}\theta_{Mc}^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\int_{\varphi_{\min}}^{\varphi_{\max}}\left\{\frac{\lambda\left[m(\varphi;a_{Rc});a_{Rc}\right]}{\psi\left[\varphi,m(\varphi;a_{Rc})\right]}\right\}^{1-\sigma}dG(\varphi)&\\
= N_c\int_{a_{\min}}^{a_{Rc}}\lambda(a;a_{Rc})\,dH(a),& \qquad (50)
\end{aligned}
$$

and the no-arbitrage condition (44) can be expressed as

$$
\frac{\dot{p}_{Rc}}{p_{Rc}} = -\frac{\sigma^{-\sigma}(\sigma-1)^{(\sigma-1)}w_{Mc}(a_{\min};a_{Rc})^{1-\sigma}\bar{X}\int_{\varphi_{\min}}^{\varphi_{\max}}\left\{\frac{\lambda[m(\varphi;a_{Rc});a_{Rc}]}{\psi[\varphi,m(\varphi;a_{Rc})]}\right\}^{1-\sigma}dG(\varphi)}{p_{Rc}\theta_{Mc}^{1-\sigma}} + \frac{\dot{C}_c}{C_c} + \rho. \tag{51}
$$

Substituting (48) and (50) into (51) then yields

$$\frac{\dot{C}_c}{C_c} - \frac{\dot{p}_{Rc}}{p_{Rc}} + \rho = \frac{\gamma \kappa_c \theta_{Rc} N_c^{\gamma} R_c^{1-\gamma} \Phi\left(a_{Rc}\right) \int_{a_{\min}}^{a_{Rc}} \lambda\left(a; a_{Rc}\right) dH\left(a\right)}{\left(\sigma - 1\right) \lambda_R\left(a_{Rc}; a_{Rc}\right)}. \tag{52}$$

This equation must be satisfied at every moment in time.

The equilibrium conditions for open economies that we have derived so far must be satisfied independently of whether capital flows internationally or not. We next examine a world economy with no international capital flows.

No International Capital Flows

In the absence of international capital flows there is trade balance at every moment in time, which means that (27) is satisfied at each point in time. On a balanced growth path the rate of growth of M_c is the same in every country and equal to g_M. Therefore K_c also grows at the rate g_M and κ_c is constant. From (48), $p_{Rc}K_c$ grows at the same rate as wages, and therefore $p_{Rc}\dot{M}_c$ also grows at the rate g_{wc}. In other words, on a balanced growth path the left-hand side of (27) grows at the rate g_{wc} and

$$\frac{\dot{p}_{Rc}}{p_{Rc}} = g_{wc} - g_M \quad \text{for all } c. \tag{53}$$

The first term on the right-hand side of (27) also grows at the rate g_{wc} and so does the second term. The latter follows from (43) and the fact that $p_{Rc}K_c$ grows at the rate g_{wc}. Using the profit equation (42), the rate of growth of the third terms is $g_M + g_{\bar{X}} - (\sigma - 1)\, g_{wc}$. However, the labor market clearing condition (**??**) implies that $g_M + g_{\bar{X}} = \sigma g_{wc}$. Therefore, the growth rate of the third term also is g_{wc}. Evidently, balanced trade implies that the rate of growth of consumption equals the rate of growth of wages

$$\frac{\dot{C}_c}{C_c} = g_{wc} \quad \text{for all } c.$$

Substituting this result together with (53) into (52) then yields

$$\rho + g_{Mc} = \frac{\gamma \kappa_c \theta_{Rc}}{\sigma - 1} N_c^{\gamma} R_c^{1-\gamma} \Phi\left(a_{Rc}\right) \frac{\int_{a_{\min}}^{a_{Rc}} \lambda\left(a; a_{Rc}\right) dH\left(a\right)}{\lambda\left(a_{Rc}; a_{Rc}\right)}, \tag{54}$$

which is the open economy AA curve.

A4.2 No International Knowledge Spillovers

The analysis in Section A4.1 up to and including equation (52) does not change when knowledge stocks are fully national, except that in this case $K_c = \theta_{Kc} M_c$ and therefore $\kappa_c = \theta_{Kc}$, where $\theta_{Kc} = \theta_{Kcc}$ is the efficiency with which country c converts its own R&D experience into usable knowledge. Under these circumstances the RR curve, described by equation (49), becomes:

$$g_{Mc} = \theta_{Kc} \theta_{Rc} N_c^{\gamma} R_c^{1-\gamma} \Phi\left(a_{Rc}\right) \int_{a_{Rc}}^{a_{\max}} \lambda_R\left(a; a_{Rc}\right) dH\left(a\right), \tag{55}$$

which is the same as in autarky. The no-arbitrage condition (52) now becomes

$$\frac{\dot{C}_c}{C_c} - \frac{\dot{p}_{Rc}}{p_{Rc}} + \rho = \frac{\gamma \theta_{Kc} \theta_{Rc} N_c^{\gamma} R_c^{1-\gamma} \Phi\left(a_{Rc}\right) \int_{a_{\min}}^{a_{Rc}} \lambda\left(a; a_{Rc}\right) dH\left(a\right)}{\left(\sigma - 1\right) \lambda_R\left(a_{Rc}; a_{Rc}\right)}. \tag{56}$$

No International Capital Flows

As in Section A4.1, trade must be balanced (27) in the absence of international capital flows. Following a similar analysis as there, (27) implies that

$$\frac{\dot{p}_{Rc}}{p_{Rc}} = g_{wc} - g_{Mc} \quad \text{for all } c$$

and

$$\frac{\dot{C}_c}{C_c} = g_{wc} \quad \text{for all } c.$$

Note that, in this case, the innovation rates on a balanced growth path, g_{Mc}, are not necessarily the same in every country. Substituting these equations into (56) yields the AA curve,

$$g_{Mc} + \rho = \frac{\gamma \theta_{Kc} \theta_{Rc} N_c^{\gamma} R_c^{1-\gamma} \Phi\left(a_{Rc}\right) \int_{a_{\min}}^{a_{Rc}} \lambda\left(a; a_{Rc}\right) dH\left(a\right)}{\left(\sigma - 1\right) \lambda_R\left(a_{Rc}; a_{Rc}\right)}, \tag{57}$$

which is the same as in autarky.

Free International Capital Flows

With international borrowing and lending, trade need not be balanced at every moment in time. But capital mobility equalizes the interest rate worldwide; i.e., $\iota_c = \iota$ for all c. This implies, via (2), that consumption grows at the same rate in every country, i.e., $g_{Cc} = g_C$. Moreover, (48) implies

$$\frac{\dot{p}_{Rc}}{p_{Rc}} = g_{wc} - g_{Mc}$$

on a balanced growth path. Substituting these results into (56) yields

$$g_C - g_{wc} + g_{Mc} + \rho = \frac{\gamma \theta_{Kc} \theta_{Rc}}{\sigma - 1} N_c^{\gamma} R_c^{1-\gamma} \Phi\left(a_{Rc}\right) \frac{\int_{a_{\min}}^{a_{Rc}} \lambda\left(a; a_{Rc}\right) dH\left(a\right)}{\lambda\left(a_{Rc}; a_{Rc}\right)}. \tag{58}$$

Next note from (25) and (41) that

$$p_X^{1-\sigma} = 1 = \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} \sum_j M_j \left[w_{Mj}\left(a_{\min}; a_{Rj}\right) \left(\theta_{Mj}\right)^{-1}\right]^{1-\sigma} \int_{\varphi_{\min}}^{\varphi_{\max}} \left\{\frac{\lambda\left[m\left(\varphi; a_{Rc}\right); a_{Rc}\right]}{\psi\left[\varphi, m\left(\varphi; a_{Rc}\right)\right]}\right\}^{1-\sigma} dG\left(\varphi\right). \tag{59}$$

Substituting (**??**) into (59) yields

$$\bar{X} = \frac{\sigma}{\sigma - 1} \sum_j N_j w_{Mj} \left(a_{\min}; a_{Rj}\right) \int_{a_{\min}}^{a_{Rj}} \lambda \left(a; a_{Rj}\right) dH\left(a\right). \tag{60}$$

Since, in equilibrium, $\bar{X} = \sum_j X_j = \sum_j C_j$ and every C_j grows at the same rate g_C, it follows that $\bar{X}$ grows at the rate g_C. Therefore, (60) implies that g_C is a weighted average of the g_{wj}'s,

$$g_C = g_{\bar{X}} = \bar{g}_w \equiv \sum_j \omega_j g_{wj}, \tag{61}$$

where $\omega_c = N_c w_{Mc} \left(a_{\min}; a_{Rc}\right) \int_{a_{\min}}^{a_{Rc}} \lambda \left(a; a_{Rc}\right) dH\left(a\right) / \sum_j N_j w_{Mj} \left(a_{\min}; a_{Rj}\right) \int_{a_{\min}}^{a_{Rj}} \lambda \left(a; a_{Rj}\right) dH\left(a\right)$. Substituting this result into (58) then yields

$$\left(\bar{g}_w - g_{wc}\right) + g_{Mc} + \rho = \frac{\gamma \theta_{Kc} \theta_{Rc}}{\sigma - 1} N_c^{\gamma} R_c^{1-\gamma} \Phi\left(a_{Rc}\right) \frac{\int_{a_{\min}}^{a_{Rc}} \lambda\left(a; a_{Rc}\right) dH\left(a\right)}{\lambda\left(a_{Rc}; a_{Rc}\right)}. \tag{62}$$

In addition, the labor market clearing condition (**??**) implies

$$g_{Mc} + g_{\bar{X}} = \sigma g_{wc}, \tag{63}$$

which is equation (30) in the main text. It follows from this equation that the growth rate of wages is higher in countries with faster rates of innovation.

If there were no differences in the growth rates of wages, then (62) would be the same as the AA curve in the closed economy, in which case the growth rate g_{Mc} would be the same as in autarky in every country. Inasmuch as g_{wc} (and therefore g_{Mc}) does vary across countries, (61) implies

$$g_C = g_{\bar{X}} = \bar{g}_w = \max_c g_{wc} \equiv \tilde{g}_w, \tag{64}$$

because the weight ω_j converges to one for the fastest growing country. In this case, (62) has $g_{Mc}+\rho$ on the left-hand side for for the country with the fastest rate of innovation, so this country retains its autarky rate of innovation. For every other country, we have $\left(\tilde{g}_w - g_{wc}\right) + g_{Mc} + \rho > g_{Mc} + \rho$, implying that its AA curve is lower than in autarky and therefore these countries have slower innovation rates g_{Mc} than in autarky and higher cutoff ability levels, a_{Rc}. Moreover, $g_{Mi} > g_{Mj}$ if and only if $g_{wi} > g_{wj}$.

It follows that in country c with $\theta_{Kc}\theta_{Rc}N_c^{\gamma}R_c^{1-\gamma} < \max_j \left\{\theta_{Kj}\theta_{Rj}N_j^{\gamma}R_j^{1-\gamma}\right\}$ the innovation rate is slower and wage inequality is less than in autarky. In other words, the country with the highest innovation capacity has the fastest innovation rate and the fastest growth rate of wages in the long run, equal to its autarky rates of innovation and wage growth. In all other countries the rate of innovation is slower than in autarky. Nevertheless, in these countries, income and consumption grow faster than in autarky. Consumption grows faster, because it grows at the rate of consumption growth in the country with the fastest growth of wages.

Note that $a_{Rc}^{trade} > a_{Rc}^{autarky}$ for any c that is not the fastest growing country. Therefore, using $g_C = \bar{g}_w$ from (64), (62) implies

$$\left(g_C^{trade} - g_{wc}^{trade}\right) + g_{Mc}^{trade} > g_{Mc}^{autarky},$$

or, using (63),

$$(\sigma - 1)\, g_{wc}^{trade} > g_{Mc}^{autarky}.$$

However, $g_{Mc}^{autarky} = (\sigma - 1)\, g_{wc}^{autarky}$ and therefore $g_{wc}^{trade} > g_{wc}^{autarky}$. In other words, despite the slower rate of innovation in these countries their wages and income grow faster than in autarky. Also note that we have described a case in which the opening of trade *reduces wage inequality* while accelerating the growth of income and consumption.

A5.2 Cross-Country Wage Levels with Differences in Innovation Capacity

Here we consider the cross-country differences in wage levels that result from asymmetries in innovation capacity. We assume equal R&D subsidy rates and complete international knowledge spillovers; i.e., $s_j = s$ and $\theta_{Kjc} = \theta_{Kc}$ for all j. Note that this allows for international differences in capacities to convert knowledge capital into new varieties, as captured by θ_{Kc}. We also allow for differences in country size, N_c, in active research projects R_c (which is proportional to the country's research capital) and for differences in research productivity, θ_{Rc}.

We have seen in Section 5.2 that, under these circumstances, the cutoff ability levels a_{Rc} are the same in all countries, and therefore so are relative wages of workers with different ability levels. We represent the wage schedule in country c by $w_c(a) = \omega_c w(a)$ and refer to ω_c as the wage level in country c. Moreover, (28) implies that, in this case, $\kappa_c \theta_{Rc} N_c^{\gamma} R_c^{1-\gamma} = \zeta$ for all countries and therefore $\zeta M_c = \theta_{Rc} N_c^{\gamma} R_c^{1-\gamma} \theta_{Kc} \sum_j M_j$. It follows that

$$\frac{M_i}{M_j} = \frac{\theta_{Ri} N_i^{\gamma} R_i^{1-\gamma} \theta_{Ki}}{\theta_{Rj} N_j^{\gamma} R_j^{1-\gamma} \theta_{Kj}}.$$

Using this result together with (39), which holds in every open economy with X replaced by $\bar{X}$, we have

$$\left(\frac{\omega_i}{\omega_j}\right)^{\sigma} = \frac{M_i/N_i}{M_j/N_j} = \frac{\left(\theta_{Ri} N_i^{\gamma} R_i^{1-\gamma} \theta_{Ki}\right)/N_i}{\left(\theta_{Rj} N_j^{\gamma} R_j^{1-\gamma} \theta_{Kj}\right)/N_j}.$$

It follows that wages are higher in country i than country j if and only if $\left(\theta_{Ri} N_i^{\gamma} R_i^{1-\gamma} \theta_{Ki}\right)/N_i > \left(\theta_{Rj} N_j^{\gamma} R_j^{1-\gamma} \theta_{Kj}\right)/N_j$, i.e., if and only if country i has a higher innovation capacity per capita.

A5.3 Spillover Effects of National R&D Subsidies

In this section, we examine the effects of changing an R&D subsidy in one country on growth and inequality in that country and in all trading partners. We suppose that international knowledge spillovers are complete and that countries are similar in all ways except in their R&D subsidies and in the proportional wage taxes used to finance these subsidies. That is, we assume $\theta_{Kcj} = \theta_K$ and $\theta_{Rc} = \theta_R$ for all c and j, and $N_c = N$ and $R_c = R$ for all c. These assumptions focus attention on variations in R&D subsidies.

The equations for the RR and AA curves, (28) and (29), can be expressed in this case as

$$g_M = \kappa_c \theta_R N^\gamma R^{1-\gamma} \Phi\left(a_{Rc}\right) \int_{a_{Rc}}^{a_{\max}} \lambda_R\left(a; a_{Rc}\right) dH\left(a\right), \tag{65}$$

$$\left(1 - s_c\right)\left(\rho + g_M\right) = \frac{\gamma}{\sigma - 1} \kappa_c \theta_R N^\gamma R^{1-\gamma} \Phi\left(a_{Rc}\right) \frac{\int_{a_{\min}}^{a_{Rc}} \lambda\left(a; a_{Rc}\right) dH\left(a\right)}{\lambda\left(a_{Rc}; a_{Rc}\right)}, \tag{66}$$

where g_M is the same in all countries in the steady state. Dividing (66) by (65) yields

$$\left(1 - s_c\right) \frac{\rho + g_M}{g_M} \Omega\left(a_{Rc}\right) = \Lambda\left(a_{Rc}\right), \tag{67}$$

where

$$\Omega\left(a_{Rc}\right) \equiv \Phi\left(a_{Rc}\right) \int_{a_{Rc}}^{a_{\max}} \lambda_R\left(a; a_{Rc}\right) dH\left(a\right)$$

is a decreasing function, as shown above (recall that RR slopes downward), and

$$\Lambda\left(a_{Rc}\right) \equiv \frac{\gamma}{\sigma - 1} \frac{\int_{a_{\min}}^{a_{Rc}} \lambda\left(a; a_{Rc}\right) dH\left(a\right)}{\lambda\left(a_{Rc}; a_{Rc}\right)} \Phi\left(a_{Rc}\right)$$

is an increasing function, as shown above (recall that AA slopes upwards). It follows from this equation that countries with higher R&D subsidies have lower cutoffs a_{Rc} and employ more workers in R&D. Moreover, multiplying (65) by M_c, recalling that $\kappa_c = \theta_K \left(\sum_{j=1}^{C} M_j\right)/M_c$, and summing up, we obtain

$$g_M = \theta_K \theta_R N^\gamma R^{1-\gamma} \sum_{j=1}^{C} \Omega\left(a_{Rj}\right).$$

Substituting this equation into (67) then yields

$$\left(1 - s_c\right) \frac{\rho + \theta_K \theta_R N^\gamma R^{1-\gamma} \sum_{j=1}^{C} \Omega\left(a_{Rj}\right)}{\theta_K \theta_R N^\gamma R^{1-\gamma} \sum_{j=1}^{C} \Omega\left(a_{Rj}\right)} \Omega\left(a_{Rc}\right) = \Lambda\left(a_{Rc}\right) \ . \tag{68}$$

There are C equations like (68), one for each country, and together they allow us to solve the ability cutoffs, a_{Rc}.

Now, proportionately differentiate this system of equations and write the (matrix) equation for the proportional changes as

$$\mathbf{A}_s \mathbf{a}_s = \mathbf{b}_s,$$

where

$$\mathbf{a}_s = \begin{pmatrix} \hat{a}_{R1} \\ \hat{a}_{R2} \\ \cdot \\ \cdot \\ \cdot \\ \hat{a}_{RC} \end{pmatrix}, \quad \mathbf{b}_s = \begin{pmatrix} \widehat{(1-s_1)} \\ \widehat{(1-s_2)} \\ \cdot \\ \cdot \\ \cdot \\ \widehat{(1-s_C)} \end{pmatrix},$$

and a "hat" over a variable represents a proportional rate of change; i.e., $\hat{a}_{Rc} = da_{Rc}/a_{Rc}$ and $\widehat{(1-s_c)} = d\,(1-s_c)\,/\,(1-s_c)$.

We note that the matrix $\mathbf{A}_s$ has positive diagonal elements and negative off-diagonal elements. In particular, in row j, the diagonal element is $\varepsilon_{\Lambda j} + \left(1-\eta_j\right)\varepsilon_{\Omega j}$, where $\varepsilon_{\Lambda j} > 0$ is the elasticity of $\Lambda\left(\cdot\right)$ evaluated at a_{Rj}, $\varepsilon_{\Omega j} > 0$ is minus the elasticity of $\Omega\left(\cdot\right)$ evaluated at a_{Rj}, and

$$\eta_j = \left[\frac{\rho}{\rho + \theta_K \theta_R N^{\gamma} R^{1-\gamma} \sum_{i=1}^{C} \Omega\left(a_{Ri}\right)}\right] \frac{\Omega\left(a_{Rj}\right)}{\sum_{i=1}^{C} \Omega\left(a_{Ri}\right)} < 1.$$

For $j \neq c$, the off-diagonal element in column j is $-\eta_j \varepsilon_{\Omega j} < 0$.

Inasmuch as $\mathbf{A}_s$ has only negative off-diagonal elements, we recognize that it is a Z-matrix. Moreover, there exists a diagonal matrix $\mathbf{D}_s$ such that $\mathbf{A}_s\mathbf{D}_s$ is diagonally dominant in its rows. To see this, consider the diagonal matrix $\mathbf{D}_s$ that has a diagonal entry in row j given by $1/\varepsilon_{\Omega j}$. Then the diagonal element in row c and column c of $\mathbf{A}_s\mathbf{D}_s$ is given by $\varepsilon_{\Lambda c}/\varepsilon_{\Omega c} + (1-\eta_c)$ and the off-diagonal element in row c and column j is given by $-\eta_j$. Summing the entries in any row c gives $\varepsilon_{\Lambda c}/\varepsilon_{\Omega c} + 1 - \sum_{j=1}^{C} \eta_j > 0$, where the inequality follows from the fact that $\sum_{j=1}^{C} \eta_j < 1$.

Having established that $\mathbf{A}_s$ is a Z-matrix and there exists a diagonal matrix $\mathbf{D}_s$ such that $\mathbf{A}_s\mathbf{D}_s$ is diagonally dominant in its rows, it follows that $\mathbf{A}_s$ is an M-matrix (see Johnson, 1982). Then its inverse, $\mathbf{A}_s^{-1}$, has only positive elements. We conclude that an increase in any subsidy rate (i.e., a reduction in any $1-s_c$) reduces every cutoff point a_{Rj}, $j = 1, \ldots, C$. Since more individuals are hired as researchers in every country, every country grows faster and experiences greater income inequality as a consequence of an increase in any subsidy rate.