Productivity Growth and Worker Reallocation*

Rasmus Lentz
Boston University

Dale T. Mortensen
Northwestern University

November 16, 2004

Abstract

Productivity dispersion across firms is large and persistent, and worker reallocation among firms is an important source of productivity growth. The purpose of the paper is to develop and study a model designed to clarify the role of worker reallocation in the growth process. The model is a modified version of the Schumpeterian theory of equilibrium firm evolution and growth developed by Klette and Kortum (2002). We show that the model is consistent with correlations between size measures and labor productivity found in Danish firm data. We also derive necessary conditions under which the reallocation of workers from less to more productive firms contribute to aggregate productivity growth in the economy modeled.

*This paper was the basis of Mortensen's Klein Lecture presented at the University of Pennsylvania on April 22, 2004.

1 Introduction

In their review article on firm productivity, Bertelsman and Doms (2000) draw three lessons from empirical studies based on longitudinal plant and firm data: First, the extent of dispersion in productivity across production units, firms or establishments, is large. Second, productivity rank of any unit in the distribution is highly persistent. Third, a large fraction of aggregate productivity growth is the consequence of worker reallocation across firms. In their recent study of wage and productivity dispersion trends in U.S. Manufacturing, Dunne, Foster, Haltiwanger, and Troske (2002) find that differences in wages across plants is an important and growing component of total wage dispersion, most of the between plant increase in wage differences is within industries, and wage and productivity dispersion between plants has grown substantially in the recent past.

Baily, Hulton and Campbell (1992) find a strong positive correlation between productivity and wages paid across plants in U.S. manufacturing and Bartelsman and Doms report that the finding is present in similar studies. Mortensen (2003) argues that dispersion in wages paid for observably equivalent workers is hard to explain unless they reflect differences in firm productivity. To the extent that wage dispersion reflects differences in firm specific labor productivity, direct voluntary flows of workers from lower to higher paying firms as well as indirect flows through unemployment from less to employment with more productive firms improve the overall allocation labor in the economy.

Although the explanations for productive heterogeneity across firms are not fully understood, economic principles suggest that its presence should induce worker reallocation from less to more productive firms as well as from exiting to entering firms. Indeed, workers should move voluntarily to capture wage gains while more productive employer have a profit incentive to expand production. There is ample evidence that workers do flows from one firm to another frequently. As Davis, Haltiwanger and Shuh (1996) and others document, job and worker flows are large, persistent, and essentially idiosyncratic in the U.S. Recently, Fallick and Fleischman (2001) and Stewart (2002) find that job to job flows without a spell of unemployment in the U.S. represent at least half of the separations and is growing. In their analysis of Danish matched employer-employee IDA data, Frederiksen and Westergaard-Nelsen (2002) report that the average establishment separation rate over the 1980-95 period was 26%. About two thirds of the outflow represents the movement of workers from one firm to another. Using firm level data based on the same source, Christensen et al. (2002) document considerable cross firm dispersion in the average wage paid. Furthermore, they show that separation rates decline steeply with a firm's relative wage suggesting that workers do move from lower to higher paying jobs.

The purpose of this paper is to develop a model that can explain productivity differences across firms and the empirical relationships between firm productivity and size. The model is an extension of that proposed by Klette and Kortum (2002), which itself builds on the endogenous growth model of Grossman and Helpman (1991). Their version of the model is designed to be consistent with stylized facts about product innovation and its relationship to the dynamics of firm size evolution found in Danish data. Specifically, the model provides an explanation for the fact that there is no correlation between labor force size and labor productivity in the data but a strong positive association between value added and labor productivity. The model also provides insight into the role of worker reallocation as a source of productivity growth.

In the model, firms are monopoly suppliers of differentiated intermediate products that serve as inputs in the production of a final consumption good. Better quality products are introduced from time to time as the outcome of R&D investment by both existing firms and new entrants. As new products displace old, the process of creative destruction induces the need to reallocate workers across productive activities. In the version of the model studied here, new product quality is a random variable and a firm's current productivity can depend on the number and quality of its past product innovations.

As a theoretical result, we show that more productive firms, those that have developed higher quality products in the past, tend to grow larger by developing more product lines in the future only if a firm's future product quality is positively correlated with it past innovation success. Conversely, if the expected quality of any future product line is the identical across firms, then investment in R&D is independent of a firm's current productivity. Interestingly, the qualitative relationship between employment size and labor productivity is ambiguous in the first case and is negative in the second because innovations are labor saving in the model. However, more productive firms can be expected to develop more product lines and enjoy larger sales volumne. If more productive firms do grow faster in this sense, then aggregate productivity growth reflects the fact that workers flow from less to more productive employers as well as from exiting to entering firms.

2 Danish Firm Data

Danish firm data provide information on productivity dispersion and the relationships among productivity, employment, and sales. The available data set is an annual panel of privately owned firms for the years 1992-1997 drawn from the Danish Business Statistics Register. The sample of approximately 6,700 firms is restricted to those with 20 or more employees. The variables observed in each year include value added measured (Y), the total wage bill (W), and full-time equivalent employment (N). In this paper we use these relationships to motivate the theoretical model studied. Both Y and W are measured in Danish Kroner while N is a body count.

Non-parametric estimates of the distributions of two alternative measures of a firm's labor productivity are illustrated in Figure 1 and Figure 2. The first measure is value added per worker (Y/N) while the second is valued added per unit of quality adjusted employment (Y/N^*). The first measure misrepresents cross firm productivity differences to the extent that labor quality differs across firms. However, if more productive workers are compensated with higher pay as would be true in a competitive labor market, one can use a wage weighted index of employment to correct for this source of cross firm differences in productive efficiency. Formally, the constructed quality adjusted employment of firm j is defined as $N_j^* = \frac{W_j}{w}$ where $w = \sum_j W_j / \left(\sum_j N_j \right)$ is the average wage paid per worker in the market. Although correcting for wage differences across firms in this manner does reduced the spread and skew of the implied productivity distribution somewhat, both distributions have high variance and skew and are essentially the same shape.

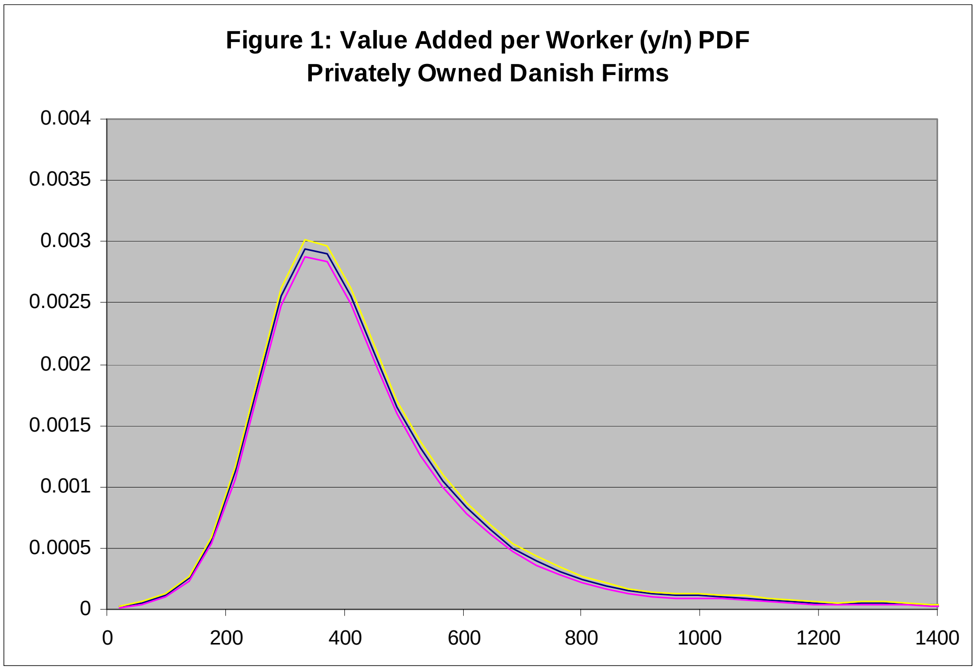
Figure 1: Value Added per Worker (y/n) PDF
Privately Owned Danish Firms
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
0
200
400
600
800
1000
1200
1400

Figure 1:

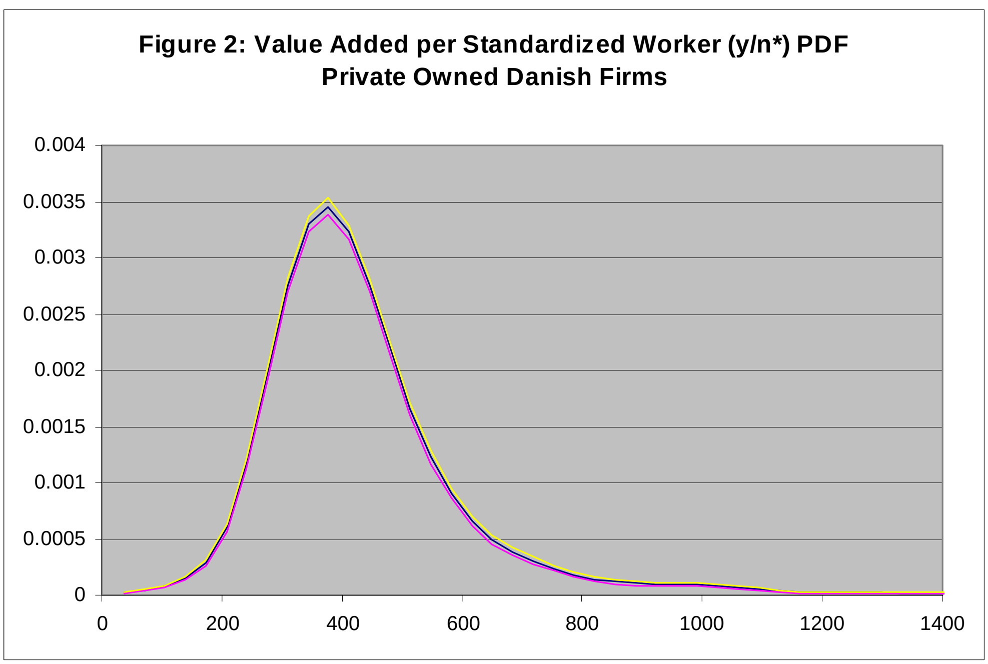
Figure 2: Value Added per Standardized Worker (y/n*) PDF
Private Owned Danish Firms
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
0
200
400
600
800
1000
1200
1400

Both distributions are consistent with those found in other data sets. (See Bertelsman and Doms (2000).) For example, the distribution is skewed to the right and very dispersed. In the case of the adjusted measure of productivity, the 5^{th} percentile is roughly half the mode while the 95^{th} percentile is approximately twice as large are the mode. The range between the two represents a four fold difference in value added per worker across firms.

There are many potential explanations for cross firm productivity differentials. A comparison of Figures 1 and 2 suggests that differences in the quality of labor inputs does not seem to be the essential one. The process of technology diffusion is a well documented. Total factor productivity differences across firms can be expected as a consequence of slow diffusion of new techniques. If technical improvements are either factor neutral or capital augmenting, then one would expect that more productive firms would acquire more labor and capital. The implied consequence would seem to be a positive relationship between labor force size and labor productivity. Interestingly, there is no correlation between the two in Danish data.

Table 2: Productivity - Size Correlations

	Employment (N)	Adjusted Employment (N*)	Value Added (Y)
Y/N	0.0331	0.1397	0.3944
Y/N*	0.0114	-0.0076	0.2618

The correlations between the two measures of labor productivity with the two employment measures and sales as reflected in value added are reported in Table 2. As documented in the table, the correlation between labor force size and productivity using either the raw employment measure or the adjusted one is zero. However, note the stong positive associate between value added and both measures of labor productivity. Figures 3 and 4 illustrate non-parametric regressions of value added on the two productivity measures. The top and bottom curves in the figures represent a 90% confidence interval for the relationship. The positive relationships illustrated in the figures are highly significant.

The theory developed in this paper is motivated by these observations. Specifically, it is a theory that postulates labor saving technical progress of a specific form. Hence, the apparent fact that more productive firms produce more with roughly the same labor input per unit is consistent with the model.

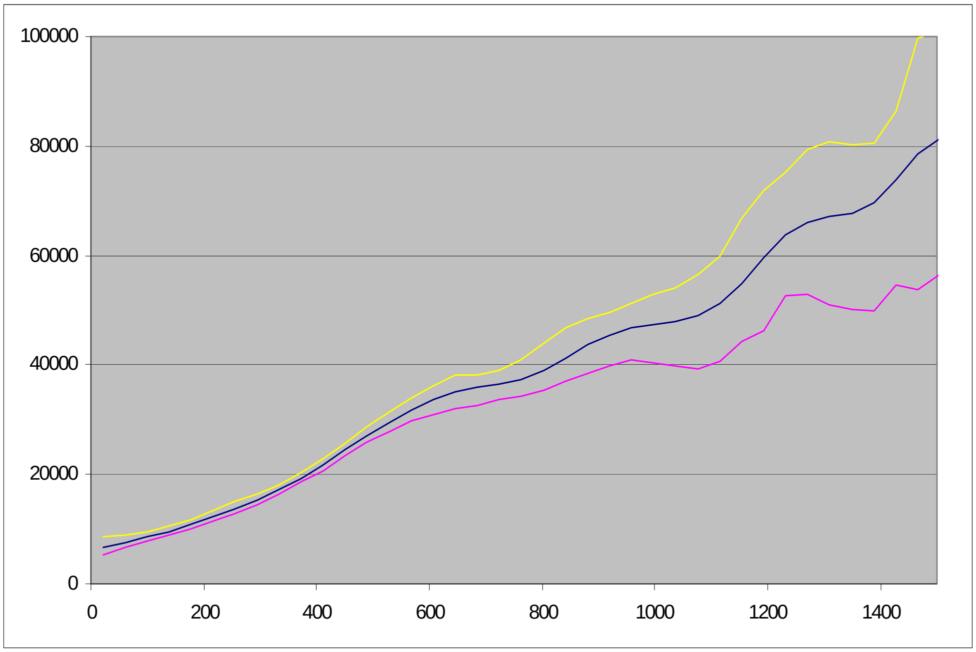

Figure 2: Productivity (Y/N) vs. Value Added (Y)

3 A Model of Creative Destruction

As is well known, firms come is an amazing range of shapes and sizes. This fact cannot be ignored in any analysis of the relationship between firm size and productivity. Futhermore, an adequate theory must account for entry, exit and firm evolution inorder to explain the size distributions observed. Klette and Kortum (2002) construct a stochastic model of firm product innovation and growth that is consistent with stylized facts regarding the firm size evolution and distribution. The model also has the property that technical progress is labor saving. For these reasons, we pursue their approach in this paper.

Although Klette and Kortum allow for productive heterogeneity across firms, firm productivity and growth are unrelated because costs and benefits of growth are both proportional to firm productivity in their model. In our version of the model, this outcome is a special case of a more general formulation in which productivity is a stochastic characteristic of new products. Allowing for a positive relationship between firm growth rates an firm productivity is necessary for consistency with the relationships found in

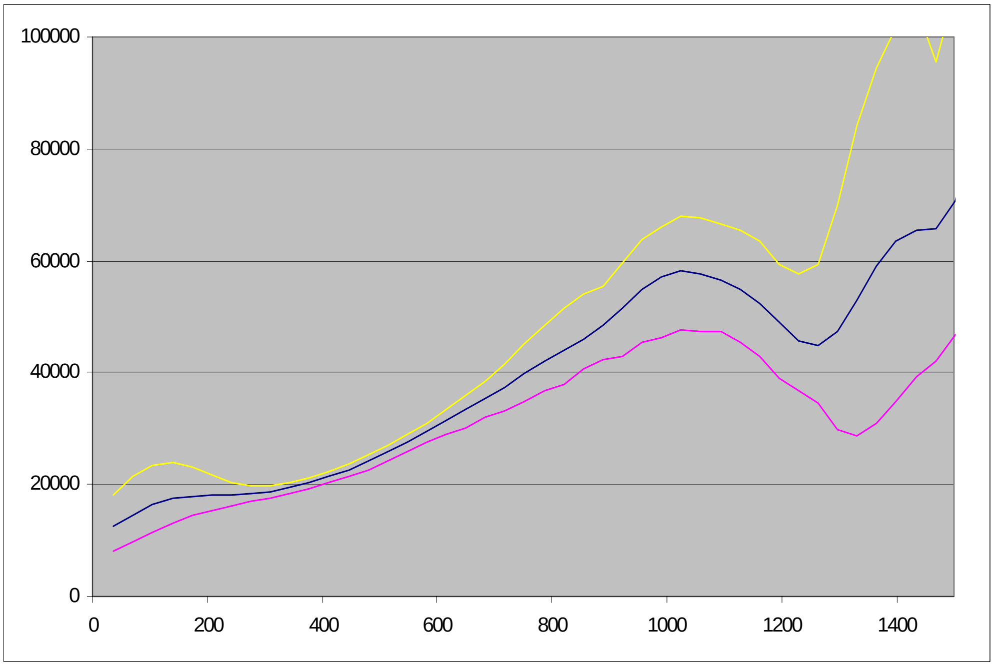

Figure 3: Standardized Productivity (Y/N*) vs. Value Added (Y)

the Danish firm data. Finally, the model is one of dynamic general equilibrium with important implication about the role of reallocation as a source of aggregate productivity growth.

3.1 Preferences and Technology

Intertemporal utility of the representative household at time t is given by

$$U_t = \int_t^{\infty} \ln C_s e^{-\rho(s-t)} ds \tag{1}$$

where $\ln C_t$ denotes the instantaneous utility of the single consumption good at date t and ρ represents the pure rate of time discount. Each household is free to borrow or lend at interest rate r_t. Nominal household expenditure at date t is $E_t = P_t C_t$. Optimal consumption expenditure must solve the differential equation $\dot{E}/E = r_t - \rho$. Following Grossman and Helpman (1991), I choose the numeraire. so that $E_t = 1$ for all t without loss of generality, which implies $r_t = \rho$ for all t.Note that this choice of the numeraire also implies that price of the consumption good, P_t, falls over time at a rate equal to the rate of growth in consumption.

The quantity of the consumption produced is determined by the quantity and quality of the economy's intermediate inputs. Specifically, there is a unit continuum of inputs and consumption is determined by the production function

$$\ln C_t = \int_0^1 \ln(A_t(j)x_t(j))dj = \ln A_t + \int_0^1 \ln x_t(j)dj \tag{2}$$

where $x_t(j)$ is the quantity of input $j \in [0,1]$ at time t, $A_t(j)$ is the productivity of input j at time t, and

$$\ln A_t \equiv \int_0^1 \ln A_t(j)dj.$$

The level of productivity of each input is determined by the number of technical improvements made in the past. Specifically,

$$A_t(j) = q^{J_t(j)} \text{ and } \ln A_t \equiv \int_0^1 \ln A_t(j)dj = \ln(q)\int_0^1 J_t(j)dj$$

where $J_t(j)$ is the number of innovations made in input j up to date t and $q > 1$ denotes the quantitative improvement (step size) in productivity attributable to any innovation. Innovations arrive at rate δ which is endogenous but the same for all intermediate products.

The model is constructed so that a steady state growth path exists with the following properties: Consumption output grows at a constant rate while the quantities of intermediate products and the endogenous innovation frequency are stationary and identical across all intermediate goods. As a consequence of the law of large numbers, the assumption that the number of innovations to date is Poisson with arrival frequency δ for all intermediate goods implies

$$\begin{aligned} \ln C_t &= \ln A_t + \int_0^1 \ln x(j)dj = \int_0^1 \ln q_j J_j(t)dj \\ &= \delta t E\ln(q). + \int_0^1 \ln x(j)dj. \end{aligned} \tag{3}$$

where $E\ln(q) \equiv \int_0^1 \ln q_j dj$ is the average or expected step size. In other words, consumption grows at the rate of growth in productivity which is the product of the creative-destruction rate and the expected log of the size of an improvement in productivity induced by each new innovation.

3.2 The Value of a Firm

Each individual firm is the monopoly supplier of the products it created in the past that have survived to the present. The price charged for each is limited by the ability of suppliers of previous versions to provide a substitute. In Nash-Bertrand equilibrium, any innovator takes over the market for its good type by setting the price just below that at which consumers are indifferent between the higher quality product supplied by the innovator and an alternative supplied by the last provider. The price charged is the product of the relative quality and the previous producer's marginal cost of production. Given the symmetry of demands for the different good types and the assumption that future quality improvements are independent of the type of good, one can drop the good subscript without confusion. Given stationarity of quantities along the equilibrium growth path, the time subscript can be dropped as well.

Labor is the only factor in the production of intermediate inputs. Labor productivity is the same across all inputs and is set equal to unity. Hence, $p = qw$ is the price in terms of the numeraire of every intermediate good as well as the value of labor productivity where w, the wage, represents the marginal cost of production of the previous supplier and $q > 1$ is the step up in quality of the innovation. As total expenditure is normalized at unity and there is a unit measure of product types, it follows that total revenue per product type is also unity, i.e., $px = 1$. Hence, product output and employment are both equal to

$$x = \frac{1}{p} = \frac{1}{wq}. \tag{4}$$

and the gross profit associated with supplying the good is

$$1 > \pi = px - wx = 1 - \frac{1}{q} > 0. \tag{5}$$

The labor saving nature of improvements in intermediate input quality is implicit in the fact that labor demand is decreasing in q.

Following Klette and Kortum (2002), the discrete number of products supplied by a firm, denoted as k, is defined on the integers and its value evolves over time as a birth-death process reflecting product creation and destruction. In their interpretation, k reflects the firm's past successes in the product innovation process as well as current firm size. New products are generated by R&D investment. The firm's R&D investment flow generates new product arrivals at frequency γk. The total R&D investment cost is $wc(\gamma)k$ where $c(\gamma)k$ represents the labor input required in the research and development process. The function $c(\gamma)$ is assumed to be strictly increasing and convex. According to the authors, the implied assumption that the total cost of R&D investment is linearly homogenous is the new product arrival rate and the number of existing product, "captures the idea that a firm's knowledge capital facilitates innovation." In any case, this cost structure is needed to obtain firm growth rates that are independent of size as typically observed in the data.

The market for any current product supplied by the firm is destroyed by the creation of a new version by some other firm, which occurs at the rate δ. Below we refer to γ as the firm's creation rate and to δ as the common destruction rate faced by all firms.[1] As product gross profit and product quality are one-to-one, the profits earned on each product reflects a firm's current labor productivity. The firm chooses the creation rate γ to maximize the expected present value of its future net profit flow conditional on information that is relevant for predicting the quality of future innovations.

Let the parameter vector θ summarize past realizations of $\pi = 1 - 1/q$. We assume that this indicator is a sufficient statistic for the distribution of the next innovation's profit. For example, the product quality sequence might be a first order Markov process, in which case θ is the profit on the last product innovation. Alternatively, we might think of the problem as one in which a firm's product profitability is initially unknown but can be learned over time by observing the past realization. In Jovanovic's original normal-normal case the sufficient statistic is a pair which include both the current estimate of the mean and its precision. In general, θ will be updated in response to the realized profitability of any new product.

Let $\Pi^k = (\pi_1, \pi_2, .., \pi_k)$ denote the firm's vector of profits for the products currently supplied, let $\Pi^{k+1} = (\Pi^k, \pi')$ represent the profits of the $k+1$ products where $\pi_{k+1} = \pi'$, and let $\Pi^k_{\langle i \rangle}$ denote Π^k excluding element $i \in \{1, ..., k\}$. In terms of this notation, the current value of the firm is a function

[1] These are in fact the continuous time job creation and job destruction rates respectively as defined in Davis et al. (1996).

of its state characterized by Π^k and θ. It solves the Bellman equation

$$rV_k(\Pi^k,\theta) = max_{\gamma\geq 0}\left\{\begin{array}{c}\sum_{i=1}^{k}\pi_i - wc(\gamma)k \\ +\gamma k[E\left\{V_{k+1}((\Pi^k,\pi'),\theta')|\theta\right\} - V_k(\Pi^k,\theta)] \\ +\delta k[\frac{1}{k}\sum_{i=1}^{k}V_{k-1}(\Pi^k_{\langle i\rangle},\theta) - V_k(\Pi^k,\theta)]\end{array}\right\}. \quad (6)$$

where $E\{\cdot|\theta\}$ is the expectation operator conditional on information about the quality of the firm's future products and θ' is the updated value of θ given the realized profit of the next innovation, denoted π'. The first term on the right side is current gross profit flow accruing to the firms product portfolio less current expenditure on R&D. The second term is the expected capital gain associated with the arrival of a new product line. Finally, because product destruction risk is equally likely across the firm's current portfolio, the last term represents the expected capital loss associated with the possibility that one among the existing product lines will be destroyed. Notice that no information about future profitability is gained or lost when a product line is destroyed.

Consider the conjecture that the solution takes the following additively separable form

$$V_k(\Pi^k,\theta) = \sum_{i=1}^{k}\frac{\pi_i}{r+\delta} + R_k(\theta). \quad (7)$$

That is, we suppose that the value of the firm is the sum of the expected present value of the firm's current products plus the value of R&D activities which depends only on expectations about the profitability of future innovations and the current number of product lines. Since $V_{k+1}((\Pi^k,\pi'),\theta') = \sum_{i=1}^{k}\frac{\pi_i}{r+\delta} + \frac{\pi'}{r+\delta} + R_{k+1}(\theta')$ under the conjecture, equation (6) and the conjecture imply

$$\begin{aligned} r\sum_{i=1}^{k}\frac{\pi_i}{r+\delta} + rR_k(\theta) &= rV_k(\Pi^k,\theta) \\ &= \sum_{i=1}^{k}\pi_i + k\max_{\gamma}\left\{\gamma E\left\{\frac{\pi'}{r+\delta} + R_{k+1}(\theta') - R_k(\theta)|\theta\right\} - wc(\gamma)\right\} \\ &\quad -\delta\sum_{i=1}^{k}\frac{\pi_i}{r+\delta} + \delta k\left[R_{k-1}(\theta) - R_k(\theta)\right] \end{aligned}$$

Because the term on the left that involve the profits of the products currently supplied cancels with the those on the right, the conjecture holds for any sequence of functions $R_k(\theta)$, $k = 1, 2, ...$ that satisfies the functional difference equation

$$\begin{aligned} rR_k(\theta) &= k \max_{\gamma} \left\{ \gamma E \left\{ \frac{\pi'}{r+\delta} + R_{k+1}(\theta') - R_k(\theta) | \theta \right\} - wc(\gamma) \right\} \\ &\quad + \delta k \left[R_{k-1}(\theta) - R_k(\theta) \right]. \end{aligned} \tag{8}$$

In words, the return on the value of the R&D department is the expected gain in future profit associated with the next innovation plus the expected capital gains and losses to the R&D operation associated with the possibility of product creation and destruction. In general, these terms are non-zero because a new innovation changes expectations about the profitability of any future innovation and because a change in scale affects future returns to and costs of R&D.

Note that equation (8) can be rewritten as

$$R_k(\theta) = k \max_{\gamma} \left\{ \frac{\gamma E \left\{ \frac{\pi'}{r+\delta} + R_{k+1}(\theta') | \theta \right\} - wc(\gamma) + \delta R_{k-1}(\theta)}{r + (\delta + \gamma) k} \right\}.$$

Because the right hand side satisfies Blackwell's sufficient conditions for a contraction that maps the set of non-negative functions defined on the product of the non-negative reals and non-negative integers into itself, a unique solution exists. If the uncertain profit of the next future innovation, π', is stochastically increasing in expected profitability as summarized by θ, the unique solution is an increasing function of θ for every value of k by the same argument. Similarly, the fact that the right hand side is strictly increasing in k, $R_{k+1}(\theta')$ and $R_{k-1}(\theta)$ also implies that the contraction maps the functions increasing in k into itself. In sum, the solution has the properties $\theta' > \theta \Rightarrow R_k(\theta') \geq R_k(\theta)$ and $R_{k+1}(\pi) > R_k(\pi)$.

As an implication of (8), a firm's optimal product creation rate maximizes the expected net return to R&D activity:

$$\begin{aligned} \gamma &= \arg\max \left\{ \gamma E\{V_{k+1}((\Pi^k, \pi'), \theta') | \theta\} - V_k(\Pi^k, \theta) - wc(\gamma) \right\} \\ &= \arg\max \left\{ \gamma E \left\{ \frac{\pi'}{r+\delta} + R_{k+1}(\theta') - R_k(\theta) | \theta \right\} - wc(\gamma) \right\}. \end{aligned} \tag{9}$$

By implication, the expected growth rate, the difference between the chosen creation rate γ and the market determined destruction rate δ, is independent of the firm's current productivity and size if the profitability of the next innovation is independent of past realization of product quality. *When past successes have no consequence for future prospects, there is no incentive for firms that are currently more productive to grow faster and to become larger.*

4 Market Equilibrium

In this section, we complete the specification of an equilibrium market model and establish that it has a solution in the special case of deterministic heterogeneity in productivity. As a corollary of the existence proof, we also find that the equilibrium is unique in the homogenous productivity case.

4.1 Product Creation

We restrict the analysis to the case of deterministic heterogeneity. Namely, assume that the profitability of the next innovation is π with probability one given that π characterizes the quality of all previous products. Since

$$\begin{aligned} rR_k(\pi) &= k\max_{\gamma}\left\{\gamma\left(\frac{\pi}{r+\delta}+R_{k+1}(\pi)-R_k(\pi)\right)-wc(\gamma)\right\} \\ &\quad +\delta k\left[R_{k-1}(\pi)-R_k(\pi)\right] \end{aligned}$$

from (8) in this case, it follows that the solution for $R_k(\pi)$ is proportional to k. Namely, $R_k(\pi)=k\Delta R(\pi)$ where by substitution

$$\Delta R(\pi)=\max_{\gamma\geq 0}\left\{\frac{\gamma\frac{\pi}{r+\delta}-wc(\gamma)}{r+\delta-\gamma}\right\} \tag{10}$$

is the value of R&D per product line for a firm of type π.

From equation (9), an interior solution for the firm's creation rate choice, denoted $\gamma(\pi)$, satisfies the following first order condition:

$$wc'(\gamma)=\frac{\pi}{r+\delta}+\Delta R(\pi)=\max_{\gamma\geq 0}\frac{\pi-wc(\gamma)}{r+\delta-\gamma} \tag{11}$$

Obviously, the optimal creation rate is a strictly increasing function of the firm's profit rate. We conjecture that the latter conclusion also holds when expected profitability is positively correlated with past realization as in the case of learning but we don't have a formal proof.

4.2 The Distribution of Firm Size

As the set of firms with k products at a point in time must either have had k products already and neither lost nor gained another, have had $k-1$ and innovated, or have had $k+1$ and lost one to destruction over any sufficiently short time period, the equality of the flows into and out of the set of firms of type π with $k > 1$ products requires

$$\gamma(\pi)(k-1)M_{k-1}(\pi) + \delta(k+1)M_{k+1}(\pi) = (\gamma+\delta)kM_k(\pi)$$

for every π where $M_k(\pi)$ is the steady state mass of firms of type π that supply k products.[2] Because an incumbent dies when it its last product is destroyed by assumption but entrants flow into the set of firms with a single product at rate η,

$$\phi(\pi)\eta + 2\delta M_2(\pi) = (\gamma(\pi)+\delta)M_1(\pi)$$

where $\phi(\pi)$ is the fraction of the new entrants that realize profit π. Births must equal deaths in steady state and only firms with one product are subject to death risk. Therefore, $\phi(\pi)\eta = \delta M_1(\pi)$ and

$$M_k(\pi) = \frac{k-1}{k}\gamma(\pi)M_{k-1} = \frac{\phi(\pi)\eta}{\delta k}\left(\frac{\gamma(\pi)}{\delta}\right)^{k-1} \tag{12}$$

by induction.

The size distribution of firms conditional on type can be derived using equation (12). Specifically, the total firm mass of type π is

$$\begin{aligned} M(\pi) &= \sum_{k=1}^{\infty} M_k(\pi) = \frac{\phi(\pi)\eta}{\delta}\sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{\gamma(\pi)}{\delta}\right)^{k-1} \\ &= \frac{\eta}{\delta}\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)\frac{\delta\phi(\pi)}{\gamma(\pi)}. \end{aligned}$$

if finite. Hence, the fraction of type π firm with k product is

$$\frac{M_k(\pi)}{M(\pi)} = \frac{\frac{1}{k}\left(\frac{\gamma(\pi)}{\delta}\right)^k}{\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)}. \tag{13}$$

[2] This equation is not enough in the general case in which an individual firm's type is transitory. In that case one must also account for type identity switches that occur as new innovations arrive.

This is the logarithmic distribution with parameter $0 < \gamma(\pi)/\delta < 1$ which is the ratio of the type's creation rate to the market wide rate of destruction.[3]

Consistent with the observations on firm size distributions, that implied by the model is highly skewed to the right. The conditional mean of the distribution,

$$E\{k|\pi\} = \sum_{k=1}^{\infty} \frac{kM_k(\pi)}{M(\pi)} = \frac{\frac{\gamma(\pi)}{\delta-\gamma(\pi)}}{\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)}$$

is increasing in $\gamma(\pi)$. Formally, because $(1+a)\ln(1+a) > a > 0$, the expected number of product produced increasing in firm profitability

$$\frac{\partial E\{k|\pi\}}{\partial \pi} = \left(\frac{(1+a(\pi))ln(1+a(\pi)) - a(\pi)}{(1+a(\pi))ln^2(1+a(\pi))}\right) \frac{\delta\gamma'(\pi)}{(\delta-\gamma(\pi))^2} > 0 \tag{14}$$

where $a(\pi) = \frac{\gamma(\pi)}{\delta-\gamma(\pi)}$ if and only if $\gamma'(\pi) > 0$.

When permanent differences in product quality exist across firms, workers move from less to more profitable surviving firms as well as from exiting to entering firms. This selection effect can be demonstrated by noting that more profitable firms are over represented (relative to their fraction at entry) among those that produce more than one product and that this "selection bias" increases with the number of products produced. Namely, the relative fraction of the more profitable firms in the surviving population, given by

$$\frac{M_k(\pi')}{M_k(\pi)} - \frac{\phi(\pi')}{\phi(\pi)} = \frac{\phi(\pi')}{\phi(\pi)}\left[\left(\frac{\gamma(\pi')}{\gamma(\pi)}\right)^{k-1} - 1\right], \tag{15}$$

is positive and increasing in k when $\pi' > \pi$.

4.3 Firm Entry and Labor Market Clearing

The entry of a new firm requires an innovation. The cost of entry is the expected cost of the R&D effort required of a potential entrant to discover and develop a new successful product. Hence, if a potential entrant obtains ideas for new products at frequency h per period, the expected opportunity cost of her effort per innovation is w/h, the expected earnings forgone during the required period of R&D activity. As no entrant knows the profitability

[3] This result is in Klette and Kortum (1992). We include the derivation here simply for completeness.

of its product a priori but all know its distribution, new firms enter if and only if the expected value of a new product exceeds the cost. Assuming that the condition holds, the endogenous equilibrium product destruction rate, δ, adjusts though entry to equate the expected cost and return. The equality of the expected return and cost of entry require that

$$\sum_{\pi} V_1(\pi,\theta)\phi(\pi) = \sum_{\pi} \max_{\gamma\geq 0}\left\{\frac{\pi - wc(\gamma)}{r+\delta-\gamma}\right\}\phi(\pi) = \frac{w}{h} \tag{16}$$

from equations (7) and (11) where $\phi(\pi)$ is fraction of entrants with product quality $q = (1-\pi)^{-1}$.[4]

Because the new product arrival rate of a firm of type π with k products is $\gamma(\pi)k$ and the measure of such firms is $M_k(\pi)$, the aggregate rate of destruction is the sum of the entry rate and the creation rates of all the incumbents given that the mass of products is fixed. That is

$$\begin{aligned}\delta &= \eta + \sum_{\pi}\sum_{k=1}^{\infty}\gamma(\pi)kM_k(\pi) = \eta + \sum_{\pi}\sum_{k=1}^{\infty}\gamma(\pi)\frac{\phi(\pi)\eta}{\delta}\left(\frac{\gamma(\pi)}{\delta}\right)^{k-1} \\ &= \eta\left(1+\sum_{\pi}\frac{\phi(\pi)\gamma(\pi)}{\delta}\sum_{k=1}^{\infty}\left(\frac{\gamma(\pi)}{\delta}\right)^{k-1}\right) = \eta\left(\sum_{\pi}\frac{\delta\phi(\pi)}{\delta-\gamma(\pi)}\right).\end{aligned}$$

where the second equality follows from (12). The last equality requires convergence of the infinite sum on the left for all values of π. This condition is equivalent to the requirement that the destruction rate exceed the creation rate of all firm types. Using the assumption that the measure of firms is unity, a direct derivation of the same relationship follows:

$$1 = \sum_{\pi}\sum_{k=1}^{\infty}kM_k(\pi) = \sum_{\pi}\frac{\eta\phi(\pi)}{\delta}\sum_{k=1}^{\infty}\left(\frac{\gamma(\pi)}{\delta}\right)^{k-1} = \eta\sum_{\pi}\frac{\phi(\pi)}{\delta-\gamma(\pi)}. \tag{17}$$

As just noted, the steady state measure of every firm type is finite if and only if the aggregate destruction rate exceeds the creation rate, $\delta > \gamma(\pi)$, for all π in the support of the distribution of firms at entry. Below, we will seek a equilibrium solution to the model that satisfies this property.

There is a fixed measure of available workers, denoted by ℓ, seeking employment at any positive wage. In equilibrium, these are allocated across

[4] For simplicity, we assume that the number of different product qualities is finite.

production and R&D activities, those performed by both incumbent firms and potential entrants. Since the number of workers employed for production purposes per product of quality q is $x = 1/wq = (1-\pi)/w$ from equations (4) and (5), the total number demanded for production activity by firms of type π with k products is $\ell_x(k,\pi) = k(1-\pi)/w > 0$. The number of R&D workers employed by incumbent firms of type π with k products is $\ell_R(k,\pi) = kc(\gamma(\pi))$. Because a potential entrant innovates at frequency h, the total number so engaged in R&D is $\ell_E = \eta/h$ given entry rate η. Hence, the equilibrium wage satisfies the labor market clearing condition

$$\begin{aligned}\ell &= \sum_\pi \sum_{k=1}^\infty \left[\ell_x(k,\pi) + \ell_R(k,\pi)\right] M_k(\pi) + \ell_E \qquad (18)\\ &= \sum_\pi \sum_{k=1}^\infty \left(\frac{1-\pi}{w} + c(\gamma(\pi))\right) k M_k(\pi) + \frac{\eta}{h} \\ . &= \sum_\pi \left(\frac{1-\pi}{w} + c(\gamma(\pi))\right) \frac{\phi(\pi)\eta}{\delta} \sum_{k=1}^\infty \left(\frac{\gamma(\pi)}{\delta}\right)^{k-1} + \frac{\eta}{h} \\ &= \eta\left(\sum_\pi \left(\frac{1-\pi}{w} + c(\gamma(\pi))\right) \frac{\phi(\pi)}{\delta - \gamma(\pi)} + \frac{1}{h}\right)\end{aligned}$$

where again the last equality is implied by equation (12) and the requirement that $\delta > \gamma(\pi)$ for all π.

4.4 Existence

Definition 1 *A steady state* market equilibrium *is a triple composed of a labor market clearing wage w, entry rate η, and creative destruction rate δ that satisfy equation (16), (17), and (18) provided that the optimal creation rate choice of any firm type is less than the rate of creative destruction, i.e., $\gamma(\pi) < \delta$ for all π in the support of the entry distribution.*

Proposition 2 *If the cost of R&D function, $c(\gamma)$, is strictly convex and $c'(0) = c(0) = 0$, then a steady state market equilibrium exists for all h sufficiently large. In the case of a single firm type, there is only one equilibrium.*

Proof. From (16), the free entry condition is

$$\sum_\pi \max_{\gamma \geq 0} \left\{\frac{\pi - wc(\gamma)}{r + \delta - \gamma}\right\} \phi(\pi) = \frac{w}{h} \qquad (19)$$

By using equation (17) to eliminate the entry rate η and the equation (19) to eliminate w/h in equation (18), one can write the result as

$$\begin{aligned} & w\ell \sum_{\pi} \frac{\delta}{\delta - \gamma(\pi)} \phi(\pi) \\ = \; & \delta \sum_{\pi} \left(\frac{1 - \pi + wc(\gamma(\pi))}{\delta - \gamma(\pi)} + \max_{\gamma \geq 0} \frac{\pi - wc(\gamma)}{r + \delta - \gamma} \right) \phi(\pi) \\ = \; & \sum_{\pi} \frac{\delta}{\delta - \gamma(\pi)} \left(1 - r \max_{\gamma \geq 0} \frac{\pi - wc(\gamma)}{r + \delta - \gamma} \right) \phi(\pi) \end{aligned}$$

where the first equality is implied by the fact that $\sum_{\pi} \phi(\pi) = 1$ and the second is a consequence of the fact that $\gamma(\pi)$ is the optimal choice of the creation rate for a type π firm. Hence,

$$1 = w\ell + \frac{r \sum_{\pi} \left(\max_{\gamma \geq 0} \frac{\pi - wc(\gamma)}{r + \delta - \gamma} \right) \frac{\phi(\pi)}{\delta - \gamma(\pi)}}{\sum_{\pi} \frac{\phi(\pi)}{\delta - \gamma(\pi)}}. \tag{20}$$

Since total value added is unity by choice of the numeraire, this expression is the income identity. Namely, the total wage bill plus the return on the values of all the operating firms in the economy is equal to value added.

In order to focus on the case in which incumbents invest in R&D, we assume the cost function, $c(\gamma)$, is increasing, strictly convex and that $c(0) = c'(0) = 0$. Under these restrictions, the optimal creation rate for each type conditional on the market wage and rate of creative destruction is uniquely determined by the first order condition stated as equation (11). Since the optimal creation rate is strictly increasing in productivity and strictly decreasing in the market wage, a necessary and sufficient condition for the optimal choice to be less than the rate of creative destruction, $\gamma(\pi) < \delta \;\; \forall\, \pi \in [\underline{\pi}, \overline{\pi}]$, at any point (w, δ) is above the curve defined by

$$wc'(\delta) = \frac{\pi - wc(\delta)}{r} \forall \pi \in [\underline{\pi}, \overline{\pi}] \Leftrightarrow w = \frac{\overline{\pi}}{rc'(\delta) + c(\delta)}. \tag{21}$$

This boundary of the admissible set if (w, δ) is labeled BB in Figure 1. As illustrated, the wage on the boundary is positive, tends to infinity as δ tends to zero, is strictly decreasing in δ, and tends to zero as δ tends to infinity given the assumed properties of the R&D cost function.

An equilibrium is any (w, δ) pair satisfying equation (19) and (20) provided that it lies above the boundary BB. Let $w = E_\pi(\delta)$ represent the locus of points implicitly defined by

$$\max_{\gamma \geq 0} \frac{\pi - wc(\gamma)}{r + \delta - \gamma} = \frac{w}{h} \tag{22}$$

and let $w = L_\pi(\delta)$ represent solution to

$$1 = w\ell + r\left(\max_{\gamma \geq 0} \frac{\pi - wc(\gamma)}{r + \delta - \gamma}\right) \tag{23}$$

in the region defined by (21). It is straight forward to show that $E'_\pi(\delta) < 0$. Because $\ell > \frac{c(\gamma)}{r+\delta-\gamma}$ when evaluated at points that satisfy (23), it also follows that $L'_\pi(\delta) > 0$. Although the curve of $E_\pi(\delta)$ lies below the boundary line BB defined by (21) at $\delta = 0$, eventually it intersects the line and remains above for all larger values of δ as drawn in the figure. For example, the curve defined by $w = E_{\overline{\pi}}(\delta)$, labled $\overline{EE}$ in the figure, intersects BB at the unique solution to $hc'(\delta) = 1$.

In Figure 1, the curves $\overline{LL}$ and $\underline{LL}$ represent $w = L_{\overline{\pi}}(\delta)$ and $w = L_{\underline{\pi}}(\delta)$ respectively Similarly, $w = E_{\overline{\pi}}(\delta)$ and $w = E_{\underline{\pi}}(\delta)$ are represented as $\overline{EE}$ and $\underline{EE}$. Because

$$\max_{\gamma \geq 0}\left\{\frac{\underline{\pi} - wc(\gamma)}{r + \delta - \gamma}\right\} \leq \max_{\gamma \geq 0}\left\{\frac{\pi - wc(\gamma)}{r + \delta - \gamma}\right\} \leq \max_{\gamma \geq 0}\left\{\frac{\overline{\pi} - wc(\gamma)}{r + \delta - \gamma}\right\} \text{ for all } \pi \in [\underline{\pi}, \overline{\pi}],$$

it follows that (22) implies $E_{\overline{\pi}}(\delta) \geq E_{\underline{\pi}}(\delta)$ and that (23) implies $L_{\overline{\pi}}(\delta) \geq L_{\underline{\pi}}(\delta)$. Furthermore, the joint solution to the equilibrium conditions (19) and (20) must lie in the intersection of the two pair of curves, the shaded area in Figure 1. Given continuity of the relationships, at least one common solution exists in that region. Finally, the shaded area lies above BB in the figure for all sufficiently large values of h because $E_\pi(\delta)$ is monotone increasing in h for any π from (22) while both $L_{\overline{\pi}}(\delta)$, defined by (23), and the BB curve, defined by (21), are independent of h. Indeed, since $\widehat{w} = h/(r + h\ell)$ at any joint solution to equations (22) and (23), the critical value of h, denoted $\widehat{h}$, and the associated rate of creative destruction at the intersection, $\widehat{\delta}$, are the unique solutions to

$$\frac{1 - \underline{\pi}}{\ell - c(\widehat{\delta})} = \widehat{w} = \frac{\widehat{h}}{r + \widehat{h}\ell} = \frac{\overline{\pi}}{rc'(\widehat{\delta}) + c(\widehat{\delta})}.$$

■

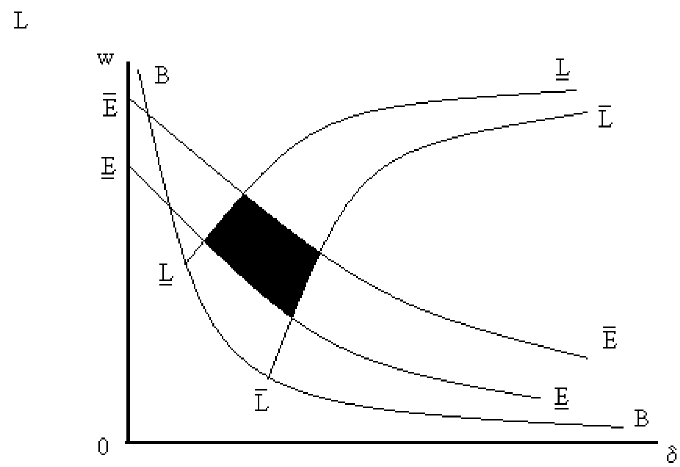

Figure 4:

5 Reallocation and Productivity Growth

Equation (14) implies that more profitable firms supply more products and sell more on average if and only if they innovate more frequently in the sense that $\gamma'(\pi) > 0$. However, because production employment per product supplied decreases with profitability, total expected employment, nEk where $n = (1-\pi)/w + c(\gamma(\pi))$, need not increase with π in general and decreases with π in the absence of firm heterogeneity. Hence, the hypothesis that firm's differ with respect to the quality of their products is consistent with dispersion in labor productivity and correlations reported above between value added, labor force size, and labor productivity.

The model developed in the paper, also implies that firm heterogeneity along this dimension has important implications for the sources of aggregate growth. Since every employed worker produces one unit of intermediate product per period, the labor productivity improvement attributable to an innovation of quality q relative to the version of the product replaced is $q-1 = \frac{\pi}{1-\pi}$. In turn, the aggregate rate of labor productivity growth is the product of the innovation rate and the average relative productivity improvement of entrants and surviving firms. Formally,

$$\frac{\dot{P}}{P}=\eta\sum_{\pi}\left(\frac{\pi}{1-\pi}\right)\phi(\pi)+\sum_{\pi}\gamma(\pi)\left(\frac{\pi}{1-\pi}\right)\sum_{k=1}^{\infty}kM_k(\pi)$$

where the first term captures the net contribution of entry and exit to productivity growth and the second is the contribution of continuing firms. The importance of entry and exit is well documented. For example, Foster, Haltiwanger, and Krizan (2001) find that 25 to 30 percent of productivity growth can be attributed to that source.

The model developed in this paper suggests the potential importance of worker reallocation from less to more profitable continuing firms as a source of productivity growth. The following decomposition,

$$\begin{aligned}\frac{\dot{P}}{P} &= \eta\sum_{\pi}\left(\frac{\pi}{1-\pi}\right)\phi(\pi)+\sum_{\pi}\gamma(\pi)\left(\frac{\pi}{1-\pi}\right)\phi(\pi) \\ &\quad +\sum_{\pi}\gamma(\pi)\left(\frac{\pi}{1-\pi}\right)\left[\sum_{k=1}^{\infty}kM_k(\pi)-\phi(\pi)\right]\end{aligned} \tag{24}$$

highlights that role. As before the first term is the net effect of entry and exist on productivity growth. The second term is the average contribution of continuing firms if there were no firm size selection and the last term can be regarded as a measure of the net contribution of worker reallocation to productivity growth. Because equations (12) and (17) imply that

$$\sum_{k=1}^{\infty}kM_k(\pi)=\frac{\eta\phi(\pi)}{\delta-\gamma(\pi)}=\phi(\pi) \tag{25}$$

if $\gamma'(\pi)\equiv 0$, the last term is zero without selection. Furthermore, because firms that grow faster supply more products, the distribution of profit over products lines, defined by the left side of (25), strictly stochastically dominates the distribution at entry, defined by the right side, when $\gamma'(\pi)>0$. Finally, because $\gamma(\pi)\left(\frac{\pi}{1-\pi}\right)$ is strictly increasing in π, the contribution of reallocation is strictly positive in this case. Note that the size of the reallocation effect, which can be written as

$$\begin{aligned}&\sum_{\pi}\gamma(\pi)\left(\frac{\pi}{1-\pi}\right)\left[\sum_{k=1}^{\infty}kM_k(\pi)-\phi(\pi)\right] \\ =\ &\sum_{\pi}\gamma(\pi)\left(\frac{\pi}{1-\pi}\right)\left[\frac{\eta}{\delta-\gamma(\pi)}-1\right]\phi(\pi),\end{aligned}$$

depends on the extent of the initial dispersion in firm profitability and the sensitivity of the innovation rate with respect to firm profit.

6 Concluding Remarks

Large and persistent differences in firm productivity and size exist. Evidence suggests that the reallocation of workers across firms and establishments is an important source of aggregate economic growth. In this paper, we explore a variant of the Schumpeterian model of firm size evolution developed by Klette and Kortum (2002) for insights regarding these and other empirical regularities. In our version of the model, entering firms that can develop more profitable products grow larger in the future and the necessary worker reallocation from less to more profitable firms contributes to aggregate productivity growth. Furthermore, the model is consistent with the observation that there is no correlation between employment size and labor productivity and a positive correlation between value added and labor productivity observed in Danish firm data.

In another paper in progress, we are estimating the structure of the model using the same Danish firm data described in this paper. The model's structural parameters are identified by the empirical firm size distribution observed, the patterns of firm sized evolution, and the correlations between firm productivity, labor force size and value added found in the data. Given these estimates, one can quantify the contribution of worker reallocation to productivity growth implied by the model. In addition, one can derive quantitative implications for growth of policies that effect entry and exist, the cost of labor, and patent rights.

References

[1] Baily, M., C. Hulton, and D. Campbell (1992) "Productivity Dynamics in Manufacturing Plants," *Brookings Papers on Economic Activity, Microeconomics,* 187-249.

[2] Bertelsman, E., and M. Doms (2000) "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economics Literature*, 38: 569-594.

[3] Christensen, B.J., R. Lentz, D.T. Mortensen, G.R. Neumann, and A. Werwatz (2002) "On the Job Search and the Wage Distribution." Forthcoming in the *Journal of Labor Economics.*

[4] Davis, S.J., and J. Haltiwanger (1991). "Wage Dispersion within and between Manufacturing Plants," *Brookings Papers of Economic Activity: Microeconomics,* 115-180.

[5] Davis, S.J., and J. Haltiwanger (1999). "Gross Job Flows," in Ashenfelter and Card (eds.), *Handbook of Labor Economics* vol 3. Elsevier Science.

[6] Dunne, T., L. Foster, J. Haltiwanger, and K.R. Troske (2002) "Wage and Productivity Dispersion in U.S. Manufacturing: The Role of Computer Investment," forthcoming in the *Journal of Labor Economics.*

[7] Fallick, B.C., and C.A. Fleischman (2001) "The Importance of Employer-to-Employer Flows in the U.S. Labor Market, Finance and Economics Discussion Paper 2001-18, Federal Reserve Board, Washing D.C.

[8] Frederiksen, A., and N. Westergaard-Nielsen (2002) "Where Did They Go." Aarhus School of Business working paper, Aarhus, DK.

[9] Foster, L., J. Haltiwanger and C.J. Krizan (2001), "Aggregate Productivity Growth: Lessons from Microeconomic Evidence" in C.R. Hulten, E.R. Dean and M.J. Harper, eds., *New Developments in Productivity Analysis.* Chicago: University of Chicago Press.

[10] Grossman, G. and E. Helpman (1991). *Innovation and Growth in the Global Economy.* Cambridge, Ma: MIT Press.

[11] Klette, J., and S. Kortum (2002). "Innovating Firms and Aggregate Innovation," NBER Working Paper 8819.

[12] Lentz, R., and D.T. Mortensen (2003). "Productivity Differences and Firm Size: Is a Firm More Than the Sum of Its Parts?", Northwestern Working Paper.

[13] Mortensen, D.T. (2003). *Wage Dispersion: Why are similar workers paid differently.* Cambridge, Ma: MIT Press.

[14] Stewart (2002) "Recent Trends in Job Stability and Job Security." U.S. BLS Working Paper 356.