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Exporting Female Labor Content or Substituting it

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Abstract

An expansion or contraction in a sector intensively using female labor must affect female labor force participation. We suggest that, whenever trade and international specialization expand sectors prone to employing females, female labor force participation actually drops, and vice versa. In general, when sectors prone to employing females expand, sectors tending towards male employment must contract. This contraction, in turn, induces male workers to migrate to the expanding sectors, which, in our specification, drives female workers out of formal employment. In this sense, a country that is exporting female labor content is, in fact, substituting male labor for female labor. Finally, we show that our mechanism also applies in a case of technological change that is biased towards female labor.

Keywords: Trade, Female labor force participation, Fertility, Technological Change.

JEL Classifications: F10, F16, J13, J16.

1 Introduction

Trade volume has increased secularly during the last century. From 1870 to 1998 growth in world trade has quadrupled growth in world income (Maddison (2001)). Another significant feature of the twentieth century was the increase in female labor force participation. The participation of married women in the U.S. labor market has been increasing from around 2% in 1880 to over 70% in 2000 (Fernández (2007)).

The focus of this study is to understand the channels through which two major economic factors, international trade and female labor force participation, are linked. Specifically, our main concern is to show how differences in capital labor ratios across economies, via international specialization, affect the trade-off in household decisions between fertility and female labor force participation and how these decisions, in turn, feed back on growth rates of per household capital stocks.

Our theory relies on three different assumptions that are consistent with empirical regularities of the labor market. First, male labor and female labor are imperfect substitutes.[1] Second, while male workers have relative advantages in brawn intensive tasks, female workers have relative advantages in brain intensive tasks. To incorporate both elements in our model, we assume that females and males have equal quantities of brains, but males have more brawn. As a direct consequence, males' wages are higher than females' wages as long as brawn is a valued input.[2] Indeed, Figure I shows that the wage ratio between female workers and male workers in the U.S. is less than one during the period $1800-1990$. According to our third assumption, child-rearing requires time that cannot be spent working.[3] Thus, the opportunity cost of raising children is proportional to mar-

[1] Acemoglu, Autor and Lyle (2004) have utilized the large positive shock to demand for female labor induced by World War II to understand the effect of an increase in female labor supply on females' and males' wages. They find that a 10% increase in female labor input decreases females' wages by about $7\%-8\%$, but reduces males' wages by only $3\%-5\%$. This suggests that the elasticity of substitution between female and male labor ranges between 2.5 and 3.5.

[2] O'Neill (2003) shows that there is still a 10% differential in female and male wages in the U.S. in 2000, that is still unexplained by gender differences in schooling, actual experience and job characteristics. For empirical evidence and theoretical explanations for the gender wage gap see Altonji and Blank (1999) and Goldin (1990) among others.

[3] Goldin (1995) provides evidence that shows that few women in the 1940's and 1950's birth cohorts were able to combine childbearing with strong labor-force attachment. Angrist and Evans (1998) and Bailey (2006) find a negative causal effect running from fertility to female labor force participation.

ket wage and, given that males and females are equally productive in raising children, mainly women with the lower market wage typically raise children. To formalize these assumptions, we adopt the model of Galor and Weil (1996) and generalize it to a trade setting.

Based on these intrinsic differences in labor endowments between the sexes, we distinguish between brain intensive sectors, hereafter "females' relative advantage sector" (FRAS) and brawn intensive sectors, hereafter "males' relative advantage sector" (MRAS). Within this framework, we address how female labor force participation is affected by an expansion or a contraction in a sector that intensively uses female labor. As a result of international trade, some economies specialize in FRAS, which expands on the expense of MRAS, while other economies experience the opposite pattern. Interestingly, our theory suggests that expanding FRAS hinders female labor force participation, while expanding MRAS generates the mirror image. The driving force of this seemingly paradoxical result is that men have higher wages and, therefore, are always formally employed. Thus, when an economy specializes on the FRAS, the MRAS contracts and male workers migrate to this first sector, driving female workers out of formal employment. Conversely, under specialization on the MRAS, male workers withdraw from the FRAS, which opens job opportunities for women and fosters female labor force participation.

Our mechanism also applies in the case of technological progress, which is biased towards female labor. In particular, technological progress biased towards FRAS increases the wages in this sector. This increase in wages attracts male workers who migrate from the MRAS, an effect that can be strong enough to drive female workers out of formal employment. In this way, technological progress biased towards female labor might curb female labor force participation.[4]

To describe the dynamics of the model, we first look at the closed economy. We follow Galor and Weil (1996) by assuming that physical capital complements brains more than brawn. Consequently, as economies accumulate physical capital, the rewards to brains increase relative to brawn and the gender wage gap declines, thus inducing higher partic-

[4]For technological progress at home and its impact on fertility see Greenwood and Seshadri (2005).

ipation of females in the labor market, a feature that is consistent with evidence [Figure I.]. Goldin (1990) writes:

> The labor market's rewards for strength, which made up a large fraction of earnings in the nineteenth century, ought to be minimized by the adoption of machinery, and its rewards for brain power ought to be increased. (*p.* 59)

Turning to the two-country model, dynamics are affected by three basic elements from trade and demographic theory. First, in a Heckscher-Ohlin framework, the relative endowments of production factors, physical capital, and labor, determine specialization patterns. Second, specialization patterns affect the gender wage gap. Third, the gender wage gap affects household choice of female labor force participation and fertility. These choices, in turn, impact household savings and population growth, which, finally, determine the per-household capital stock for the subsequent generation. Adding the complementarity assumption between physical capital and female labor described above, it is the capital abundant economy which specializes in the FRAS and *vice versa*.

Thus, our model suggests that international trade fosters female labor force participation and decreases fertility in the capital scarce economy; two effects that enhance growth in per-household capital. The impact of trade on the capital abundant economy, however, is ambiguous. While international trade hinders female labor force participation and increases fertility, these adverse effects on per-household capital accumulation may or may not be dominated by the positive gains from trade. In either case, our model suggests that trade cannot accelerate capital accumulation in the rich country by more than it accelerates it in the poor country and, thus, our theory predicts convergence of per-household capital stocks.

The model connects to various strands in the literature. The vast work connecting international trade and labor markets typically analyzes the impact of trade on unemployment and labor reallocation (e.g., Davis (1998), Wacziarg and Wallack (2004) and Helpman and Itskhoki (2007)). Related articles reveal labor market friction as a determinant of comparative advantage and international trade (Saint-Paul (1997), Cunat and Melitz (2007)).

Other scholars investigate whether to include labor market standards in trade agreements (Brown (2001), Brown, Deardorff and Stern (1998) and Bagwell and Staiger (2001)). The link between trade, the gender wage gap and female labor force participation, however, is understudied. A noteworthy exception is Becker (1971) who argues that trade increases competition among firms and, thus, reduces costly discrimination and closes the gender wage gap. Tests of this hypothesis have generally produced mixed support (see Black and Brainerd (2004), Artecona and Cunningham (2002), Hazarika and Otero (2004), Berik, van der Meulen and Zveglich (2004) for some of the scarce empirical investigations). Our mechanism, in contrast, operates through the differential demand for gender labor across sectors and international specialization under perfectly competitive goods and factor markets.

The reduction in the gender wage gap and the increase in women's labor force participation has been the subject of much debate. Welch (2000) and Black and Spitz-Oener (2007) focus on the role of primary attributes. While Welch (2000) attributes the reduction in the gender wage gap to the expansion in the value of brains relative to brawn, Black and Spitz-Oener (2007) addresses the importance of the relative increases in non-routine analytic tasks and non-routine interactive tasks, which are associated with higher skill levels.[5] Our paper is close to this literature by taking primary attributes as the source of the gender wage gap.

The link between women's relative wages and fertility is relatively established.[6] In our framework, the pure effect of an increase in household income, holding the price of children constant, is to raise the demand for children. If all child-rearing is done by females, an increase in females' wages raises both household income and the price of children, and thus have offsetting income and substitution effects on the demand for children.[7] In our

[5]See also Mulligan and Rubinstein (2005) who attribute the reduction in the gender wage gap to a positive selectivity bias and Fernández (2007) who addresses the role of culture and learning. For gender wage gap in the U.S., see Goldin (1990) and for the evolution of female labor force participation, see Goldin (2006).

[6]The analysis of fertility in the context of relative wages dates back to Becker (1960), Mincer (1963), Becker (1985) and Becker (1991).

[7]Pencavel (1986) finds a positive association between fathers' labor supply and the number of children. This is consistent with our framework assuming that fathers' wage has a purely income effect on the number of children.

model, if both males' and females' wages proportionately increase, then the substitution effect driven by the increase in the cost of raising children negates the income effect and leaves fertility unchanged. In such a framework, closing the genders wage gap causes fertility to decline.[8]

There is little research on the links and interaction between demography and international trade. Using the Heckscher-Ohlin model, Findlay (1995) shows that developing countries see a decline in the incentive to invest in education, so that, in the long run, the accumulation of human capital is negatively affected. The developed economies, on the other hand, start with a higher skill level and therefore tend to specialize in high-skilled production. This expands the demand for high-skilled labor and therefore provides an incentive to further accumulate human capital.[9] In a Ricardian model, Galor and Mountford (2008) endogenizes educational choice and fertility choice, arguing that the gains from trade are channeled towards population growth in non-industrial countries while in industrial countries they are directed towards investment in education and growth in output per-capita.[10] Our theory predicts the opposite effect: trade reduces fertility in developing countries and enhances capital accumulation and growth of income per-capita, simultaneously highlighting its impact on female labor force participation.

The rest of the paper is organized as follows. Section 2 formalizes our argument and Section 3 presents some concluding remarks.

[8] For other contributions regarding fertility choice, see Razin and Ben-Zion (1975) and Eckstein, Stern and Kenneth (1988). For a comprehensive discussion on the demographic transition see Galor (2005).

[9] In an econometric analysis of data on approximately 90 countries during 1960-90, Wood and Ridao-Cano (1999) finds that greater openness tends to cause divergence of secondary and tertiary enrollment rates between more-educated and less-educated countries, and also between land-scarce and land-abundant countries.

[10] Their theory suggests that international trade enhanced the specialization of industrial economies in the production of skill intensive goods. The increase in demand for skilled labor induced an investment in the quality of the population, expediting demographic transition, stimulating technological progress and further enhancing the comparative advantage of these industrial economies in the production of skill intensive goods. Thus, the pattern of trade enhances the initial pattern of comparative advantages and disadvantages.

2 The Model

In our modeling strategy we follow Galor and Weil (1996) by adopting a standard OLG model with endogenous choice of fertility.

The economy is populated by a mass of L_t households, each containing one adult man (a husband) and one adult woman (a wife). Individuals live for three periods: childhood, adulthood and old age. In childhood, each individual consumes a fixed quantity of time from her parents. In adulthood, individuals raise children, supply labor to the market, and save their wages. In old age, individuals do not work but consume their savings. The capital stock in each period is equal to the aggregate savings of the previous period.

A key assumption is that men and women differ in their labor endowments. While men and women have equal endowments of mental labor units, men have more physical labor units than women. These differences translate into a gender wage gap, which, in turn, governs the trade-off between female labor force participation and fertility.

2.1 Production

2.1.1 Technologies

Two intermediate goods, X_1 and X_2 are assembled into a final good Y by the CES-technology:

$$Y_t = \left(\theta X_{1,t}^{\rho} + (1-\theta) X_{2,t}^{\rho}\right)^{1/\rho} \qquad \rho, \theta \in (0,1). \tag{1}$$

Intermediate goods are produced using three factors: capital K, physical labor L^p, and mental labor L^m. We want to reflect the fact that sectors vary in their factor intensity, in particular, in their intensity of mental and physical labor. This, in turn, generates differences in demand for male and female labor across sectors. Thus, we impose the following structure on production of intermediate goods[11]

$$
\begin{aligned}
X_1 &= aK_t^{\alpha}(L_t^m)^{1-\alpha} + bL_{1,t}^p \\
X_2 &= bL_{2,t}^p.
\end{aligned} \tag{2}
$$

Here, the variables $L_{i,t}^p$ stand for the physical labor employed in sector i at time t, while L_t^m is the amount of mental labor in the first sector at time t.

2.1.2 Labor Supply

Men and women are equally efficient in raising children. On the labor market, however, each woman supplies one unit of mental labor L^m while men supply one unit of mental labor L^m plus one unit of physical labor L^p. Thus, as long as physical labor has a positive price, men receive a higher wage than women and therefore the opportunity cost of raising children is higher for a man than for a woman. Consequently, men only raise children when women are doing so full-time.

Finally, we assume that male workers cannot divide mental and physical labor and must allocate both units to one sector. This means, in particular, that men employed in the X_2-sector waste their mental endowment.

2.2 Preferences

Households of period t derive utility from the number of their children n_t and their old-age consumption c_{t+1} of a final good Y according to[12]

$$
u_t = \gamma \ln(n_t) + (1-\gamma)\ln(c_{t+1}). \tag{3}
$$

It is assumed that parents' time is the only input required to raise children and thus the opportunity cost of raising children is proportional to the market wage. Let w_t^F and w_t^M

[11] As shown in an earlier version of this paper, assuming that physical capital is a production factor of X_2 does not change the spirit of our results.

[12] Note that since the basic unit is a household which consists a husband and wife, n_t is in fact the number of pairs of children that a couple has.

be the hourly wage of female and male workers, respectively. Normalizing the hours per period to unity, the full income of a household is $w_t^M + w_t^F$, which is spent on consumption and raising children. Further, let z be the fraction of the time endowment of one parent that must be spent to raise one child. If the wife spends time raising children, then the marginal cost of a child is zw_t^F. If the husband spends time raising children, then the marginal cost of a child is zw_t^M. The household's budget constraint is therefore

$$\begin{aligned} w_t^F zn_t + s_t \leq w_t^M + w_t^F \qquad & if \qquad zn_t \leq 1 \\ w_t^F + w_t^M(zn_t - 1) + s_t \leq w_t^M + w_t^F \qquad & if \qquad zn_t \geq 1 \end{aligned} \tag{4}$$

where s_t is the household's savings. In the second period, the household consumes their savings

$$c_{t+1} = s_t(1 + r_{t+1}) \tag{5}$$

where r_{t+1} is the net real interest rate on savings.

2.3 Optimality

It will prove useful to conduct the analysis in terms of per-household variables. We therefore define:

$$k_t = K_t/L_t \qquad\qquad m_t = L_t^m/L_t \qquad\qquad l_{i,t} = L_{i,t}^p/L_t$$

as capital, productive mental labor and sectorial physical labor *per-household*, respectively. Finally, we define

$$\kappa_t = k_t/m_t \tag{6}$$

as the ratio of capital to mental labor employed in the first sector. This ratio will play a central role in the following analysis.

2.3.1 Firms

Profit maximization of decentralized intermediate goods firms implies, by (2), that relative prices are:

$$\frac{p_{2,t}}{p_{1,t}} = \frac{1-\theta}{\theta}\left(\frac{X_1}{X_2}\right)^{1-\rho} = \frac{1-\theta}{\theta}\left(\frac{a\kappa_t^{\alpha}m_t + bl_{1,t}}{bl_{2,t}}\right)^{1-\rho}, \tag{7}$$

where we write $p_{i,t}$ as X_i's price in period t. Given $p_{i,t}$, cost minimizing final good producers leads us to the usual final good price index P_t, which we normalize to one

$$P_t = \left(\left(\frac{\theta}{p_{1,t}^{\rho}}\right)^{1/(1-\rho)} + \left(\frac{1-\theta}{p_{2,t}^{\rho}}\right)^{1/(1-\rho)}\right)^{-(1-\rho)/\rho} = 1. \tag{8}$$

From equation (2) the return to capital in the first sector is

$$r_t = p_{1,t}\alpha a\kappa_t^{\alpha-1} \tag{9}$$

Wages are derived from (2) and reflect the marginal productivity of labor. For males we have

$$w_t^M = p_{1,t}b[(1-\alpha)a/b\kappa_t^{\alpha}+1] \qquad if \qquad L_{1,t}^p > 0 \tag{10}$$

$$w_t^M = p_{2,t}b \qquad if \qquad L_{2,t}^p > 0, \tag{11}$$

which reflects mental and physical labor productivity in the first sector, and only physical labor productivity in the second sector. Similarly, female wage is

$$w_t^F = p_{1,t}(1-\alpha)a\kappa_t^{\alpha} \qquad if \qquad zn_t < 1, \tag{12}$$

which reflects mental labor productivity in the first sector.

2.3.2 Households

The household maximizing problem yields

$$zn_t = \begin{cases} \gamma(1 + w_t^M/w_t^F) & if \quad \gamma(1 + w_t^M/w_t^F) \leq 1 \\ 2\gamma & if \quad 2\gamma \geq 1 \\ 1 & otherwise. \end{cases} \tag{13}$$

Equation (13) implies that in the case in which $\gamma \geq 1/2$ women raise children full time regardless of their wages. We rule out this scenario by imposing $\gamma < 1/2$. Under this restriction, women raise children full-time only under very high gender wage gaps. But as the gender gap decreases women join the labor force and fertility decreases. In the limit when w_t^F approaches w_t^M, women spend a fraction 2γ of their time raising children. Finally, under $\gamma < 1/2$ the budget constraint (4) collapses to

$$s_t = (1 - zn_t)w_t^F + w_t^M \tag{14}$$

and (13) becomes

$$zn_t = \min\left\{\gamma\left(1 + w_t^M/w_t^F\right), 1\right\}. \tag{15}$$

2.4 Closed Economy

2.4.1 Static Equilibrium

The equilibrium of the integrated economy will be determined by looking at two regimes separately. The first is a regime in which women do not participate in the formal labor market, and the second is a regime in which women participate. To simplify the analysis, we assume that the second sector is too small to accommodate all male labor in equilibrium. Specifically, we assume[13]

$$2-\alpha \geq 1/\theta \tag{16}$$

to be satisfied throughout the following analysis. Under this assumption, $L_{1,t}^{p}>0$ holds and the ratio of male to female wage can be computed by the marginal productivities in the first sector

$$\frac{w^M}{w^F}=1+\frac{b}{(1-\alpha)a\kappa_t^{\alpha}}. \tag{17}$$

This ratio determines female labor force participation $1-zn_t$ through (15)

$$zn_t=\min\left\{\gamma\left(2+\frac{b}{(1-\alpha)a\kappa_t^{\alpha}}\right),1\right\}. \tag{18}$$

To determine equilibrium κ_t, combine male wages (10) and (11), prices (7), and the resource constraint for male labor $1=l_{1,t}+l_{2,t}$ to get

$$(1-\alpha)\frac{a}{b}\kappa_t^{\alpha}+1=\frac{1-\theta}{\theta}\left(\frac{\frac{a}{b}\kappa_t^{\alpha}m_t+l_{1,t}}{1-l_{1,t}}\right)^{1-\rho}. \tag{19}$$

Further note that

$$l_{1,t}=m_t-(1-zn_t) \tag{20}$$

so that equation (19) becomes

$$(1-\alpha)\frac{a}{b}\kappa_t^{\alpha}+1=\frac{1-\theta}{\theta}\left(\frac{\frac{a}{b}\kappa_t^{\alpha}m_t+m_t-(1-zn_t)}{1-m_t+(1-zn_t)}\right)^{1-\rho}. \tag{21}$$

Equations (6), (18), and (21) determine m_t and zn_t and thus the equilibrium. There are two qualitatively different types of equilibria to distinguish.

The First Regime $zn_t=1$. In the case in which $zn_t=1$, equation (21) can be written

[13] A sufficient condition for $l_{i,t}>0$ is that the relative price (7) falls short of the ratio of marginal rates of transformation at $l_{1,t}=0$ and $zn_t=0$ i.e. $(1-\alpha)\kappa_t^{\alpha}a/b+1>(1-\theta)/\theta\,(\kappa_t^{\alpha}a/b)^{1-\rho}$. If $\kappa_t^{\alpha}a/b\geq 1$ then this sufficient condition is implied by $(1-\alpha)\geq(1-\theta)/\theta$, or (16). If $\kappa_t^{\alpha}a/b<1$ instead, the sufficient condition is implied by $1>(1-\theta)/\theta$ and hence, again, by (16).

in terms of κ_t as: (substitute $m_t = k_t/\kappa_t$)

$$(1-\alpha)\frac{a}{b}\kappa_t^{\alpha} + 1 = \frac{1-\theta}{\theta}\left(\frac{\frac{a}{b}\frac{k_t}{\kappa_t^{1-\alpha}} + \frac{k_t}{\kappa_t}}{1-\frac{k_t}{\kappa_t}}\right)^{1-\rho}. \tag{22}$$

The Second Regime $zn_t < 1$. In case in which $zn_t < 1$ we use $m_t = k_t/\kappa_t$ and zn_t from (18) to write (21) as

$$(1-\alpha)\frac{a}{b}\kappa_t^{\alpha} + 1 = \frac{1-\theta}{\theta}\left(\frac{\frac{a}{b}\frac{k_t}{\kappa_t^{1-\alpha}} + \frac{k_t}{\kappa_t} - 1 + \gamma\left(2 + \frac{b}{a}\frac{\kappa_t^{-\alpha}}{1-\alpha}\right)}{1-\frac{k_t}{\kappa_t} + 1 - \gamma\left(2 + \frac{b}{a}\frac{\kappa_t^{-\alpha}}{1-\alpha}\right)}\right)^{1-\rho}. \tag{23}$$

Equations (22) and (23) determine the equilibrium κ_t in the first and second regime, respectively. Notice that expressions on the left of both equations are increasing in κ_t, while both terms on the right are decreasing in κ_t. This implies that κ_t is unique in both regimes. Moreover, the expressions on the right of (22) and (23) are increasing in k_t and we can write $\kappa_t(k_t)$ as an increasing function.

This means that, quite intuitively, a capital-rich economy has a higher capital-mental labor share than a capital scarce economy. When going back to equation (18), this observation shows also that the higher the capital stock k_t of an economy, the lower fertility zn_t is. As $\kappa_t(k_t)|_{k_t=0} = 0$, (18) further implies that there is a $k_o > 0$ so that the economy is in the first regime when its capital stock falls short of k_o, while the economy is in the second regime if not. By combining condition $\gamma\left(2 + b/\left[(1-\alpha)a\kappa_o^{\alpha}\right]\right) = 1$ with equation (22) and $\kappa_o = k_o/m_o$, this threshold can be shown to be

$$k_o = \theta\,(1-\gamma)\left(1 - 2\gamma + \gamma\frac{1-\alpha\theta}{1-\alpha}\right)^{-1}\left[\frac{(1-\alpha)(1-2\gamma)}{\gamma}\frac{a}{b}\right]^{-1/\alpha}. \tag{24}$$

At capital stocks below the threshold k_o all women raise children full-time. When capital is gradually accumulated and this threshold is passed, women integrate into the labor market and, as the variable κ_t keeps increasing, the gender wage gap closes and female labor force rises. At the same time, and as a mirror image, fertility declines.

These observations regarding the impact of the capital stock on fertility and on female labor force participation brings us to the dynamics of the model.

2.4.2 Dynamics

The dynamics of the model are governed by two endogenous variables: savings s_t and fertility n_t. With the notation in per-household terms, the ratio of saving and fertility gives the next period's capital stock, i.e. $k_{t+1} = s_t/n_t$. Combining the budget constraint (14) and fertility (15) and distinguishing the two regimes, we can write

$$k_{t+1} = \frac{s_t}{n_t} = \begin{cases} zw_t^M & if \quad k_t < k_o \\ z\frac{1-\gamma}{\gamma}w_t^F & if \quad k_t \geq k_o. \end{cases} \tag{25}$$

Equations (10) and (11) give the price ratio

$$\frac{p_{2,t}}{p_{1,t}} = (1-\alpha)\frac{a}{b}\kappa_t^{\alpha} + 1 \tag{26}$$

which, combined with the normalization (8), renders the price of the first intermediate good

$$p_{1,t} = \left(\theta^{1/(1-\rho)} + (1-\theta)^{1/(1-\rho)}\left(\frac{1}{(1-\alpha)\frac{a}{b}\kappa_t^{\alpha}+1}\right)^{\rho/(1-\rho)}\right)^{(1-\rho)/\rho}.$$

With (10), (12) and (25) we thus have

$$k_{t+1} = \begin{cases} zb\left(\theta^{\frac{1}{1-\rho}}\left((1-\alpha)\frac{a}{b}\kappa_t^{\alpha}+1\right)^{\frac{\rho}{1-\rho}} + (1-\theta)^{\frac{1}{1-\rho}}\right)^{\frac{1-\rho}{\rho}} & if \quad k_t < k_o \\ zb\frac{1-\gamma}{\gamma}\left(\theta^{\frac{1}{1-\rho}}\left((1-\alpha)\frac{a}{b}\kappa_t^{\alpha}\right)^{\frac{\rho}{1-\rho}} + (1-\theta)^{\frac{1}{1-\rho}}\left(\frac{(1-\alpha)\frac{a}{b}\kappa_t^{\alpha}}{(1-\alpha)\frac{a}{b}\kappa_t^{\alpha}+1}\right)^{\frac{\rho}{1-\rho}}\right)^{\frac{1-\rho}{\rho}} & if \quad k_t \geq k_o. \end{cases} \tag{27}$$

These expressions show that in both regimes, k_{t+1} is increasing in κ_t and thus, since κ_t is an increasing function in k_t, the schedule $k_{t+1}(k_t)$ of the dynamic system is described by an increasing function.

We can now make two observations, which jointly imply the existence of a steady state under the second regime. First, the variable κ_t determined by (22) or (23) as well as the threshold capital stock (24), is independent of z. Thus, given that z is sufficiently large, an economy with per-household capital stock $k_t = k_o$ from (24) experiences positive capital growth due to capital accumulation (27): its capital stock in period $t+1$ exceeds its capital stock of the previous period, i.e. $k_{t+1} > k_t$ holds. Second, as k_t grows unbounded, the ratio $\kappa_t / k_t = 1/m_t$ is bounded above[14]. Thus, dividing the second line on the right hand side of equation (27) by k_t shows that k_{t+1}/k_t approaches zero as k_t grows unbounded. Together, these findings imply that, if z is sufficiently large, the dynamic system has a steady state in the second regime.

Our knowledge about the dynamics and the steady state of the system is sufficient to tell a simple story about economic development and female labor force participation. In an economy where capital is scarce, female labor force participation is zero. As time passes and per-household capital stock gradually accumulates, the rewards of formal employment for female workers increase relative to rewards for male workers. This closing of the gender wage gap fosters female labor force participation and curbs fertility. Both effects accelerate per-household capital accumulation, which continues under the second regime up to the point where the economy reaches its steady state.

2.5 International Trade

International trade in goods induces specialization at the country level so that countries expand some sectors while contracting others. If, as in the current model, sectors differ in factor intensity, international specialization affects relative factor prices within each country. In the following paragraphs, we explore these effects of trade, particularly its impact on the gender gap and hence on fertility and female labor force participation.

We assume that the world consists of two countries, Home (no *) and Foreign (*). In addition, the superscript ^{A} indicates autarky variables, while its absence indicates variables of the free trade equilibrium. Moreover, we denote the relative price of the two goods by $\pi_t = p_{2,t}/p_{1,t}$, the ratio of male to female wage by $\omega_t = w_t^M / w_t^F$, and the relative population size of Foreign to Home by $\lambda_t = L_t^* / L_t$. Without loss of generality Home will represent the capital scarce and Foreign the capital abundant country, i.e., we assume

[14] See Appendix.

that $k_t < k_t^*$ for the initial period t. For later use, we define the set of all possible factor distributions in a world as:

$$FD_t = \left\{ (\lambda_t, k_t, k_t^*) \mid \lambda_t \in [0, \infty];\ k_t, k_t^* \geq 0 \text{ and } \left(k_t + \lambda_t k_t^*\right) / (1 + \lambda_t) = \bar{k}_t \right\}, \tag{28}$$

where $\bar{k}_t$ is the average per household capital stock of the world economy.

2.5.1 Factor Price Equalization

A good starting point for analysis of the free trade equilibrium is the Factor Price Equalization Set

$$FPES_t = \left\{ (\lambda_t, k_t, k_t^*) \in FD_t \mid w^M = w^{*,M},\ w^F = w^{*,F},\ r^F = r^* \right\}. \tag{29}$$

(Remember that the absence of superscript ^{A} indicates equilibrium variables under free trade – *e.g.* at w^M, $w^{*,M}$ etc.) Among all possible distributions of factors across countries, the $FPES_t$ comprises those that lead to free trade equilibria characterized by identical factor prices across countries. In terms of prices and output, these equilibria then replicate the equilibrium of an integrated world economy where factors are not restricted by national borders.[15] Thus, the $FPES_t$ describes the conditions on factor distributions under which borders do not affect the world efficiency frontier. Loosely conceptualized, a factor allocation is an element of the $FPES_t$ if relative factors are distributed "not too unevenly".

The following proposition conveniently characterizes the $FPES_t$ of the present model.

Proposition 1

Under costless trade, the following statement holds: Factor prices equalize $\Leftrightarrow \kappa_t^* = \kappa_t$.

Proof. See Appendix. ■

[15] If the equilibrium of the integrated economy is replicated, factors in all countries must equalize. Conversely, if factor and good prices equalize in both countries, the world equilibrium is an equilibrium of the integrated economy.

The proposition shows that $\kappa_t = \kappa_t^*$ implies $\omega_t = \omega_t^*$, a regime in which fertility, determined by (15), equalizes in both countries: $zn_t = zn_t^* = z\bar{n}_t$.[16] Combined with $\kappa_t = \kappa_t^* = \bar{\kappa}_t$ this leads to:

$$\bar{\kappa}_t = \frac{k_t}{l_{1,t} + 1 - z\bar{n}_t} = \frac{k_t^*}{l_{1,t}^* + 1 - z\bar{n}_t}. \tag{30}$$

By the definition of the $FPES_t$ $\bar{\kappa}_t$ and $\bar{n}_t$ are also the capital-mental labor ratio and fertility of the integrated world economy. The constraints $l_{1,t}, l_{2,t}^* \in [0, 1]$ lead to a restriction on capital stock conditions for factor price equalization:

$$(1 - z\bar{n}_t)\bar{\kappa}_t \leq k_t, k_t^* \leq (2 - z\bar{n}_t)\bar{\kappa}_t \tag{31}$$

by the resource constraint. Capital stocks of both countries must add up to the aggregate world capital stock, i.e., $\bar{k}_t = (k_t + \lambda_t k_t^*)/(1 + \lambda_t)$. Thus, the $FPES_t$ is described by (31) and

$$k_t = (1 + \lambda_t)\bar{k}_t - \lambda_t k_t^*. \tag{32}$$

Using the concise graphical representation from Helpman and Krugman (1985), Figure III illustrates the $FPES_t$. Each point A on the plane represents a partition of world labor and world capital: the distance between the vertical axis and A represents Home's male labor L_t, while the distance between the horizontal axis and A represents Home's capital K_t; Foreign's variables are $L_t^* = \bar{L}_t - L_t$ and $K_t^* = \bar{K}_t - K_t$, respectively. The upper panel of Figure III shows the case $z\bar{n}_t < 1$, where a minimum amount of capital is required in each country to keep female labor force productive in the first sector. The lower panel shows the case $z\bar{n}_t = 1$. In this case, a country may entirely lack capital while the world economy is still at its efficiency frontier, replicating the equilibrium of the integrated economy.

We can now readily determine the specialization pattern of both economies under the assumption that factor prices equalize. Recalling assumption $k_t < k_t^*$, we observe:

$$m_t = k_t/\bar{\kappa}_t < k_t^*/\bar{\kappa}_t = m_t^*,$$

[16] Upper bars indicate variables of the integrated economy.

while

$$l_{2,t} = 1 - [m_t - (1 - z\bar{n}_t)] > 1 - [m_t^* - (1 - z\bar{n}_t)] = l_{2,t}^*.$$

Confirming Heckscher-Ohlin-based intuition, the capital scarce Home country specializes in production of the labor intensive good X_2 while capital abundant Foreign specializes in X_1-production.

We can further compare the trade equilibrium with the respective autarky equilibria: notice that $1 - z\bar{n}_t \leq m_t < m_t^*$ implies $l_{1,t}^* > 0$ so that $\omega_t^* = 1 + b/(a(1-\alpha))\bar{\kappa}_t^{-\alpha}$ and (18) applies for Foreign. As $\omega_t^* = \omega_t$ and since $\kappa_t(k_t)$ is an increasing function, we use (18) again to conclude:

$$zn_t^A \geq z\bar{n}_t \geq zn_t^{*,A}.$$

These inequalities are strict if $1 > zn_t^A$ holds. Consequently, relative to autarky, trade increases female labor force participation in the capital scarce country and decreases it in the capital abundant country.

Both observations combined imply that the country which, by international specialization, *contracts* the sector that is particularly suitable for female labor, experiences an *increase* in female labor force participation. Conversely, the country which *expands* the sector suitable for female labor, experiences a *decrease* in female labor force participation.

The reason for this seemingly paradoxical finding is the following. For each economy, the key determinant of female labor force participation is the wage gap $\omega_t^{(*)}$. In autarky and under factor price equalization, this wage gap is determined by the relative productivities in the X_1-sector via (18) and ultimately by the capital-mental labor ratio $\kappa_t^{(*)}$. When international specialization induces Home to contract its X_1-sector and expand its X_2-sector, male workers migrate from the first to the second sector, taking their mental labor with them. Thus, they increase the ratio κ_t and hence female labor force participation $(1 - zn_t)$. Conversely, when Foreign workers react to trade-induced international price shifts and migrate from the second to the first sector, they dilute the capital-mental labor share κ_t^*, which increases the wage gap and decreases female labor force participation.[17]

In sum, under factor price equalization, we get sharp results on the impact of trade on female labor force participation in the capital scarce and abundant countries, respectively. The key mechanism for the result described above, however, depends on the fact that the wage gap is a function of only the capital-mental labor ratio $\kappa_t^{(*)}$. It may occur to the reader that international trade can induce male workers of one country to entirely abandon the first sector, while, at the same time, factor prices and the wage gap in particular do not equalize in both countries. If this is the case, the one-to-one relationship between κ_t and zn_t described by (18) does not hold and the mechanism described above ceases to apply. Consequently, our results under factor price equalization cannot be expected to hold under each and every factor distribution $(\lambda_t, k_t, k_t^*) \in FD_t$. The extent to which they generalize beyond factor price equalization is the subject of the next subsection.

2.5.2 Beyond Factor Price Equalization

Let us begin the general case of international trade by focusing on one country, for example, Home, with exogenous relative world prices π_t – i.e., assume, for the moment, Home to be a small open economy. For this exercise, we abandon Home's role as the capital scarce country. When world prices coincide with Home's autarky price π_t^A, we have $l_{1,t}, l_{2,t} > 0$, as argued in the case of the closed economy. Thus, by wages (10), (11), and (12) we find that:

$$\omega_t = \pi_t \frac{b/a}{1-\alpha} \kappa_t^{-\alpha} \tag{33}$$

$$\pi_t = (1-\alpha) \frac{a}{b} \kappa_t^{\alpha} + 1 \tag{34}$$

[17] The effect of relative productivities on the gender wage gap, which is the core of our mechanism operates under substantial generalizations. If $F(K, M, L)$ represents a standard constant return to scale production function in the first sector, it is sufficient to assume that capital K complements mental labor M relatively more than physical labor L (*i.e.* , $F_{KM}/F_M > F_{KL}/F_L \geq 0$, in line with Goldin (1990)) in order to generate the effect discussed. In particular, under these conditions, higher male employment in the first sector increases the gender wage gap.

hold for π_t in a small neighborhood of π_t^A. Combine (33) and (34) to verify that in this neighborhood, the wage gap

$$\omega_t = \frac{\pi_t}{\pi_t - 1} \tag{35}$$

is decreasing in π_t and zn_t is also decreasing by (15). Since κ_t is increasing in π_t by (34), $m_t = l_{1,t} + 1 - zn_t$ must be decreasing in π_t, which finally means that $l_{1,t}$ is decreasing in π_t. These relations hold as long as $l_{1,t}, l_{2,t} > 0$ apply. Thus, by the constraints $l_{1,t} \in [0, 1]$, there are thresholds $\underline{\pi}$ and $\bar{\pi}$ with $\underline{\pi} < \pi_t^A < \bar{\pi}$ so that for $\pi_t < \underline{\pi}$, we have $l_{1,t} = 1$ and κ_t as well as the wage gap ω_t defined by (17) are constant. Conversely, for $\pi_t > \bar{\pi}$, we have $l_{1,t} = 0$ in which case (33) holds and $\kappa_t = k_t/(1 - zn_t)$ and (15) imply:

$$\frac{\omega_t}{(1 - \gamma(1 + \omega_t))^\alpha} = \pi_t \frac{b/a}{1 - \alpha} k_t^{-\alpha}. \tag{36}$$

This equation defines ω_t as an increasing function of π_t. Finally, at $\pi_t \to \infty$ equation (36) implies $\omega_t \to (1 - \gamma)/\gamma$.

Figure IV summarizes these findings of the function $\omega_t(\pi_t)$. For small π_t, the wage gap ω_t is constant. For the intermediate range $\pi_t \in (\underline{\pi}, \bar{\pi})$, the wage gap $\omega_t(\pi_t)$ is decreasing but for $\pi_t > \bar{\pi}$ it is increasing. By the generic relation (15), these swings in ω_t are paralleled by swings in zn_t.

Now consider the Home economy facing relative world prices $\pi_t < \pi_t^A$. This means that, relative to autarky, the wage gap ω_t increases and, hence, fertility n_t rises while female labor participation $(1 - zn_t)$ drops. At the same time trade expands the X_1-sector and contracts the X_2-sector.[18] If, instead, $\pi_t > \pi_t^A$, there are two possible outcomes. First, if π_t is not too large, then the effect of trade is a reduction in the wage gap ω_t and thus a decrease in fertility n_t and an increase in female labor force participation $(1 - zn_t)$. Second, if π_t is sufficiently large, then trade induces an increase in ω_t and n_t and a decrease in $(1 - zn_t)$. In Figure IV, the threshold that separates the two cases is labeled π_u. In either case, trade contracts the X_1-sector and expands the X_2-sector.[19]

[18] To see this, notice that $\pi_t < \pi_t^A$ implies $l_{1,t} > l_{1,t}^A$ and, as (34) holds, $\kappa_t < \kappa_t^A$. This, in turn leads to $m_t > m_t^A$ so that total output in the first sector $ak_t^\alpha m_t^{1-\alpha} + bl_{1,t}$ rises relative to autarky. Output of the second sector $b(1 - l_{1,t})$ drops.

[19] Observe that $\pi_t > \pi_t^A$ implies $l_{1,t} < l_{1,t}^A$ so output in the second sector $b(1 - l_{1,t})$ expands in both

Now, return to the trade equilibrium between capital scarce Home and capital abundant Foreign. The autarky prices of both countries satisfy (34), implying $\pi_t^A < \pi_t^{*,A}$, while the world price under free trade π_t must lie between the respective autarky prices:

$$\pi_t^A \leq \pi_t \leq \pi_t^{*,A}. \tag{37}$$

Thus, trade (weakly) increases relative prices π_t in Home while it (weakly) decreases them in Foreign. With this observation, we can apply the insights of the analysis above. For capital abundant Foreign, trade unambiguously causes a (weak) increase in the wage gap ω_t and thus a drop in female labor force participation. We can therefore generalize the first part of our result derived under factor price equalization. The country which, by international specialization, expands the sector suitable for female employment experiences a decrease in female labor force participation.

For capital scarce Home, however, trade induces a decrease in the wage gap ω_t and an increase in female labor force participation if and only if π_t is not too high (i.e., $\pi_t \leq \pi_u$ holds). In this restricted case, we recover the second part of the result derived under factor price equalization. The country which contracts the sector suitable for female labor experiences an increase in female labor force participation.

This second observation is a non-trivial generalization of the parallel result under factor price equalization. To verify this statement, use that under free trade $l_{1,t}^* > 0$ and $l_{2,t} > 0$ hold so that, by (10) and (11)

$$(1-\alpha)\frac{a}{b}(\kappa_t^*)^\alpha + 1 \geq \pi_t \geq (1-\alpha)\frac{a}{b}\kappa_t^\alpha + 1 \tag{38}$$

holds. Proposition 1, however, states that factor price equalization requires $\kappa_t = \kappa_t^*$, implying $\pi_t = (1-\alpha)\frac{a}{b}\kappa_t^\alpha + 1$. By construction of $\bar{\pi}$, however, all world equilibria with $\pi_t \in (\bar{\pi}, \pi_u)$ are characterized by equality $\pi_t > (1-\alpha)\frac{a}{b}\kappa_t^\alpha + 1$, implying that factor prices do not equalize. Since finally, by construction of π_u we have $\omega_t > \omega_t^A$ for all equilibria with

cases. Further, for $\pi_t < \bar{\pi}$ (34) holds, implying $\kappa_t > \kappa_t^A$ or $m_t < m_t^A$. Any increase in π_t above $\bar{\pi}$ reduces female labor $1 - zn_t$ while $l_{1,t} = 0$ continues to hold. Thus, $m_t < m_t^A$ in this range, too. Together, this means that output in the first sector $ak_t^\alpha m_t^{1-\alpha} + bl_{1,t}$ falls.

$\pi_t \in (\bar{\pi}, \pi_u)$ we conclude that trade induces an increase of female labor force participation in Home for a set of factor endowments that is strictly larger than the $FPES_t$.

Summarizing, we use the definitions (28) and (29) to state the following proposition.

Proposition 2

(i) In Foreign, trade expands the sector that uses female labor intensively, but unambiguously reduces female labor force participation.

(ii) There is a set $S_t \subset FD_t$ with $FPES_t \subsetneqq S_t$ and the following property: for each element of S_t trade contracts the sector that uses female labor intensively in Home, but increases Home's female labor force participation.

2.5.3 Dynamics under Trade

The dynamics of the model under free trade are again driven by two key variables, savings s_t and fertility n_t. Per-household capital stocks of either country follow the generic dynamic system equivalent to (25), now expanded to:

$$k_{t+1}^{(*)} = \begin{cases} z w_t^{M,(*)} & if \quad z n_t^{(*)} = 1 \\ z \frac{1-\gamma}{\gamma} w_t^{F,(*)} & if \quad z n_t^{(*)} < 1 \end{cases} \tag{39}$$

To calculate the respective wages (10) - (12), we can use the final good normalization (8) and the definition of π_t to derive:

$$p_{1,t} = \left(\theta^{\frac{1}{1-\rho}} + (1-\theta)^{\frac{1}{1-\rho}} \pi_t^{\frac{-\rho}{1-\rho}} \right)^{(1-\rho)/\rho} \quad \text{and} \quad p_{2,t} = \left(\theta^{\frac{1}{1-\rho}} \pi_t^{\frac{\rho}{1-\rho}} + (1-\theta)^{\frac{1}{1-\rho}} \right)^{(1-\rho)/\rho} \tag{40}$$

These defined wages and dynamic system, (39), give rise to the following observations

Proposition 3

(i) $z n_t^* \leq z n_t$.

(ii) $k_{t+1}^* \geq k_{t+1}$.

(iii) If $\alpha(\theta/(1-\theta))^{\frac{-1}{1-\rho}} \geq (1-2\gamma)/\gamma$ *holds then* $k_{t+1} \geq k_{t+1}^{A}$.

(iv) $k_{t+1}^{*}/k_{t+1} \leq k_{t+1}^{*;A}/k_{t+1}^{A}$.

Proof. See Appendix. ■

Proposition 3 (i) and (ii) show that trade cannot reverse the order of countries regarding population growth or capital abundance. The capital rich country has always weakly lower fertility rates, higher female labor force participation and faster pace of per-household capital accumulation.

Proposition 3 (iii) shows that, given that the first sector is sufficiently large (i.e., $1-\theta$ is sufficiently small), trade unambiguously accelerates the pace of capital accumulation in the capital scarce country. It is worth emphasizing that this result also holds in the case where world prices π_t are very large and all men in Home work in the X_2-sector while female labor participation drops relative to autarky ($\pi_t > \pi_u$ in Figure IV). Even in this case, where a reduced female labor force participation depresses savings and increased population growth dilutes the following period's capital stock, the gains from trade are sufficient to grant a net increase in per-household capital accumulation relative to autarky. We cannot, however, make a parallel statement for the capital rich economy, for which the effect of trade on capital accumulation is ambiguous. Indeed, it can be shown that for capital accumulation in the rich economy, the positive forces stemming from the gains of trade might either dominate or be dominated by the adverse effect of reduced female labor force participation and higher fertility.

Finally, Proposition 3 (iv) makes a relative statement about the countries' capital accumulation. Trade cannot accelerate capital accumulation in the rich country by more than it accelerates it in the poor country. In particular, the proposition shows that trade spurs convergence of per-household capital stocks. At the same time, using Proposition 3 (ii) and (iv), a simple induction argument leads to $k_{t+\tau}^{*}/k_{t+\tau} \leq k_{t+\tau}^{*,A}/k_{t+\tau}^{A}$ for all $\tau \geq 0$ and hence:

$$\lim_{t\to\infty} k_t = \lim_{t\to\infty} k_t^{*} = \tilde{k}.$$

Since in the limit, factor endowments between countries equalize, the motives to trade disappear. Consequently, the limit $\tilde{k}$ is equal to the limit of the closed economy: $\tilde{k} = k$, where k is the steady state capital stock of the closed economy.

Summarizing Proposition 3, international trade fosters convergence in fertility, labor force participation, and per-household capital stocks.

2.6 Technological Progress

The reduction in the gender wage gap is often attributed to technological change. Thus, Welch (2000) and Black and Spitz-Oener (2007) argue that the increase in the market price for women's labor was brought about by an increase in the valuation of skill, which is, at least in part, explained by technological change. Galor and Weil (1996) show how technological change can eliminate poverty traps, characterized by high fertility, low female labor force participation and low per-household capital stocks. They argue that "technological progress will eventually eliminate such a development trap, leading to a period of rapid output growth and a rapid fertility transition" (p. 383).

Another popular hypothesis rests on demand shifts in favor of goods whose production is more intensive in skill or, more generally, in female labor inputs. The mechanism outlined above, in which, male workers searching for the highest return to their labor crowd out women in the labor market sheds some doubt on the generality of these pro-growth effects. Indeed, we show next that the effect that leads to a decrease in female labor force participation and an increase in fertility in response to the expansion of the females' comparative advantage sector operates under technological change and shifts in demand as well.

For the formal analysis of technological change and demand shifts, we return to the closed economy. To incorporate technological change biased towards the sectors that generate demand for female labor, we rewrite the production functions (2) as:

$$\begin{aligned} X_1 &= \mu \left[a K_t^{\alpha} (L_t^m)^{1-\alpha} + b L_{1,t}^p \right] \\ X_2 &= b L_{2,t}^p \end{aligned} \tag{41}$$

so that growth of the parameter $\mu \geq 1$ mimics technological progress that is biased towards the first sector. As a result of incorporating μ into our framework (23) becomes[20]

$$\frac{\theta}{1-\theta}\mu^{\rho}\left[(1-\alpha)\frac{a}{b}\kappa_t^{\alpha}+1\right]=\left(\frac{\frac{a}{b}\frac{k_t}{\kappa_t^{1-\alpha}}+\frac{k_t}{\kappa_t}-1+\gamma\left(2+\frac{b}{a}\frac{\kappa_t^{-\alpha}}{1-\alpha}\right)}{1-\frac{k_t}{\kappa_t}+1-\gamma\left(2+\frac{b}{a}\frac{\kappa_t^{-\alpha}}{1-\alpha}\right)}\right)^{1-\rho} \tag{42}$$

While the right hand side of (42) is decreasing in κ_t, the left hand side of (42) is, $\rho \in (0,1)$, increasing in κ_t and in μ. This implies that an increase in μ decreases the equilibrium level of κ_t, which, in turn, decreases female's productivity relative to male productivity, widens the gender wage gap and curbs female labor force participation.

After reading the previous subsections, the intuition for this result is straight forward. An increase in μ increases male productivity in the first sector relative to the second sector. As long as the elasticity of substitution between X_1 and X_2 is greater than one, the relative price π decreases but the decrease ia less than the increase in μ. As a result, male wage increases in the first sector, inducing male workers to migrate from the second sector to the first sector. This increases mental labor employed in the first sector and dilutes κ so that women's relative productivity declines, driving women out of formal employment into the child-rearing.

A similar mechanism applies under demand shifts towards the first good, equivalent to an increase in the parameter θ (compare (1)). Again, equation (42) shows that an increase in θ is followed by a decrease in κ_t, which curbs women's productivity by more than men's, widens the wage gap and thus decreases female labor force participation while fostering fertility.

Thus, our model shows that neither a technological change biased towards sectors with a high demand for female labor nor demand shift towards goods of these sectors necessarily generates increases in female labor participation. The resulting increase in fertility generally counters the pro-growth effects.

[20] Under $\mu \geq 1$ condition (16) is sufficient for $l_{1,t}^p > 0$ to hold, i.e., male employment in the first sector is positive.

3 Concluding Remarks

This paper analyzes the impact of an expansion or contraction in sectors prone to employing females on female labor force participation. We argue that when international trade expands sectors conductive to female employment, female labor force participation drops and *vice versa*. This effect operates as follows. Male workers earn higher wages than women and are therefore always formally employed. Thus, when an economy specializes in sectors prone to employing females, other sectors contract and male workers migrate to expanding sectors, driving female workers out of formal employment. Alternatively, when international trade expands sectors that are prone to employing males, these sectors expand, which attracts male workers away from sectors conductive to female employment and, thus, foster female labor force participation.

Interestingly, our mechanism also applies to the case of technological progress that is biased towards female labor. As this type of technological progress increases the wages in this sector, it attracts male workers, and drives female workers out of formal employment. Surprisingly, our theory suggests that this type of technological progress may curb female labor force participation.

Turning to the dynamics, our model suggests that international trade fosters per-household capital growth in the capital scarce economy. In the capital abundant economy, however, the impact of international trade on capital growth is ambiguous. Although international trade hinders female labor force participation and increases fertility, domination of these adverse by positive forces stemming from gains from trade may occur. In both cases, our model suggests that trade cannot accelerate capital accumulation in the rich country by more than it accelerates capital accumulation in the poor country and, thus, our theory predicts convergence of per-capital stocks.

A Appendix

Proof that $1/m_t$ is bounded above. First observe that $k_t \to \infty$ means $k_t > k_o$ so that the second regime applies. Use (23) to confirm that $\kappa_t \to \infty$ as $k_t \to \infty$ (else the denominator in the brackets of the expression on the right turns negative). Finally, divide equation (21) by κ_t^α to get

$$\frac{1-\theta}{\theta}\frac{1}{\kappa_t^{\alpha\rho}}\left(\frac{\frac{a}{b}m_t + [m_t-(1-zn_t)]\,\kappa_t^{-\alpha}}{1-m_t+(1-zn_t)}\right)^{1-\rho} \to (1-\alpha)\frac{a}{b} \qquad (k_t\to\infty).$$

Since this limit is positive, the term in brackets must approach infinity as $k_t \to \infty$ so that, as $\lim_{\kappa_t\to\infty} zn_t = 2\gamma$, $\lim_{k_t\to\infty} m_t = 2(1-\gamma)$ must hold. This proves that $1/m_t$ is bounded above. ∎

Proof of Proposition 1. The proof of "$\Rightarrow$" is immediate by $r_t = r_t^*$ and (9).

For "$\Leftarrow$" assume that $\kappa_t^* = \kappa_t$, which implies $r_t = p_{1,t}\alpha a\kappa_t^{\alpha-1} = p_{1,t}\alpha a\left(\kappa_t^*\right)^{\alpha-1} = r_t^*$ and $w_t^F = p_{1,t}(1-\alpha)a\kappa_t^\alpha = p_{1,t}(1-\alpha)a\left(\kappa_t^*\right)^\alpha = w_t^{F,*}$. By $X_{2,t}>0$ we have $l_{2,t}+l_{2,t}^*>0$. In case $l_{2,t}^*, l_{2,t}>0$ $w_t^M = w_t^{M,*}$ follows from (10). In case $l_{2,t}^*=0$ this implies

$$w_t^M = p_{2,t}b \le w_t^{M,*}.$$

At the same time $l_{1,t}^* = 1$ implies

$$w_t^{M,*} = p_{1,t}((1-\alpha)a(\kappa_t^*)^\alpha + b) = p_{1,t}((1-\alpha)a\kappa_t^\alpha + b) \le w_t^M$$

so that $w_t^M = w_t^{M,*}$. In case $l_{2,t}=0$ switching Home and Foreign variables leads to $w_t^M = w_t^{M,*}$ again. ∎

Proof of Proposition 3. (i) By (15) it is sufficient to show $\omega_t^* \le \omega_t$. Since free trade implies $l_{1,t}^*>0$ and $l_{2,t}>0$ we have $\omega_t = \pi_t b/\left[a\left(1-\alpha\right)\kappa_t^\alpha\right] \ge 1 + b/\left[a\left(1-\alpha\right)\kappa_t^\alpha\right]$ and $\omega_t^* = 1 + b/\left[a\left(1-\alpha\right)\left(\kappa_t^*\right)^\alpha\right] \ge \pi_t b/\left[a\left(1-\alpha\right)\left(\kappa_t^*\right)^\alpha\right]$. Combining these relations gives

$$\frac{\omega_t^*}{\omega_t} \le \frac{\pi_t+\omega_t^*}{\pi_t+\omega_t}$$

and proves statement (i).

(ii) By (i) and (15) we have $zn_t^* \leq zn_t$ and can distinguish two cases. The first where $zn_t = 1$ gives with (39) and $l_{2,t} > 0$

$$\frac{k_{t+1}^*}{k_{t+1}} \geq \frac{w^{M,*}}{w^M} \geq \frac{p_{2,t}b}{p_{2,t}b} = 1$$

If instead $zn_t < 1$ (i) implies $zn_t^* < 1$ so that (39)

$$\frac{k_{t+1}^*}{k_{t+1}} = \frac{w^{F,*}}{w^F} = \frac{\omega_t}{\omega_t^*}\frac{w^{M,*}}{w^M} \geq \frac{w^{M,*}}{w^M} = 1$$

where we used (i) in the first inequality and the second inequality follows as above.

(iii) If $zn_t^A = 1$ we have

$$\frac{k_{t+1}^A}{k_{t+1}} \leq \frac{w^{M,A}}{w^M} = \frac{p_{2,t}^A b}{p_{2,t}b} \leq 1$$

If, instead, $zn_t^A < 1$ then $zn_t < 1$ (from (35) as long as $l_{1,t} > 0$ and $m_t > 0$ otherwise) and

$$\frac{k_{t+1}^A}{k_{t+1}} \leq \frac{w^{F,A}}{w^F} = \frac{\omega_t}{\omega_t^A}\frac{w^{M,A}}{w^M}$$

For the case $\omega_t \leq \omega_t^A$ (or $\pi_t \leq \pi_u$ in Figure IV) this proves the claim. If instead $\omega_t > \omega_t^A$ we use $\kappa_t = k_t/(1 - zn_t)$ and (15) to write

$$\kappa_t \left(1 - \gamma\left(1 + \pi_t \frac{b/a}{1-\alpha}\kappa_t^{-\alpha}\right)\right) = k_t$$

and take implicit derivatives

$$\frac{d\kappa_t}{d\pi_t} = \kappa_t \frac{1}{1-\alpha}\frac{\gamma}{(1-\gamma)a/b\kappa_t^\alpha - \gamma\pi_t}$$

At the same time (40) leads to

$$\frac{dp_{1,t}}{d\pi_t} = -p_{1,t}^{1-\frac{\rho}{1-\rho}}\left(\frac{1-\theta}{\pi_t}\right)^{\frac{1}{1-\rho}}$$

Thus,

$$\frac{d}{d\pi_t}\ln\left(p_{1,t}\kappa_t^{\alpha}\right) = \frac{\alpha}{1-\alpha}\frac{\gamma}{(1-\gamma)a/b\kappa_t^{\alpha}-\gamma\pi_t} - \left(\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}\pi_t^{\frac{\rho}{1-\rho}}+\pi_t^{\frac{-\rho}{1-\rho}}\right)^{-1}\pi_t^{-1}$$

A sufficient condition for this expression to be positive is

$$\frac{\alpha}{1-\alpha}\frac{\gamma}{\pi_t^{-1}(1-\gamma)a/b\kappa_t^{\alpha}-\gamma} > \frac{1}{\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}\pi_t^{\frac{\rho}{1-\rho}}+1}$$

or with $\omega_t = \pi_t b/\left[a\left(1-\alpha\right)\kappa_t^{\alpha}\right]$

$$\frac{\alpha}{1-\alpha}\frac{\gamma}{\frac{1-\gamma}{1-\alpha}\frac{1}{\omega_t}-\gamma} > \frac{1}{\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}\pi_t^{\frac{\rho}{1-\rho}}+1}$$

Since $\omega_t > 1$ and $\pi_t > 1$ this condition is satisfied whenever

$$\alpha\frac{\gamma}{1-\gamma-(1-\alpha)\gamma} > \frac{1}{\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\rho}}+1}$$

or $(\theta/(1-\theta))^{\frac{1}{1-\rho}} \geq (1-2\gamma)/(\alpha\gamma)$ holds, proving the statement (iii).

(iv) Notice with Proposition 2 (i) that $zn_t^* < 1$ implies $k_{t+1}^*/k_{t+1}^{*,A} = p_{1,t}(\kappa_t^*)^{\alpha}/(p_{1,t}^{*,A}(\kappa_t^{*,A})^{\alpha})$. If $zn_t^* = 1$, instead, $k_{t+1}^*/k_{t+1}^{*,A} = p_{1,t}\left((1-\alpha)a\kappa_t^*+b\right)/(p_{1,t}^{*,A}((1-\alpha)a\kappa_t^{*,A}+b))$. Now, inequality (37) and expression (40) for the price $p_{1,t}^{(*,A)}$ imply $p_{1,t}/p_{1,t}^{*,A} \leq 1$. Further, by $m_t^{*,A} \leq m_t^*$ we have $\kappa_t^{*,A} \geq \kappa_t^*$ and thus

$$k_{t+1}^*/k_{t+1}^{*,A} \leq \left(\kappa_t^*/\kappa_t^{*,A}\right)^{\alpha}$$

Similarly, we compute for $zn_t < 1$ that $k_{t+1}/k_{t+1}^A = p_{1,t}\kappa_t^{\alpha}/(p_{1,t}^A(\kappa_t^A)^{\alpha})$ while for $zn_t = 1$ $k_{t+1}/k_{t+1}^A = p_{1,t}\left((1-\alpha)a\kappa_t^{\alpha}+b\right)/(p_{1,t}^A((1-\alpha)a(\kappa_t^A)^{\alpha}+b))$ holds. By (37) and expression (40) we have $p_{1,t}/p_{1,t}^A \geq 1$. Further, by $m_t^A \geq m_t$ we have $\kappa_t^A \geq \kappa_t$ and thus

$$k_{t+1}/k_{t+1}^A \geq \left(\kappa_t/\kappa_t^A\right)^{\alpha}$$

Combining both inequalities leads to

$$\frac{k^*_{t+1}/k^{*,A}_{t+1}}{k_{t+1}/k^A_{t+1}} \leq \left(\frac{\kappa^*_t/\kappa^{*,A}_t}{\kappa_t/\kappa^A_t}\right)^\alpha = \left(\frac{m^{*,A}_t/m^*_t}{m^A_t/m_t}\right)^\alpha$$

Using again $m^{*,A}_t \leq m^*_t$ and $m^A_t \geq m_t$ shows that the expression on the right falls weakly short of unitiy, which proves the statement. ■

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Figure I

Relative Wages, United States 1800-1990. Source: Galor 2005.

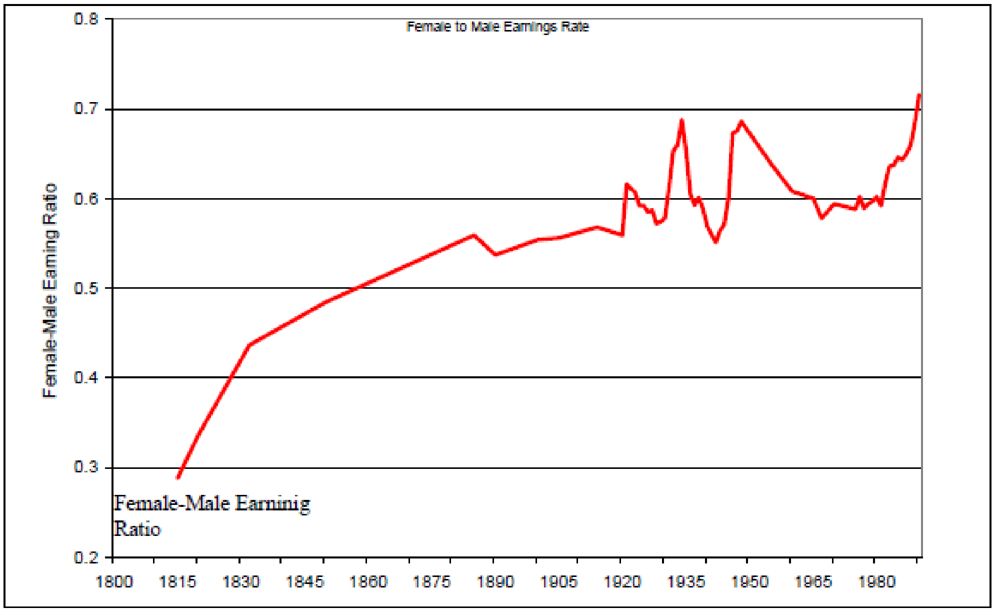

Figure II

Dynamic System of the Integrated Economy

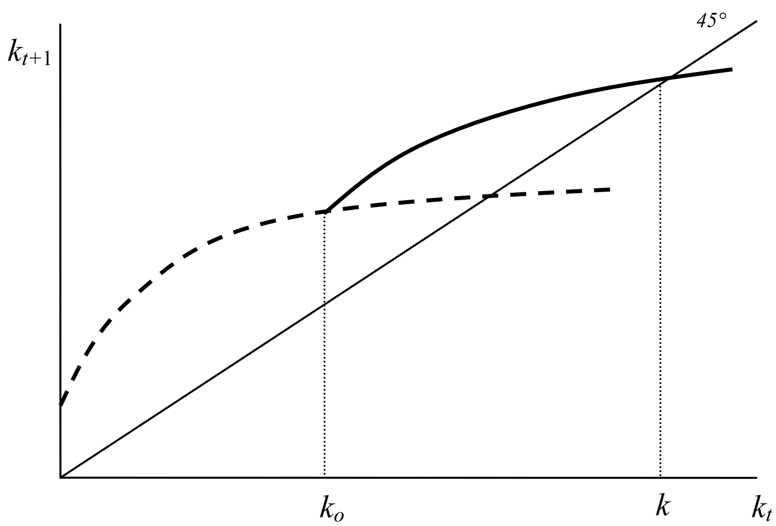

Figure III

Factor Price Equalization Set

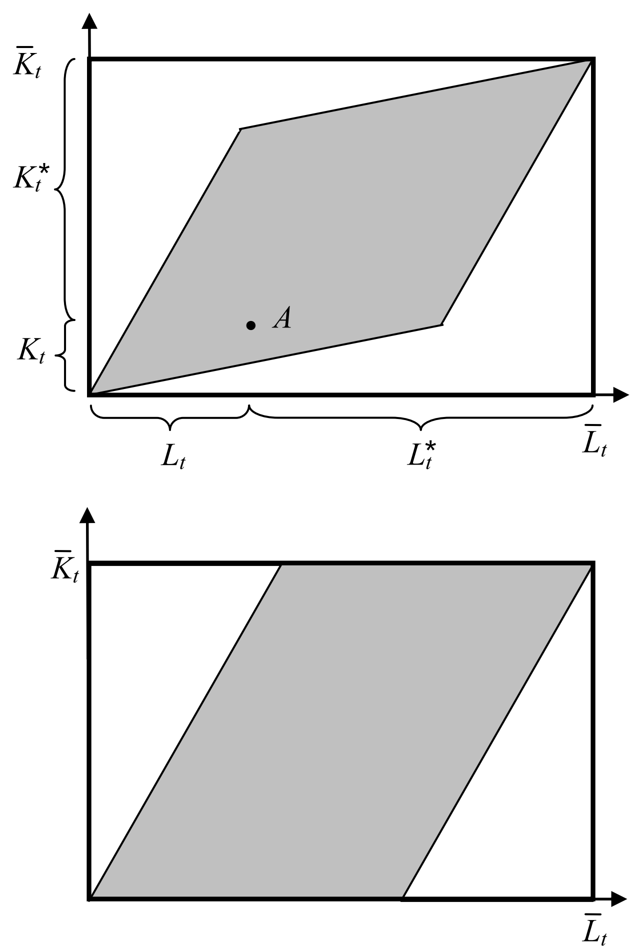

Figure IV

Wage Gap and World Price