Resource extraction, capital accumulation and time horizon*

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Abstract

This paper shows that a seemingly simple assumption, that of the time horizon of economic agents, has important consequences when modeling exhaustible resources but hardly makes any difference when modeling capital. It does so by exploring a common observation, namely, that economic agents have a progressive finite time horizon, meaning that they make a plan over a finite number of years but update this plan on a regular basis. This behavior can be observed in the business plans of firms, in US social security and in the extraction decisions of natural resource owners. Compared to an infinite horizon assumption, progressive finite time yields virtually identical results when used in a standard model of capital accumulation. However, used in models of natural resources, this behavior has the effect of removing the scarcity consideration of resource owners, thus letting only operating costs and demand determine the extraction rate. This implies that extraction will be non-decreasing and resource prices non-increasing for a long period of time – in line with the trends of a majority of exhaustible resources in the last century. Using data on how resource prices react to changes in resource stocks, the infinite horizon hypothesis is consistently rejected in favor of the progressive finite time hypothesis. A calibration of the model to the oil market yields a price which closely fits the gradually falling real price up until 1998 and the sharply increasing price thereafter.

JEL classification: D81, D91, O40, Q31

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1 Introduction

How foresighted are economic agents? Macroeconomics tends to gravitate towards the assumption that they have an infinite time horizon. The rationale for this assumption is that also finitely lived agents may behave as if they are taking the infinite future into account. This can either be because people care about their offspring or because today's agents care about the value of their asset tomorrow which, in itself, depends on the value of the asset the day after and so on. However, the infinite horizon assumption also requires that people have some basic information about possible things to come many years from now.

This paper departs from the infinite horizon assumption and instead explores a common observation, namely, that people tend to make plans for only a finite future and that the plan is revised regularly. In each time period, a plan is formulated treating the distant future as so uncertain that it might as well be ignored. The first period of the plan is executed and a new plan is then formulated for an equally long future and so on. I call this behavior progressive finite time[1].

The first question that arises is whether progressive finite time is a good description of economic behavior. This will be elaborated upon in the next section. The second question is if it really matters for the outcome whether a progressive finite time horizon rather than an infinite horizon is assumed. It is shown in the paper that while this type of behavior makes virtually no difference in a standard capital accumulation model, it has a qualitative effect on natural resource extraction and prices. This also provides an explanation for why we observe non-increasing exhaustible resource prices and exponentially increasing extraction and why the empirical literature generally does not find any support for the infinite horizon resource model[2]. The intuition is as follows.

In a basic infinite time exhaustible resource model with extraction costs, the logic is that resource use will fall over time due to discounting of the future. Hence, the resource price (the marginal product) will increase over time. More precisely, in a general equilibrium, resource owners need to be indifferent between keeping the resource in the ground or extracting it and putting the money in the bank. This leads to the conclusion that the resource price (net extraction costs) should rise at the rate of interest or, at the very least, that there should be a strong correlation between price growth and the real interest. This implication, which is maintained also under several extensions, scarcely has any support in the data[3].

Progressive finite time changes these results since total depletion may not

[1] This is similar to what Goldman (1968) refers to as continual planning revision and Easly & Spulber (1981) call rolling plans.

[2] For the basic model without extraction costs, see Hotelling (1931) and Dasgupta & Heal (1974) and for the basic model with extraction costs, see Weinstein & Zeckhauser (1976). Appendix C displays the trends of some resources. Examples of empirical studies are Heal & Barrow (1981), Abgeyebge (1989) and Halvorsen & Smith (1991).

[3] See section 6 for a review of this literature and the previous footnote for some empirical studies.

at all be optimal within the current time horizon if the resource stock is large enough or the time horizon short enough. The resource price would simply not cover the marginal cost of extraction. Hence, since total depletion is not optimal within the finite plan anyway, this essentially removes the scarcity consideration. Then, the extraction cost and the demand for the resource alone will determine the price and the rate of extraction. The outcome, which is an implementation of the first instant of a sequence of plans, will yield non-decreasing extraction and thus non-increasing prices for a long period of time. Furthermore, it will decouple price growth from the interest rate. This is shown in the paper, using the simplest possible model of exhaustible resources. By then adding technical change that pushes demand upwards we get exponentially increasing extraction, and by improving mining technology that reduces extraction costs, the resource price may well remain constant or even fall over long time periods. These results are in line with the factual trends of a majority of resources and are difficult to obtain with models of natural resources when agents have an infinite horizon.

In contrast to the qualitative effects of the time horizon on resource models, it is shown that deviating from infinite time to progressive finite time has almost no effect on standard models of capital accumulation. The reason is that progressive finite time really is a compromise between infinite and finite time models. As is well known, a finite time model of capital initially displays qualitatively close results to an infinite time model. Since in progressive finite time only the first period of the plan is executed before a new plan is written, also the long-run outcome will be qualitatively similar to an infinite time horizon. The difference between progressive finite time and infinite time will be in the steady state level and for reasonably long time horizons, also this difference will be marginal.

By using data on resource stocks over a large number of commodities during several years, the infinite horizon hypothesis is tested empirically against the progressive finite time hypothesis. Essentially, progressive finite time implies that the price should react to revisions in the stock only if exhaustion is within the market's horizon. In contrast, an infinite horizon implies that the price should react to revisions no matter how many years are left to exhaustion. Testing for a structural break in the data, the infinite horizon hypothesis is consistently rejected in favor of progressive finite time and suggests that the horizon length is somewhere in the range of 20-25 years. A further calibration of the model to the oil market closely replicates the falling real oil price after WWII and accurately predicts the sharp price increase after 1998 up until the recent financial crisis. However, it does not replicate the oil price shocks during the 1970:s which arguably were due to other factors than scarcity[4]. In comparison, a calibration of an infinite horizon model may replicate the falling price for a few initial years but predicts an increase in the price occurring about 35 years earlier than what is observed in the data.

Interestingly, even though progressive finite time has a qualitative impact on exhaustible resource models and not on capital accumulation models, it is the resource model that is time consistent while the capital model is not. In

[4] See e.g. Barsky & Kilian (2002) and Hamilton (2003).

a progressive finite time world, resource prices and extraction are expected to be constant and this is indeed also the realized outcome for many years. So if rational expectations is an ability that grows out of a trial and error process, then progressive finite time resource owners will find no reason to alter their forecasting procedure for a long time. Also the welfare losses using progressive finite time instead of an infinite horizon need not be that great. This is due to that later losses of a lower stock are largely discounted away and compensated for by higher consumption in the early days.

An important question is what value, if any, economic agents assign to holding the resource stock at the end of their horizon. Under full uncertainty, such a value need to be formed through rules of thumb or guesses which are not necessarily correct even on average and cannot be verified ex ante. Roughly speaking, in terms of the model, adding a continuation value to the stock has no effect on the outcome as long as the value is not based on rational expectations. In fact, it may even strengthen the explanatory power of the model.

The paper starts by introducing the concept of progressive finite time in more detail and presenting some microeconomic observations of such behavior. Section 3 shows that standard capital accumulation models are hardly affected by the choice of time horizon. Section 4 presents, by way of a simplistic model of exhaustible resources, the qualitative difference between a progressive finite time and a standard infinite horizon approach. It also addresses potential welfare losses and how a final stock value affects the results. Section 5 shows what the possibilities are of learning to improve one's planning over time. Section 6 first reviews the exhaustible resource literature and contends that an infinite horizon assumption is hard to reconcile with observed outcomes in the realm of exhaustible resources. Second, it shows that a progressive finite time assumption solves this disparity by using it in a model of exhaustible resources, capital accumulation and technical change. Section 7 uses data on resource stocks to test the model empirically. Section 8 calibrates the model to the oil market in order to investigate how it fits the historical price and extraction of oil. Finally, section 9 concludes by relating progressive finite time to other areas of economics and discusses some welfare implications.

2 Introducing progressive finite time

The idea that agents continuously update finite plans was first formalized by Goldman in 1968. The concept has been further explored in a few papers since, mainly dealing with risk and stationarity (Easly & Spulber, 1981) and optimality in settings of capital accumulation (Kaganovich,1985; Spulber, 1991)[5]. It has also been used extensively within supply chain management research, usually referred to as rolling horizons (e.g. Clark, 1998; Perea-López et al, 2003).

A finite planning horizon is meant to catch the notion that the further into the future we look the more uncertain we are of what the possible outcomes are

[5] Generally, progressive finite time has been found to yield results fairly close to the optimal infinite time horizon when it comes to capital. This is also what is shown in the next section.

and what probabilities to assign to them. At a far enough future the knowledge is so limited that we perceive to be under full uncertainty. The exact reason for this can possibly be modeled in many ways[6]. In this paper I will use the simplest possible assumption that enables analyzing the macroeconomic effects of having finite plans. Namely that agents have fully certain and correct knowledge of exogenous factors for the next T years, and have no information whatsoever afterwards. This is admittedly a very crude way of modelling, but it contains no obvious bias (compared to explicitly micro-modelling this behavior) since it catches the driving mechanism that plans are finite. For a further discussion see appendix A.

Graphically, progressive finite time is presented in figure 1. At a point in time (q) the economic agent receives perfect information for the coming T time periods (by perfect information is meant that they know the true outcome of exogenous variables, parameters and functional forms). No information is available, and the agent ignores forming beliefs over events beyond T. The agent makes a plan for the next T time periods with the aim of maximizing the aggregated discounted utility from consumption subject to some intertemporal constraints, today's state variable(s) and some terminal conditions. As part of this plan, a forecast of endogenous variables up to time $q + T$ is made (most notably prices) and used in solving the finite maximization problem. The agent then implements the first instant of the plan which determines also the state variable(s) for the next instant. A new plan is then written encompassing the time interval from $q + \varepsilon$ to $q + \varepsilon + T$ taking the state variable(s) as given. This way, writing of plans and executing the first instant of each plan, continuously progresses into infinity[7]. An important detail to note is that today's plan does not rely on the plans to be made in the future[8].

A subtlety in modelling progressive finite time is that the terminal conditions may be as important as the finite time horizon itself. In both a social planner and a decentralized setting this can be manifested through the beliefs on the continuation value of the assets at the end of the horizon (in terms of market value or in terms of welfare).

Now, when having no information about the distant future, whether and how to include an unverifiable future asset value must essentially be based on rules of thumb. To the extent that agents want to include the continuation value in their business plan, their estimation of this must be based on a guess, which will not necessarily be correct even on average. For most of the analysis the imposed final asset value will be zero. But also the case of a positive final asset value will be analyzed. The price forecasts within the planning horizon then have to be consistent with this terminal constraint in a general equilibrium.

[6] E.g. through ambiguity aversion, maximizing over the worst possible outcomes, gradually increasing uncertainty or increasing costs to forecasting. For a longer description see appendix A.

[7] For a formal description see appendix A.

[8] To make plans contingent on future plans would indeed be impossible given that the reason for making a finite plan in the first place is the complete uncertainty about the distant future.

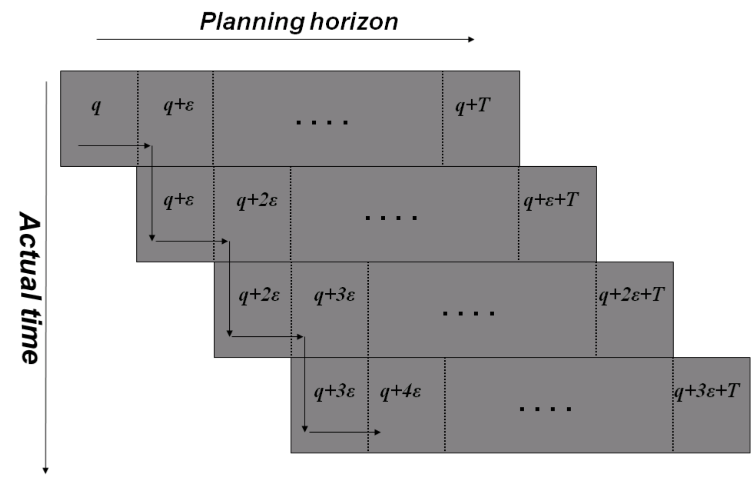

Figure 1: Schematic of a progressive finite time horizon. Each horizontal rectangle is a plan. The outcome is the diagonal formed by the first instant of each plan.

The essence of progressive finite time is perhaps best caught by the classical proverb "We'll cross that bridge when we get to it". It expresses the idea that people know that things will change in the future, but not exactly how, and that therefore there is no point in dealing with it now when the picture is unclear. Casual observations of this behavior among economic agents abound. Business plans of firms are exclusively stated finitely and updated regularly. Another example is of US social security. On an annual basis the solvency of the social insurance system over the next 75 years is updated in a report (Board of trustees, 2009)[9]. These forecasts and business plans are of course written today in knowing that they will be updated again next year. Also most government budgets are specified for a year or two at a time[10]. Another suggestive piece of evidence is that future markets for commodities such as oil seldom span more than a few years into the future.

Furthermore, in interviews I have conducted with Scandinavian resource owners, they state that the distant future is so uncertain that there is no point in including it in a business plan. They further say that it would be bad policy to deliberately save resources for an uncertain future and that they extract as long as the price covers their (marginal) costs of extraction. This can also be verified in some contract schemes. E.g. the Norwegian government, who is the owner of several North Sea oil and gas fields, gives firms the right to extract, at a more or less freely chosen rate, for ten years at a certain field. The rights are then given

[9] The report also includes an infinite horizon analysis. But, for policymakers the 75 year analysis is the most used.

[10] For further examples see Easly & Spulber (1981).

to a new firm through a bidding process. This type of contract should indeed induce extraction at the maximum possible speed with little consideration on the side of the firm of the continuation value of the field after the ten years. The choice of opening a new field for extraction is based on technical factors[11].

There is also plenty of experimental research suggesting that there are limitations to how far ahead people plan and that, when making finite plans, they assign no or low and certainly not a rational value to outcomes beyond their horizon. Most conducive to the subject of this paper is a recent study by van Veldhuizen & Sonnemans (2011). They show experimentally that when subjects have access to a large stock of exhaustible resources they tend to ignore the dynamic issue of resource allocation more than what rationality prescribes. For a review of other experiments and a further discussion of issues related to progressive finite time see appendix A.

3 Capital accumulation with progressive finite time

As a benchmark, consider a standard Ramsey-Cass-Koopmans capital accumulation economy consisting of a mass 1 of capital owners with an infinite time horizon and competitive firms owned by the agents.

$$\begin{aligned} & \max \sum_{0}^{\infty} \beta^t U(C_t)\, dt \\ K_{t+1} + C_t &= r_t K_t + (1-\delta) K_t + w_t \\ & K_0 \text{ given, } F_t = F(K_t) \end{aligned}$$

With appropriate assumptions on utility (U) and production (F) and a transversality condition, the results of this problem are well known. There is a unique path converging to a steady state of capital no matter which initial capital level is assumed. In particular, if initial capital is lower than that of the steady state, it will be increasing monotonically towards the steady state.

Now consider the finite time equivalent of this problem stretching from time q to $q+T$.

$$\begin{aligned} & \max \sum_{0}^{\infty} \beta^t U(C_t)\, dt \\ K_{t+1} + C_t &= r_t K_t + (1-\delta) K_t + w_t \\ & K_0 \text{ given, } K_{q+T} \geq 0,\ F_t = F(K_t) \end{aligned}$$

It is well established that the solution to this problem displays turnpike properties where (if K_q is small) capital initially increases and then falls as time approaches $q+T$ while consumption increases monotonically[12].

[11] For more information see NMPE (2008).

[12] For reference see Cass (1966).

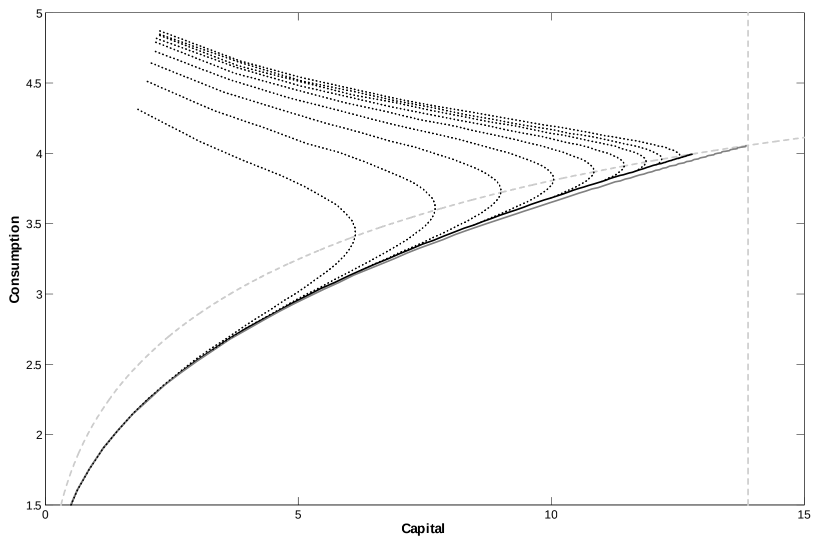

Figure 2: Phase diagram comparing a capital accumulation model of progressive finite time horizon of 35 years with an infinite time horizon model. Grey dashed lines = Equilibrium lines; Full grey line = Infinite time saddlepath; Full black line = Progressive finite time outcome ; Dotted black lines = Progressive finite time plans.

The progressive finite time alternative in effect is a sequence of finite time problems. In every instant of time (q) the representative agent makes a finite time *plan* from q to $q+T$ taking K_q as given. It executes the first instant (ε) of this plan and then makes a new plan based on new information from $q+\varepsilon$ to $q+\varepsilon+T$ taking $K_{q+\varepsilon}$ as given. This procedure is repeated until infinity[13]. The result is an infinite sequence of plans, each one having turnpike properties, and the *realized outcome* being an implementation of the first instant of each plan.

The results may be better understood with two simple figures. Figure 2 is a phase diagram showing the results of the infinite time problem and the progressive finite time problem when initial capital is low. The figure shows a saddle path of the infinite time problem representing the convergence to the steady state. Additionally there is a sequence of progressive finite time plans. The outcome of the progressive finite time case comes from the execution of the first instant of each plan. We then get convergence to a steady state, like in the infinite time version, but with a slightly lower level of capital and consumption.

The economy's evolution over time is presented in figure 3. With progressive finite time, each plan diverges from the infinite horizon case only at the end of the plan - early on in each plan capital is scheduled to increase just like with the infinite horizon. Since it is only the earliest part of each plan that is

[13] Note that every single plan is autonomous, in the sense that it is independent of what the future plans look like. Thus, a single plan is exactly equivalent to a finite horizon solution.

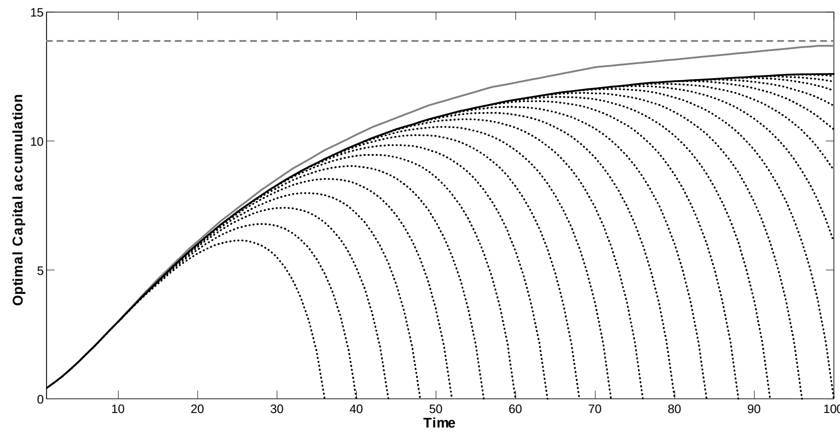

Figure 3: Time paths of capital with a progressive finite time horizon of 35 years and infinite time horizon. Grey dashed line = Infinite horizon steady state; Full grey line = Infinite time outcome; Full black line = Progressive finite time outcome ; Dotted black lines = Progressive finite time plans.

ever implemented, the falling parts of the plans never really bite. This way the outcome always lies on the upwards sloping part of the plan which has dynamics very similar to the infinite horizon case.

If we were to impose a final constraint so that $K_{q+T} \geq K_{\min} > 0$ that would make little difference to the results shown here. The same goes if we attach a final value to the capital stock. It would only lift the realized progressive finite time path and if anything make progressive finite time and an infinite time horizon look even more similar. For a large enough $K_{\min}$ (or capital stock value) we may even get more capital accumulation than in the infinite horizon model.

4 Natural resource extraction with progressive finite time

4.1 Basic results

To understand why the choice of time horizon matters in resource models, consider now a simplistic model of natural resource extraction with a mass 1 of identical resource owners having an infinite time horizon.

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{1}$$

$$C_t = p_t E_t - M(E_t) + w_t \tag{2}$$

$$S_{t+1} = S_t - E_t \tag{3}$$

$$S_{t+1} \geq 0 \tag{4}$$

$$S_0 \text{ given, } F = F(E_t, 1) \tag{5}$$

Now production of the consumption good is done using an exhaustible resource E by competitive firms with w denoting the labor wage and p the resource price after extraction. The cost of extracting the resource is $M(E)$ and is borne by the owners where M is an increasing and weakly convex function. The stock of the resource S is depleted at the rate of extraction, and there is a resource constraint stating that the stock cannot go below zero. Taking the first order conditions this problem yields

$$\begin{aligned} \frac{p_{t+1} - M'(E_{t+1})}{p_t - M'(E_t)} &= 1/\beta \frac{U'(C_{t+1})}{U'(C_t)} \\ p_t &= F'(E_t) \end{aligned}$$

which together with equations (2)-(4) gives the classic Hotelling result. That is, due to discounting of future utility, the solution displays falling extraction and consumption while the resource price (in equilibrium determined by the marginal productivity) , is increasing over time.

The formulation of the finite time version of this problem is seemingly similar. Let q denote the current year and T the number of future years to maximize over.

$$\max \sum_{t=0}^{T} \beta^t U(C_t) \tag{6}$$

$$C_{q+t} = p_{q+t} E_{q+t} - M(E_{q+t}) + w_{q+t} \tag{7}$$

$$S_{q+t+1} = S_{q+t} - E_{q+t} \tag{8}$$

$$S_{q+t+1} \geq 0 \tag{9}$$

$$S_q \text{ given, } F = F(E) \tag{10}$$

In contrast to the infinite horizon case, now there can be two possible outcomes of the problem. If the stock is large enough, or the time horizon short enough, or the cost of extraction high enough, then it will not be optimal to deplete the whole stock within T years. This is since the marginal cost of extraction will surpass the marginal productivity of the resource. In effect the resource constraint ($S_{q+t+1} \geq 0$) will not be binding. In this case it is optimal to extract such that the marginal mining cost equals the price of the resource in every period which in equilibrium implies that

$$p_t = M'(E_t) = F'(E_t). \tag{11}$$

This implies that extraction will be constant and so will the price. The other alternative is if the stock is small enough to be depleted fully within T years. Then the resource constraint will be binding and the solution will be qualitatively similar to the infinite time problem where extraction is falling and the resource price is increasing over time.

$$\frac{p_{q+t+1} - M'(E_{q+t+1})}{p_{q+t} - M'(E_{q+t})} = 1/\beta \frac{U'(C_{q+t+1})}{U'(C_{q+t})} \tag{12}$$

The progressive finite time version, again, is a sequence of finite time plans where the first year of each plan is executed before a new plan is made taking the current stock level as given. If the stock is initially high enough (supposedly when $q = 0$), then the resource constraint will not be binding and the *plan* will be to extract so that the marginal cost of extraction is equal to the resource price forecasting that the resource price equals its marginal productivity - as given by (11). Similar plans and forecasts will annually be formulated and the first year executed, which implies the *outcome* of constant extraction and prices for possibly a very long time. So it will continue until the stock becomes small enough to be depleted within T years. At this point the new *plan* will change character to include the binding resource constraint implying falling extraction - as in (12). In the period after, the stock will be smaller necessitating lower planned extraction and so on. This phase therefore yields the *outcome* of falling extraction and consumption and increasing prices.

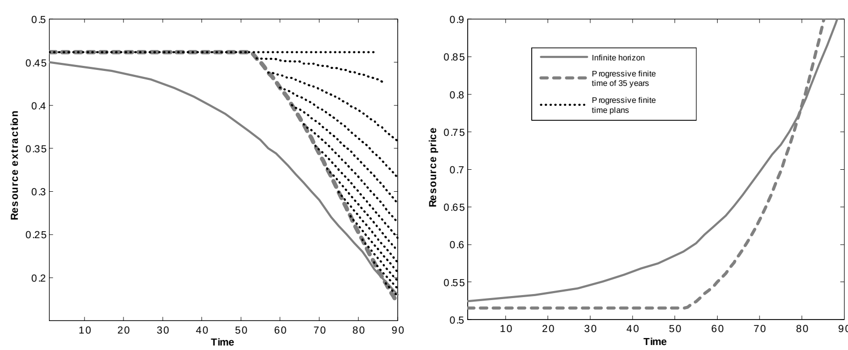

Figure 4: An exhaustible resource model with progressive finite time horizon of 35 years compared to infinite time horizon.

These results are visualized in figure 4. In total, the results of the infinite horizon and the progressive finite time horizon version of the problem are qualitatively different. This is because, in the latter, extraction and prices may be constant for a significant length of time. Once the shift to the second phase occurs the results are qualitatively similar to those of infinite horizon. But the progressive finite time problem has faster decreasing extraction and faster increasing resource prices. This is largely since the economy now has to cope with a smaller stock of resources due to the overextraction in the early years.

The empirical observations speak of an extraction which is increasing over time and a price which may be decreasing. To obtain such results it is enough to add some technological improvements in the extraction technology. This, and a few more enrichments of the model will be explored in section 6, in order to analyze under what conditions the extraction is increasing and the price is decreasing over time.

4.2 Empirical predictions

In preparation for the empirical tests, this section will outline formally some additional results that contrast the infinite and progressively finite resource extraction models. To help fix ideas, it will be assumed that the owner of the resource is profit maximizing (i.e. linear utility) and that the extraction costs are linear with B marginal extraction costs.

$$\begin{aligned} U(C) &= C \\ M(E) &= BE \end{aligned}$$

These two assumptions are sufficient, but far from necessary, to obtain the basic results described earlier. For the purpose of empirical testing it also helps to measure the resource scarcity in terms of years left to exhaustion. Now, if the exhaustible resource is truly necessary for production the remaining years are always infinite. However, a slight and, for most cases, realistic extension leads to exhaustion in finite time - the existence of a renewable substitute at some strictly positive level. Now the production function is

$$F(E_t + R).$$

Assuming that F is increasing and concave, a competitive equilibrium of the infinite horizon model (equations 1-5) will yield the following result.

$$\frac{p_{t+1} - B}{p_t - B} = 1/\beta \ \forall t \text{ such that } S_{t+1} > 0 \tag{13}$$

$$p_t = F'(E_t + R). \tag{14}$$

This is a form of the classic Hotelling result which states that the scarcity rent is increasing at the rate of interest $(1/\beta)$. Clearly, from equation (13), it follows that $p_{t+1} > p_t$ as long as the stock is not exhausted. Additionally, through concavity of F, it must hold that $E_{t+1} < E_t$. To analyze what happens after exhaustion, define the remaining years to exhaustion at time t as $\tau(S_t)$. Implicitly it is given by

$$\frac{F'(R) - B}{F'\left(S_{t+\tau(S_t)} + R\right) - B} = 1/\beta, \tag{15}$$

i.e. in the period of exhaustion, extraction is equal to the remaining stock. For all time periods prior to this, equation (13) holds. A few additional results can be derived.

Proposition 1 *In an infinite horizon model with a renewable substitute,* $\frac{dp_t}{dS_t} < 0$.
Proof. $\frac{dp_t}{dS_t} = \frac{dp_t}{dE_t}\frac{dE_t}{dS_t}$. *From concavity of* F *follows that* $\frac{dp_t}{dE_t} < 0$. *Equation (13) and concavity of* F *imply that* $\frac{dE_{t+i}}{dS_t} > 0$ $\forall i \in \{0, ..., \tau(S_t)\}$. *The result then follows.* ■

The proposition implies that an unexpected increase in the stock will lead to a decrease in the price when the market has an infinite time horizon. The effect of the remaining years to exhaustion on the price is outlined in the following proposition.

Proposition 2 *In an infinite horizon model with a renewable substitute, if* $\tau(S_t)$ *is increasing then* p_t *is decreasing .*
Proof. *Backward induction of (15) and (13), with concavity of* F, *imply that* $\tau(S_t)$ *is weakly increasing in* S_t *(weakly because* τ *is an integer). Proposition 1 implies that* $\frac{dp_t}{dS_t} < 0$.*Thus, since* $\tau(S_t)$ *is increasing only if* S_t *is increasing and* p_t *is decreasing iff* S_t *is increasing,* p_t *is decreasing if* $\tau(S_t)$ *is increasing.* ■

The proposition implies that a resource with more remaining years to exhaustion should, ceteris paribus, have a lower price.

A final proposition connected to the infinite horizon model regards the growth rate of the price and how it is affected by the size of the stock. Let p_t^* denote the price at time t given S_t^*, and p_t^{**} denote the price at t given S_t^{**}.

Proposition 3 *In an infinite horizon model with a renewable substitute, iff* $S_t^* > S_t^{**}$ *then* $\frac{p_{t+1}^*-p_t^*}{p_t^*} < \frac{p_{t+1}^{**}-p_t^{**}}{p_t^{**}}$.
Proof. *Define* $\lambda(S_t) \equiv p_t - B$. *Then* $\frac{p_{t+1}-p_t}{p_t} = \frac{\lambda(S_{t+1})+B}{\lambda(S_t)+B} - 1$. *Using (3) in this expression and the result that* $\lambda(S_{t+1})/\lambda(S_t) = 1/\beta$ *(from equation 13) the price growth inequality can be rewritten to* $\frac{\lambda(S_t^*)/\beta+B}{\lambda(S^*)+B} < \frac{\lambda(S_t^{**})/\beta+B}{\lambda(S^{**})+B}$. *This inequality holds since* $\frac{d\lambda(S_t)}{dS_t} = \frac{dp_t}{dS_t} < 0$ *(from proposition 1).* ■

Corollary 4 *In an infinite horizon model with a renewable substitute, if* $\tau_t^* > \tau_t^{**}$ *then* $\frac{p_{t+1}^*-p_t^*}{p_t^*} < \frac{p_{t+1}^{**}-p_t^{**}}{p_t^{**}}$.
Proof. *Follows from proposition 3 and that only if* $S_t^* > S_t^{**}$ *then* $\tau_t^* > \tau_t^{**}$ *(see the proof of proposition 2).* ■

Following the proposition and the corollary we should observe an acceleration of the price growth as time progresses and that the closer we are to exhaustion of the resource stock the higher should the price growth be. Also, an unexpected stock increase should have a negative effect on the price growth.

Let's contrast these results with the same model but with a progressive finite time horizon (equations 6-10), again, with linear utility, linear extraction costs and a renewable substitute. First the plans will be characterized after which the sequence of plans, which also form the outcome, will be described. With competitive markets and a concave F, the *plan* may take two qualitatively different forms. The first alternative is a counterpart of equation (11).

$$\begin{aligned} F'\left(E_{q+t}+R\right) &= B \ \forall t \in [0,T] \text{ if } TE_{\max} \leq S_q. \\ E_{\max} &\equiv \left[\left(F'\right)^{-1}(B) - R\right] \end{aligned} \quad (16)$$

Here $\left(F'\right)^{-1}(B)$ is the inverse of F' with respect to B and thus $\left(F'\right)^{-1}(B) - R$ is the extraction level where marginal productivity of the resource is exactly equal to the marginal extraction costs[14]. That way, $E_{\max}$ can be thought of as the maximum profitable extraction level in one period[15]. Equation (16) then states that if the endowment at q is larger than what can profitably be extracted in T years, then the extraction plan will be constant. If, on the other hand, $TE_{\max} > S_q$, the plan takes the second alternative form, which is the counterpart of equation (12).

$$\begin{aligned} \frac{F'\left(E_{q+t+1}+R\right)-B}{F'\left(E_{q+t}+R\right)-B} &= 1/\beta \quad t \in \{0,...,\min\{\tau\left(S_q\right),T\}\}, && (17) \\ E_{q+t} &= 0 \ t \in \{\min\{\tau\left(S_q\right),T\}+1,...,T\} \\ \sum_{t=0}^{T} E_{q+t} &= S_q && (18) \end{aligned}$$

where $\tau\left(S_q\right)$, the remaining years to exhaustion, is implictly defined by

$$\frac{F'(R)-B}{F'\left(S_{q+\tau(S_q)}+R\right)-B} = 1/\beta. \quad (19)$$

If the endowment is smaller than the maximum profitable extraction level over T years then we will get an extraction plan which is decreasing and a price forecast which is increasing over time up until $\tau\left(S_q\right)$ - in similarity to the infinite horizon model. Practically speaking, if T is large and S_q is small, the plan will dictate a falling extraction and increasing price for $\tau\left(S_q\right)$ periods and a constant price thereafter.

Letting $\tilde{x}$ denote a realized variable and S_0 be the resource stock at time zero, the observed outcome will go through three phases. Firstly, as follows directly from equation 16, for the initial $q = 0...S_0/E_{\max}$ periods

$$\begin{aligned} \tilde{E}_q &= E_{\max} && (20) \\ \tilde{p}_q &= F'\left(R+E_{\max}\right) && (21) \\ \tilde{S}_{q+1} &= S_0 - qE_{\max}. \end{aligned}$$

[14] Implicitly it is assumed that F' is invertible.

[15] Or more exactly, the maximum marginally profitable extraction level.

Secondly, for the periods $q = S_0/E_{\max}+1...q(S_q)$, where S_q solves $\frac{F'(R)-B}{F'(S_q+R)-B} = 1/\beta$,

$$\begin{aligned} \tilde{E}_q &= E_q \\ \tilde{p}_q &= F'(R+E_q) \\ \tilde{S}_{q+1} &= \tilde{S}_{q+1} - E_q \end{aligned} \tag{22}$$

where E_q is obtained by solving equations (17) and (18). Thirdly, for the periods $q = q(S_q) + 1...\infty$,

$$\begin{aligned} \tilde{E}_q &= 0 \\ \tilde{p}_q &= F'(R) \\ \tilde{S}_{q+1} &= 0 \end{aligned}$$

In one sentence, the progressive finite time model predicts that we should observe a constant price for a number of initial years, then an increasing price until the resource stock is exhausted whereby the price will be constant until infinity. To see why $\tilde{E}_q$ is decreasing in the second phase we can note that equations (17) and (18) imply an E_q which is decreasing with the stock (see proof of proposition 1).

In order to perform the empirical tests, the equivalents of propositions 1 to 3 and corollary 4 will now be derived for the progressive finite time model.

Proposition 5 *In a progressive finite time model with a renewable substitute*

1. $\frac{d\tilde{p}_q}{d\tilde{S}_q} = 0$ *if* $\tilde{S}_q > TE_{\max}$
2. $\frac{d\tilde{p}_q}{d\tilde{S}_q} < 0$ *if* $\tilde{S}_q < TE_{\max}$

Proof. *The first part follows from equation 21 which implies that $\tilde{p}_q$ is independent of the stock when $\tilde{S}_q > TE_{\max}$. The second part holds iff $\frac{d\tilde{p}_q}{d\tilde{S}_q} = \frac{d\tilde{p}_q}{d\tilde{E}_q}\frac{d\tilde{E}_q}{d\tilde{S}_q}$. From concavity of F follows that $\frac{d\tilde{p}_q}{d\tilde{E}_q} < 0$. Equation (17) and concavity of F imply that the planned extraction has $\frac{dE_{q+t}}{dS_q} > 0\ \forall t \in \{0, ..., \min\{\tau(S_q), T\}\}$. In particular, $\frac{dE_q}{dS_q} > 0$ which by (22) also implies that $\frac{d\tilde{E}_q}{d\tilde{S}_q} > 0$.* ■

This proposition expresses that unanticipated resource findings will have an effect on the price if and only if the previous stock was small. Or, put differently, if and only if the remaining years to exhaustion before the finding was less than the market's horizon. It also follows that when $\tilde{S}_q = TE_{\max}$ the price is sensitive to stock decreases but not to stock increases. The next proposition is the progressive finite time equivalent of proposition 2.

Proposition 6 *In a progressive finite time model with a renewable substitute then, iff* $\tau\left(\tilde{S}_q\right) < T$, $\tilde{p}_q$ *is decreasing if* $\tau\left(\tilde{S}_q\right)$ *is increasing.*
Proof. *See appendix.* ■

The proposition expresses that a resource with fewer remaining years to exhaustion should have a higher price. But, this should only occur when the remaining years are less than the markets time horizon.

As a comparison to proposition 3 and corollary 4 the upcoming proposition and corollary characterize how the price growth in the progressive finite time model is affected by the stock and remaining years. As before, $\tilde{p}_q^*$ will be the solution of the progressive finite time model with $\tilde{S}_q^*$, and $\tilde{p}_q^{**}$ will be the solution with $\tilde{S}_q^{**}$.

Proposition 7 *In a progressive finite time model with a renewable substitute, iff* $\tilde{S}_{q+1}^{**} \geq TE_{\max}$, *then* $\frac{\tilde{p}_{q+1}^* - \tilde{p}_t^*}{\tilde{p}_q^*} = \frac{\tilde{p}_{q+1}^{**} - \tilde{p}_q^{**}}{\tilde{p}_q^{**}} = 0$ *if* $\tilde{S}_{q+1}^* > \tilde{S}_{q+1}^{**}$.
Proof. *See appendix.* ■

Corollary 8 *In an infinite horizon model with a renewable substitute,*

1. *if* $T \geq \tau_t^* > \tau_t^{**} > 0$ *then* $\frac{\tilde{p}_{q+1}^* - \tilde{p}_t^*}{\tilde{p}_q^*} < \frac{\tilde{p}_{q+1}^{**} - \tilde{p}_q^{**}}{\tilde{p}_q^{**}}$.

2. *if* $\tilde{S}_{q+1}^*/E_{\max} > \tilde{S}_{q+1}^{**}/E_{\max} \geq T$ *then* $\frac{\tilde{p}_{q+1}^* - \tilde{p}_t^*}{\tilde{p}_q^*} = \frac{\tilde{p}_{q+1}^{**} - \tilde{p}_q^{**}}{\tilde{p}_q^{**}} = 0$.

3. $\lim_{\tilde{S}_{q+1} \overset{+}{\rightarrow} TE_{\max}} \frac{d\frac{\tilde{p}_{q+1} - \tilde{p}_t}{\tilde{p}_q}}{dS_{q+1}} = 0 < \lim_{\tilde{S}_{q+1} \overset{-}{\rightarrow} TE_{\max}} \frac{d\frac{\tilde{p}_{q+1} - \tilde{p}_t}{\tilde{p}_q}}{d\tilde{S}_{q+1}}$

Proof. *See appendix.* ■

The proposition and corollary express that, in a progressive finite time world, we should expect the price growth to be higher the smaller is the resource stock only if exhaustion is nearer than the market horizon. On the contrary, if the infinite horizon is right, then we should observe an increasing price growth no matter how many years are left to exhaustion.

4.3 Terminal constraints and end values

By letting T grow the results of progressive finite time will eventually converge to the case of an infinite horizon. This can be seen in, for example, proposition 7 and corollary 8 which, as T increases, converge to their infinite horizon counterparts. This is since when T grows sufficiently, the initial endowment S_0 must necessarily be smaller than $TE_{\max}$. Also if we use a resource constraint $S(t) \geq S_{\min}$ where $S_{\min}$ is large enough it is possible to get a progressive finite time environment which is similar or even more conservationist than in an infinite horizon case. If, on the other hand, $S_{\min}$ is small enough or if the rule of thumb is to leave a certain, small enough, fraction of the stock at the end, the original progressive finite time results go through.

A more subtle result is if we let there be a value to the final stock. Agents may use this value as a rule of thumb to proxy for scarcity or a longer horizon. For clearness of the ensuing results it will be modelled, again, with a competitive production market and atomistic and identical and profit maximizing resource owners of mass 1 each owning an equally sized resource stock. Now a representative household with a progressive finite time horizon makes a plan for extraction covering the next T years also considering there is some price, P_{q+T+1}, to every unit of the stock left in the ground at the end. Thus, in creating the plan, the following problem will be solved, where w is the wage and p is the resource price after extraction.

$$\max \sum_{t=0}^{T} \beta^t \left[p_{q+t} E_{q+t} - M\left(E_{q+t}\right)\right] + \beta^{T+1} P_{q+T+1} \left[S_q - \sum_{t=0}^{T} E_{q+t}\right]$$
$$S_{q+t+1} = S_{q+t} - E_{q+t}$$
$$S_q \text{ given, } S_{q+t} \geq 0,\ F\left(t\right) = F\left(E\left(t\right), 1\right),\ \beta \in\ ]0,1[$$

To solve this problem we need to make an assumption on how P_{q+T+1} is determined. Given that progressive finite time comes from lack of information regarding events beyond T, the case of an exogenous unit price will be analyzed first. Then the case of an endogenous unit price which is determined by the size of the aggregate stock will be analyzed.

When the continuation value is exogenous there may be two types of outcomes from this problem. The first type is just a straightforward maximization which yields the following *plan* and forecast.

$$\begin{aligned} p_{q+t} - M'\left(E_{q+t}\right) &= \beta^{T-t+1} P_{q+T+1} \quad \forall t \leq T \\ p_{q+t} &= F'\left(E_{q+t}\right) \end{aligned}$$

For standard functional forms this implies an extraction plan that is decreasing over time. What the unit end value $\beta^{T-t+1} P_{q+T+1}$ does is pushing a wedge between marginal productivity and marginal extraction. A wedge that is growing with t. However, and perhaps more importantly to note, is that the planned extraction for a certain period is determined independently of planned extraction in other periods. The reason for this comes from the assumption that the unit price P_{q+t+1} is independent of the size of the stock left behind. Thus, the progressive finite time *outcome*, i.e. the implementation of the first period of each plan is simply

$$F'\left(\tilde{E}_q\right) - M'\left(\tilde{E}_q\right) = \beta^{T+1} P_{q+T+1}\ ,\ q = 0...\infty.$$

From this expression it is clear that the time path of realized extraction is determined by the time path of P_{q+T+1}. As an example consider the case of a constant final unit value[16]. In this case the realized extraction will stay

[16] This can be the counterpart of believing there is some fixed price tag for selling the mine at the end of the horizon as a rule of thumb to roughly capture that there is some scarcity.

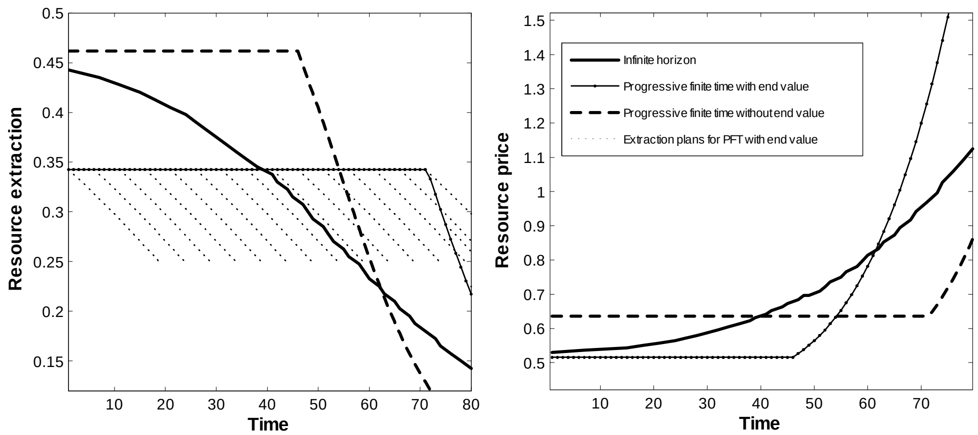

Figure 5: Simulation of an exhaustible resource model with a progressive finite time horizon of 20 years and an end value of the stock.

constant over time, just like in the case of no final value of the stock. Naturally this implies also a fixed realized resource price. If P_{q+T+1} is decreasing with q then extraction will increase over time and vice versa. Moreover, the level of P_{q+T+1} has no effect on the time trend. Thus, even if the guess on P_{q+T+1} is a gross overestimation of the value that will eventually be observed, this has no effect on the trend.

A constant extraction from a finite resource cannot continue indefinitely. Eventually, the nature of the solution has to change, which leads to the second type of outcome. Previously, the non-negativity constraint ($S_{q+t} \geq 0$) was not binding since the final value kept extraction low enough. Eventually, however, the *plan* will imply leaving no stock at the end. Thus, every plan will look like a standard Hotelling model with a finite horizon and a binding resource constraint. Since the *realized* stock is falling over time the realized extraction will fall and the price will increase.

Overall the total realized outcomes are displayed in figure 5. Here a constant and fairly high end value is set and thus extraction is initially lower than what would materialize in an infinite horizon case. However, extraction stays constant and only starts falling after many years. To note is that it stays constant for substantially longer than in the case of progressive finite time with no end value. In fact, the higher the end value is set the longer constant extraction will be upheld. A qualitative difference of adding an end value is that the plans in the first phase now dictate falling extraction over time while the realized extraction is constant[17].

Up until now the representative resource owner took the end value as an

[17] Recall that with no end value both plans and realized outcomes are constant in the first phase.

exogenous guess. If instead the end value is endogenous to the problem the progressive finite time result may no longer hold. In particular if P_{q+T+1} is based on perfect information until infinity, then P_{q+T+1} becomes a function of the stock[18]. This function perfectly incorporates all future profits of the resource and the progressive finite time and infinite horizon cases therefore perfectly align. More generally, if agents believe the end unit price is a function of the remaining stock, $P_{q+T+1}(S_{q+T+1})$, then if they believe that $P'_{q+T+1} < 0$ extraction will decrease over time since the value of saving the resource for the future increases as the stock falls. This can be seen in the first order condition determining the realized outcome.

$$\begin{aligned} U'(C(E_q))[F'(E_q) - M'(E_q)] &= \beta^T P_{q+T+1}(S_{q+T+1}) \ , \ q = 0...\infty. \quad (23) \\ S_{q+T+1} &= S_q - \sum_{t=0}^{T} E_{q+t} \end{aligned}$$

The left hand side of (23) is independent of S_q and decreasing in E_q[19]. The right hand side is decreasing in S_q and increasing in E_q. Thus, as S_q falls with time also E_q must fall. If agents instead believe $P'_{q+T+1} > 0$ then these results are reversed. The realized extraction will increase over time with a falling price, until the resource constraint is binding, whereby extraction will fall and prices increase.

An overall conclusion from the previous analysis is that adding a continuation value does not by itself undermine the main results of constant extraction and prices. Rather it depends on how this value is changing over time and thus how it is formed. If agents rationaly calculate the continuation value then we get falling prices. But if they routinely just attach some final unit value, to roughly catch that there is a future market, then constant prices and extraction may be realized during a long time period. One can note that when progressive finite time is coupled with a zero continuation value it is not the zero value by itself that drives the results but rather the consequential assumption that when the value is zero it is also constant.

A final discussion is that of potential welfare losses. To compare the welfare of a world using progressive finite time with one of infinite horizon is analytically hard. A numerical analysis however suggests that one either needs a very short time horizon or unrealistic parameter settings for losses to be large[20]. For example, a time horizon of 35 years consistently yields losses on the level of less

[18] The total continuation value is equal to all discounted future profits of the resource, $P_{q+T}S_{q+T} = \sum_{t=1}^{\infty} \beta^{t-1} \left[p_{q+T+t}E_{q+T+t} - M(E_{q+T+t})\right]$

[19] Since we know that now $F'(E_q) > M'(E_q)$.

[20] In the simulations I have used a CRRA utility function varying $\sigma \in [0, 5]$; the cost function $M = \frac{E^\theta}{Am}$ varying $\theta \in [1, 4]$; a concave production function $F = AE^\alpha$ letting $\alpha = .3$ mainly but vary the technology ratio $A_m/A \in [1, 4]$; a discount factor of 0.95 so that every period should be interpreted as a year; vary the horizon $T \in [5, 35]$. Then I vary the initial stock so that the phase of constant prices lasts between 0 and 100 years.

than 2 percent. The reason for losses being small is that with progressive finite time, consumption is initially higher than with an infinite horizon. For example, in figures 5 and 4 it takes over 60 years for extraction to fall below the infinite horizon counterpart. For subsequent losses to have an effect, economic agents need to be either very patient or very inclined to consumption smoothing[21]. For standard values of discounting and concavity of the utility function and reasonable horizon lengths[22] and for initial resource stocks such that extraction is non-decreasing for 50 years or more, the welfare losses stay under three percent and mostly well below[23].

Another way of addressing welfare is to ask what gains a single agent with an infinite horizon can make given that (s)he knows that everyone else has a progressively finite horizon. For specific parameter settings it is possible to get very large gains. For most parameter combinations however, and for those that are the most realistic, gains stay at levels of 0.5-5%[24]. The main reason for this is, again, that the additional profits of an infinite horizon can only be realized far into the future. When prices are constant for nearly a century, the later gains of having a larger stock once prices do start rising, are effectively discounted away. This also implies that, in the more complete model, of the upcoming section 6 , that an agent that is outsmarting the market by having a longer horizon will nevertheless extract at a growing rate. This is demonstrated in appendix B. Considering that anything that approaches a reasonable accuracy about the infinite horizon requires insights about the very far future and extreme computational ability, to properly asses the gains and losses computed above one has to compare them to the costs of making such plans. Furthermore, since an agent cannot know the gains of infinite plans without actually making them it may be interesting to analyze what signals agents receive regarding the quality of their plans. This will be dealt with in the next section.

5 Is progressive finite time immune to learning?

A common discussion in economics is whether certain behavior and beliefs are rational. One argument for imposing rationality on beliefs and behavior is that, over time, agents would learn from their mistakes and eventually form beliefs which are rational (i.e. stand the test of time) and thus also choose behavior that, at least on average, maximizes their objective function[25]. Since the motivation for modeling progressive finite time comes mainly from information

[21] E.g. $\sigma > 5$ in a CRRA utility function.

[22] Annual discounting between 3 and 7 percent, $\sigma \leq 3$, $T > 20$.

[23] Most likely this is an upper bound since the losses are driven by the convergence of consumption to zero over time - i.e. the progressive finite time model converges to zero faster than does the infinite horizon model. In a model with a renewable substitute there will be a lower bound for consumption implying losses should be more limited than here. Resource augmenting technical change should have a similar effect.

[24] For a typical result, see appendix B.

[25] In other settings time inconsistency may be embedded in the preferences of agents. For example, as modeled by hyperbolical discounting leading to "games against selves" (see Krusell & Smith, 2003).

constraints (rather than lack of care of the future), this notion of rationality is important to address. Put differently, if agents constantly notice that their forecasts are wrong they may learn from this and over time improve the accuracy of their forecasts - for example by extending the time horizon. So the question to be answered in this section is to what extent progressive finite time agents will get the opportunity to learn from potential mistakes they make[26].

If beliefs (i.e. the price forecast) and planned actions (i.e. state variables) are in line with later observed outcomes I will refer to it as progressive finite time being "immune to learning". In a sense this catches the spirit of self-confirming equilibria (e.g. Fudenberg & Levine, 1993). The main idea is to capture that agents on an aggregate level may be on a non-optimal path but that they may not have any reason to suspect that some other actions are better since their beliefs are always fulfilled. To see whether the requirements above are fulfilled one essentially asks if and after how long time agents get the opportunity to realize they have been wrong.

A capital accumulation model with progressive finite time is immune to learning only up until a new plan is made. This can be seen in figure 3. Since the first period of a plan is implemented, also the state variables of that next period and thus the interest rate will be consistent with the forecast. However, the new plan will differ from the previous regarding all ensuing periods, implying that later outcomes will differ from what was forecast and planned in the first plan.

This is contrasted by the exhaustible resource model. As long as the economy stays within the first paradigm (when the resource constraint is not binding) agents will not be proven wrong and will thus be immune to learning. This is since prices are forecast to equal marginal extraction which is also what is realized. This comes clear in figure 4, where the outcomes are equal to the plans for the first 55 years. It implies that agents will not get signals that they need to improve their forecasting since they turn out to be right. This will proceed as long as the economy is in a state where the resource constraint is not binding[27]. It does not, however, continue when the resource constraint eventually starts binding. Then, progressive finite time is only immune to learning up until a new plan is made. This is since in the next plan extraction is revised downwards and the price forecast revised upwards. Also this can be seen in figure 4 - when the extraction rate falls the plan lies above the realization[28].

A noteworthy and possibly counter-intuitive implication of the results above is the reversed connection between outcomes under different horizons and opportunities to learn from mistakes. When the observable outcomes of progressive finite time and infinite horizon are qualitatively similar (in the capital model

[26] A formal treatment can be found in an online appendix on my website: http://people.su.se/~dask4398/.

[27] Note that this holds also for an exhaustible resource model that incorporates exogenous technical change in extraction and/or production, as long as agents have correct expectations of technical advancement within the time horizon.

[28] These results of the resource model do not hold if agents asign an exogenous end value to the stock. Then, the plans dictate falling extraction and increasing prices also initially, while the realization is constant extraction and prices.

and in the late years of the resource model) then agents will have the chance to realize that plans are biased when using progressive finite time. But when the qualitative results *do differ* between progressive finite time and infinite horizon (in the early years of the resource model) then agents will not get the opportunity to learn since they are not proven wrong[29].

As the economy reaches the change of phases in the resource model, agents will start receiving signals that a short horizon yields inaccurate predictions. This also coincides with an increase in welfare losses[30]. It then seems reasonable that over time agents may try lengthening their forecasts to horizons such that welfare losses are small.

6 Explaining the observed price and extraction trends

The models outlined in the previous sections were deliberately kept simplistic to emphasize the main difference between how the time horizon affects capital accumulation and exhaustible resources respectively. This section will present a more complete model in order to explain the observed trends of extraction and prices of exhaustible resources. The model results will be presented verbally and graphically, but all claims have analytical support which is presented in appendix B. But first a review of the resource literature and the empirical observations to motivate why the observed trends need an explanation.

6.1 The resource extraction and price puzzle

During the last century a large majority of exhaustible resources have displayed exponentially increasing extraction and constant or decreasing price trends (see appendix C). Even though the price volatility has been substantial there are only few examples of resource prices actually increasing over a longer period of time.

The benchmark models of exhaustible resources without extraction costs were developed by Hotelling (1931) and Dasgupta & Heal (1974) and with extraction costs by Weinstein & Zeckhauser (1975), Solow & Wan (1976) and Heal (1976). The focus of these papers was mainly to analyze the intertemporal trade-off between using a resource today and saving it for later days. A central result is then that the price of an exhaustible resource will contain two elements - the marginal cost of extraction and the scarcity rent. The former may possibly be decreasing over time but the latter must be increasing at the rate of interest. Like was shown in figure 4, the benchmark model predicts that extraction will

[29] A striking example of this is Simon's (1996) claims. Historically, people that have ignored scarcity in their forecasts have turned out to be more right than people that have considered resource scarcity a factor. Thus, according to Simon, the best way of making forecasts on the resource markets is by ignoring scarcity.

[30] The numerical analysis suggests that the closer one is to the change of phases, the larger the losses are.

be falling and prices increasing over time, i.e. the opposite of the empirical observations. The model also implies that there should be a direct correlation between the interest rate level and the growth of the resource price.

By introducing a quickly improving mining technology one may get the infinite horizon model to exhibit non-decreasing extraction and non-increasing prices for some time. But remember, the faster the mining technology will improve the faster its effect wears off and the shorter the period of non-decreasing extraction will be. Graphically, in figure 4, improving mining technology implies a pivoting of the extraction graphs upwards and the price graphs downwards. But this would also pivot the progressive finite time graphs in the same direction giving even more rapidly increasing extraction and falling resource prices. So a progressive finite time assumption yields the observed results under much weaker parameter, functional and technological conditions, than does the infinite horizon assumption. To get falling resource prices over any longer period of time, in an infinite horizon model, one would need to assume very specific exogenous settings. These main theoretical results are also robust to some extensions[31]. Meanwhile, it has been notoriously hard to find the empirical link between the growth of resource prices and the level of the interest rate. Several empirical papers reject that there is any correlation as implied by an infinite horizon theory of exhaustible resources (Heal & Barrow, 1981; Abgeyebge, 1989; Halvorsen & Smith, 1991).

Arrow & Chang (1982) model how new findings relax scarcity. They show that a constant price trend can only be obtained if the true resource stock is unlimited. If it is bounded (which should be the case for many important resources such as oil, gas and coal) the price trend must be increasing, albeit possibly at an initially slow pace. In these models it is indeed not possible to get a falling trend of the resource price. The intuitive reason is that it is the expected rather than the verified amount that determines scarcity and thus extraction and prices. Unexpected discoveries may lower the price temporarily but likewise the absence of discoveries should increase the price. If agents have rational expectations about the true stock, this cannot be a consistent explanation for non-increasing prices of a broad range of resources over a long period of time[32].

Another explanation to the extraction and price trends has been proposed by Tahvonen & Salo (2001). Here a renewable substitute and endogenous technical change put a cap on resource prices. While this indeed is an alternative explanation that holds for cases where there is a renewable substitute present all along, the model results that now will be presented hold also without a

[31] E.g. the risk of a renewable substitute making the resource worthless (as in Kamien & Schwartz,1978; Davison, 1978) in effect makes discounting of the future stronger, only reinforcing the effect of decreasing extraction and increasing prices.

[32] Another model that explores the exploration aspect is that of Pindyck (1978). Here there is a cost to finding new reserves, but the total reserves are known. The result of a potentially falling resource price critically hinges on two assumptions. That the per unit extraction cost is falling with the number of wells and that extraction costs are not convex within one well. This is in opposite to the common understanding that new findings are usually costlier to extract from and that there are usually physical limits to extraction per time unit, for example due to well pressure and infrastructure limits.

renewable substitute.

6.2 The model

Consider a representative agent with a discrete progressive finite time horizon[33]. For each time period $q = 0, 1, ...\infty$ (s)he aims to write a plan by maximizing the following objective.

$$\max \sum_{t=0}^{T} \beta^t U\left(C_{q+t}\right) \quad , \beta \in \;]0, 1] \;\; , \; U' > 0, U'' < 0 \tag{24}$$

The economy consists of competitive firms whose production facilities (H) display CES properties.

$$\begin{aligned} H_{q+t} &= \left[\gamma F_{q+t}^{(\sigma-1)/\sigma} + (1-\gamma)\, G_{q+t}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \sigma > 0 & (25) \\ F_{q+t} &= K_{q+t}^{\alpha} \left(A_{L,q+t} L_{q+t}\right)^{1-\alpha} & (26) \\ G_{q+t} &= A_{NR,q+t}\left(E_{q+t} + R\right) & (27) \end{aligned}$$

Here σ represents the substitutability between the production capacity (F) and the resource capacity (G). The production capacity is based on the amount of machines (capital, K), workers ($L = 1$) and how efficient these workers are (A_L). The production capacity is assumed to be of Cobb-Douglas type. The resource capacity (G) is determined by the amount of a constant and exogenously given renewable resource ($R \geq 0$) and how much of an exhaustible resource (E) that is being extracted in each time period. For tractability the renewable and exhaustible resource are assumed to be perfect substitutes and there is a resource augmenting technology (A_{NR}) making their use more efficient. Furthermore, at a time period q there is a stock ($S_q \geq 0$) of exhaustible resources from which to extract. Thus, for a plan to extract E_{q+t} we get the law of motion of the resource stock and the resource constraint as follows.

$$\begin{aligned} S_{q+t+1} &= S_{q+t} - E_{q+t} & (28) \\ S_{q+t+1} &\geq 0 & (29) \end{aligned}$$

The process of extracting the exhaustible resource is subject to a cost.

$$\begin{aligned} M\left(E, A_M\right), \frac{\partial^2 M}{\partial E^2} &> 0 \; \forall \; E, \;\; M\left(0, A_m\right) = 0, \\ \lim_{E\to\infty} \frac{\partial M}{\partial E} &= \infty, \;\; \frac{\partial M}{\partial A_M} < 0 \text{ if } E > 0. & (30) \end{aligned}$$

Thus, the marginal cost is strictly increasing with scale which can be motivated by infrastructure investments (for transport or digging) that are needed if extraction rates are to be increased. Moreover, as is the case for oil, there is a flow

[33] Discrete time is used for clarity of exposition. The continous time counterpart of course yields the same results but requires a bussier notation.

of the resource given by the geological structure that effectively determines the maximum extraction rate[34]. A_M is a technology making the extraction process more efficient. Suppressing the variables of technology we can now state the budget constraint[35].

$$C_{q+t} + K_{q+t+1} = p_{q+t}\left(E_{q+t} + R\right) + r_{q+t}K_{q+t} + w_{q+t} - M\left(E_{q+t}\right) + \left(1-\delta\right)K_{q+t} \tag{31}$$

At time q an *equilibrium plan* is defined as a set of prices $\{p_{q+t}, r_{q+t}, w_{q+t}\}_{t=0}^{T}$ and resource stocks, factor inputs and consumption $\{S_{q+t}, E_{q+t}, K_{q+t}, C_{q+t}\}_{t=0}^{T}$ such that markets clear and the problem (24)-(31) is solved. The outcome will be the realized first period control variables and the second period state variables from each plan $\{E_q, C_q, S_{q+1}, K_{q+1}\}_{q=0}^{\infty}$.

6.3 Non-scarcity phase

The economy described above will go through two phases, if the initial resource stock is large enough (see proposition 11 in the appendix). Initially there will be a finite but potentially very long phase with no perceived scarcity of the exhaustible resource. This will be taken over by a phase where the resource constraint is binding. The results are visualized in figure 6. In this particular simulation the non-scarcity phase lasts for 66 time periods.

The initial phase where no scarcity is perceived occurs since total depletion is not optimal within the time horizon of the economic agents, in essence implying that the exhaustible resource will be treated as non-exhaustible. In this phase the marginal productivity and marginal extraction costs alone determine the extraction rate (equation 34). Since there is no perceived bound to the extraction - other than the extraction costs - any change that either increases demand or decreases the extraction cost will lead to increased extraction. Furthermore, any change that increases demand will increase the resource price and any change that decreases the extraction cost will decrease the resource price (proposition 13 and corollary 14). More precisely, labor augmenting technology will increase extraction and the price; mining technology will increase extraction and lower the price; the effect of resource augmenting technology depends on parameters. Under fairly general conditions we can then get extraction which is increasing exponentially and a resource price which is non-increasing for a long period of time. This happens if capital is initially low and the mining technology improves fast enough to offset the increased cost when extraction increases.

During this phase capital accumulation, output, consumption and the real wage will be increasing over time. The returns to capital will be roughly constant and profits of resource owners will be positive (equation 37) - all in line with the stylized macroeconomic facts of the last century[36]. Furthermore, if production

[34] For a summary see Witze (2007) or Davidson (1963).

[35] It is assumed that the representative agent owns the capital, the resource stock and performs the extraction.

[36] These results hold also for other production functions and for more weakly specified

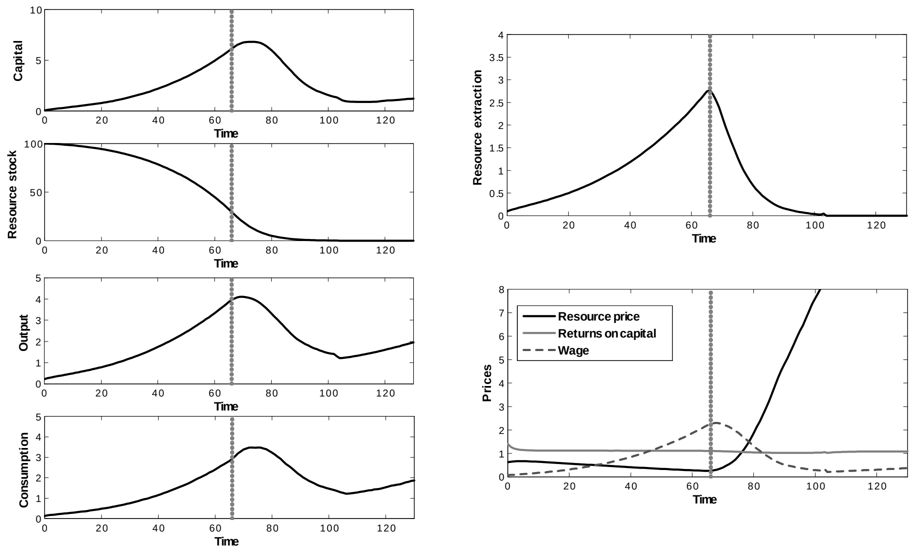

Figure 6: A ten year horizon economy's evolution over time. The vertical line marks the shift from "Non-scarcity" to the "Shadow of exhaustion" phase.

is of Cobb-Douglas type there exists a balanced growth path of capital, output, consumption, real wage and extraction but where the resource price is non-increasing if the extraction cost function is not "too" convex[37]. The intuition for this is that when extraction is increasing over time the cost function cannot be so convex as to outweigh the improvement in extraction technology (propositions 16 and 17).

6.4 Shadow of exhaustion phase

Increasing extraction cannot be sustained forever with a finite resource. Eventually, as the stock becomes small enough, total depletion will be possible within the time horizon of the representative agent. At this point the resource owners will start treating the resource as exhaustible implying that also the scarcity rent will start affecting the extraction rate. I call this phase "the Shadow of exhaustion" as the economic agents act based on the resource limits. In figure 6 this phase is represented by period 67 onwards.

mining cost functions. The reason for not stating all functions completely general is that it would yield a plethora of subcases to prove. Essentially, any functional combination where the marginal producitivity of resources intersects the marginal extraction costs from above (in a schedule of extraction on the x-axis and price on the y-axis), will yield similar results. This way we can even have a marginal extraction cost which is decreasing with scale. This arguably covers most of the reasonable cases of production and mining. The way labor and capital are specified in production plays little role here.

[37] A rough calibration yields the extraction cost function should not be more than quadratic.

Here the economy will behave more or less as described by the classical Hotelling model with extraction costs (see for example Weinstein & Zeckhauser's 1975 article or equations 42, 43 and 44 in the appendix). In this phase extraction can be increasing for a few time periods but soon enough it will start falling and the resource price start to increase. In this model, with a single type of an exhaustible resource, the economy will experience a downturn. This is since previously, the production process relied on large quantities of a "cheap" resource that was perceived to be limitless, which now falls in supply and increases in price. A detail to note is that this shift between the economic phases will appear smoothly, where the change will manifest itself through the trend of variables, but there will be no discrete level effects. This is because, even though agents do not have a perfect foresight, they do get an early warning to consider scarcity before total depletion becomes factual. Of course, the shorter the time horizon is the more abrupt the change will be. In a sense, this model predicts an unfolding of events somewhere in between the doomsday scenario of total economic collapse, when we for example start running out of oil, and the optimistic take that resource scarcity will never be an issue.

During this phase the technologies are assumed to continue evolving over time. If there exists a renewable substitute or if resources are not essential in production, the exhaustible resource will loose its role over time (equations 45-48). The improvements in production efficiency will then eventually lead the economy onto a new growth path based on renewable resources only.

7 Empirical tests

This section has two objectives. Firstly, it will test whether there is empirical support for the infinite horizon or the progressive finite time assumption in the resource extraction model. It will pit the predictions (outlined in section 4.2) of the two alternative assumptions against each other. Secondly, and to some extent simultaneously, it will assess what the market's time horizon is, contingent on the progressive finite time assumption being correct.

7.1 Data

Data over remaining reserves has been collected from the US geological survey (USGS) for each year in 1996-2011[38]. They publish a yearly mineral commodities summary which reports the remaining "Reserve base" and "Reserves" for roughly 80 commodities. In the report, the reserve base is defined as "...encompassing those parts of the resources that have a reasonable potential for becoming economically available within planning horizons beyond those that assume proven technology and current economics". Reserves are defined as "that part of the reserve base which could be economically extracted or produced at the time of determination". Thus, the reserves estimation is a subset of the reserve base. After excluding some resources where data is lacking and

[38] See www.usgs.gov.

some where reserves are sufficient for any conceivable time horizon (like stone, sand and salt) a dataset of 52 commodities was constructed. Reserve data was then added for oil and gas which goes back to 1980 (from BP, 2011), i.e. a total of 54 commodities. Extraction and price data for these commodities was also collected from the USGS (and BP's Statistical review of World Energy in 2011, for oil and gas)[39].

The predictions from the progressive finite time model depend on whether the resource constraint is binding within the market's time horizon. In order to know whether this is the case for a specific observation, a measure of the remaining years to exhaustion is needed. Denote the remaining years by $\tau_{i,q}$ where $q \in \{1996...2011\}$ is the observation year and $i \in \{1...54\}$ is the commodity number. $\tau_{i,q}$ should be based on the production prognosis made at year q for the specific commodity. In absence of actual market forecasts, τ needs to be proxied in some way. The method here will be to make a production prognosis and compare it to the known reserves at the time. For this purpose, the production trend from the years preciding q was extrapolated forward. The base method was to use the previous 15 years and give more weight to recent years when calculating the trend. Now, it is hard to know whether this is good approximation of how the market makes forecasts. A number of other methods were therefore tested for robustness. Namely, varying the previous years used for the trend; giving all previous years the same weight; using the simple reserve to production ratio; using the growth in the upcoming years for all resources combined, to proxy for the trend. Generally, the exact method used had no significant effects on the results, except for the reserve to production ratio which undermined the results for both the infinite and progressive finite time assumptions.

7.2 The effect of stock revisions on the price growth

The main empirical test employed will be to use changes in stocks and see how that affects the price. The change in the stock is plausibly an exogenous event. Although market participants may expect that the reserve estimates will be revised from time to time, it seems reasonable to assume that they don't know exactly when, to what extent and in what direction. This is the reason why, also theoretically, new reserve findings will have short run effects on the price but no long run effects (see Arrow & Chang, 1982). In practice reserve revisions may follow from new findings, revisions of current stocks or from access to better data from some, quite often less developed, countries. A problem here is that, with yearly data, the exact timing of the news is a bit vague. Whether they arrived early or late in the year will affect how much the average price of that year will be affected. Therefore it should be expected that the data contains large amounts of noise. Furthermore, one could expect the reserves to be revised every year purely as a consequence of extraction. However, a closer look at the data shows that far from all observations have an update of the stock. A guess

[39] www.bp.com

to why this is the case could be that often the extraction is not extensive enough to put a dent in the reserves and that only major surprises, such as a new finding or a substantial up- or downgrading of the current stock will lead to a revision. Thus, in the upcoming tests revised stocks will interpreted as news arriving beyond what could have been expected in the previous period.

A potential source of endogeneity is that as exhaustion is nearing more exploration efforts are made. Thus there may be a reversed causality where price increases lead to more exploration. This should not affect the results directly since the price growth following a revision is always compared to the price growth before. It may, however, imply that there are fewer observations where the stocks are revised and there are still many remaining years to exhaustion. In practice it should blow up the standard errors when the remaining years are long. It will therefore be important to also look at the sign of the coefficient when evaluating the results. Another source of worry may be that somehow market participants are better at foreseeing or have better information about revisions that occur when the remaining years are short. This should imply that the effect on the price growth will occur before the time when the is it logged in the data. Thus, this will push down the coefficients when the remaining years are short which will be unfavorable to the progressive finite time hypothesis.

Propositions 3 and 7 express the effect from reserve revisions on the price growth. In essence the infinite horizon assumption (proposition 3) implies that revisions upwards should have a negative effect on the price growth while the progressive finite time assumption (proposition 7) implies this effect to occur only if the remaining years are less than the market's horizon[40]. To discriminate between these two alternatives the observations will be grouped according to how many years are left to exhaustion, i.e. $\tau_{q,i}$. The following equation will be estimated using OLS.

$$\begin{aligned}\frac{\triangle p_{q+,i}}{p_{q+,i}} - \frac{\triangle p_{q-,i}}{p_{q-,i}} &= a + b_1\left(\frac{S_{q,i} - S_{q-1,i}}{S_{q-1,i}}\right) \\ &\quad + b_2\left(\frac{S_{q,i} - S_{q-1,i}}{S_{q-1,i}}\right)^2 + b_3\left(\frac{\triangle p_{q+,comp}}{p_{q+,comp}} - \frac{\triangle p_{q-,comp}}{p_{q-,comp}}\right) + \varepsilon_{q,i}\end{aligned} \tag{32}$$

$\frac{\triangle p_{q+,i}}{p_{q+,i}}$ is the growth rate of the price from year q onwards and $\frac{\triangle p_{q-,i}}{p_{q-,i}}$ is the price growth rate leading up to to year q. The coefficient of interest is here b_1 which catches the first order effect of a one percent increase in the stock on the growth rate of the price after the change compared to before. $p_{q,comp}$ is a composite price index of all commodities. Thus, the term connected to b_3 is a control variable for price changes of commodities in general - a sort of time fixed effects. If the infinite horizon assumption is right we should expect b_1 to be negative no matter how data is grouped, i.e. for all $\tau_{q,i}$, while if the progressive finite time assumption is right we should expect b_1 to be negative if and only if $\tau_{q,i} < T$ and ambiguous otherwise.

Figure 7 displays the coefficient b_1 and its 95% confidence interval (y-axis)

[40] An equivalent reasoning applies to reserve downgrades.

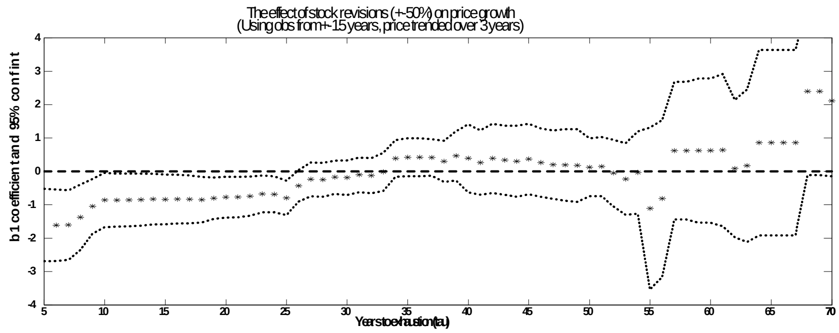

Figure 7:

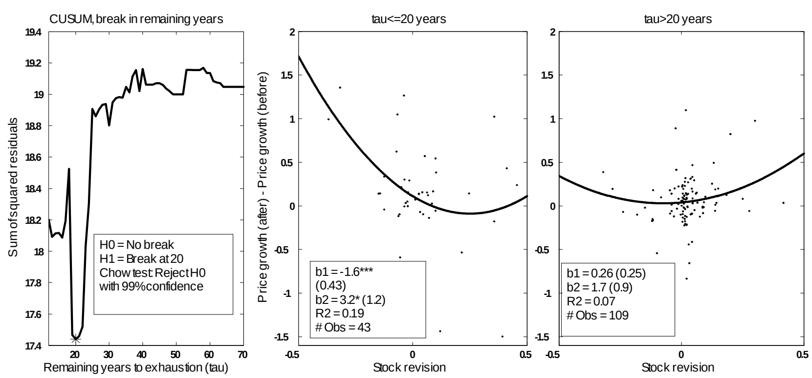

Figure 8:

when grouping observations by $\tau_{q,i}$ (x-axis). I.e. equation 32 is estimated once for every subset of the remaining years to exhaustion[41]. Clearly, from the figure, when using observations with more than 25 remaining years b_1 is insignificant and when there are more than 34 remaining years b_1 even has the wrong sign. This lends support to the progressive finite time assumption compared to an infinite horizon. Where the exact cutoff for b_1 is is hard to judge from the figure. Indeed, although the qualitative result of the existance of a cutoff remains, the exact placement may vary up or down by a few years depending on the exact values chosen for grouping, maximum revision size and prior and posterior years used when calculating the price growth.

To better pin-point the cutoff and also test whether it is significant, a

[41] In this specific figure $\frac{\triangle p_{q+,i}}{p_{q+,i}}$ was calculated using the average growth rate 3 years after q. $\frac{\triangle p_{q-,i}}{p_{q-,i}}$ used the 3 years preceeding q. Furthermore, observations with revisions exceeding $\pm 50\%$ of the stock were discarded. Finally, observations were grouped according to $\tau_{q,i} \pm 15$ years.

CUSUM test of structural breaks was used. The test checks for what cutoff of remaining years to exhaustion (τ) it is most likely to have a structural break in the data. Practically, the sample was partitioned into one pool with $\tau_{q,i} \leq \tau$ and another pool with $\tau_{q,i} > \tau$. The regression (32) was then run on each pool separately and the sum of square residuals of both regressions together was calculated. The results are displayed in figure 8. The leftmost graph displays the sum of squared residuals when varying the cutoff τ from 10 to 70. The structural break is most likely to occur at $\tau = 20$ remaining years as this is the min point of the residuals. A Chow test rejects, at 1% level, the null hypothesis (no structural break) in favor of the hypothesis of a break occuring at $\tau = 20$ years. The middle schedule of figure 8 displays the observations and the estimated coefficients b_1 and b_2 for the pool with $\tau_{q,i} \leq 20$. As predicted by proposition 7, an increase in the stock will have a first order effect of lowering the growth rate of the price (and vice versa) since b_1 is negative. It is significant at the 99% level. The second order effect implies that this effect is not linear since $b_2 > 0$ although it is less significant and less strong than the first order effect. The rightmost schedule shows the equivalent results for observations with $\tau_{q,i} > 20$. In line with proposition the results for b_1 and b_2 here are not significant at any standard level of confidence and b_1, most importantly, even shows the wrong sign from what the infinite horizon model predicts in proposition 3. Thus, proposition 7 and a progressive finite time horizon gets stronger support from these empirical tests compared to the competing proposition 3.

As tests of robustness, changes were made to the modeling choices. I.e. the number of years used for calculating the price growth, the number of years used for making production forecasts, what size of revisions to include and whether to include the quadratic term (b_2) and the composite price index (b_3). Generally, the main results survive these tests although the exact cutoff may change, usually upwards but below 30 years. What can be noted is that in the occasional empirical test where no support is found for the progressive finite time assumption this also discredits the infinite horizon assumption. An interesting observation is that when using the "reserve base" as a measure for the stock instead of the "reserves" all the results turn ambiguous. An interpretation of this is that the market operates on the basis of the reserves rather than the reserve base.

At what horizon the market reacts to news about the stock was tested in an additional way. Now the theoretical predictions of interest are expressed in proposition 3 and its corollary (for the infinite horizon) and proposition 7 and its corollary (for the progressive finite time horizon).

$$\frac{\triangle p_{q+,i}}{p_{q+,i}} - \frac{\triangle p_{q-,i}}{p_{q-,i}} = a + b_1 \tau_{q,i} \times (S_{q,i} > S_{q-1,i}) + b_2 \left(\frac{\triangle p_{q+,comp}}{p_{q+,comp}} - \frac{\triangle p_{q-,comp}}{p_{q-,comp}} \right) + \varepsilon_{q,i}$$

As before, the dependent variable is the change of the price growth after compared to before a revision of the stock. But the variable of interest, b_1, now represents what an additional year to exhaustion implies for the price growth given that the stock has been revised downwards. Thus, stock revisions is now

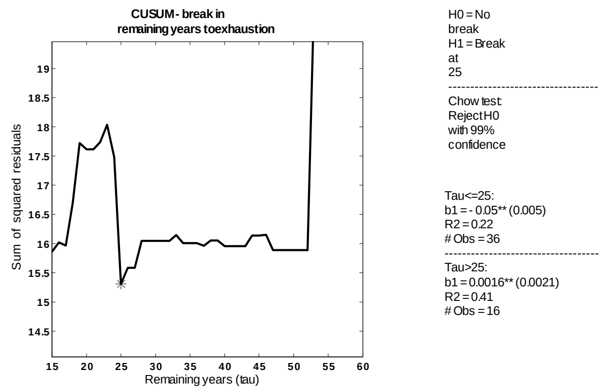

Figure 9:

a dummy variable. Since the size of the revision is likely to play a role (like was shown in the previous tests) one can expect large amounts of noise to seep through in this test as all downward revisions are considered equal. As before, a CUSUM test of structural breaks was made and thus, choosing a cutoff τ, the sample was divided into a pool of observations with $\tau_{q,i} \leq \tau$ and a pool of observations with $\tau_{q,i} > \tau$. Following the theoretical predictions, to support the infinite horizon assumption we should find no significant break in the data while support for the progressive finite time assumption will be given if there is a break and $b_1 < 0$ for $\tau_{q,i} \leq \tau$ and ambiguous otherwise.

The results are displayed in figure 9. The structural break is most likely to occur at a horizon of 25 years (left schedule). Indeed, the Chow test rejects the null hypothesis of no break in the data with 99% confidence, although one cannot reject that the structural break may occur at a longer horizon than 25 years. Thus the accuracy of this test seems less reliable. For the pool of observations with $\tau_{q,i} \leq 25$, b_1 is negative and significant (95%) while it is positive and insignificant for observations with $\tau_{q,i} > 25$. Also these results were tested for robustness by varying the modelling specifications and the central findings mainly survive.

7.3 The effect of remaining years to exhaustion on price growth

A word of caution is in place regarding a potential selection bias in the previous tests. Practically, the market's horizon may vary between the commodities. More precisely, one could expect that important commodities with large revenues such as oil, gas and iron ore should have a longer horizon than other resources with more niche usage. Furthermore, it seems reasonable to assume that the reserves for these important resources are more precisely measured implying that they are revised more regularly but with smaller steps. Also, fewer major surprises should occur for these resources. The results in the previous tests were largely driven by major revisions and surprises rather than the small increments made for the important resources. It therefore seems plausible that the results will mainly reflect those resources which have a rather short time horizon and that there will be a downward bias in τ. With the limited amounts of observations, where reserves are revised, there is little hope to deal with this within the same identification framework as above. Therefore, this section will estimate how the price growth *correlates* with the remaining years to exhaustion for those observations where no or only small (<10%) revisions to the stock are made. The word correlates is in italics to emphasize that we no longer have any plausibly exogenous variation to rely on for the identification. Although this test may be more representative of the more important commodities it is less sharp in terms of picking up the direction of causality.

Again, propositions 3 (for the infinite horizon) and 7 (for the progressive finite time) will be put against each other. Proposition 3 expresses that the price should be growing in the absence of any news about the stock, due to increasing scarcity. In comparison, proposition 7 predicts that the price growth should have no component of increasing scarcity if $\tau_{q,i} > T$. Thus, if progressive finite time is a correct assumption then, holding other things equal, there should be an additional component in the price growth when we come closer to exhaustion than the market's horizon. This additional component should be roughly constant and equal to the rate of return of comparative investments. It is represented by the coefficient a in the following regression.

$$\frac{\triangle p_{q+,i}}{p_{q+,i}} = a + b_2 \frac{\triangle p_{q+,comp}}{p_{q+,comp}} + \varepsilon_{q,i}$$

The dependent variable $\left(\frac{\triangle p_{q+,i}}{p_{q+,i}}\right)$ is simply the growth rate of the price in the years following the observation. b_2 represents the effect from growth in the commodity price index. Once again a structural breaks test was run to see where and if there is any place, as exhaustion is approaching, where "suddenly" there is an additional component to the price growth.

The results are presented in figure 10. This test suffers substantially from the yearly volitility in the price (see appendix C). Thus, to smooth the price growth the average was calculated using the eight years following the observation. This also implies that observations late in history could not be used as price data, for obvious reasons, only is available for up to 2010. Of most interest is the left schedule showing the CUSUM test. There seems to be a break in the data at around 40 remaining years to exhaustion whereby the price growth rate goes from 3% to 12% anually. The Chow test rejects the hypothesis of no break with 90% confidence. This can also be verified by occular inspection in the right schedule where, below 30-40 remaining years, there are many observations which exhibit a rather high price growth. Whereas, for the observations with more remaining years the price growth is more centered around zero. A T-test also confirms this difference.

To see whether these results are robust, a few other methods were used

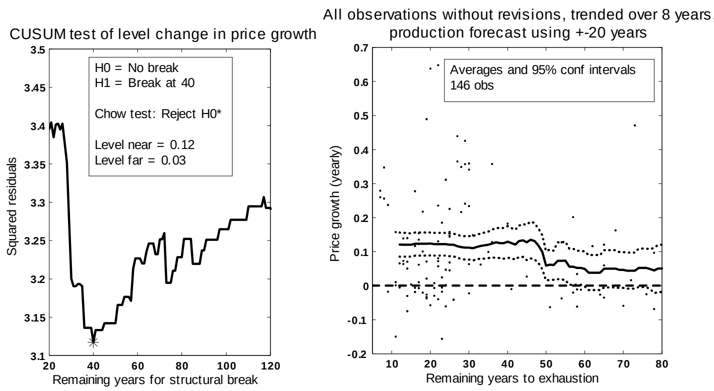

Figure 10:

to indicate what the time horizon may be. These are more numerically and less econometrically oriented as they rely on Monte Carlo simulations where modelling choices are varied. They are described in more detail in appedix C. Generally, it is found that a ball-park guess for the time horizon lies somewhere around 40 years.

8 Calibration to the oil market

Section 6 showed under what conditions a progressive finite time model replicates the overall historical trends of most resource markets - i.e. a long period of increasing extraction and non-increasing prices. This section aims to test whether the model fits the price and extraction of oil for the period 1949-2009. For this purpose the larger model of section 6 will be applied. Data will be used for all parameters and variables letting only the oil extraction and price be determined endogenously by the model. The only parameters which are not available from previous research and public data are the time horizon, the technological improvements in mining and the curvature of the mining cost function. The mining technology and the cost curvature will be chosen in order for the model to match the extration and price data as well as possible, while the time horizon will be based on the empirical results in the previous section.

8.1 Data and parameter values

In order to perform the calibration a number of parameter values and data are needed. A difficulty here is that while oil prices and scarcity are determined on the world market many parameter values, most notably on the demand side, are not available on an aggregate global level. To circumvent this problem the model is calibrated to the US economy and then, based on the US share of world oil consumption, the labor and capital inputs are scaled accordingly. E.g.

if the US consumes 50% of the world supply of oil in a certain year then the US labor and capital inputs are multiplied by two to get the world economy. The implicit assumption underlying this procedure is that capital and labor are equally substitutable with energy (like in the CES specification in equation 25) and that the energy and labor technologies evolve equally fast in the rest of the world as in the US. Stated differently, the oil input for a unit of output is assumed to be the same globally as in the US.

To get the initial oil reserves in 1949 the oil consumption from 1949-2009 is added to the reserves left in 2010 - i.e. $2.56 * 10^{12}$ barrels[42]. Thus it is assumed that the beliefs of the reserves in 1949 were about equal to those manifested in 2010 and that the reserves reported in 2010 are representative of the true beliefts of the market[43].

For most parameters and time series the calibration uses the same values as used by Hassler et al (2011)[44]. Using US data on energy consumption, labor, capital and output they calibrate the time series for energy saving technology and labor/capital augmenting technology[45]. They also calibrate the elasticity of substition between energy and capital/labor to be close to zero which means that production is close to a Leontief specification. Hence, I will use $\sigma = 0.05$[46]. To match the US energy share of output, γ is chosen to 5% and to get the capital to labor ratio $\alpha = 30\%$.

The subjective discount factor β is chosen to a 0.95, i.e. a yearly discount rate of 5%, and a CRRA utility function is used with a risk aversion constant of 1.

The three most important calibration choices pertain to the time horizon, to the amount of alternative energy inputs (R in equation 27) and to the extraction costs. The time horizon is important since it will determine when the shift to the scarcity phase will occur and thus when prices will start to rise.

Following the previous empirical tests and guessing that the oil market has a longer horizon than the average commodity the time horizon T is chosen to 40 years.

The amount of alternative energy inputs is important mainly since it will affect how hard scarcity of oil will hit the economy when the oil reserves even-

[42] Data from BP's statistical review of world energy from 2011, can be obtained at www.bp.com. Consumption data is not available on the US level prior to 1965. To get US oil consumption from 1949 the trend from 1965 onwards is extrapolated backwards. The estimated reserves in 2010 are roughly equal in size to the amount consumed from 1949 to 2009.

[43] Assuming instead that beliefs in 1949 equal those in 1980 (the earliest year where oil reserves are reported by BP) makes no difference. As will be seen in the results, the size of the oil reserves are only binding from about 1998 onwards, thus the assumption really boils down to agents in 1998 having the same beliefs on the reserves as those in 2010.

[44] I'm grateful to Conny Olovsson for supplying their data and results. The prices, costs, reserves and inputs obtained from other sources were deflated appropriately to match the units from Hassler et al's paper.

[45] Energy consumption from the US Energy Information Administration (EIA), labor from the Bureau of Labor Statistics (BLS) and capital and output from the Bureau of Economic Analysis (BEA).

[46] Hassler et al get a match of σ between 0 and 0.05.

tually start binding. A smaller amount of alternative energy implies a steeper increase in price when scarcity hits. Data for the oil share of energy in the US in each year is taken from the Energy Information Administration[47].

Finally, the extraction costs need to be calibrated. The functional form will be $M_t = E_t^{\theta}/A_{m,t}$. Three parameter choices need to be made here. Firstly, θ catches the curvature of costs with respect to the amount extracted in a single year. Secondly, $g_{m,t} = (A_{m,t+1} - A_{m,t})/A_{m,t}$, represents how fast the mining technology evolves. Data on these two parameters is not readily available or easily calibrated. For simplicity I will assume that both of them are constant. The model will be simulated multiple times to see which parameter combination - θ and g_m - that best fits the data and how sensitive the results are for changes in these parameters. As it will turn out, the choice of these parameters is not very important, within reasonable bounds. Thirdly, the mining technology level need to be pinned down. Also this value is scarcly attainable. But, given that we have chosen a value for θ and g_m, it is enough to know the average mining costs and the amount extracted in one year and from that deduct the mining technology level in all other years. Although far from perfect, the best available source for the average mining cost for various regions that I have found comes from a recent online news article by Reuters[48]. They give estimates of the average extraction costs (operating and capital costs) in the most important oil producing countries[49]. Weighing the average extraction costs by the size of the reserves in that country the average global extraction cost is set to 12$/barrel in 2008[50].

8.2 Results

In figure 11 the oil consumption and price are compared to data[51]. The best results are obtained with $\theta = 1.1$ and $g_m = 1.5\%$. A first remarkable result is that the model's oil consumption path closely follows the one obtained from data. This is largely due to the choice of near Leontief production which essentially does not leave much freedom in choosing oil consumption to be anything else than what matches the data on capital, labor and the technologies. Thus, the real test of the model comes in how well it does in matching the price of oil. As can be seen, the model's price matches closely the downward sloping price trend in the years before the first oil crisis. It also accuratelly predicts the sudden change of trend observed in 1998 and closely follows the sharply increasing prices thereafter. What the model cannot explain is the sharp price increase in 1972-1981 and the subsequent decrease in 1982-1986 - also known as the oil crisis. However, although there is an ongoing debate about what the

[47] Yearly data was used but in general it oscilates around 40% with a downward trend, i.e. the US share of world oil consumption is falling over time.

[48] http://www.reuters.com/article/2009/07/28/oil-cost-factbox-idUSLS12407420090728

[49] They base their estimates on the International Energy Agency world report from 2008 and on interviews they have conducted with various oil companies.

[50] E.g. the average cost per barrel was estimated to 4-6$ in Saudi Arabia, 15-30$ in Nigeria and 20$ in Venezuela.

[51] The real domestic First Purchase Price for the US is used.

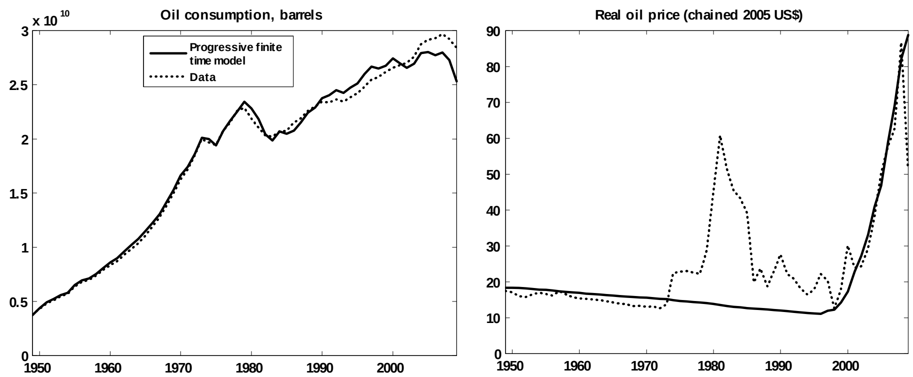

Figure 11: The progressive finite time model calibrated to the oil market. $\theta = 1.1$, $g_m = 1.5\%$, Extraction cost per barrel in 2008 =12\$, $T = 40$.

most important reasons for the oil crisis were, there seems to be a consensus that it had little to do with resource scarcity. Rather, a mix of failing price limitations, monpoly power, monetary expansion and wars in the middle east seem to explain these particular historical events[52]. As none of these market imperfections are included in the model of this paper I do not attempt to match the model to the data of these years when choosing θ and g_m.

A sensitivity analysis is now in place. Firstly, varying the extraction cost parameters of θ and g_m does not alter the model results much as long as θ stays between 0.7 and 1.3 and the technological improvement in mining is kept between zero and three percent annually. Depending on the exact values we may get a price which is closer to constant in the earlier years and a slight change of the slope after 1998. Thus, it seems that extraction costs are roughly linear and that mining technology has improved at a low to medium rate[53]. If instead we change the average cost production to be below 12$ in 2008, the main effect is to shift the price schedule uniformly down while the broad shape of first slowly decreasing prices and then sharply increasing around 1998 remains.

Finally, in regard to the choice of the time horizon, it mainly affects the timing of the price increase. Choosing a time horizon of 30 years will roughly delay the break in the price trend by about ten years and choosing a horizon of 50 years will make the break happen about ten years earlier. An interesting comparison can be made with a model of infinite horizon. Simulating such a model and varying the parameters θ and g_m yields a price path which is increasing slowly from about 1965 and onwards (see figure 12). The infinite horizon model thus predicts that the price increase should happen about 30

[52] See for example Barsky & Kilian (2002), Hamilton (2003), Barsky & Kilian (2004) and Kilian (2009) and references therein.

[53] Implicitly, when matched with data, the parameter g_m catches also the possibility that extraction costs may go up as extraction is made deeper in the ground or at sea or at more remote locations.

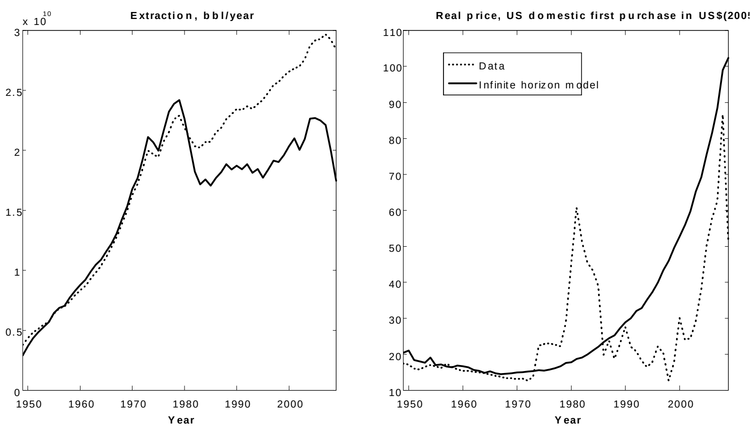

Figure 12: Calibration results of the the infinite horizon model. Average extraction cost in 2008 = 12\$/barrel. Best fit obtained using $\theta = 0.9$ and $g_m = 0\%$.

years too early. For such a story to be reasonable one would need to believe that the oil crisis was indeed an indication of scarcity and that the U-shape of the oil price in 1981 to 2008 was due to some annomally.

9 Concluding discussion

This paper has shown that a seemingly simple assumption, that of the time horizon of economic agents, has important consequences when modelling exhaustible resources but hardly makes any difference when modelling capital. The initial models are kept as simplistic as possible in order to highlight this point. In a more complete model, it is shown that progressive finite time can explain why the extraction of resources has been increasing over the last century while prices have been decreasing or remained constant. This model is also consistent with all standard observations of capital accumulation, output, consumption, interest rates and labor wage. Empirical tests using sudden changes in resource stocks lend support to the progressive finite time assumption and consistently reject the infinite horizon assumption. Furthermore, the model shows significant predictive power when calibrated to the oil market.

One question that may arise is how economic agents value keeping reserves past their forecasting horizon. It was shown that attaching a final value to the stock does not by itself alter the main results. Rather it depends on how this final value changes over time. Furthermore, it was shown that even though there is a qualitative difference between progressive finite time and an infinite horizon in exhaustible resource models, progressive finite time expectations will be time-consistent since constant prices are both expected and realized. Resource owners will not get the opportunity to learn from experience that making finite plans is not optimal.

It is not only capital accumulation models that show similar results with both infinite and progressive finite time. Also a model where agents consume some wealth with which they are initially endowed will have this property. Here, the difference will be that consumption will fall slightly faster within a progressive finite time model compared to one of infinite horizon. The outcome of the progressive finite time model will look much like an infinite time model with slightly more impatient agents. It may be so that in earlier evaluations of capital and wealth models, progressive finite time behavior has been confused with impatience since, as far as outcomes go, they are usually hard to tell apart.

Finally, it was shown that the direct welfare losses of having a progressive finite time horizon are possibly small for a resource owner, especially considering what it would entail to gather and process information about anything that resembles infinity. However, a common expectation is that if prices were to rise, a substitute for the resource would be searched for and eventually found. But if the trend and level of the resource price do not reflect the scarcity of the resource, which is the case with progressive finite time, then this search will be initiated too late. Scarcity may then become a serious limitation to the economy before a substitute resource or technology has been found.

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10 Appendix A - Further discussion of progressive finite time

10.1 A formal description

For each $q \in [0, \infty)$ the representative agent forms a plan and executes its first period. Let C denote the control variable of consumption and $\mathbf{X}$ a possibly singular set of state variables. Also, let the subscript letter denote the date the plan is formulated from and the letter within parenthesis denote the date within the plan. Let $\varepsilon \to 0$, a plan formulated at q is then a set $\left[C_q^* (t), \mathbf{X}_q^* (t + \varepsilon)\right]_{t=q}^{T}$ that satisfies $\max \sum_{0}^{T} \beta^t U(C_q(t))$ subject to $\dot{\mathbf{X}}_q (C_q(t), \mathbf{X}_q (t))$, the initial condition $\mathbf{X}_q (q)$ and appropriate terminal conditions applied to $\mathbf{X}_q (T)$. The actual outcome then is a set $\left[\tilde{C}_q (q), \tilde{\mathbf{X}}_q (q + \varepsilon)\right]_{q=0}^{\infty}$, i.e. a sequential implementation of the first period of each plan.

10.2 Experimental evidence

Shortsightedness and underestimation of future values has support by experimental research. Hey & Knoll (2007) show that many people do not plan ahead also when the intertemporal problem is simple to solve. Instead, they maximize their current payoff and attribute no value to future outcomes (Hey & Knoll, 2007; Bone et al, 2009). Regarding the problem of time inconsistency, this research shows that people are unable to predict their own behavior (Hey, 2002) and that they themselves do not assume that their own future behavior will be rational (Carbone & Hey, 2002). Also the ability to backward induct properly is limited (Johnson et al, 2002) and the subjects in these experiments do not necessarily learn from experience (Bone et al, 2009).

10.3 Possible behavioral explanations

Do agents really have full information up until a certain point in time and then no information beyond? This is obviously a simplification. This section will discuss behavioral mechanisms pointing towards a progressive finite time horizon and roughly sketch a few more elaborate ways of modelling it. However, the simplification made in this paper carries no obvious bias, since the driving mechanism is the plan being finite and not whether there is some uncertainty within the plan.

This said, it may be more realistic to have uncertainty increasing gradually with time but going to infinity within finite time. Another alternative mechanism is to have agents ignore all outcomes that have a low enough probability to endogenize progressive finite time.

A further option is the concept of ambiguity aversion which also points towards agents using progressive finite time. It has been shown that people generally avoid gambling on, and assign low values to, lotteries where they cannot assess the probability distribution (e.g. Ellsberg, 1961; Gilboa & Schmeidler, 1989). In a progressive finite time context this corresponds to preferring to use the assets in the near future, where the outcomes are either known or can be assessed by probabilities, than to use them in the far future where one does not have the information to even form expectations. This is at least numerically easily verified in a simple two-period model of resource extraction with ambiguity aversion, fearing a backstop will make the resource worthless. If the probability distribution of the backstop appearing is unknown the resource owner will want to leave no resources for later.

Related to this is a study by Gneezy et al (2006). They show that the certainty equivalence of a lottery can be lower than the worst outcome of that same lottery. In a progressive finite time setting this would be equivalent to believing that the continuation value of the asset is negative.

Another possible explanation for a finite horizon may be that finding and processing information is costly. When working on forecasts, at some point an agent has to decide to stop looking further into the future and how to treat the unknown beyond. One could possibly model the additional gains from information and compare them to the costs.

If, instead, one would assume that agents do not at all care about the utility in the very long run but are *not* constrained by information, the natural resource results of progressive finite time collapse. This is since these agents do care about the price of their assets tomorrow which in itself is determined by the price the day after and so on. This way the outcomes of the infinite future unfold back to today even though today's agents may not directly care about it. Thus, the distinction in progressive finite time is of agents being myopic due lack of information rather than lack of care.

On a theoretical basis one may well point out the time inconsistency in a person having a plan that (s)he knows might change tomorrow. The question is then why the change is not made already today. But, it would indeed be impossible to know how the plan will be updated tomorrow if one does not have tomorrow's information already today. For a discussion on this see Dequech (2001) or Dunn (2001). The notion of rationality mostly used in economics works well in the setting of decision making under risk, meaning agents know the possible outcomes and can assign probabilities to them, as described by Friedman & Savage (1948). However, if economic decisions need to be made under Knightian uncertainty (Knight, 1921), i.e. without knowing the outcome space and/or the probability distribution, then the standard notion of rationality is hardly applicable. In addition, the dynamic inconsistency will, as this paper shows, be fairly small in most cases making progressive finite planning a fairly accurate rule of thumb for most, also in hindsight.

11 Appendix B - Analytical results

11.1 Empirical predictions

Proof of proposition 6

As a guide to how to understand the upcoming proof, we can note that the proposition pertains to how the price reacts to changes to the forecast years to exhaustion. Thus it has to do with variables within the plan. The proof will therefore use the plans rather than the outcomes.

We start with the "if" part of $\tau\left(\tilde{S}_q\right) < T$. Backward induction of (19) and (17), with concavity of F, imply that $\tau\left(\tilde{S}_q\right)$ is weakly increasing in $\tilde{S}_q$ (weakly because τ is an integer). Proposition 5 gives that $\frac{d\tilde{p}_q}{d\tilde{S}q} < 0$. Thus, since $\tau\left(\tilde{S}_q\right)$ is increasing only if $\tilde{S}_q$ is increasing and $\tilde{p}_q$ is decreasing iff $\tilde{S}_q$ is increasing, $\tilde{p}_q$ is decreasing if $\tau\left(\tilde{S}_q\right)$ is increasing. Now for the "only if" part. Suppose $\tau\left(\tilde{S}_q\right) \geq T$, then equation (16) applies. Here $\tilde{E}_q = E_{\max}$ is independent of $\tilde{S}_q$ and thus $\tilde{p}_q$ is independent of $\tilde{S}_q$.■

Proof of proposition 7

Start with the "if" part of $\tilde{S}^{**}_{q+1} \geq TE_{\max}$, then by equation (16) $\tilde{p}^{*}_{q+1} = \tilde{p}^{*}_t = \tilde{p}^{**}_{q+1} = \tilde{p}^{**}_t = B$ which implies that $\frac{\tilde{p}^{*}_{q+1}-\tilde{p}^{*}_t}{\tilde{p}^{*}_q} = \frac{\tilde{p}^{**}_{q+1}-\tilde{p}^{**}_q}{\tilde{p}^{**}_q} = 0$. Now for the "only if" part. Suppose first that $TE_{\max} > \tilde{S}^{*}_{q+1} > \tilde{S}^{**}_{q+1}$. Define $\lambda\left(\tilde{S}_q\right) \equiv \tilde{p}_q - B$. Then $\frac{\tilde{p}_{q+1}-\tilde{p}_t}{\tilde{p}_q} = \frac{\tilde{\lambda}\left(\tilde{S}_{q+1}\right)+B}{\tilde{\lambda}\left(\tilde{S}_q\right)+B} - 1$. We know that $\frac{d\lambda\left(\tilde{S}_{q+1}\right)}{d\tilde{S}_{q+1}} = \frac{d\tilde{p}_{q+1}}{d\tilde{S}_{q+1}} < 0$ when $TE_{\max} \geq \tilde{S}^{*}_{q+1}$(proposition 5, second statement). Thus, $D \equiv \lambda\left(\tilde{S}_{q+1}\right)/\lambda\left(\tilde{S}_{q+1}+\tilde{E}_q\right) > 1$ (note that this holds also in the special case of $\tilde{S}_{q+1}+\tilde{E}_q > TE_{\max}$, then $\lambda\left(\tilde{S}_{q+1}+\tilde{E}_q\right) = 0$, with the convention that $\lim_{\lambda_q \to 0} \lambda_{q+1}/\lambda_q = \infty$). Using D, the price growths are unequal, $\frac{\lambda\left(\tilde{S}^{*}_{q+1}\right)D+B}{\lambda\left(\tilde{S}^{*}_{q+1}\right)+B} < \frac{\lambda\left(\tilde{S}^{**}_{q+1}\right)D+B}{\lambda\left(\tilde{S}^{**}_{q+1}\right)+B}$, since $\frac{d\lambda\left(\tilde{S}_{q+1}\right)}{d\tilde{S}_{q+1}} < 0$ (proposition 5, second statement). Secondly suppose $\tilde{S}^{*}_{q+1} \geq TE_{\max} > \tilde{S}^{**}_{q+1}$, then by the same steps as previously $\frac{\tilde{p}^{**}_{q+1}-p^{**}_q}{\tilde{p}^{**}_q} > \frac{\tilde{p}^{*}_{q+1}-\tilde{p}^{*}_t}{\tilde{p}^{*}_q} = 0$. This concludes the "only if" part and thus the proof.■

Proof of corollary 8

The first part follows from the "only if" part of the proof of proposition 7 and that only if $\tilde{S}^{*}_t > \tilde{S}^{**}_t$ then $\tau^{*}_t > \tau^{**}_t$ (see the proof of proposition 6). The second part follows from the "if" part of the proof of proposition 7. The third part follows directly from propostion 7 and its proof where it's shown that, at $\tilde{S}_{q+1} = TE_{\max}$, $\frac{d\frac{\tilde{p}_{q+1}-\tilde{p}_t}{\tilde{p}_q}}{d\tilde{S}_{q+1}} = 0$ for increases of $\tilde{S}_{q+1}$ and $\frac{d\frac{\tilde{p}_{q+1}-\tilde{p}_t}{\tilde{p}_q}}{d\tilde{S}_{q+1}} < 0$ for decreases of $\tilde{S}_{q+1}$.■

11.2 Full model - basic results

Lemma 9 *If $K > 0$ then $\lim_{E \to 0} H_E(E) > 0$.*

Proof. *There are a few cases depending on R and σ. When $R = 0$, for $\sigma < 1$ we have that $\frac{\sigma-1}{\sigma} < 0$ which implies that $\lim_{E\to 0} H_E = \lim_{E\to 0} H(E)^{1/\sigma} G(E)^{\frac{\sigma-1}{\sigma}} \frac{1-\gamma}{E} =$*

$$\lim_{E\to 0} \underbrace{(1-\gamma) A_{NR}^{\frac{\sigma-1}{\sigma}}}_{0< \quad <1} \underbrace{\left[\overbrace{\underbrace{\gamma \left(\frac{F}{E}\right)^{\frac{\sigma-1}{\sigma}}}_{=0} + \underbrace{(1-\gamma) A_{NR}^{\frac{\sigma-1}{\sigma}}}_{0<}}^{0< \quad <\infty} \right]^{\overbrace{\frac{1}{\sigma-1}}^{<0}}}_{>0}$$

which is larger than 0. For $\sigma = 1$ we have that $\lim_{E\to 0} H_E = \lim_{E\to 0} H(E) \frac{1-\gamma}{E} = \infty$ since $H(E)$ is Cobb-Douglas. For $\sigma > 1$ we have that $\frac{\sigma-1}{\sigma} > 0$ which implies that $\lim_{E\to 0} H_E = \lim_{E\to 0} (1-\gamma) \underbrace{H(E)^{1/\sigma}}_{>0} \underbrace{A_{NR}^{\frac{\sigma-1}{\sigma}}}_{>0} \underbrace{E^{-1/\sigma}}_{=\infty} = \infty$. When $R > 0$, $H(0) > 0$, $G(0) > 0$, $0 + R > 0$ which implies that $\lim_{E\to 0} H_E = \lim_{E\to 0} H(E)^{1/\sigma} G(E)^{\frac{1-\sigma}{\sigma}} \frac{1-\gamma}{E+R} > 0$. Thus, for all possible cases $\lim_{E\to 0} H_E(E) > 0$. ■

Lemma 10 *If $K > 0$ then $\lim_{E\to\infty} H_E(E) < \infty$.*

Proof. *There are a few cases depending on σ. For $\sigma < 1$ we have that $\frac{\sigma-1}{\sigma} < 0$ which implies that $\lim_{E\to\infty} G(E)^{\frac{\sigma-1}{\sigma}} = 0$, $\lim_{E\to\infty} \frac{H(E)^{1/\sigma}}{E+R} =$*

$$= \lim_{E\to\infty} \underbrace{\left[\underbrace{\gamma F^{\frac{\sigma-1}{\sigma}} (E+R)^{1-\sigma}}_{=\infty} + \underbrace{(1-\gamma) A_{NR}^{\frac{\sigma-1}{\sigma}} (E+R)^{\frac{-(\sigma-1)^2}{\sigma}}}_{=0} \right]^{\overbrace{\frac{1}{\sigma-1}}^{<0}}}_{=0} = 0 \text{ which}$$

implies that $\lim_{E\to\infty} \frac{H(E)^{1/\sigma}}{E+R} G(E)^{\frac{1-\sigma}{\sigma}} (1-\gamma) = 0$. For $\sigma = 1$ we have that $\frac{\sigma-1}{\sigma} = 0$ which implies that $\lim_{E\to\infty} H_E = \lim_{E\to\infty} H(E) \frac{1-\gamma}{E+R} = 0$ since $H(E)$ is Cobb-Douglas. For $\sigma > 1$ we have that $\frac{\sigma-1}{\sigma} > 0$ which implies that

$$\lim_{E\to\infty} H_E = \lim_{E\to 0} \underbrace{(1-\gamma) A_{NR}^{\frac{\sigma-1}{\sigma}}}_{0<\ <\infty} \underbrace{\left[\overbrace{\underbrace{\gamma \left(\frac{F}{\underbrace{E+R}_{=0}} \right)^{\frac{\sigma-1}{\sigma}}}_{=0} + \underbrace{(1-\gamma) A_{NR}^{\frac{\sigma-1}{\sigma}}}_{0<\ <\infty}}^{0<\ <\infty} \right]^{\overbrace{\frac{1}{\sigma-1}}^{>0}}}_{0<\ <\infty} \in\ ]0,\infty[.$$

Thus, for all possible cases $\overset{E\to\infty}{\lim_{E\to\infty}} H_E(E) < \infty$. ■

Proposition 11 *If S_0 is sufficiently large, then there exist two economic phases. The first phase is defined by $\sum_{t=0}^{T} E^*_{q+t} < S_q$. The second phase is defined by $\sum_{t=0}^{T} E^*_{q+t} = S_q$.*

Proof. *For the first phase, letting $S_q \to \infty$, $\lim_{E\to\infty} \frac{\partial M}{\partial E} = \infty$ and Lemma 9 imply that if $\sum_{t=0}^{T} E^*_{q+t} = S_q$ then the marginal extraction cost is higher than the resource price (which equals marginal productivity since the production sector is competitive) which clearly violates optimality, thus $\sum_{t=0}^{T} E^*_{q+t} < S_q$. For the second phase, letting $S_q \to 0$, $M'(0, A_M) = 0$ and Lemma 10 imply that the marginal extraction cost is below the resource price and that is would be profitable to extract more, thus the resource constraint must be binding $\sum_{t=0}^{T} E^*_{q+t} = S_q$.* ■

11.3 The non-scarcity phase

For the problem (24-31) and q such that $\sum_{t=0}^{T} E_{q+t} < S_q$ the terminal condition is given by $C_{q+T} = \max H(E_{q+T}, K_{q+T}) - M(E_{q+T})$. With competitive firms ($p = H_E$) it has a unique solution when

$$H_E(E_{q+T}, K_{q+T}) = M'(E_{q+T}) \tag{33}$$

This equation can be solved to get $\widetilde{E}_{q+T} = \widetilde{E}_{q+T}(K_{q+T})$ (where ˜ indicates that $\widetilde{E}_{q+x}$ is a function of K_{q+x}). Using backward induction, planning for time $T-1$ and backwards until $t = 0$, the full plan at time q is characterized by the following set of equations for $t = 0...T-1$.

$$
\begin{aligned}
K_{q+T+1} &= 0 \\
C_{q+T} &= p_{q+T}\left(\widetilde{E}_{q+T} + R\right) + r_{q+T}K_{q+T} + w_{q+T} - M\left(\widetilde{E}_{q+T}\right) + (1-\delta)\,K_{q+T} \\
M'\left(\widetilde{E}_{q+T}\right) &= H_E\left(\widetilde{E}_{q+T}, K_{q+T}\right) && (34) \\
U'\left(C_{q+t}\right) &= \beta U'\left(C_{q+t+1}\right)\left[r_{q+t+1} + (1-\delta)\right], && (35) \\
M'\left(\widetilde{E}_{q+t}\right) &= H_E\left(\widetilde{E}_{q+t}, K_{q+t}\right) = p_{q+t} \\
C_{q+t} &= p_{q+t}\left(\widetilde{E}_{q+t} + R\right) + r_{q+t}K_{q+t} + w_{q+t} - M\left(\widetilde{E}_{q+t}\right) - K_{q+t+1} + (1-\delta)\,K_{q+t} && (36) \\
p_x &= H_E\left(\widetilde{E}_x, K_x, L_x\right),\ r_x = H_K\left(\widetilde{E}_x, K_x, L_x\right),\ w_x = H_L\left(\widetilde{E}_x, K_x, L_x\right)
\end{aligned}
$$

One such set of equations will be solved for each current time period (q) as long as $\sum_{t=0}^{T} \tilde{E}_{q+t} < S_q$. For $T-1 \geq t > 0$ the equations describe a plan for how to choose $\tilde{E}_{q+t}$, K_{q+t} and C_{q+t} while for $t = 0$ they describe the true outcome, i.e. $\tilde{E}_q$, K_q and C_q.

For the use of exhaustibles the actual evolution over time is $H_E\left(E_q, K_q\right) = M'\left(E_q\right)$, $H_E\left(E_{q+1}, K_{q+1}\right) = M'\left(E_{q+1}\right)$ and so on. From these expressions we can note that the extraction of the exhaustible resource is determined fully by the current state of capital (and technology). Thus, during this paradigm there is no intertemporal choice of how much of the exhaustible resource to use. Using this and the next lemma we can establish the subsequent results.

Lemma 12 *$H_{EE} \leq 0 \;\forall\; E$.*
Proof. *It can be shown that*

$H_{EE} = -\frac{(1-\gamma)\gamma}{\sigma} A_{NR}^{\frac{\sigma-1}{\sigma}} \left[\gamma\left(\frac{F}{E+R}\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\,A_{NR}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{2-\sigma}{\sigma-1}} F^{\frac{\sigma-1}{\sigma}} \left(E+R\right)^{\frac{\sigma-1}{\sigma}} \leq 0 \;\;\forall\; R,\, E$ *and* σ. ■

Proposition 13 *If at time q the plan is such that $\sum_{t=0}^{T} E_{q+t} < S_q$, then there exists a unique equilibrium where $E_q > 0$.*
Proof. *That there exists an equilibrium follows from $\lim_{E\to 0} M'(E) = 0$, $\lim_{E\to\infty} M'(E) = \infty$, $\lim_{E\to 0} H_E(E) > 0$, $\lim_{E\to\infty} H_E(E) < \infty$, together with H_E and M' being continuous in $\mathbb{R}_+$ and the intermediate value theorem. Uniqueness follows from $M'' > 0 \;\forall\; E$ and $H_{EE} \leq 0 \;\forall\; E$ (Lemma 12). That there is no equilibrium when $E = 0$ follows from $\lim_{E\to 0} M'(E) = 0$ and $\lim_{E\to 0} H_E(E) > 0$.* ■

Let p_q denote the price of exhaustibles at time q.

Corollary 14 *All else equal;* $\frac{\partial E_q}{\partial A_{M,q}} > 0$, $\frac{\partial p_q}{\partial A_{M,q}} < 0$; $\frac{\partial E_q}{\partial A_{L,q}} > 0$, $\frac{\partial p_q}{\partial A_{L,q}} > 0$; $\frac{\partial E_q}{\partial K_q} > 0$, $\frac{\partial p_q}{\partial K_q} > 0$; *if* $\sigma \geq 1$ *then* $\frac{\partial E_q}{\partial A_{NR,q}} > 0$ *and* $\frac{\partial p_q}{\partial A_{NR,q}} > 0$.

Proof. We can start by noting in proposition 13 that in the two dimensional plane of E and prices, H_E will be intersecting M' from above. Thus, any downward shift of H_E will cause E to increase and the unique intersection (the equilibrium price) to decrease and vice versa. Any upward shift of M' will cause E to increase and the unique intersection (equilibrium price) to increase and vice versa.

$\frac{\partial E_q}{\partial A_{M,q}} > 0$, $\frac{\partial p_q}{\partial A_{M,q}} < 0$: Follows from $\frac{\partial M'}{\partial A_M} < 0$ and the above statement;

$\frac{\partial E_q}{\partial A_{L,q}} > 0$, $\frac{\partial p_q}{\partial A_{L,q}} > 0$: When $E > 0$,

$$\frac{\partial H_E}{\partial A_L} = \underbrace{(1-\gamma)\frac{\gamma}{\sigma}A_{NR}^{\frac{\sigma-1}{\sigma}}}_{>0}\underbrace{\left[\gamma\left(\frac{F}{E+R}\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)A_{NR}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{2-\sigma}{\sigma-1}}}_{>0}\underbrace{\frac{F^{\frac{-1}{\sigma}}}{(E+R)^{\frac{\sigma-1}{\sigma}}}\frac{\partial F}{\partial A_L}}_{>0} >$$

0. The statement follows from this and the initial statement about shifts in M' and H_E;

$\frac{\partial E_q}{\partial K_q} > 0$, $\frac{\partial p_q}{\partial K_q} > 0$: Same approach as the previous proof;

That if $\sigma \geq 1 \frac{\partial E_q}{\partial A_{NR,q}} > 0$, $\frac{\partial p_q}{\partial A_{NR,q}} > 0$: Firstly, we can rewrite H_E in the following form $H_E = (1-\gamma)\left[\gamma\left(\frac{F}{E+R}\right)^{\frac{\sigma-1}{\sigma}} A_{NR}^{\frac{(\sigma-1)^2}{\sigma}} + (1-\gamma)A_{NR}^{\frac{(\sigma-1)^3}{\sigma^2}}\right]^{\frac{1}{\sigma-1}}$. Secondly, when $E > 0$, the derivative is $\frac{\partial H_E}{\partial A_{NR}} = \frac{(1-\gamma)}{\sigma A_{NR}}[\cdot]^{\frac{1}{\sigma-1}-1}\left\{\gamma\left(\frac{F}{E+R}\right)^{\frac{\sigma-1}{\sigma}} A_{NR}^{\frac{(\sigma-1)^2}{\sigma}}(\sigma-1) + (1-\gamma)\frac{(\sigma-1)^2}{\sigma}A_{NR}^{\frac{(\sigma-}{\sigma^2}}\right.$ 0 if $\sigma \geq 1$ but ambiguous otherwise. The statement follows from this and the initial statements about shifts in M' and H_E.■

In equilibrium the price of resources is equal to the marginal extraction cost. This implies that the profits for resource owners will be given by

$$\pi = EM'(E, A_M) - M(E, A_M) \tag{37}$$

Proposition 15 *In equilibrium, there will be strictly positive profits from extracting resources.*
Proof. *Follows from (37) and convexity of M with respect to E.* ■

For balanced growth to occur we need to restrict the functional forms such that production is of Cobb-Douglas type (i.e. $\sigma = 1$) and that there is no renewable resource, $R = 0$, thus $H = K^{\gamma\alpha}A_L^{(1-\alpha)\gamma}E^{1-\gamma}A_{NR}^{1-\gamma}$. Formally proving the convergence to balanced growth is generally hard since one would have to analyze a sequence of plans. But, intuitively, since progressive finite time is a hybrid of finite and infinite time it is hard to imagine why the economy would not converge to balanced growth. For the extraction cost a tractable functional form that abides by the restrictions in (30) is $M(E_{q,t}) = \frac{E_{q,t}^{\theta}}{A_{M,q,t}}$, $\theta > 1$. In equilibrium $H_E = M'$, thus

$$E = \left[\frac{K^{\gamma\alpha} A_L^{(1-\alpha)\gamma} A_{NR}^{1-\gamma} A_M (1-\gamma)}{\theta} \right]^{\frac{1}{\theta-1+\gamma}} \quad (38)$$

Since $\theta > 1$, it is immediate to see from (38) that extraction E is increasing in all state variables. Assuming $U = \ln(c)$ and letting $g_L \equiv \frac{A_{L,q+1}}{A_{L,q}}$, $g_{NR} \equiv \frac{A_{NR,q+1}}{A_{NR,q}}$ and $g_M \equiv \frac{A_{M,q+1}}{A_{M,q}}$ and using (35), (36) and (38) we get expressions for the growth rate of the endogenous variables.

$$\frac{E_{q+1}}{E_q} = g_L^{\frac{(1-\alpha)\gamma}{(1-\alpha\gamma)\theta-(1-\gamma)}} g_{NR}^{\frac{(1-\gamma)}{(1-\alpha\gamma)\theta-(1-\gamma)}} g_M^{\frac{(1-\alpha\gamma)}{(1-\alpha\gamma)\theta-(1-\gamma)}} \quad (39)$$

$$\frac{C_{q+1}}{C_q} = \frac{M_{q+1}}{M_q} = \frac{H_{q+1}}{H_q} = \frac{K_{q+1}}{K_q} = g_L^{\frac{(1-\alpha)\gamma\theta}{(1-\alpha\gamma)\theta-(1-\gamma)}} g_{NR}^{\frac{(1-\gamma)\theta}{(1-\alpha\gamma)\theta-(1-\gamma)}} g_M^{\frac{1-\gamma}{(1-\alpha\gamma)\theta-(1-\gamma)}} \quad (40)$$

Proposition 16 *In the first paradigm, if the economy is on a BGP and g_L, g_{NR}, $g_M \geq 1$ then the extraction rate, capital, output and consumption will be increasing exponentially.*
Proof. *Follows directly from the exponents in equations (38) and (40).* ∎

The trend of the resource price on a BGP in equilibrium is given by

$$\frac{M'_{q+1}}{M'_q} = g_L^{\frac{(1-\alpha)\gamma(\theta-1)}{(1-\alpha\gamma)\theta-(1-\gamma)}} g_{NR}^{\frac{(1-\gamma)(\theta-1)}{(1-\alpha\gamma)\theta-(1-\gamma)}} g_M^{\frac{-\gamma(1-\alpha)}{(1-\alpha\gamma)\theta-(1-\gamma)}} \quad (41)$$

Proposition 17 *In the first paradigm, if the economy is on a BGP where $g_L = g_M = g_{NR} > 1$ then the price of exhaustibles is decreasing if $\frac{\gamma-\alpha\gamma}{(1-\alpha\gamma)} > (\theta-1)$, constant if $\frac{\gamma-\alpha\gamma}{(1-\alpha\gamma)} = (\theta-1)$ and increasing if $\frac{\gamma-\alpha\gamma}{(1-\alpha\gamma)} < (\theta-1)$.*
Proof. *Follows directly from the exponents on equation (41) .* ∎

11.4 Scarcity phase

For q such that the plan dictates $\sum_{t=0}^{T} E_{q+t} = S_q$, i.e. the resource constraint (29) will be binding at $t = T$. Using backward induction applied to (24), (28), (29) and (31), imposing a terminal condition of where no resources or capital are left for $T+1$, we get the following set of equations defining the plan for each current time period (q) within the paradigm.

$$\begin{aligned}
C_{q+T} &= p_{q+T}\left(E_{q+T}+R\right)+r_{q+T}K_{q+T}+w_{q+T}-M\left(S_{q+T}\right)+(1-\delta)\,K_{q+T} \\
S_{q+T+1} &= 0,\ E_{q+T}=S_{q+T},\ K_{q+T+1}=0 \\
C_{q+t}+K_{q+t+1} &= p_{q+t}\left(E_{q+t}+R\right)+r_{q+t}K_{q+t}+w_{q+t}-M\left(E_{q+t}\right)+(1-\delta)\,K_{q+t} \quad (42) \\
\frac{U'\left(C_{q+t}\right)}{U'\left(C_{q+t+1}\right)} &= \beta\frac{p_{q+t+1}-M'\left(E_{q+t+1}\right)}{p_{q+t}-M'\left(E_{q+t}\right)-\mu_{q+t}} \quad (43) \\
r_{q+t+1}+(1-\delta) &= \frac{p_{q+t+1}-M'\left(E_{q+t+1}\right)}{p_{q+t}-M'\left(E_{q+t}\right)-\mu_{q+t}} \quad (44) \\
p_x &= H_E\left(E_x,K_x,L_x\right),\ r_x=H_K\left(E_x,K_x,L_x\right),\ w_x=H_L\left(E_x,K_x,L_x\right)
\end{aligned}$$

Here equations (42), (43) and (44) are for $t=0...T-1$. The actual outcome in this paradigm is S_q, K_q and C_q for a sequence of q, $q+1$ etc. μ_{q+t} is the multiplier on the resource constraint (29). For most cases the resource constraint will be binding only in the last period, $t=T$. However, there are cases when the resource constraint is binding also before. In particular, this happens when there is a renewable substitute ($R>0$) and T is large.

11.5 Beyond scarcity

We can begin by noting that as the use of exhaustibles decreases and extraction technology improves the cost of extraction will become irrelevant for the total evolution. It is then immediate that with high substitutability ($\sigma>1$) the long run growth of the economy as a whole depends on the progress of the fastest of either labor augmenting or resource augmenting technical change. This can be seen in the following expression of output growth, where the larger of the resource capacity (G) or production capacity (F) shapes the output growth.

$$\frac{H_{q+1}}{H_q}=\left[\frac{\gamma F_{q+1}^{\frac{\sigma-1}{\sigma}}+(1-\gamma)\,G_{q+1}^{\frac{\sigma-1}{\sigma}}}{\gamma F_q^{\frac{\sigma-1}{\sigma}}+(1-\gamma)\,G_q^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}} \quad (45)$$

When $\sigma\leq 1$ there exist two cases - with or without a renewable substitute R.

When $R>0$ and $E\rightarrow 0$ the model essentially becomes one of capital accumulation in the presence of a fixed resource - e.g. land. Given $g_L\equiv\frac{A_{L,q+1}}{A_{L,q}}>1$ and $g_{NR}\equiv\frac{A_{NR,q+1}}{A_{NR,q}}>1$ the economy will grow at whichever of these rates that is the lowest if $\sigma<1$. While if $\sigma=1$ and assuming $U=\ln(c)$ there exists a balanced growth path where the following growth rates apply.

$$g_k\equiv\frac{k_{q+1}}{k_q}=g_H=g_c=g_{NR}^{\frac{1-\gamma}{1-\gamma\alpha}}g_L^{\frac{(1-\alpha)\gamma}{1-\gamma\alpha}} \quad (46)$$

When instead $R=0$ the prospect for long run growth is more ambiguous, since the lowering of extraction has a decreasing effect on the economy. For $\sigma=1$ there exists a balanced growth path where growth or decline depends on if the technological progresses are fast enough to outweigh the discount rate.

Note that an implicit assumption here is that the mining technology progresses fast enough so that the mining cost plays no role, even if the economy displays negative growth.

$$g_E \equiv \frac{E_{q+1}}{E_q} = (1-\gamma)\,\beta \tag{47}$$

$$g_k = g_H = g_c = \left[[(1-\gamma)\,\beta]^{1-\gamma} \, g_L^{(1-\alpha)\gamma} g_{NR}^{1-\gamma} \right]^{\frac{1}{1-\alpha\gamma}} \tag{48}$$

If $\sigma < 1$ the chances for growth are even smaller. In this case the effect from the discount rate needs to be outpaced by the progress of resource augmenting technology alone.

12 Appendix C - Empirical observations and numerical results

12.1 A small infinite horizon agent in a progressive finite time world

A possible perturbation to the model is to have a small agent who has an infinite (or at least significantly longer) time horizon and therefore can foresee that resource prices will eventually rise. Would it save its resources for the future? The answer is that also it would extract as much as it can for a very long initial phase. This is because it knows that prices will not increase for a long time and any possible additional profits will be realized only far into the future and are therefore largely discounted away today. It is only when the market is approaching sufficiently near the change of phases that an infinite horizon agent will lower its extraction, since only then arbitrage profits have a high enough present value. Below is how such an agent would extract in the full model simulation.

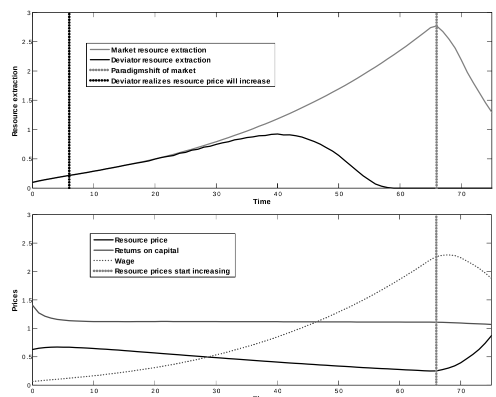

Figure 13: Simulation of the extraction decision of a market with a ten year horizon and how an atomistic resource owner with a 60 year horizon (deviator) will choose to extract given this market. The simulation uses the full model of section 6.

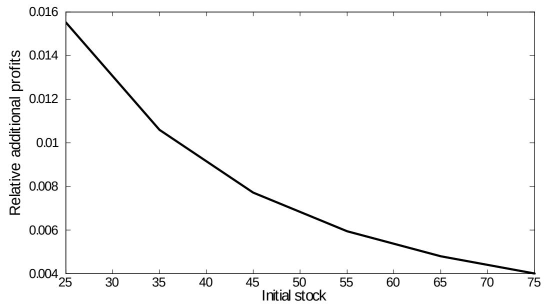

Figure 14: The additional relative profits a single atomistic infinite horizon agent can make given that everyone else has a progressive finite time horizon of 35 years. The simulation uses the basic resource model of section 4 with ρ=2. Note that an initial stock of 25 in this setting implies a constant price for 20 years and an initial stock of 75 implies a constant price for 60 years.

12.2 Time horizon calibration using Monte Carlo

The model predicts that when the resource stock is binding within the owners time horizon the price should start rising (or rise more than before). Thus, in this exercise, a rising price in the data will indicate that the stock is binding. By then extrapolating the production path into the future and comparing with the remaining reserves, a number for the remaining years of production before exhaustion is obtained. This value will indicate a minimum time horizon[54]. Likewise, a non-increasing price in the data will indicate that the stock is not binding within the owners' time horizon. Then a comparison of the reserves and the production trend yields the number of remaining years of production which will be interpreted as the maximum time horizon of the resource owner[55]. Repeating this procedure for a large number of resources and over many years yields one distribution for the minimum horizon and one distribution for the maximum horizon with the "true" horizon being somewhere in between.

Now, there are many ways of defining what manifests a price increase and there are many potential methods that the market may use to make production forecasts. Which price rule and forecast method that are the "right" ones to use in the estimation is therefore unknown. To circumvent this the estimation was performed multiple times using different price rules and forecast methods (i.e. a Monte Carlo procedure).

To calculate the remaining years the reserves will last a production prognosis needed to be made. A number of methods were used. One method was to take the trend of the last $z \in [5, 15]$ and extrapolate it forward. I.e. varying

[54] A minimum horizon since we do not know if the price would have risen had the time horizon been longer.

[55] A maximum time horizon since we do not know if the price would have been non-increasing had the remaining years of production been even shorter.

the number of years that firms look backwards when extrapolating forwards. A potential problem with this method is that a humped extraction path may be extrapolated as constant which may be highly inaccurate. Therefore a higher weight was given to the trend in the later years. A second method was to use the standard reserve to production ratio, i.e. the current production is assumed to continue unchanged. This method was not used in the main specification as it created a lot of noise in the data, possibly due to it being an inaccurate description of how the market makes forecasts. The third method was to use the average growth rate of all resources combined in the last z years and extrapolate forward. The main benefit of this method is that it washes out short run fluctuations arising from a single commodity.

To indicate when a price is defined as increasing the requirement was that the price should increase in a share $x \in [0.5, 1]$ out of the upcoming $y \in [3, 8]$ years. By varying x and y various price increase indicators were tested. Another price increase indicator was to require that the price in a share x of the upcoming y years be higher than it is at the year of observation, varying x and y similarly as in the previous method. Optimally, one would like to use a large number of future years to determine when a price is increasing - the fewer years the more noise, due to short run factors such as bottlenecks in extraction and distribution. However, since the number of years in the dataset is only sixteen. Using many years to indicate price increases implies cutting many observations towards the end of the time series.

Each production forecast method was combined with each price increase indicator. Taking one specific price increase indicator and one specific production forecast method a distribution of minimum time horizon was obtained (i.e. those observations where the price rises) and a distribution of maximum time horizons was obtained (i.e. those observations where the price does not rise). Combined, the two distributions consist of roughly 400 observations[56]. The means of these two distributions was then saved to a meta-distribution consisting of all the rounds of the Monte Carlo estimation[57].

A few caveats are worth mentioning. First of all it seems plausible that not all resource markets have the same time horizon. Owners of important and large revenue resources such as fossil fuels or iron ore probably have longer horizons than owners of more niche resources. This means that the maximum and minimum horizon distribution from each estimation round can be expected to overlap considerably. Furthermore, most resource prices are very volatile and react sharply to short run demand and supply side shocks. Hence, whichever rule is used for determining that a price is rising, large amounts of noise can be expected to seep through making the rule inaccurate.

In figure 15 the results are reported when using the first production method[58].

[56] 53 commodities during 15 years, net of some missing data and the number of years towards the end that fall out due to the price indicator requiring y future years.

[57] In total, the Monte Carlo method consisted of about 300 rounds.

[58] Similar results are obtained when using the third production forecasting method (averaging the growth rates of all commodities) - a slightly longer time horizon is obtained but with a clearer division between min and max horizon. The second method (reserve to production

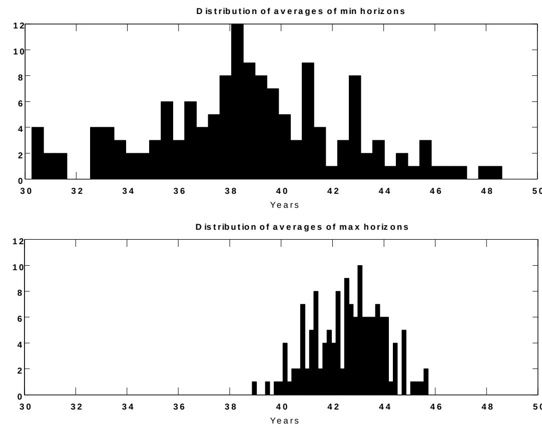

Figure 15: Distributions of means from the Monte Carlo simulation using the first production forecast method.

As can be seen the distribution of the means is significantly different between the minimum horizon and the maximum horizon. The mean of the minimum horizon distribution is 38.7 years and the mean of the maximum horizon distribution is 42.6 years[59]. A T-test confirms the difference, showing that we can expect the minimum horizon to be shorter than the maximum horizon at all conventional levels of confidence. To see that this is not driven by some outliers, a T-test was run separately on each Monte Carlo round. Although not as conclusive as the T-test for the mean, the separate tests show a positive value in more than 85% of the rounds (i.e. the minimum horizon is smaller than the maximum horizon) but at much lower levels of confidence each time. Finally, putting together all the observations from all the Monte Carlo rounds into a maximum and a minimum distribution and performing a T-test yields a significant difference between the two at all conventional levels.

As a test to see whether the results are driven by some structure of the data the above procedure was performed using the data for reserve base instead of the more narrowly defined reserves. Generally, the reserve base for most commodities will last for more than 100 years. Thus, if agents use a progressive finite time horizon, the remaining reserve *base* should be a bad predictor of price increases. Running the above procedure yields no significant difference between the maximum and the minimum horizon (in fact, the T-tests even show the wrong sign).

ratio), gives inconclusive results.

[59] The standard deviations are 3.9 and 1.4 years respectively.

12.3 Empirical observations

Extraction and price paths of a few exhaustible resources (data from the US geological survey and BP). For each extraction path a dashed trend line of the type $E = Ae^{Bt}$ or $E = At^{B}$ has been fitted to the real values through calibration of A and B. For the prices seven year averages have been calculated (dashed). For tantalum only four year averages were used due to data constraints.