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# "Competing for Consumer Inattention"

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**Abstract**

Consumers purchase multiple types of goods, but may be able to examine only a limited number of markets for the best price. We propose a simple model which captures these features, conveying new insights. A firm's price can deflect or draw attention to its market, and consequently, limited attention introduces a new dimension of cross-market competition. We characterize the equilibrium, and show that having partially attentive consumers improves consumer welfare. With less attention, consumers are more likely to miss the best offers; but enhanced cross-market competition decreases average price paid, as leading firms try to stay under the consumers' radar.

# 1 Introduction

Classic models of price competition assume that consumers have unlimited ability to track down the best deals. The wide array of goods and services in the marketplace casts doubt that this is a faithful description of the average consumer. With only limited attention to devote to finding cheaper substitutes, consumers may pay close attention to some purchases while neglecting to find the best price in others. This paper investigates the price and welfare implications of allocating limited attention across markets. Our simple model conveys some new insights: (i) a firm's price can deflect or draw attention to its market; and consequently, (ii) limited attention introduces a new dimension of competition across (even otherwise independent) markets.

We convey these insights in a simple framework, but they should remain important considerations in more general settings. Consumers in our model have unit demand for each of $M$ different goods. To make point (ii) as starkly as possible, each consumer's utility is separable across goods, which ensures these markets would be independent if attention were unlimited. Reservation prices are assumed to be one for all consumers and all goods. Each good is offered by two sellers whose constant marginal cost is normalized to zero, and who set prices independently. For each market, consumers have a default seller who is interpreted as the most visible provider of that good or service. Consumers share the same default set of sellers, who are thought of as the *market leaders.* Confronted with market leaders' prices, consumers decide which markets to examine further, to see whether the competing firm (the *market challenger*, whose identity and price they do not know) offers a better deal. Consumers may have only limited attention to devote to comparison-shopping, with the ability to investigate at most $k \in \{0, \ldots, M\}$ markets. The distribution of attention in the population is captured by a probability distribution $(\alpha_0, \ldots, \alpha_M)$.

Our model captures the view that limited attention introduces an auditing component into consumption decisions. Given his budget of attention, a consumer uses what he knows (in this case, the price offered by market leaders) to decide which dimensions of his consumption decision are worthiest of further investigation. For instance, when buying groceries online, which items does a consumer buy from his saved list, and which does he check for better bargains? In a sense, a consumer's problem under limited attention is akin to that of maintenance scheduling in operations research: only a subset of items can be served, and those that are neglected may suffer from poor performance. For a consumer with limited attention, inspecting one market means overlooking another. The cost associated with this tradeoff is *endogenous*, equal to the expected equilibrium savings foregone by neglecting that other market.

Our setting is one of imperfect information, since consumers do not observe challengers' prices when allocating their attention. The analysis focuses on partially symmetric, perfect Bayesian Nash equilibria (henceforth *equilibria*). These preserve the symmetry of the model, with firms in the same position (as leaders or challengers) using the same pricing strategy. In that case, consumers expect the most savings to be found in markets with the most expensive leaders. Hence firms' profits may vary discontinuously with the leaders' prices, as consumers shift their attention between markets. A more standard form of discontinuity also arises when firms in a market quote the same price. Despite these discontinuities, we constructively establish that a partially symmetric equilibrium exists for any distribution of attention, and moreover, that only one such equilibrium exists. In this equilibrium, all firms employ atomless pricing strategies, but leaders systematically charge a wider range of prices than challengers. The support of the leaders' strategy has no gap. However, depending on the distribution of attention, challengers may avoid charging some intermediate prices. Constructing the unique equilibrium then requires an ironing procedure.

What is the equilibrium effect of (in)attention on consumer welfare? As might be expected, an increase in the proportion $\alpha_0$ of fully inattentive consumers is detrimental. However, varying the distribution of partially attentive consumers has perhaps surprising implications. Any change in the distribution of attention which *decreases* the average level of attention (holding $\alpha_0$ constant) is *beneficial*. This may seem unintuitive at first, since consumers inspecting fewer markets are more likely to miss the best deals. But this intuition does not take into account the countervailing effect of partial inattention on firms' behavior. [1] Consumers' limited capacity to search for better deals induces cross-market competition for their *in*attention: by lowering its price, a leader can increase the chance his market remains under the consumers' radar. The overall effect could, at least in theory, be determined by computing the consumer surplus directly using our expressions for the equilibrium strategies. Our argument follows a different route, taking advantage of the fact that total surplus remains constant and that firms' equilibrium profits turn out to be much simpler to calculate. We delve further into the mechanics of competition for inattention, exploring how the leaders' pricing strategy adjusts.

This paper proceeds as follows. In the next subsection we discuss how our paper fits within the literature. Section 2 presents the model. Section 3 presents the main results and their intuition, including how consumers allocate attention, the equilibrium characterization, and comparative statics with respect to partial attention. We also illustrate these results for the special case of two markets. The constructive proof of the unique equilibrium is presented in Section 4. Concluding remarks, and possible directions for future research, are given in Section 5. Some proofs are relegated to the appendix.

### Related literature

Our setting builds on the seminal literature on price dispersion (Salop and Stiglitz, 1977; Rosenthal, 1980; Varian, 1980), which explains observed variation in prices by introducing "captive" consumers who purchase from a randomly selected firm, without engaging in price comparisons. Among other differences with that literature, we consider multiple markets and introduce partially attentive consumers, which are driving forces behind our results. These

[1] As an analogy, think of auctions under asymmetric information. Fixing the bids, first price gives a strictly higher profit than second price. However, this does not mean that equilibrium profits are necessarily higher with a first-price auction, as individuals' bidding behavior responds to the auction format.

and other features of our framework, such as the endogenous cost of neglecting a market and the asymmetric positions of firms, also depart from the standard approach taken in the search and rational inattention literatures. In the search literature, consumers incur a fixed, exogenous cost of sampling prices of a product sold by multiple firms; classic references include Burdett and Judd (1983), where consumers decide in advance how many prices to simultaneously sample, or Stahl (1989), where consumers search sequentially. Recent papers aim to capture sluggish price adjustments by introducing a cost for firms to review its price policy or gather information about current market conditions. Some authors assume a fixed cost of review (e.g., Mankiw and Reis (2002) and more recently, Alvarez, Lippi and Paciello (2011)). Others, in the "rational inattention" literature, model an exogenous cost of information processing using entropy measures (e.g., Sims (2003) and Woodford (2009)). The decision makers' dilemma in that literature is whether to obtain any information, and if so, how much. In our approach, prices serve as cues to determine which markets are worthiest of attention, which introduces an element of competition across sellers of different goods.

Market interaction between profit-maximizing firms and consumers with limited attention is, of course, more intricate than the stylized environment we analyze. Our model isolates an aspect of the feedback between consumer attention and firm behavior that has not been studied in the literature. One strand of this literature has focused on a different aspect of attention: when firms offer a multi-dimensional product, consumers may take only a subset of these dimensions into consideration. This approach is exemplified by Spiegler (2006), where a consumer samples one price dimension from each firm selling a product with a complicated pricing scheme (e.g., health insurance plans); Gabaix and Laibson (2006), where some consumers do not observe the price of an add-on before choosing a firm; Armstrong and Chen (2009), who extend the notion of "captive" consumers to those who always consider one dimension of a product but not another (say, price but not quality); and Bordalo, Gennaioli and Shleifer (2013), who study a duopoly model where firms decide on price and quality, taking into account that the relative weights consumers give to these attributes is determined endogenously by the choices of both firms. The above works study symmetric pricing equilibria for firms in a single market, with some differing implications for welfare. In Gabaix and Laibson (2006), for instance, prices increase as more consumers notice add-ons; while in Armstrong and Chen (2009), reducing the proportion of captive consumers reduces the incentive to offer low quality, but has an ambiguous effect on consumer welfare.

Taking a different approach to attention, Eliaz and Spiegler (2011a,b) formalize a model of competition over consumers who only consider a subset of available products. They abstract from prices and analyze firms who compete over market share only by offering a menu of products together with a payoff irrelevant marketing device (e.g., packaging). Consumers in their model are characterized by a preference relation and a consideration function, which determines, given firms' choices, whether a consumer pays attention only to its (exogenously determined) default firm or whether he also considers the competitor. They show that consumer welfare need not be monotonic in the amount of attention implied by the consideration function.

# 2 The model

We propose a simple model capturing the feature that consumers purchase multiple types of goods and services, but may have the capacity to examine only a limited number of markets in search of the best price. The market for each good or service consists of two firms, a *leader* and a *challenger*, who compete in prices. All consumers know the market leaders' prices, but need to pay attention to a market to identify the challenger and learn his offer. Consumers differ in the number of markets to which they can pay attention. The leader in a market is interpreted as the most visible provider of the good or service, and is the default provider for a consumer who chooses not to allocate the time or capacity to search that market further.

There is a unit mass of consumers, each of whom desires at most one unit of any given good. For simplicity, we assume that the consumers' reservation price for each type of good is one. Letting $M$ denote the number of markets (one per good), a consumer's utility from purchasing the bundle $(x_1, x_2, \ldots, x_M) \in \{0,1\}^M$ at prices $(p_1, p_2, \ldots, p_M)$ is $\sum_{m=1}^{M}(1 - p_m)x_m$.

The distribution of attention in the consumer population is captured by a probability distribution $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_M)$, where $\alpha_k$ is the proportion of consumers who can inspect up to $k$ markets to find the best price. Consumers optimally decide which markets to inspect. If a consumer inspects a market, then he can choose whether to purchase from the market leader, the challenger, or not at all. If he does not inspect a market, then his only decision for that market is whether to purchase from its leading firm. The distribution of attention is common knowledge among firms. We assume throughout a positive measure of fully attentive consumers ($\alpha_M > 0$), inattentive consumers ($\alpha_0 > 0$), and partially attentive consumers ($\alpha_0 + \alpha_M < 1$). We further assume that a positive fraction of fully attentive consumers insist on inspecting a market when indifferent.[2]

The game unfolds over two periods. First, all firms independently set prices to maximize (expected) profit. We normalize marginal costs to zero, so realized profit is simply the product of the firm's price and its market share. Upon observing all the leaders' offers, consumers decide how to allocate their attention, and make their purchasing decisions, to maximize (expected) utility.

**Equilibrium.** Because consumers have only imperfect information when allocating their attention, the equilibrium notion applied is that of Perfect Bayesian equilibrium. We restrict attention throughout to *partially symmetric* equilibria where market leaders follow a common pricing strategy, as do market challengers. The leaders' strategy may differ from that of the challengers, and we do not impose restrictions on the consumers' strategies. We note that

[2]One could instead assume any positive measure of consumers who are standard, that is, aware of all firms and prices. Without either assumption, the model admits Diamond-type equilibria (Diamond, 1971), as in many search models, in addition to the partially symmetric equilibrium we characterize. Indeed, for any $p \in [\alpha_0, 1]$, there would be an equilibrium where all firms charge $p$, and consumers inspect none of the markets on the equilibrium path.

equilibrium existence is nontrivial, since firms' profits are discontinuous.[3]

**Notation and definitions.** The leaders' and challengers' pricing strategies are described by (right-continuous) cumulative distribution functions $F_\ell : \mathbb{R} \to [0,1]$ and $F_c : \mathbb{R} \to [0,1]$, respectively. A price $p$ is said to be in the support of the pricing strategy $F$ if $F(p+\varepsilon) > F(p-\varepsilon)$, for all $\varepsilon > 0$. A price $p$ is said to be an atom of the strategy $F$ if $F$ is discontinuous at $p$, that is, $F(p) > F(p^-)$, where $F(p^-) = \lim_{p' \uparrow p} F(p')$. We do not put *a priori* restrictions on the presence of atoms or gaps in the support of the pricing strategies.

# 3 Main results and intuitions

In this section, we first present our characterization of partially symmetric equilibria and some of the intuitions behind it, leaving the complete equilibrium analysis to Section 4. We then examine how the equilibrium and consumer welfare change with the distribution of attention among consumers.

## 3.1 Consumer attention and its implications

Suppose the leading firm in market $i$ quotes a price $p_i$. The expected gain from inspecting market $i$ is the expected savings from finding a cheaper price by the challenger, i.e.,

$$\int_0^{\min\{p_i,1\}} (\min\{p_i, 1\} - x) dF_c(x). \tag{1}$$

[3]Firms' payoffs exhibit two forms of discontinuity. The first, related to how a leader and a follower in a market share consumers when quoting the same price, appears in many models of competition. Existence in such cases follows from results by Dasgupta and Maskin (1986) or Reny (1999). The second form of discontinuity is related to how consumer attention is allocated across markets, and its impact on challengers' profits, when some leaders quote the same price. For each price he may quote, a challenger's profit is discontinuous over a continuum of leaders' prices, which prevents a direct application of Dasgupta and Maskin (1986). It also implies that challengers cannot secure themselves a positive payoff in the sense of Reny (1999). While alternative methods may be used to show existence, we provide a constructive proof that also establishes uniqueness.

Note that the above expression relies on the symmetry in the challengers' pricing strategies. Optimality requires the following. If a consumer inspects market $i$, and inspecting market $j$ gives strictly higher expected savings, then he also inspects market $j$. If a consumer inspects fewer markets than his capacity allows, then any market left uninspected has zero expected savings. The next proposition formalizes these statements using (1).

**Proposition 1.** *Suppose market leaders post the prices $p_1, \ldots, p_M$, and let $q_i = \min\{p_i, 1\}$, for each $i$. If a consumer inspects market $i$, $q_i < q_j$ and $F_c(q_j^-) > 0$, then he also inspects market $j$. If a consumer inspects fewer markets than his capacity allows, then $F_c(q_i^-) = 0$ for any uninspected market $i$.*

Proposition 1 takes a simple form when leaders' prices are all distinct, are no higher than the consumers' reservation price, and there is positive probability that each market's challenger posts a cheaper price than the leader: *the consumer inspects the $k$ markets with the highest leader prices.* Through a series of results in Section 4, we show that these properties hold in equilibrium for almost all prices quoted by leaders. In the remainder of this section, we use this simple characterization of attention allocation to express firms' incentives.

**Market leaders.** We begin by computing the probability that a leader's market is paid attention to by a consumer with $k$ units of attention, assuming that leader charges the price $p$ and that all other market leaders follow the pricing strategy $F_\ell$. Letting $x = F_\ell(p)$, we denote this probability by $\pi_k^\ell(x)$. Observe that his market receives attention from such a consumer if there are *no more than $k-1$ other markets* whose price turns out to be higher than $p$. Since the probability that another leader charges above $p$ is $1-x$ (which follows from the symmetry of leaders' strategies), we find that[4]

$$\pi_k^\ell(x) := \sum_{i=0}^{k-1} \binom{M-1}{i} x^{M-1-i}(1-x)^i. \tag{2}$$

[4]This amounts to having at most $k-1$ "successes" in $M-1$ trials that are i.i.d., where the probability of "success" (which means finding a price higher than $p$) is $1-F_\ell(p)$.

As expected, $\pi_0^\ell(x) = 0$ and $\pi_M^\ell(x) = 1$. In addition, the probability of being inspected by a given consumer is increasing in his capacity for attention $k$, and increasing with one's price (as captured by $x$).

**Market challengers.** Consider a challenger's probability of selling to a consumer with $k$ units of attention, assuming that he himself charges the price $p$ and that all market leaders follow the pricing strategy $F_\ell$. Letting $x = F_\ell(p)$, we denote this probability by $\pi_k^c(x)$. If a consumer is only partially attentive (that is, $k < M$), then $\pi_k^c(x)$ is not simply $1 - x$, the *ex-ante* probability that the leader's price is higher than $p$. For the challenger, selling requires the consumer to pay attention to the market, an event whose probability is itself impacted by the leader's price. We may compute $\pi_k^c(x)$ as follows. The challenger has zero probability of making a sale if the leader in his market quotes a price strictly less than $p$. If the leader quotes a price $q > p$, then the consumer will purchase from the challenger so long as he inspects the market, which occurs with probability $\pi_k^\ell(F_\ell(q))$. Integrating over the possible prices of the market leader, the desired probability is given by $\int_p^\infty \pi_k^\ell(F_\ell(q))dF_\ell(q)$. This probability depends only on $x = F_\ell(p)$ and not the entire distribution $F_\ell$, as can be seen using the change of variables $t = F_\ell(q)$:

$$\pi_k^c(x) := \int_x^1 \pi_k^\ell(t)dt. \tag{3}$$

As expected, $\pi_0^c(x) = 0$ and $\pi_M^c(x) = 1 - x$. In addition, the probability of selling to a given consumer is increasing in his capacity for attention $k$, and decreasing with the probability $x$ that the leader's price is better.

## 3.2 Equilibrium characterization

It will be helpful to define the total probability that a leader's market draws attention if he charges a price $p$, and the total probability that a market challenger sells if he charges a price $p$. Recalling that $\alpha$ is the distribution of attention among consumers, and letting $x = F_\ell(p)$, those probabilities are

$$\Pi_\ell(x) := \sum_{k=1}^{M} \alpha_k \pi_k^\ell(x) \text{ and } \Pi_c(x) := \sum_{k=1}^{M} \alpha_k \pi_k^c(x),$$

respectively. Since there is a positive measure of partially attentive consumers, $\Pi_\ell$ is strictly increasing and $\Pi_c$ is strictly decreasing; hence their inverses $\Pi_\ell^{-1}$ and $\Pi_c^{-1}$ are well-defined.

**Deriving indifference conditions.** Propositions 2 through 7 in Section 4 show that any equilibrium, if one exists, must satisfy the following properties. First, $\alpha_0$ is the lowest price in the support of both the leaders' and challengers' strategies. Second, both leaders' and challengers' pricing strategies must be atomless. Third, the leaders' strategy has full support over the interval $[\alpha_0, 1]$, while the challenger's highest price $\overline{p}_c$ must be strictly smaller than 1. Given these properties, we can derive firms' equilibrium profits.

A leader is sure to sell to captive consumers as long as his price is less than one. When charging arbitrarily close to one, however, he is nearly certain to lose all non-captive consumers to the challenger (because $F_\ell$ is atomless, and $\bar{p}_c < 1$). Hence a leader's equilibrium profit must be $\alpha_0$. Since the leader sells at the price $p$ either when a consumer does not pay attention, or when he pays attention but the challenger's price is higher, we must have

$$p\Big(1 - \Pi_\ell\big(F_\ell(p)\big) + \Pi_\ell\big(F_\ell(p)\big)\big(1 - F_c(p)\big)\Big) = \alpha_0, \tag{4}$$

for prices $p$ in the support of the leaders' strategy.

Next, a challenger's profit from each price in its support must equal its profit from quoting $\alpha_0$. This profit is given by $\alpha_0 \Pi_c(0)$ , which in turn equals $\alpha_0 EA(\alpha)/M$, where

$$EA(\alpha) := \sum_{k=1}^{M} \alpha_k k$$

is the expected level of attention in the consumer population. Indeed, because the leaders' strategy is atomless and prescribes only prices above $\alpha_0$, the challenger is sure to sell to consumers who pay attention; and given that market leaders all use the pricing strategy $F_\ell$, there is a $k$ out of $M$ chance that his market leader's price will be among the $k$-highest.[5] Therefore, for any price $p$ in the support of $F_c$, it must be that

$$p\Pi_c\big(F_\ell(p)\big) = \frac{\alpha_0 EA(\alpha)}{M}. \tag{5}$$

For any price in the support of a challenger's strategy, the leader's strategy is derived from the indifference condition (5); for all other prices, it is derived from the indifference condition (4), as a function of the (constant) level of $F_c$. In other words,

$$F_\ell(p) = \begin{cases} \Pi_c^{-1}\left(\frac{\alpha_0 EA(\alpha)}{Mp}\right) & \text{for all } p \text{ in the support of } F_c, \\ \\ \Pi_\ell^{-1}\left(\frac{p-\alpha_0}{pF_c(p)}\right) & \text{for all other } p \in [\alpha_0, 1]. \end{cases} \tag{6}$$

The challenger's strategy is also derived from the indifference condition ( 4) for any price in its support. Solving for $F_c$ in (4) and applying the expression for $F_\ell$ above, we see that for each price in the support of the challengers' pricing strategy, $F_c$ must coincide with the function $\tilde{F}_c$ defined by

$$\tilde{F}_c(p) := \frac{p-\alpha_0}{p\Pi_\ell\Big(\Pi_c^{-1}\big(\frac{\alpha_0 EA(\alpha)}{Mp}\big)\Big)}, \text{ for all } p \in [\alpha_0, 1]. \tag{7}$$

**Which prices does a challenger charge?** The difficulty lies in knowing the support of the challengers' strategy, since $\tilde{F}_c$ may be *nonmonotonic* without further restrictions on the attention distribution. Such an example is illustrated in Figure 1. If an equilibrium exists, then any nonmonotonicity in $\tilde{F}_c$ must be "ironed" by introducing one or more gaps in the support of the challengers' strategy.

[5] This can also be seen by applying the Euler integral $\int_0^1 t^{a-1}(1-t)^{b-1}dt = \frac{(a-1)!(b-1)!}{(a+b-1)!}$ in the definition of $\pi_k^c(0)$ to show that it simplifies to $k/M$.

Due to the absence of atoms, $F_c$ must be continuous. Hence any single gap in $F_c$ must be an interval between two prices whose $\tilde{F}_c$ values coincide. In Figure 1, for instance, a gap cannot start at a price lower than $p_1$. On the other hand, there is a range of prices larger than $p_1$ which can serve as the leftmost endpoint of a gap. Remember that the leaders' pricing strategy $F_\ell$ is defined piecewise in (6) according to the challengers' support. Can $\tilde{F}_c$ be ironed in a way that ensures $F_\ell$ is increasing and atomless, as we know it must be? These requirements turn out to be unrestrictive: $F_\ell$ satisfies them whenever $\tilde{F}_c$ is ironed in the continuous manner described above. Thus there are infinitely many ways to construct valid distribution functions $F_\ell$ and $F_c$ which leave the leaders and challengers indifferent over all prices in their respective supports. However, there is a *unique* way to iron $\tilde{F}_c$ that yields *equilibrium* pricing strategies $F_\ell$ and $F_c$. Using any other approach, the challenger would have a profitable deviation to some price outside his support, as explained further below.

**Theorem 1.** *For any distribution of attention $\alpha$, there exists a unique partially symmetric equilibrium. The challengers' pricing strategy $F_c$ is atomless and given by*

$$F_c(p) = \min_{\tilde{p} \in [p,1]} \tilde{F}_c(\tilde{p}), \text{ for all } p \in [\alpha_0, \overline{p}_c], \tag{8}$$

*where $\overline{p}_c \in (\alpha_0, 1)$ is the smallest price for which the above expression equals one, and $\tilde{F}_c$ is given by Equation (7). The leaders' pricing strategy $F_\ell$ has full support on $[\alpha_0, 1]$, is atomless, and given by Equation (6).*

Figure 1

Theorem 1 is proved in Section 4 . There we provide a complete equilibrium analysis, covering some important steps (e.g., ruling out the presence of atoms, characterizing the support of the leader) that have been glossed over in this section when deriving necessary equilibrium conditions. Moreover, we resolve the question of existence by verifying that the construction indeed yields an equilibrium.

To state the characterization of $F_c$ a bit differently, note that among all pricing strategies which lie below the graph of $\tilde{F}_c$, the challengers' strategy is the one which is pointwise highest.

Hence it prescribes the "cheapest" price distribution among those, in the sense of first-order stochastic dominance. Graphically, this means $\tilde{F}_c$ must be ironed as illustrated in Figure 1, by starting any gap at the smallest possible price while still preserving continuity. To understand why this must be the case, consider a price $p$ which is in a gap of the challenger's pricing strategy. In this case, $F_\ell(p)$ is found using the leaders' indifference condition (4). If the challenger charges $p$, his expected profit is

$$p\Pi_c\Big(\Pi_l^{-1}\left(\frac{p-\alpha_0}{pF_c(p)}\right)\Big). \tag{9}$$

Contrary to Theorem 1, suppose that $F_c(p) > \tilde{F}_c(p)$ for the gap price $p$. Since $\Pi_c$ is decreasing and $\Pi_\ell^{-1}$ is increasing, the expression in Equation (9) decreases when replacing $F_c(p)$ with the lower value $\tilde{F}_c(p)$, with the resulting expression simplifying to $\alpha_0 EA(\alpha)/M$, the challengers' equilibrium profit.[6] Hence the challenger would obtain strictly higher profit by charging the gap price $p$ than any price in the support of his strategy.

The presence of a gap in the challengers' strategy depends on the way attention is distributed among consumers. For any attention distribution $\alpha$, the distribution of *partial attention* is

$$(\frac{\alpha_1}{1-\alpha_0}, \ldots, \frac{\alpha_M}{1-\alpha_0}).$$

This is simply $\alpha$ conditioned on consumers being at least partially attentive, that is, $k \geq 1$. The lack of monotonicity in Figure 1 can be attributed to having multiple peaks in the partial attention distribution. Gaps can be ruled out when, given the proportion of consumers with attention span $k$ and the proportion with attention span $k+2$, there are sufficiently many consumers falling in between. More formally, the partial attention distribution is *log-concave* if $\alpha_k^2 \geq \alpha_{k-1}\alpha_{k+1}$ for each $k \in \{2, \ldots, M-1\}$, or equivalently, the likelihood ratio $\alpha_{k+1}/\alpha_k$

[6] For some intuition, note from indifference condition (4) that the more likely are challengers to be cheaper than $p$, the more likely are leaders to be more expensive than $p$. Indeed, leaders' prices increase so that a leader charging $p$ is better shrouded from consumer attention, and can maintain its equilibrium profit against the more competitive challenger. Hence $F_c(p) > \tilde{F}_c(p)$ implies a challenger's market share when charging $p$ is larger than that yielding his equilibrium profit.

is decreasing in $k$. Note that this is trivially satisfied when there are only two markets, and is implied whenever the entire attention distribution is log-concave. When partial attention has this feature, the form of the equilibrium pricing strategies simplifies.

**Theorem 2.** *When $\tilde{F}_c$ is strictly increasing, the challengers' pricing strategy $F_c$ has full support on $[\alpha_0, \bar{p}_c]$ and the leaders' pricing strategy $F_\ell$ simplifies to*

$$F_\ell(p) = \max\left\{\Pi_c^{-1}\Big(\frac{\alpha_0 EA(\alpha)}{Mp}\Big)\ ,\ \Pi_\ell^{-1}\Big(1 - \frac{\alpha_0}{p}\Big)\right\}$$

*for all $p \in [\alpha_0, 1]$. A sufficient condition for $\tilde{F}_c$ to be strictly increasing is log-concavity of the partial attention distribution.*

Theorem 2 is proved in the appendix. Many distributions (and their truncations) satisfy log-concavity. For example, the property is satisfied by a positive binomial distribution, where consumers start with $M$ units of attention but can lose up to $M-1$ of them due to independent, exogenously occurring emergencies (e.g., the consumer's washing machine breaks down, his child gets the flu, his boss asks for overtime, etc.). We note that gaps can also be ruled out under other assumptions on partial attention, such as when the distribution is increasing (that is, $\alpha_k \leq \alpha_{k+1}$ for each $k \geq 1$).[7]

## 3.3 The comparative statics of attention

A natural question that arises from our analysis is how partial attention affects consumer surplus. One can think of at least two reasons why less attention could be detrimental for consumers as a whole. First, leaders might have an incentive to take advantage of less comparison shopping, thereby quoting higher prices. Second, fixing leaders' prices, the challengers realize that partially attentive consumers who approach them do so because

[7] We show in the appendix that gaps can be ruled out when $\Pi_c(0) - \Pi_c(x)$ is strictly log-concave. While $\Pi_c(0) - \Pi_c(x)$ can be written as the sum of log-concave functions, log-concavity is not necessarily preserved by aggregation. We show that log-concavity is preserved if the sequence $\beta_1 = \alpha_M$, $\beta_k = \beta_{k-1} + \sum_{i=1}^{k} \alpha_{M-i+1}$ is log-concave. This is implied, for instance, by log-concavity of partial attention, or by increasingness.

their market's leader is expensive. However, these intuitions ignore a countervailing effect: partial attention introduces a new form of cross-market competition, as each market leader has an incentive to lower its price in order to better deflect consumer attention. Thus, the comparative statics of attention involve subtle interactions between cross-market and within-market competition. In this section, we start by investigating the effects on consumer welfare, before examining the effects on equilibrium pricing.

**Theorem 3.** *Consider two distributions of attention $\alpha$ and $\hat{\alpha}$ which share the same proportion of fully inattentive consumers ($\alpha_0 = \hat{\alpha}_0$). Then consumer welfare is higher under $\alpha$ than $\hat{\alpha}$ if, and only if, the expected level of attention under $\alpha$ is lower than under $\hat{\alpha}$.*

*Proof.* As argued in Section 3.2, a leader's equilibrium expected profit is equal to the proportion of fully inattentive consumers, and is thus the same under both $\alpha$ and $\hat{\alpha}$. As also argued there, a challenger's equilibrium expected profit is equal to the proportion of fully inattentive consumers, multiplied by the expected level of attention, divided by $M$. Hence producer surplus is lower under $\alpha$ than $\hat{\alpha}$ if, and only if, the expected level of attention under $\alpha$ is lower than under $\hat{\alpha}$. The result then follows from the fact that total surplus remains constant (equal to $M$). ∎

Neither fully attentive nor fully inattentive consumers generate competition for inattention. While fully attentive consumers do generate within-market competition, fully inattentive consumers are simply captive to market leaders. As might be expected, increasing the proportion $\alpha_0$ of captive consumers has a negative effect on consumer surplus.[8] At the opposite end of the attention spectrum, Theorem 3 means that making fully attentive consumers less attentive benefits consumers as a whole. In particular, the closer the attention distribution is to the limit distribution $(\alpha_0, 1-\alpha_0, 0, \ldots, 0)$, the better off consumers are; similarly, the closer is the distribution to the limit distribution $(\alpha_0, 0, \ldots, 0, 1-\alpha_0)$, the worse off consumers are.

[8]Increasing $\alpha_0$ at the expense of reducing $(\alpha_1, \ldots, \alpha_M)$ by the infinitesimal amounts $(\varepsilon_1, \ldots, \varepsilon_M)$ has a total effect on producer surplus of $\sum_{i=1}^{M} \varepsilon_i (M + EA(\alpha) - \alpha_0 i) > 0$.

To gain some intuition for Theorem 3, remember that in equilibrium, leaders are willing to quote prices that are more expensive than what a challenger would ever charge. When charging such a price $p$, a leader's profit, given by $p(1-\Pi_\ell(F_\ell(p)))$, relies on *not* drawing too much consumer attention. Suppose partial attention decreases. If the other leaders' pricing strategy were to remain unchanged, then the leader's profit from quoting $p$ would rise above $\alpha_0$. Yet competition implies that no leader can make a profit that large. Hence the likelihood of having other leaders quote prices smaller than $p$ must go up, so that the leader quoting $p$ "sticks out" with sufficient probability.

The pricing effects of a change in partial attention may be more ambiguous for lower prices, as leaders become competitive against the challengers. Building on the insight from Theorem 2, we focus on cases where $\tilde{F}_c$ is strictly increasing and show that the leaders' pricing strategies are comparable under first-order stochastic dominance when the change in partial attention can be ranked in the monotone likelihood ratio order. Given two attention distributions $\alpha$ and $\hat{\alpha}$, we say that the partial attention distribution under $\alpha$ dominates the partial attention distribution under $\hat{\alpha}$ in the *monotone likelihood ratio order (MLR)* if $\hat{\alpha}_k/\alpha_k$ is increasing in $k \in \{1, \ldots, M\}$, with at least one strict inequality. The MLR ordering has a long tradition in economics, starting with Milgrom (1981), and is known to be stronger than first-order stochastic dominance.

**Theorem 4.** *Let $\alpha$ and $\hat{\alpha}$ be two attention distributions with $\alpha_0 = \hat{\alpha}_0$ and log-concave partial attention distributions. If the partial attention distribution under $\hat{\alpha}$ dominates that under $\alpha$ in the MLR order, then market leaders' equilibrium prices are first-order stochastically higher under $\hat{\alpha}$ than under $\alpha$.*

More generally, Theorem 4 remains true when replacing the log-concavity requirement with any conditions on $\alpha$ and $\hat{\alpha}$ guaranteeing that the challengers' strategy has no gap (e.g., as in footnote 7). For intuition on why the result holds, remember that $\Pi_\ell$ and $\Pi_c$ (the probabilities that a leader's market receives attention and that a challenger makes a sale) depend on the attention distribution. In what follows, $\Pi_\ell$ and $\Pi_c$ correspond to the attention distribution $\alpha$, while $\hat{\Pi}_\ell$ and $\hat{\Pi}_c$ correspond to the attention distribution $\hat{\alpha}$. Recall from Theorem 2 that the probability $F_\ell(p)$ that a leader charges a price lower than $p$ under attention distribution $\alpha$ is simply

$$\max\Big\{\Pi_c^{-1}\big(\frac{\alpha_0 EA(\alpha)}{Mp}\big)\ ,\ \Pi_\ell^{-1}\big(1-\frac{\alpha_0}{p}\big)\Big\}, \tag{10}$$

when the partial attention distribution is log-concave. An analogous expression describes the probability $\hat{F}_\ell(p)$ that a leader charges a price lower than $p$ under attention distribution $\hat{\alpha}$. This is illustrated in Figure 2. The theorem is proved by showing that each of the two expressions on the right-hand side of (10) shifts *downwards* when consumer attention increases from $\alpha$ to $\hat{\alpha}$. Consequently, market leaders charge first-order stochastically higher prices when attention increases. The downward shift for the second expression in (10), which relates to the intuition given earlier for prices above the challengers' support, actually holds for any first-order stochastic increase in partial attention. The downward shift in the first expression in (10) is less obvious, and holds for MLR shifts.[9] Figure 2

Changes in partial attention have a more ambiguous effect on the challengers' pricing strategy. Since consumer welfare increases when there is less attention, it is clear that challengers cannot increase their prices by too much. As we next illustrate, when there are just two markets, log-concavity of the partial attention distribution is trivially satisfied, and MLR-dominance reduces to first-order stochastic dominance. In that case, one can show that *both* leaders' and challengers' prices decrease when partial attention decreases. More generally, however, it is unclear whether the challengers' strategy shifts according to first-order stochastic dominance.

[9]This sufficient condition is not necessary, as the attention distributions used for Figure 2 do not have the MLR property but do have the critical feature that $\hat{\Pi}_c/\Pi_c$ decreases.

## 3.4 Illustration: the case of two markets

We illustrate the equilibrium pricing strategies in the case $M = 2$. A leader's probability of drawing the attention of a consumer with $k$ units of attention (defined in Equation (2)), and a market challenger's probability of selling to such a consumer (defined in Equation (3)), take a simple form:

$$\pi_1^\ell(x) = x, \quad \pi_2^\ell(x) = 1, \quad \pi_1^c(x) = \frac{1 - x^2}{2} \text{ and } \pi_2^c(x) = 1 - x.$$

The challengers' indifference condition (5) reduces to

$$\alpha_1 \frac{1 - F_\ell(p)^2}{2} + \alpha_2(1 - F_\ell(p)) = \frac{\alpha_0 EA(\alpha)}{Mp},$$

$$\text{or} \quad F_\ell(p) = \frac{-\alpha_2 + \sqrt{\alpha_2^2 + \alpha_1 EA(\alpha)(1 - \frac{\alpha_0}{p})}}{\alpha_1}$$

for any price $p$ in the support of $F_c$. Similarly, plugging $F_\ell(p)$ into the leaders' indifference condition (4) and solving for $F_c(p)$ gives:

$$F_c(p) = \frac{1 - \frac{\alpha_0}{p}}{\sqrt{\alpha_2^2 + \alpha_1 EA(\alpha)(1 - \frac{\alpha_0}{p})}},$$

for any price $p$ in the support of $F_c$. It is easy to check that the right-hand side, which is $\tilde{F}_c$, is increasing for any $p$ greater than $\alpha_0$. This is consistent with Theorem 2, since the log-concavity condition is satisfied for any distribution of attention when $M = 2$. The challengers' strategy therefore has no gap, and the maximal price in the support of $F_c$ is the price $\bar{p}_c \in (\alpha_0, 1)$ at which $F_c$ reaches 1.[10] Straightforward algebra gives

$$\bar{p}_c = \frac{2\alpha_0}{2 - \alpha_1 EA(\alpha) - \sqrt{\alpha_1^2 EA(\alpha)^2 + 4\alpha_2^2}}.$$

All that remains is to find the leaders' strategy $F_\ell$ for prices between $\bar{p}_c$ and 1. This follows from the leaders' indifference condition (4), which gives:

$$F_\ell(p) = \frac{1 - \frac{\alpha_0}{p} - \alpha_2}{\alpha_1}$$

for each $p \in [\bar{p}_c, 1]$.

To perform comparative statics, note that when $M = 2$, increasing attention while keeping the proportion of captive consumers fixed simply amounts to shifting weight from $\alpha_1$ to $\alpha_2$. All such shifts are comparable in the MLR order. By Theorem 4, $F_\ell(p)$ must decrease when shifting weight from $\alpha_1$ to $\alpha_2$. This can also be checked directly given the expression of $F_\ell$ in the previous paragraph. While the effect on $F_c$ is ambiguous for general $M$, in the case $M = 2$ the challengers' prices also first-order stochastically increase when shifting weight from $\alpha_1$ to $\alpha_2$. Indeed, one can check that

$$\frac{dF_c(p)}{d\alpha_2} - \frac{dF_c(p)}{d\alpha_1} = -\frac{\alpha_0\alpha_2}{p} \frac{1 - \frac{\alpha_0}{p}}{\left(\alpha_2^2 + \alpha_1 EA(\alpha)(1 - \frac{\alpha_0}{p})\right)^{3/2}},$$

which is negative for $p \in [\alpha_0, 1]$.

# 4 Complete equilibrium analysis

Building on the characterization of consumer attention in Proposition 1, we first develop a series of necessary conditions on firms' equilibrium pricing strategies that uniquely pin down the equilibrium, if one exists. We then resolve the matter of existence by checking that the

[10] Notice that $F_c$ is increasing, $F_c(\alpha_0) = 0$, and $F_c(1) = \frac{1-\alpha_0}{\sqrt{\alpha_2^2+\alpha_1 EA(\alpha)(1-\alpha_0)}} = \frac{1-\alpha_0}{\sqrt{(1-\alpha_0)^2-\alpha_0\alpha_1 EA(\alpha)}} > 1$, where the second equality follows from $\alpha_2^2 + \alpha_1 EA(\alpha) = (1-\alpha_0)^2$.

construction works.

## 4.1 Necessary conditions

We begin with a useful observation about the supports of the challengers' and leaders' strategies.

**Proposition 2.** *The lowest price in the support of $F_\ell$ and $F_c$ coincide, and is greater than or equal to $\alpha_0$. The highest prices in the support of $F_\ell$ and $F_c$ are both smaller than or equal to one.*

*Proof.* A market leader is sure to sell to inattentive consumers, even when charging the reservation price of 1. He can thus guarantee himself a profit of at least $\alpha_0$. Any price below $\alpha_0$ or above 1 generates a profit strictly less than $\alpha_0$. Hence a leader would not choose a strategy for which $F_\ell(1) < 1$ or $F_\ell(p) > 0$, for some $p < \alpha_0$.

Let $p_\ell$ be the lowest price in the support of $F_\ell$ and let $p_c$ be the lowest price in the support of $F_c$. Suppose $p_\ell < p_c$. Consider a deviation by some leader to a pricing strategy $F'_\ell$ that puts an atom equal to $F_\ell(p')$ on some price $p' \in (p_\ell, p_c)$ and coincides with $F_\ell$ for all $p > p'$. To see that this deviation increases the leader's profit note that for each price $p \in [p_\ell, p')$ in the support of the original strategy $F_\ell$ the deviant leader will now sell at a higher price. This is true whether or not its market is inspected, since at a price of $p'$ it still undercuts the challenger. Suppose next that $p_c < p_\ell$ and that a challenger deviates to a strategy $F'_c$ that puts an atom equal to $F_c(p')$ on some price $p' \in (p_c, p_\ell)$ and coincides with $F_c$ for all $p > p'$. Then conditional on being inspected, for each price $p \in [p_c, p')$ in the support of the original strategy $F_c$ the challenger would sell at a strictly higher price. Since there are fully attentive consumers who will inspect the market, this deviation raises the challenger's expected profits.

It remains to show that the largest price in the support of $F_c$ is smaller or equal to 1. Any price above 1 does not yield a sale, as it is higher than the consumers' reservation price. In this case, as he can sell to at least some fully attentive consumers, any positive price below $\alpha_0$ constitutes a profitable deviation for the challenger. ∎

We next argue that $F_\ell$ is atomless. If leaders have an atom at a price strictly above the lowest price $\underline{p}$ in their support, then some *leader* could profitably deviate by moving mass from this price to one which is "slightly" below it. This small price decrease is more than compensated by the decreased attention to the leader's market. However, if the leaders' atom is on $\underline{p}$, we must distinguish between two cases. If the challenger's strategy does not have an atom at $\underline{p}$, or if some consumers favor the leader in case of a tie at $\underline{p}$, then the *challenger* could profitably deviate by shifting weight to prices slightly below $\underline{p}$. Otherwise, a *leader* can profitably deviate for the same reasons as given above.

**Proposition 3.** *The leaders' pricing strategy $F_\ell$ is atomless.*

*Proof.* Let $\underline{p}$ be the smallest price in the support of $F_\ell$, and suppose $F_\ell$ has an atom at $p \in (\underline{p}, 1]$. For any small $\varepsilon > 0$, consider the alternate pricing strategy for the leader which equals $F_\ell(p)$ for all $q \in [p - \varepsilon, p]$, and coincides with $F_\ell$ elsewhere. This deviation has two opposite effects on the leader's profit. There is a negative effect from selling at a price $p - \varepsilon$ compared to those prices $q \in (p - \varepsilon, p]$. This loss is of order $\varepsilon$ and can be made as small as desired by decreasing $\varepsilon$. In view of Proposition 1, there is also a positive effect from the decrease in attention when charging $p - \varepsilon$ rather than a price in $(p - \varepsilon, p]$. This gain admits a strictly positive lower bound that is independent of $\varepsilon$ for the leader of at least one market. To see this, consider a leader whose market is inspected with positive probability when all leaders quote $p$. The probability that all other leaders charge $p$ and that one's challenger has a price strictly less than $p$ occurs with probability $F_c(p^-)(F_\ell(p) - F_\ell(p^-))^{M-1}$, which is positive since $p$ is an atom and $F_c(p^-) > 0$ by Proposition 2. For the fraction $1 - \alpha_0 - \alpha_M$ of partially attentive consumers, if the deviator charges $p - \varepsilon$ they surely do not pay attention to his market, while if he charges $p$ the probability of drawing attention is strictly positive (and independent of $\varepsilon$). Hence this deviation is strictly profitable for $\varepsilon > 0$ small enough.

Suppose now that $F_\ell$ has an atom at $\underline{p}$, which is also the lowest price in the support of $F_c$ by Proposition 2. For any small $\varepsilon > 0$, consider the alternate pricing strategy for the challenger which equals $F_c(\underline{p}+\varepsilon)$ for all $q \in [\underline{p}-\varepsilon, \underline{p}+\varepsilon]$, and coincides with $F_c$ elsewhere. This deviation has two opposite effects on the challenger's profit. There is a negative effect from selling at a lower price $\underline{p}-\varepsilon$ compared to those $q \in (\underline{p}, \underline{p}+\varepsilon]$; this results in a decrease in profit of no more than $2\varepsilon F_c(\underline{p}+\varepsilon)$. There is also a positive effect occurring in the event that the market draws attention when the leader's price is $\underline{p}$, which occurs with strictly positive probability (independent of $\varepsilon$). In this event, the deviation yields a sale at the price $\underline{p}-\varepsilon$ with probability $F_c(\underline{p}+\varepsilon)$, while the original strategy yields a sale at the price $\underline{p}$ with probability $F_c(\underline{p})\beta$, where $\beta \in [0,1]$ is the proportion of consumers who purchase from the challenger when there is a tie at $\underline{p}$. The challenger's alternate strategy is a profitable deviation for $\varepsilon > 0$ small enough if either $F_c(\underline{p}) = 0$ (that is, the challenger does not have an atom at $\underline{p}$), or there is an atom at $\underline{p}$ and $\beta < 1$. To conclude the proof, suppose that both $F_c$ and $F_\ell$ have an atom at $\underline{p}$ and $\beta = 1$. In that case, consider the alternate pricing strategy for the leader which equals $F_\ell(\underline{p})$ for all $q \in [\underline{p}-\varepsilon, \underline{p}]$, and coincides with $F_\ell$ for higher prices. The loss from selling at a lower price can be made arbitrarily small, while the gain in winning against the challenger is bounded from zero. Indeed, there is positive probability that the market is inspected, and $\beta = 1$ implies the challenger wins in the event of a tie at $\underline{p}$, an event occurring with probability $F_\ell(\underline{p})F_c(\underline{p}) > 0$ under $F_\ell$. This is thus a profitable deviation from $F_\ell$, a contradiction. ∎

As a consequence of the previous two propositions, Proposition 1 now takes the simpler form, "the consumer inspects the $k$ markets with the highest leader prices." Our next result is concerned with the highest prices firms could charge. Challengers, who make their profit by underbidding their market leader, certainly would not charge more than a leader's highest price. We show, furthermore, that challengers charge strictly less. Since we have not yet ruled out the possibility that $F_c$ has an atom at its highest price (or elsewhere), the strict ranking of highest prices is helpful to derive the leaders' highest price. Whenever a leader charges his highest price, any consumer who is at least partially attentive will inspect his market, and find a cheaper alternative, with probability one. As such, the leader may as well take full advantage of the remaining consumers' inattention, by charging all the way up to their reservation price.

**Proposition 4.** *The highest price in the support of $F_c$ is strictly smaller than the highest price in the support of $F_\ell$, which is one.*

*Proof.* Let $\overline{p}_\ell$ ($\overline{p}_c$) be the highest price in the support of $F_\ell$ (respectively, $F_c$). Since $F_\ell$ is atomless, there exists $\varepsilon > 0$ small enough that the probability a leader charges more than $\overline{p}_\ell - \varepsilon$ is strictly smaller than $\alpha_0$. Thus the challenger's profit from charging any price above $\overline{p}_\ell - \varepsilon$ is strictly smaller than the profit obtained by charging $\alpha_0$, given that he cannot affect the attention to his market. Since the challenger would have a profitable deviation if $F_c(\overline{p}_\ell - \varepsilon) < 1$, we conclude that $\overline{p}_c < \overline{p}_\ell$.

We now show that the leaders' highest price is one. If $\overline{p}_\ell < 1$, then for each $\varepsilon > 0$, consider the alternate pricing strategy for a market leader which equals $F_\ell(\overline{p}_\ell - \varepsilon)$ for all $q \in [\overline{p}_\ell - \varepsilon, 1 - \varepsilon)$ and coincides with $F_\ell$ elsewhere. For $\varepsilon$ small enough, $\overline{p}_\ell - \varepsilon$ is larger than the highest price in the support of $F_c$. By charging $1 - \varepsilon$ instead of $p \in [\overline{p}_\ell - \varepsilon, \overline{p}_\ell)$, the leader has a gain of at least $\alpha_0(1 - \varepsilon - \overline{p}_\ell)$, since fully inattentive consumers buy from the leader at any price below their reservation level. The leader's loss from this deviation is proportional to the increase in probability of having partially attentive consumers check his market (thereby finding a cheaper price). As $F_\ell$ atomless, when $\varepsilon$ is small then it is almost certain that when charging $p \in [\overline{p}_\ell - \varepsilon, \overline{p}_\ell)$ his market was already being checked by these consumers. Thus, there is $\varepsilon$ sufficiently small that for any $p \in [\overline{p}_\ell - \varepsilon, \overline{p}_\ell)$, the loss is strictly less than the gain $\alpha_0(1 - \varepsilon - \overline{p}_\ell)$. The expected change in profit from deviating, obtained by integrating gains minus losses over $p \in [\overline{p}_\ell - \varepsilon, \overline{p}_\ell)$, is thus strictly positive. ∎

The above results allow us to derive the leaders' equilibrium profit, as shown in Section 3.2.

**Corollary 1.** *The leaders' equilibrium profit is* $\alpha_0$.

With Corollary 1 in mind, it becomes possible to identify the common lowest price of challengers and leaders.

**Proposition 5.** *The lowest price in the support of both* $F_\ell$ *and* $F_c$ *is* $\alpha_0$.

*Proof.* We know that $F_\ell$ and $F_c$ share a common lowest price $\underline{p} \geq \alpha_0$. Suppose by contradiction that $\underline{p} > \alpha_0$. Consider a deviation where the leader charges $(p + \alpha_0)/2$ with probability one. In this case, the leader sells to all consumers, whether or not they pay attention to his market. This delivers a profit of $(p + \alpha_0)/2$. Since equilibrium profit is $\alpha_0$ by Corollary 1, the deviation is strictly profitable. ∎

As explained in Section 3.2, the above results can be used to derive the equilibrium profit of challengers.

**Corollary 2.** *The challengers' equilibrium profit is* $\alpha_0 EA(\alpha)/M$.

The following result rules out atoms for the challenger. Of course, Propositions 3 and 6 imply that no firm can use a pure strategy in equilibrium.

**Proposition 6.** *The challengers' pricing strategy* $F_c$ *is atomless.*

*Proof.* Suppose that $F_c$ has an atom at some price $p > \alpha_0$. We begin by pointing out that there cannot exist $\varepsilon > 0$ for which $F_\ell(p + \varepsilon) - F_\ell(p) = 0$. Otherwise, $F_\ell$ has a gap in its support to the right of $p$, and the challenger could profitably deviate by shifting his atom from $p$ to $p + \varepsilon$.

Consider an alternate strategy for the leader which equals $F_\ell(p + \varepsilon)$ for $q \in [p - \varepsilon, p + \varepsilon]$ and is given by $F_\ell$ elsewhere. For each $q \in [p - \varepsilon, p + \varepsilon]$, the only loss associated with this deviation is the decrease in price, which is at most $2\varepsilon$. Among the various gains in profit from switching is the increased probability of selling by underbidding the challenger when the market is examined. Notice that the leader's market is examined with a probability bounded from below by $\alpha_M$. Thus there is positive probability, bounded from zero, both that the market is inspected and that the challenger quotes $p$. In this joint event, the gain by charging $p - \varepsilon$ instead of any $q \in (p, p + \varepsilon)$ is strictly positive, since the leader sells to an inspecting consumer when charging $p - \varepsilon$, but not when charging $q$. Hence, for $\varepsilon$ small enough, this deviation is strictly profitable for the leader, a contradiction. We conclude $F_c$ is atomless for prices above $\alpha_0$.

Finally, suppose by contradiction that $F_c$ has an atom at $\alpha_0$. Since $\alpha_0$ also belongs to the support of $F_\ell$, the leader must get a profit $\alpha_0$ by charging any price $p \in (\alpha_0, \alpha_0 + \varepsilon)$. However, for any such price there is probability larger than $\alpha_M F_c(\alpha_0) > 0$ that the leader does not sell. Hence the profit from any such $p$ is bounded away from $\alpha_0$ for small $\varepsilon$, a contradiction. ∎

We now examine whether firms necessarily use *strictly* increasing strategies. While for market leaders the answer is a clear yes, for market challengers the answer depends on the distribution of consumer types. This contrasts with the previous literature on competition with mixed strategies over prices, in which all firms use strictly increasing cumulative distribution functions.

**Proposition 7.** *The leaders' strategy $F_\ell$ cannot have any gaps in its support.*

*Proof.* Suppose that $F_\ell$ has a gap in its support, that is, $F_\ell$ is constant over an interval inside $[\alpha_0, 1]$. Consider then $p'$ and $p''$ with $F_\ell(p') = F_\ell(p'')$ such that for all $\varepsilon > 0$, $F_\ell(p') > F_\ell(p' - \varepsilon)$ and $F_\ell(p'' + \varepsilon) > F_\ell(p'')$. In other words, $p'$ is the left-most point of the gap, and $p''$ is the right-most point of the gap. We know that $\alpha_0 < p' < p'' < 1$ since $\alpha_0$ and 1 belong to the support of $F_\ell$, which is atomless. Notice that $F_c$ must also be constant on $[p', p'')$. Otherwise, any mass placed on that interval by $F_c$ can be moved to an atom at $p''$. Indeed, this deviation does not change the set of events where the challenger sells (which has positive measure), and only increases the price of sale.

Because $F_\ell$ and $F_c$ have been shown to be atomless, the profit a leader obtains when charging the price $p$ is

$$p[1 - \Pi_\ell(F_\ell(p)) + \Pi_\ell(F_\ell(p))(1 - F_c(p))].$$

Notice that this expression is strictly larger at $p = p''$ than it is at $p = p'$, since $p'' > p'$, $F_\ell(p') = F_\ell(p'')$ and $F_c(p') = F_c(p'')$. This contradicts the fact that both $p'$ and $p''$ are in the support of $F_\ell$. ∎

There were two key steps in proving Proposition 7. First, if leaders have a gap extending from $p'$ to $p''$, then the support of $F_c$ does not contain that interval either. Next, if $F_c(p') = F_c(p'')$, then a leader would strictly prefer to charge $p''$ than prices equal to or nearby $p'$, contradicting that $p'$ is in the support. One might think that we could analogously prove that $F_c$ has no gap, simply by starting the argument at the second step. However, the contradiction relied on there being a gap in the leaders' strategy ($F_\ell(p') = F_\ell(p'')$) to show that charging $p''$ does not significantly increase attention relative to charging $p'$, and that more profits can therefore be made. Indeed, if other leaders do charge prices in $(p', p'')$ with positive probability, then charging $p''$ instead of $p'$ can yield a significant increase in attention. The resulting loss might overwhelm the gains from a higher price.

The above results allow us to now complete our characterization of the leaders' and challengers' equilibrium pricing strategies. Given that there is zero probability of ties, and given our knowledge of equilibrium profits and firms' highest and lowest prices, the indifference conditions for equilibrium indeed correspond to (4) and (5). Consequently, $F_\ell$ must be given by

$$F_\ell(p) = \begin{cases} \Pi_c^{-1}\left(\frac{\alpha_0 EA(\alpha)}{Mp}\right) & \text{for all } p \text{ in the support of } F_c, \\ \\ \Pi_\ell^{-1}\left(\frac{p-\alpha_0}{pF_c(p)}\right) & \text{for all other } p \in [\alpha_0, 1], \end{cases}$$

as claimed in Theorem 1. Moreover, for any price in the challengers' support, $F_c$ must coincide with the function $\tilde{F}_c$, which is defined in (7) by

$$\tilde{F}_c(p) = \frac{p - \alpha_0}{p\Pi_\ell\Big(\Pi_c^{-1}\big(\frac{\alpha_0 EA(\alpha)}{Mp}\big)\Big)}.$$

Proving that $F_c(p) = \min_{\tilde{p}\in[p,1]} \tilde{F}_c(\tilde{p})$ for all prices in $[\alpha_0, \bar{p}_c]$, as claimed in Theorem 1, requires one more result.

**Proposition 8.** *If the price $p$ is in the support of the challengers' strategy, then $\tilde{F}_c(p) \leq \tilde{F}_c(\tilde{p})$ for all $\tilde{p} \in [p, 1]$.*

*Proof.* This is immediate if $\tilde{p}$ is also in the support of $F_c$, since in that case $\tilde{F}_c(p) = F_c(p) \leq F_c(\tilde{p}) = \tilde{F}_c(\tilde{p})$, with the inequality following from $p < \tilde{p}$. Suppose then that $\tilde{p}$ is not in the support of $F_c$ and, by contradiction, that $\tilde{F}_c(\tilde{p}) < \tilde{F}_c(p)$. The challenger's profit when charging $\tilde{p}$ is given by

$$\tilde{p}\Pi_c(F_\ell(\tilde{p})) = \tilde{p}\Pi_c\Big(\Pi_\ell^{-1}\big(\frac{\tilde{p} - \alpha_0}{\tilde{p}F_c(\tilde{p})}\big)\Big).$$

Since $p < \tilde{p}$ and $p$ is in the support of the challengers' strategy, $\tilde{F}_c(p) = F_c(p) \leq F_c(\tilde{p})$. Hence $\tilde{F}_c(\tilde{p}) < F_c(\tilde{p})$. Since $\Pi_\ell$ is strictly increasing and $\Pi_c$ is strictly decreasing,

$$\tilde{p}\Pi_c(F_\ell(\tilde{p})) > \tilde{p}\Pi_c\Big(\Pi_\ell^{-1}\big(\frac{\tilde{p} - \alpha_0}{\tilde{p}\tilde{F}_c(\tilde{p})}\big)\Big).$$

Applying the definition of $\tilde{F}_c$, we conclude that

$$\tilde{p}\Pi_c\Big(\Pi_\ell^{-1}\big(\frac{\tilde{p} - \alpha_0}{\tilde{p}\tilde{F}_c(\tilde{p})}\big)\Big) = \frac{\alpha_0 EA(\alpha)}{M},$$

which is the challenger's equilibrium profit. Hence $F_c$ could not be part of an equilibrium, since charging $\tilde{p}$ would be a strictly profitable deviation. ∎

The characterization of $F_c$ in Theorem 1 now follows. Indeed, for any price $p$ in the support of $F_c$, we know that $F_c(p) = \tilde{F}_c(p)$. By Proposition 8, it must be that $\tilde{F}_c(p) \leq \tilde{F}_c(\tilde{p})$ for all $\tilde{p} > p$, proving the desired characterization for those prices that the challenger employs.

But the characterization also holds for any price $p < \bar{p}_c$ which is part of a gap in the support of $F_c$. To see this, observe that the leftmost endpoint of the gap (denoted $p_1$) and the rightmost endpoint of the gap (denoted $p_2$) do belong to the support of $F_c$, and so the desired characterization holds for them. Because $F_c$ is atomless, $F_c(p_1) = F_c(p) = F_c(p_2)$, which squeezes $F_c(p)$ to the desired value.

## 4.2 Establishing existence

The above results establish that $F_\ell$ and $F_c$ are the unique candidates for an equilibrium. To prove existence, we begin with a technical result whose proof appears in the appendix.

**Proposition 9.** *The pricing strategies $F_\ell$ and $F_c$ are well-defined, atomless cumulative distribution functions. Moreover, $F_\ell$ is strictly increasing over $[\alpha_0, 1]$, and $\alpha_0$ is the lowest price in the support of $F_\ell$ and $F_c$.*

It remains to show that neither leaders nor challengers have a profitable deviation given consumers' optimal allocation of attention (which is described in Proposition 1). The construction of $F_c$ ensures that quoting prices in the support of $F_\ell$ gives the leader a profit of $\alpha_0$. Quoting a price above 1 or a price below $\alpha_0$ thus yields the leader a strictly smaller profit. The construction of $F_\ell$ ensures that quoting any price in the support of the challenger's strategy yields a profit of $\alpha_0 EA(\alpha)/M$. Since $EA(\alpha)/M$ is the expected proportion of consumers checking his market, the challengers' profit is clearly larger than that attained by quoting a price smaller than $\alpha_0$. We now prove that quoting any price $p \geq \alpha_0$ which is not in the support of $F_c$ also yields a smaller profit. Consider any $p$ outside the support of $F_c$. Since $F_c(p) \leq \tilde{F}_c(p)$,

$$F_\ell(p) = \Pi_\ell^{-1}\Big(\frac{p - \alpha_0}{pF_c(p)}\Big) \geq \Pi_\ell^{-1}\Big(\frac{p - \alpha_0}{p\tilde{F}_c(p)}\Big).$$

Applying the decreasing function $\Pi_c$ on both sides, multiplying by $p$, and plugging in the definition of $\tilde{F}_c$, we find that $p\Pi_c(F_\ell(p)) \leq \alpha_0 EA(\alpha)/M$. In other words, the challenger cannot obtain a higher profit by deviating to $p$.

# 5 Conclusion

This paper proposes a stylized model of price competition, with consumers optimally deciding which components of their expenses to audit given bounds on their attention. In the classic framework, where consumers are fully attentive, the cross-market implications of prices are limited to income and substitution effects. Limited attention brings a new dimension to competition, with the prices of the most visible firms exerting an externality on other markets by deflecting or drawing consumers' attention. Taking into account the firms' equilibrium response, decreasing the average attention level benefits consumers through competition for their inattention.

Our model suggests interesting new avenues for exploration. A first direction would be to embed the model into a dynamic framework to determine endogenously which firms serve as default providers. Competition for inattention may be exacerbated, with default providers further lowering their prices, as the benefit of remaining in their position increases the incentive to be under the consumers' radar. A second direction would be to further investigate consumers' optimal allocation of attention in heterogeneous markets. Inspecting markets with the highest expected savings may translate into more intricate attention strategies.[11] A third direction would be to include multiple challengers in each market. Our assumption of a single challenger is a reduced-form representation of friction in identifying challengers and learning their offers. In a more general model, sampling each additional challenger's price would deplete some of the consumer's budget for attention. One can then study the tradeoff between allocating attention across markets versus within markets. A consumer would allocate each additional unit of attention to the market with the highest expected

[11] Note that the notion of partially symmetric strategies may extend in some circumstances to markets with heterogeneity. Indeed, suppose that firms know characteristics (such as cost or reservation price) which are relevant for their own market, but all face the same uncertainty regarding those characteristics in other markets. A strategy for a firm is a function that maps the characteristics of its market into a price cdf. Partial symmetry of such strategies means that firms in the same market position use the same function.

savings given the prices he has observed so far. A fourth direction would be to consider general preferences, allowing for complementarity and non-satiation, to investigate the effect that competition for inattention has on the total surplus.

We hope that the present paper motivates researchers to investigate these questions, and will be useful for further analysis of consumers' optimal allocation of attention and the implications for price theory.

# Appendix

### Proof of Proposition 9

We begin with $F_c$. Observe that $\alpha_0 EA(\alpha)/Mp$ belongs to the range of $\Pi_c$ for any $p \in [\alpha_0, 1]$, and that the domain of $\Pi_\ell$ is $[0,1]$. Hence both $\tilde{F}_c$ and $F_c$ are well-defined at any such $p$. We argue that $F_c$ is a valid distribution function. It is increasing and continuous by construction. Moreover, it is easy to see that $\tilde{F}_c(\alpha_0) = 0$, as the numerator is zero and the denominator is nonzero: observe that $\Pi_c^{-1}(EA(\alpha)/M) = 0$ and $\Pi_\ell(0) = \alpha_M > 0$. It remains to show that $F_c(\bar{p}_c) = 1$ for some $\bar{p}_c \in (\alpha_0, 1)$, which itself follows if there exists a largest price strictly smaller than one such that $\tilde{F}_c$ equals one. Such a price exists by the Intermediate Value Theorem, because $\tilde{F}_c$ is continuous, with $\tilde{F}_c(\alpha_0) < 1$ and $\tilde{F}_c(1) > 1$. To see the last fact, suppose to the contrary that $\tilde{F}_c(1)$ were less than or equal to one. In that case, we would have $\Pi_c^{-1}(\alpha_0 EA(\alpha)/M) \geq \Pi_\ell^{-1}(1-\alpha_0) = 1$, which is impossible because $\Pi_c$ is strictly decreasing and satisfies $\Pi_c(1) = 0$. It can be checked by elementary calculus that $\tilde{F}_c'(\alpha_0) > 0$, so $\alpha_0$ is in the support of $F_c$.

We next show that $F_\ell$ is well-defined. Again, because $\alpha_0 EA(\alpha)/Mp$ belongs to the range of $\Pi_c$ for any $p \in [\alpha_0, 1]$, we know that $F_\ell$ is well-defined whenever $p$ belongs to the support of $F_c$. Consider then a price $p \in [\alpha_0, 1]$ that does not belong to the support of $F_c$. Since $F_c(p) \leq \tilde{F}_c(p)$, we have

$$\frac{p-\alpha_0}{pF_c(p)} \geq \frac{p-\alpha_0}{p\tilde{F}_c(p)} = \Pi_\ell\Big(\Pi_c^{-1}\big(\frac{\alpha_0 EA(\alpha)}{Mp}\big)\Big),$$

which is greater than or equal to $\alpha_M$, as desired. Moreover, we claim that $(p-\alpha_0)/pF_c(p) \leq 1-\alpha_0$. This is obvious if $F_c(p) = 1$. If $F_c(p) < 1$, then there exists some $p' > p$ in the support of $F_c$ such that $F_c(p') = F_c(p)$. Hence

$$\frac{p-\alpha_0}{pF_c(p)} \leq \frac{p'-\alpha_0}{p'F_c(p')} = \Pi_\ell\Big(\Pi_c^{-1}\big(\frac{\alpha_0 EA(\alpha)}{Mp'}\big)\Big),$$

which is less than or equal to $1 - \alpha_0$, as desired. Therefore, $(p - \alpha_0)/pF_c(p)$ also belongs to the range of $\Pi_\ell$, ensuring that $F_\ell$ is also well-defined for prices $p \in [\alpha_0, 1]$ outside of the support of $F_c$.

Finally, we show that $F_\ell$ is an atomless and gapless cumulative distribution function. Since $\alpha_0$ is in the support of $F_c$, we have $F_\ell(\alpha_0) = 0$. Since 1 is not in the support of $F_c$, we conclude that $F_\ell(1) = \Pi_\ell^{-1}(1 - \alpha_0) = 1$. We complete the proof by showing that $F_\ell$ as defined in (6) is continuous and strictly increasing over $[\alpha_0, 1]$, which also proves that $\alpha_0$ is in its support. Since $\Pi_c^{-1}$ is strictly decreasing and $\Pi_\ell^{-1}$ is strictly increasing, each of the two functions defining $F_\ell$ in (6) is strictly increasing within any interval of prices for which they are applied. Moreover, each of these functions is continuous. The argument is complete if we show that $F_\ell$ is itself continuous. Let $p$ be a boundary point of the support of $F_c$, and let $(p_n)_n$ be a sequence which is not in the support of $F_c$ but which converges to $p$. Since the support of a distribution is closed, $p$ is in the support of $F_c$ and so $F_\ell(p) = \Pi_c^{-1}(\alpha_0 EA(\alpha)/Mp)$. Moreover, because $p$ is in the support of $F_c$, the minimum in (8) is achieved by $\hat{p} = p$, or $F_c(p) = (p - \alpha_0)/p\Pi_\ell(F_\ell(p))$. Since $F_c$ is continuous, $F_c(p_n)$ converges to $F_c(p)$, and so $F_\ell(p_n)$ converges to $\Pi_\ell^{-1}((p - \alpha_0)/pF_c(p))$. But simple algebra shows

$$\Pi_\ell^{-1}((p - \alpha_0)/pF_c(p)) = \Pi_c^{-1}(\alpha_0 EA(\alpha)/Mp)$$

if and only if

$$F_c(p) = (p - \alpha_0)/p\Pi_\ell(F_\ell(p)),$$

completing the proof. ∎

**Proof of Theorem 2**

The result is established in four steps.

**Step 1.** *If $\tilde{F}_c$ is strictly increasing, then the support of $F_c$ is $[\alpha_0, \bar{p}_c]$ and*

$$F_\ell(p) = \max\Big\{\Pi_c^{-1}\big(\frac{\alpha_0 EA(\alpha)}{Mp}\big)\ ,\ \Pi_\ell^{-1}\big(1-\frac{\alpha_0}{p}\big)\Big\}.$$

*Proof.* We know $\tilde{F}_c(\alpha_0) < 1 < \tilde{F}_c(1)$ from Proposition 9. Since $\tilde{F}_c$ is strictly increasing and continuous, there is a unique $\bar{p}_c \in (\alpha_0, 1)$ solving $\tilde{F}_c(p_c) = 1$. Using Theorem 1 and increasingness of $\tilde{F}_c$, we know that $F_c(p) = \tilde{F}_c(p)$ for all $p \in [\alpha_0, \bar{p}_c]$, and hence the support of $F_c$ is $[\alpha_0, \bar{p}_c]$. By construction, $F_c(p) < 1$ if and only if $p < \bar{p}_c$. Using the definition of $F_c$, this means that

$$\Pi_\ell^{-1}\big(1-\frac{\alpha_0}{p}\big) < \Pi_c^{-1}\big(\frac{\alpha_0 EA(\alpha)}{Mp}\big)$$

for $p \in [\alpha_0, \bar{p}_c)$, with the reverse inequality holding for $p \in [\bar{p}_c, 1]$. The construction of $F_\ell$ in Theorem 1 then implies that $F_\ell$ is given by the maximum of these two functions. ∎

**Step 2.** *If $\Pi_c(0) - \Pi_c(x)$ is strictly log-concave with respect to $x \in [0,1]$, except perhaps at a finite number of points, then $\tilde{F}_c$ is strictly increasing for $p \in [\alpha_0, \bar{p}_c]$.*

*Proof.* We know that $\Pi_c(0) = \frac{EA(\alpha)}{M}$. Subtracting

$$\Pi_c\big(\Pi_c^{-1}(\frac{\alpha_0 EA(\alpha)}{Mp})\big) = \frac{\alpha_0 EA(\alpha)}{Mp}$$

from the previous equation and dividing by $\Pi_\ell\big(\Pi_c^{-1}(\frac{\alpha_0 EA(\alpha)}{Mp})\big)$ yields[12]

$$\frac{\frac{EA(\alpha)}{M}\frac{p-\alpha_0}{p}}{\Pi_\ell\big(\Pi_c^{-1}(\frac{\alpha_0 EA(\alpha)}{Mp})\big)} = \frac{\Pi_c(0) - \Pi_c\big(\Pi_c^{-1}(\frac{\alpha_0 EA(\alpha)}{Mp})\big)}{\Pi_\ell\big(\Pi_c^{-1}(\frac{\alpha_0 EA(\alpha)}{Mp})\big)}.$$

The LHS is a positive constant times $\tilde{F}_c(p)$. Hence $\tilde{F}_c(p)$ is strictly increasing for $p \in [\alpha_0, 1]$ if and only if the RHS is. By assumption, the derivative of

$$\frac{\Pi_c(0) - \Pi_c(x)}{\Pi_\ell(x)} = 1/(\log(\Pi_c(0) - \Pi_c(x))'$$

[12]We thank Xiaosheng Mu for pointing out this identity.

is strictly positive on $[0,1]$, except perhaps at finitely many points. Continuity of $(\Pi_c(0) - \Pi_c(x))/\Pi_\ell(x)$ implies that it is strictly increasing on $[0,1]$. This concludes the proof, using the change of variable $x = \Pi_c^{-1}(\alpha_0 EA(\alpha)/Mp)$, which is a strictly increasing function of $p$. ∎

**Step 3.** *The following equivalence holds:*

$$\Pi_c(0) - \Pi_c(x) = \frac{1}{M}\sum_{j=0}^{M}\binom{M}{j}x^j(1-x)^{M-j}\sum_{k=0}^{M}\alpha_k \max\Big\{j - M + k, 0\Big\}.$$

*Proof.* We first recall some standard definitions and identities. The beta function is $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt$. The Euler integral of the first kind implies $B(a,b) = (a-1)!(b-1)!/(a+b-1)!$ for integers $a,b$. The incomplete beta function is $B(x;a,b) = \int_0^x t^{a-1}(1-t)^{b-1}dt$, and the regularized incomplete beta function is $I_x(a,b) = B(x;a,b)/B(a,b)$, which satisfies $I_x(a,b) = \sum_{j=a}^{a+b-1}\binom{a+b-1}{j}x^j(1-x)^{a+b-1-j}$. Next, observe that for each $k$,

$$\begin{aligned}
\pi_k^c(0) - \pi_k^c(x) &= \int_0^x \sum_{i=0}^{k-1}\binom{M-1}{i}(1-t)^i t^{M-1-i}dt \\
&= \sum_{i=0}^{k-1}\binom{M-1}{i}B(x; M-i, i+1) \\
&= \sum_{i=0}^{k-1}\binom{M-1}{i}B(M-i, i+1)I_x(M-i, i+1) \\
&= \frac{1}{M}\sum_{i=0}^{k-1}I_x(M-i, i+1) \\
&= \frac{1}{M}\sum_{i=0}^{k-1}\sum_{j=M-i}^{M}\binom{M}{j}x^j(1-x)^{M-j} \\
&= \frac{1}{M}\sum_{j=M-k+1}^{M}\Big(j-(M-k)\Big)\binom{M}{j}x^j(1-x)^{M-j},
\end{aligned}$$

since in the penultimate summation, $j = M$ appears $k$ times, $j = M-1$ appears $k-1$ times, …, and $j = M-k+1$ appears one time.

Using the above result and interchanging the order of summation,

$$\begin{aligned}\Pi_c(0) - \Pi_c(x) =& \frac{1}{M}\sum_{k=1}^{M}\alpha_k \sum_{j=M-k+1}^{M}\Big(j-(M-k)\Big)\binom{M}{j}x^j(1-x)^{M-j} \\ =& \frac{1}{M}\sum_{k=1}^{M}\alpha_k \sum_{j=0}^{M}\max\Big\{j-(M-k),0\Big\}\binom{M}{j}x^j(1-x)^{M-j} \\ =& \frac{1}{M}\sum_{j=0}^{M}\binom{M}{j}x^j(1-x)^{M-j}\sum_{k=0}^{M}\alpha_k\max\Big\{j-(M-k),0\Big\} \ \blacksquare\end{aligned}$$

**Step 4.** *If $(\alpha_1,\dots,\alpha_M)$ is a log-concave sequence, or if $\alpha_i \le 2\alpha_j$ for all $i<j$, then $\Pi_c(0) - \Pi_c(x)$ is strictly log-concave in $x \in [0,1]$, excepts perhaps at finitely many points.*

*Proof.* Theorem 2 of Mu (2013) shows that if $(\beta_0,\dots,\beta_M)$ is a non-constant log-concave sequence, then $\sum_{j=0}^{M}\binom{M}{j}x^{M-j}(1-x)^j\beta_j$ is log-concave in $x\in[0,1]$. Moreover, it can be seen from Mu's Equation (10) that the log-concavity holds strictly, except at perhaps finitely many points. Using a change of variable and symmetry of binomial coefficients, observe that

$$\sum_{j=0}^{M}\binom{M}{j}x^{M-j}(1-x)^j\beta_j = \sum_{j=0}^{M}\binom{M}{j}x^{j}(1-x)^{M-j}\beta_{M-j}.$$

If a sequence is log-concave, then it is also log-concave when read backwards. Thus, Mu's theorem holds also when replacing $\sum_{j=0}^{M}\binom{M}{j}x^{M-j}(1-x)^j\beta_j$ by $\sum_{j=0}^{M}\binom{M}{j}x^{j}(1-x)^{M-j}\beta_j$. Using Step 3, to ensure the desired property of $\Pi_c(0)-\Pi_c(x)$, it thus suffices to show that each of the above properties of $(\alpha_1,\dots,\alpha_M)$ implies that $(\beta_0,\dots,\beta_M)$ is log-concave, where we define

$$\beta_j := \sum_{k=0}^{M}\alpha_k\max\{j-(M-k),0\}.$$

(Notice that $\beta$ is non-constant since $\alpha \neq 0$.) Defining $\hat{\alpha}_{M-k} := \alpha_k$, observe that $\beta_j = \sum_{i=0}^{M}\hat{\alpha}_i\max\{j-i,0\}$.

Consider first the case that $\alpha$ is a log-concave sequence (hence so is $\hat{\alpha}$). Since $\max\{i,0\}$ is a log-concave sequence, then so is $\max\{j-i,0\}$. Because each $\beta_j$ is the convolution of two log-concave sequences, $\beta$ is log concave itself. Next, consider the case that $\alpha_i \leq 2\alpha_j$ when $i < j$. Applying the identity $\beta_k = \beta_{k-1} + \sum_{i=1}^{k} \alpha_{M-i+1}$, and rearranging terms, $\beta$ is log-concave if and only if

$$\begin{aligned} \beta_k(\beta_{k-1} + \sum_{i=1}^{k} \alpha_{M-i+1}) &\geq \beta_{k-1}\beta_{k+1} \\ \Leftrightarrow \qquad \beta_k \sum_{i=1}^{k} \alpha_{M-i+1} &\geq \beta_{k-1} \sum_{i=1}^{k+1} \alpha_{M-i+1} \\ \Leftrightarrow \qquad \Big(\sum_{i=1}^{k} \alpha_{M-i+1}\Big)^2 &\geq \beta_{k-1}\alpha_{M-k} \\ \Leftrightarrow \qquad \sum_{i=1}^{k} \alpha_{M-i+1}^2 + 2\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \alpha_{M-i+1}\alpha_{M-j+1} &\geq \beta_{k-1}\alpha_{M-k}. \end{aligned}$$

Note that $2\alpha_{M-j+1} \geq \alpha_{M-k}$ when $j \leq k$. Hence the left-hand side of the last expression is at least $\alpha_{M-k} \sum_{i=1}^{k-1}(k-i)\alpha_{M-i+1}$ which is precisely $\alpha_{M-k}\beta_{k-1}$, as desired. This concludes the proof of this step and of Theorem 2 ∎

**Proof of Theorem 4**

From the discussion that follows the statement of the theorem, it suffices to show that each of the two functions in Equation (10) strictly decreases when replacing $\alpha$ with $\hat{\alpha}$. It will be convenient to prove this in a more general setting, where $\alpha$ and $\hat{\alpha}$ come from a family of distributions parametrized by $\lambda$, a real-valued scalar taking values in either a continuous or discrete set; the case where $\lambda$ can take one of two values corresponds to the presentation in Section 3.3. Let $\alpha(\lambda)$ denote the distribution from this family given $\lambda$. The family satisfies (i) $\alpha_0(\lambda) = \alpha_0(\hat{\lambda}) = \alpha_0$ for all $\hat{\lambda} \neq \lambda$; (ii) log-concavity of the sequence $\alpha_1(\lambda), \ldots, \alpha_M(\lambda)$ for each $\lambda$; and (iii) the monotone likelihood ratio property

$$\frac{\alpha_k(\hat{\lambda})}{\alpha_k(\lambda)} \leq \frac{\alpha_{k+1}(\hat{\lambda})}{\alpha_{k+1}(\lambda)}$$

for $\hat{\lambda} > \lambda$ and $k \in \{1, \ldots, M-1\}$, with at least one strict inequality.

For any $\lambda$, let $\Pi_\ell(\lambda, x) = \sum_{k=1}^{M} \alpha_k(\lambda)\pi_k^\ell(x)$ and $\Pi_c(\lambda, x) = \sum_{k=1}^{M} \alpha_k(\lambda)\pi_k^c(x)$. The notations $\Pi_\ell^{-1}(\lambda, x)$ and $\Pi_c^{-1}(\lambda, x)$ refer to the inverse with respect to $x$, holding $\lambda$ fixed. Let $EA(\lambda)$ be the expected level of attention under $\alpha(\lambda)$. Note that the MLR ranking implies log-supermodularity of $\alpha_k(\lambda)$ in $k, \lambda$. Similarly, note that for $\hat{\lambda} > \lambda$, decreasingness of $\Pi_c(\hat{\lambda}, x)/\Pi_c(\lambda, x)$ in $x$ amounts to log-submodularity of $\Pi_c(\lambda, x)$ in $\lambda, x$. The proof proceeds as follows. Step 1 shows that $\Pi_\ell^{-1}(\lambda, 1 - \frac{\alpha_0}{p})$ strictly decreases in $\lambda$. Steps 2-4 show that $\Pi_c^{-1}(\lambda, EA(\lambda)/Mp)$ strictly decreases in $\lambda$.

**Step 1.** $\Pi_\ell^{-1}(\lambda, 1 - \frac{\alpha_0}{p})$ *is strictly decreasing in* $\lambda$.

*Proof.* Remember that $\Pi_\ell(\lambda, x)$ is a weighted average, under $\alpha(\lambda)$, of the probability $\pi_k^\ell(x)$ that a leader's market is inspected by a consumer with $k$ units of attention, when there is probability $x$ that other market leaders are cheaper. The higher a consumer's attention level $k$, the higher is this probability $\pi_k^\ell(x)$. So a first-order increase in the attention distribution means that given any $x$, a leader now faces a higher total probability of drawing consumers' attention. The conclusion immediately follows. ∎

**Step 2.** $\Pi_c^{-1}(\lambda, EA(\lambda)/Mp)$ *is strictly decreasing in* $\lambda$ *if and only if* $\Pi_c(\hat{\lambda}, x)/\Pi_c(\lambda, x)$ *is strictly below its value at* $x = 0$ *whenever* $x > 0$ *and* $\hat{\lambda} > \lambda$.

*Proof.* Let $x$ and $\hat{x}$ satisfy $\Pi_c(\lambda, x) = \alpha_0 EA(\lambda)/Mp$ and $\Pi_c(\hat{\lambda}, \hat{x}) = \alpha_0 EA(\hat{\lambda})/Mp$, where $\hat{\lambda} > \lambda$. Showing $x > \hat{x}$ amounts to proving $\Pi_c(\hat{\lambda}, x) < \Pi_c(\hat{\lambda}, \hat{x})$, as the probability a challenger makes a sale, $\Pi_c(\hat{\lambda}, \cdot)$, is decreasing in the probability $x$ that his leader is cheaper. Consider the ratio of these expressions, which we can multiply and divide by $\Pi_c(\lambda, x)$, and simplify using the definitions of $x$ and $\hat{x}$:

$$\frac{\Pi_c(\hat{\lambda}, x)}{\Pi_c(\hat{\lambda}, \hat{x})} = \frac{\Pi_c(\hat{\lambda}, x)}{\Pi_c(\lambda, x)} \frac{\Pi_c(\lambda, x)}{\Pi_c(\hat{\lambda}, \hat{x})} = \frac{\Pi_c(\hat{\lambda}, x)}{\Pi_c(\lambda, x)} \frac{EA(\lambda)}{EA(\hat{\lambda})}.$$

This ratio is smaller than 1 if and only if

$$\Pi_c(\hat{\lambda}, x)/\Pi_c(\lambda, x) < EA(\hat{\lambda})/EA(\lambda).$$

Notice that $EA(\hat{\lambda})/EA(\lambda)$ is the value of $\Pi_c(\hat{\lambda},\cdot)/\Pi_c(\lambda,\cdot)$ at $x=0$. ∎

**Step 3.** $\Pi_c(\lambda, x)$ *is log-submodular in* $\lambda, x$ *(and hence* $\Pi_c(\hat{\lambda}, x)/\Pi_c(\lambda, x)$ *is decreasing in* $x$*).*

*Proof.* It is well-known that if the function $t(i,y)$ is log-supermodular in $i, y$ and the function $s(i,z)$ is log-supermodular in $i,z$, then $\int_i t(i,y)s(i,z)di$ is log-supermodular in $y,z$ (see, for example, Corollary 1 in Quah and Strulovici, 2011). This preservation of log-supermodularity result extends to discrete summations (e.g., $i$ comes from the set $\{1,2,\ldots,n\}$).[13] To see this, apply the preservation result to the functions $\tilde{t}(j,y)$ and $\tilde{s}(j,z)$, which are defined with $j \in [0,1)$ as follows: if $\frac{i-1}{n} \le j < \frac{i}{n}$, then $\tilde{t}(j,y) = t(i,y)$ and $\tilde{s}(j,y) = s(i,y)$. Below, we iteratively apply the preservation result to prove that $\Pi_c(\lambda,x)$ is log-submodular in $\lambda, x$. Consider the function

$$\int_0^1 1_{(t\le x)} \left( \sum_{k=1}^{M} \alpha_k(\lambda) \sum_{i=1}^{M} 1_{(i\le k-1)} \binom{M-1}{i} t^i (1-t)^{M-i-1} \right) dt, \tag{11}$$

which is simply $\Pi_c(\lambda, 1-x)$ using a change of variables from $t$ to $1-t$ (note that $1(\cdot)$ is the indicator function which is equal to 1 if its argument is true). We first show that $1_{i\le k-1}$ is log-supermodular in $i$ and $k$. Indeed, consider $(\bar{i},\bar{k}) \ge (\underline{i},\underline{k})$. Then

$$1_{(\bar{i}\le\bar{k}-1)} 1_{(\underline{i}\le\underline{k}-1)} \ge 1_{(\bar{i}\le\underline{k}-1)} 1_{(\underline{i}\le\bar{k}-1)},$$

since if the right-hand side is one, then so is the left-hand side. Next, we show that $\binom{M-1}{i} t^i(1-t)^{M-i-1}$ is log-supermodular in $i,t$. Indeed, the ratio

$$\frac{\binom{M-1}{i} t^i (1-t)^{M-i-1}}{\binom{M-1}{i-1} t^{i-1}(1-t)^{M-i}} = \frac{(M-i)t}{i(1-t)}$$

is increasing in $t$ for $t\in[0,1)$. Applying the preservation result, this implies that the inner sum in (11) is log-supermodular in $k,t$. By assumption, $\alpha_k(\lambda)$ is log-supermodular in $k,\lambda$. Applying the preservation result again, the expression inside the large parentheses in (11)

[13] We thank Bruno Strulovici for pointing this out.

is log-supermodular in $t, \lambda$. Since $1_{(t \leq x)}$ is log-supermodular in $t, x$ (the argument is the same as before), the entire expression in (11) is log-supermodular in $\lambda, x$, by applying the standard preservation result. But since that sum is $\Pi_c(\lambda, 1-x)$, we obtain that $\Pi_c(\lambda, x)$ is log-submodular in $\lambda, x$ as desired. ∎

**Step 4.** *The derivative of* $\Pi_c(\hat{\lambda}, x)/\Pi_c(\lambda, x)$ *with respect to* $x$ *is* strictly *negative at* $x = 0$ *for* $\hat{\lambda} > \lambda$.

*Proof.* The sign of this derivative is the same as the sign of

$$\Pi_c(\hat{\lambda}, 0)\Pi_\ell(\lambda, 0) - \Pi_c(\lambda, 0)\Pi_\ell(\hat{\lambda}, 0) = \frac{EA(\hat{\lambda})}{M}\alpha_M(\lambda) - \frac{EA(\lambda)}{M}\alpha_M(\hat{\lambda}).$$

This expression is proportional to

$$\alpha_M(\lambda)\sum_{k=1}^{M} k\alpha_k(\hat{\lambda}) - \alpha_M(\hat{\lambda})\sum_{k=1}^{M} k\alpha_k(\lambda) = \sum_{k=1}^{M-1} k[\alpha_M(\lambda)\alpha_k(\hat{\lambda}) - \alpha_M(\hat{\lambda})\alpha_k(\lambda)],$$

which is indeed strictly negative because $\alpha_k(\hat{\lambda})/\alpha_k(\lambda) \leq \alpha_M(\hat{\lambda})/\alpha_M(\lambda)$ for all $k \geq 1$, with at least one strict inequality. ∎

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# Figure Captions

Figure 1. The construction of $F_c$ in an example where $\tilde{F}_c$ is not increasing.

Figure 2. Comparative Statics on $F_\ell$. The bold curves depict the market leaders' pricing strategies, which are the upper envelope of the corresponding two functions from (10). $F_\ell$ corresponds to attention distribution $\alpha$, while $\hat{F}_\ell$ corresponds to $\hat{\alpha}$.

# Figures

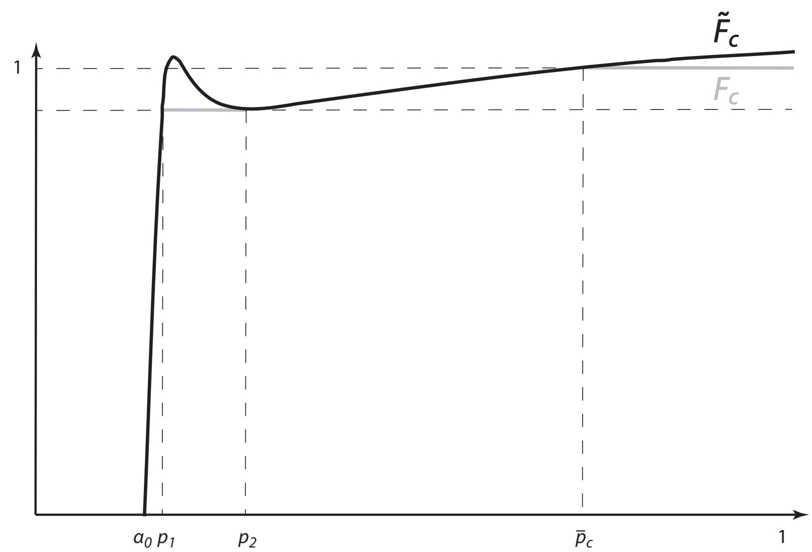


FIGURE 1: The construction of $F_c$ in an example where $\tilde{F}_c$ is not increasing.

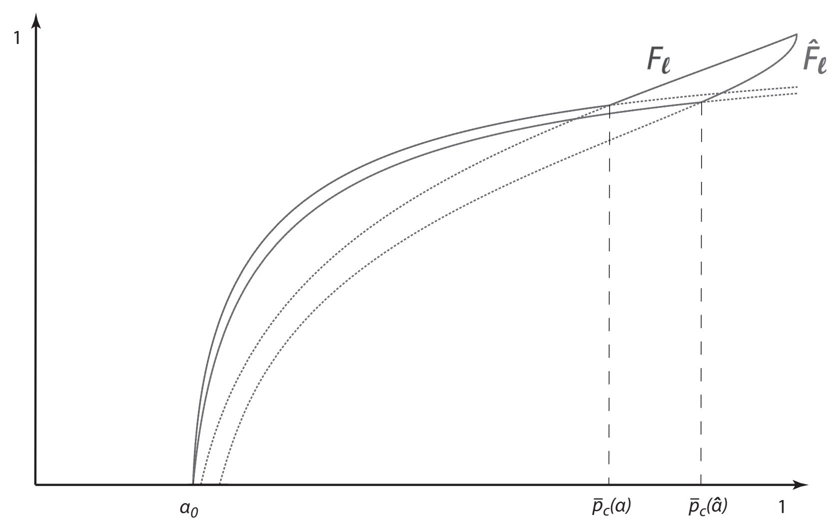


FIGURE 2: Comparative Statics on $F_\ell$. The bold curves depict the market leaders' pricing strategies, which are the upper envelope of the corresponding two functions from (10). $F_\ell$ corresponds to attention distribution $\alpha$, while $\hat{F}_\ell$ corresponds to $\hat{\alpha}$.