What Does the Corporate Income Tax Really Tax?

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Abstract

More than four decades have passed since Arnold Harberger wrote his seminal paper on the corporate income tax. Harberger's model has proved to be an excellent tool for studying taxation of industry-specific factor inputs. But it hasn't passed the test of time with respect to understanding the corporate income tax. Specifically, Harberger's model equates "corporateness" with the production of particular products. This modeling approach is at odds with the data, which show essentially all goods being produced by both corporate and non-corporate firms. To address this disconnect with the facts, Gravelle and Kotlikoff (1989) developed a model with quite different incidence and efficiency properties in which corporate and non-corporate firms produce the same goods using the same technology. In their model, entrepreneurs can't utilize their managerial talents unless they operate on their own. Corporations, in contrast, can expand managerial input, but doing so limits the effectiveness of managers. The relative pluses and minuses of the two types of firms permit their co-existence.

Unfortunately, the Gravelle-Kotlikoff model misses a number of interconnected factors that seem critical to the choice of corporate form and to understanding the impact of the corporate tax. These include risk, the problem of multiple owners free-riding on each other's oversight of the firm, the choice of limited liability, the capital structure decision, the distribution of share ownership, the relative asset positions, abilities, and risk tolerances of owners who are overseeing their firm's operations, and bankruptcy costs.

Our model incorporates each of these factors. Our highly preliminary and initial findings suggest that firms' capital structures, level of debt, borrowing rate, number of owners, and choice of business form are highly sensitive to owners' risk aversion and asset levels. We are not yet in a position to understand the incidence of the corporate tax, but speculate that it falls on risk averse agents with low asset levels.

1 Introduction

This paper provides a new model of the corporate income tax plus some highly preliminary calculations from the model. Our model considers the decision of entrepreneurs to establish their firms as a) untaxed proprietorships and partnerships, with unlimited liability and no corporate tax liability, b) subchapter S corporations and limited partnerships with limited liability and no corporate tax liability, or c) as C-corporations, with limited liability, which do face the corporate income tax.

In deciding which of these three forms to adopt, entrepreneurs presumably consult Internal Revenue Service regulation 301.7701-2(a)(1), which states that "an organization will be taxed as a corporation if its characteristics are such that it more closely resembles a corporation than a partnership or trust." The IRS makes this definition less circular by describing corporate characteristics as including 1) associates, 2) an objective to carry on business and divide the profits, 3) continuity of life, 4) centralization of management, 5) liability of corporate debts limited to corporate property, and 6) the free transferability of interests.

Of these corporate characteristics, the most important seems to be the choices of limited liability and associates, which refers to the number of associates. In particular, if the enterprise wants to have a large number of owners as well as enjoy limited liability, it must pay the corporate tax. On the other hand, if the enterprise is willing to operate without one of these two characteristics it can avoid the corporate tax. Thus, partnerships can have as many partners as they'd like, but still avoid the tax by accepting unlimited liability. And subchapter S corporations as well as limited partnerships can opt for limited liability protection and still avoid the tax by accepting limits on the number of owners. In the case of S corporations, the limit is 75 or fewer owners. In the case of limited partnerships, the limit is 35 or fewer owners.

Note that S corporations and limited partnerships are able to transfer ownership rights without major limitations. They also readily meet characteristics 2, 3, and 4. Hence, the key to modeling the corporate tax would seem to be to understand the decision of firms in deciding whether or not to opt for a large number of owners as well as for limited liability. The number of owners connects to the scale of the enterprise and its need for capital and management. Stated differently, very large enterprises have large capital and management needs, which may require large numbers of owners.

There are clear synergies in having multiple owners. One is simply having more people to keep track of the company's interests and oversee the company's operations. Another is having more ideas about how to move the firm forward. A third is limiting mistakes in decision making. A fourth is diversifying over idiosyncratic levels of effectiveness in particular owners' oversight and management. A fifth is being able to spread the firm's investment risk over more players. But too many cooks can also spoil the broth. The ability of owners to monitor each others' oversight activities is limited, which gives rise to free riding. And there is no principal here to impose penalties on free riding agents a la Holmstrom (1982).

Limiting owner's liability has its own advantages and disadvantages, some of which interact with the number of owners. For example, being able to safely invest some of one's assets outside the firm requires limited liability, but doing so raises the need for making up the loss in equity by bringing on more owners. Or consider the issue of free riding. With unlimited liability, owners will have more incentive to oversee the firm's operations because they risk losing all their assets. This mitigates the free-riding problem.

To our knowledge, joint consideration of the number of owners and the choice of liability has played no direct role to date in the analysis of the corporate income tax. Indeed, in Harberger's (1962) classic model of the tax, there is no choice whatsoever by firm owners of any of the characteristics associated with corporate form. Instead, Harberger's two-product, general equilibrium model assumes that corporations and only corporations can produce one of the two goods and that non-corporations and only non-corporations can produce the other good. A similar model is studied by Shoven (1976), but with more sectors that are exclusively corporate and non-corporate.

In equating "corporateness" with the production of particular products, Harberger ends up studying the incidence and efficiency effects of taxing the use of capital in producing particular commodities. This approach generates a set of very interesting incidence outcomes depending on assumed elasticities of substitution in production and demand. Consequently, the Harberger model is a great tool to teach tax incidence and has become a mainstay of public finance reading lists. That said, the model is not a great way to learn about the impact of the corporate income tax. The reality is that almost all goods are produced by both corporate and non corporate firms. Ignoring this fact ignores the potential for the corporate tax to shift production not across sectors, but from corporate to non-corporate producers within the same sector.

To deal with this shortcoming, Gravelle and Kotlikoff (1989) present a two-good (sector) model with corporate and noncorporate production of both goods. Their model predicts a much larger deadweight loss from corporate taxation than Harberger's. It also generates much different incidence results depending on particular assumptions. The Gravelle-Kotlikoff model has three productive factors: capital, labor, and managerial input (entrepreneurial input in the case of non-corporate firms). Each agent is free to be a corporate manager, an entrepreneur, or a worker. While agents are equally productive as corporate managers or workers, they are not equally productive as entrepreneurs. In equilibrium, those agents who are most productive as entrepreneurs will establish their own firms, with the marginal entrepreneur just indifferent between establishment of his or her own firm and working either as a corporate manager or worker.

If an agent chooses to be an entrepreneur, she must manage her enterprise solely on her own. I.e., there can't be more than one cook making the broth. Corporations, on the other hand, can have as many managers as they want. But when agents manage corporations, they can no longer utilize their individual entrepreneurial talents. Instead, they become homogenized and have to operate within standardized norms in which all managers have the same, rather low productivity. Thus Gravelle and Kotlikoff (1989) touch on the issue of number of owners (managers) and the degradation of managerial input associated with more managers. But they do so in a rather mechanical way and in a model that features no uncertainty, no treatment of borrowing and limited liability, and no consideration of scale economies in production.

Chamley (1983) provides a signaling theory of the choice of limited versus unlimited liability. In his model, entrepreneurs differ in their abilities, which determine the success probability of a common risky investment. Lenders do not observe the entrepreneurs' abilities. As a result, more able entrepreneurs accept the risk of unlimited liability in order to signal to lenders that they are more able and, thus, more credit worthy. Chamley doesn't examine the impact of the corporate tax in his study. But it seems clear that an increase in the corporate tax in his model would erode the value of the signal being provided by high ability entrepreneurs. It would do so by leading less able entrepreneurs, who were otherwise be indifferent or close to indifferent between limited and unlimited liability, to choose unlimited liability. This reduces the average quality of the pool of entrepreneurs that choose unlimited liability as well as that of the pool that continues to choose limited liability. Consequently, the borrowing rates for both groups go up, visiting a portion of the incidence of the tax on the high ability entrepreneurs.

Including asymmetric information about owners' skills is not our first order of business in the proposed study. But we believe Chamley has identified an important reason for firms to opt for limited liability, and hope to be able to incorporate this issue in our framework as described below. A paper that comes somewhat closer to our proposed initial approach is that of Farrell and Scotchmer (1988), who provide a theory of partnership formation. In their coalition-formation game, agents who differ in ability join together to exploit economies of scale, but must share their output equally. In teaming up with high ability agents, low ability agents can free ride more effectively on their partners. Farrell and Scotchmer's main finding is that high ability agents will form partnerships with other high ability agents or the highest ability agents available and, thereby, limit the free riding.

Holmstrom (1982) combines the assumption of unobservable effort implicitly assumed by Farrell and Scotchmer with more explicit analysis of the economies from partnering, specifically the gains from risk sharing. As indicated, he provides mechanisms for overcoming free riding, but only if a principal is available to enforce penalties. Kandel and Lazear (1992) introduce peer pressure as a way of limiting free riding. Their paper seems, however, to assume that effort is observable, but not contractable.

Lang and Gordon (1995) and Gaynor and Gertler (1995) provide evidence for free riding in settings in which effort, while not directly observable, is correlated with individual-specific output. The former study shows that compensation arrangements within law partnerships are formulated taking into account the gains from risk sharing. Specifically, larger law firms, being better able to share risk, base partners' income less on their own billing and more on total firm billings. The latter study shows that more risk averse physicians join partnership with better income-sharing arrangements, but then proceed to free ride on their partners as measured by their work effort.

Finally, Roger Gordon and several co-authors have considered the tax rationale for choosing organizational form. As estimated by Graham (2003), investors in the top federal tax bracket, receive about 60 cents in after-tax income for each dollar earned within a partnership and about 50 cent for each dollar earned via an equity investment in a chapter C corporation. Given the current and past tax advantage to not paying corporate taxes, Gordon and Mackie-Mason (1994) ask why investors would invest in C corporations, which bear that liability. Their answer is the ability to publicly trade C-corp shares, which they model as a portfolio diversification gain. They estimate this gain at about 4 percent of equity value. In Gordon and Mackie-Mason (1997), the two authors test the proposition that profitable firms shift out of the corporate sector when the tax penalty to incorporating is larger, and conversely for firms with tax losses. Their empirical results, like those of Gordon and Mackie-Mason (1991), provide support for this hypothesis. Taxes surely play a key role in the choice of organizational form, but we question whether having a liquid market in C-corp shares is an important incentive in of itself for establishing C-corps and, thereby, opting to pay the corporate tax. Recall that

S-corps and limited partnerships are also able to trade their shares and ownership rights, albeit in somewhat thinner markets and with somewhat higher transactions costs.

Our model captures many of the elements we've just raised. We begin by assuming that owners/entrepreneurs/agents have equal amounts of assets, and then discuss ways to make the model more realistic as well as embed it in a general equilibrium setting.

2 The Model

2.1 Unlimited liability

Consider identical agents seeking to establish a firm assuming, for the moment, that they will operate under unlimited liability. The firm's production function depends on capital and labor inputs and is denoted by Ffirm's borrowing rate, r for the riskless rate of return, z for the firm's uncertain total factor productivity, and G for the cumulative density of z.We use the

superscripts u to denote variables in the unlimited liability case.

Denote the bankruptcy default threshold by z_d^u. The unlimited liability firm borrows at the interest rate $\tilde{r}^u$, defined by

$$(1+r)\,D^u = \int_0^{z_d^u} \alpha z F\left(\sum_i a_i + D^u, \sum_i e_i^u\right) dG\,(z) + \left[1 - G\,(z_d^u)\right](1+\tilde{r}^u)\,D^u. \qquad (1)$$

The first argument of the production function F is K^u– the firm's capital stock, consisting of the sum of each agent's assets plus the firm's debt. The second argument is L^u – the firm's total supply of effort, consisting of the sum of effort by the individual owners. In the event of default, that the bank obtains a fraction α of firm's value $zF\,(K^u, L^u)$. The remaining fraction represents non-recoverable bankruptcy costs.

The default threshold z_d^u is defined by

$$z_d^u F\left(\sum_i a_i + D^u, \sum_i e_i\right) - (1+\tilde{r}^u)\, D^u = 0, \tag{2}$$

Thus, we have

$$(1+r)\, D = F(K^u, L^u)\left\{\alpha \int_0^{z_d^u} z dG(z) + [1 - G(z_d^u)]\, z_d^u\right\}. \tag{3}$$

Each agent/owner is assumed to take her firm's level of debt and borrowing rate as given in choosing her level of effort. Specifically, owner i chooses e_i to solve

$$\max_{e_i} U(0, e_i)\, G(z_d^u) + \int_{z_d^u}^{\infty} U\left(\frac{zF(N^u a + D^u, \sum e_i^u) - (1+\tilde{r}^u)\, D^u}{N^u}, e_i^u\right) dG(z) \tag{4}$$

subject to (2). In considering (2), each owner realizes that her own effort can affect the firm's probability of default, even though her effort won't affect its debt level or borrowing rate, which are set before her effort is applied.

We consider the symmetric NE $e_i^u = e(D^u, \tilde{r}^u)$. The owners take this function and the free riding it entails as given in choosing a level of debt to maximize their common expected utility. In their first stage maximization, the owners also consider how their choice of debt affects the rate at which they can borrow. I.e., they take into account (1).

2.2 Limited liability

We use l to indicate variables in the unlimited liability case, θ_i to reference the share of owner $i's$ assets she elects to invest in the firm, and τ to denoted the corporate income tax rate. Assuming interest payments are deductibile from the corporate tax, the limited liability firm borrows at the interest rate $\tilde{r}^l$, defined by

$$(1+r)\,D^l = \int_0^{z_d^l} \alpha z\,(1-\tau)\,F\left(\sum \theta_i a + D^l, \sum e_i^l\right) dG\,(z) + \left[1 - G\left(z_d^l\right)\right]\left(1 + (1-\tau)\,\tilde{r}^l\right) D^l. \tag{5}$$

θ_iThe default threshold z_d^l satisfies

$$\left(z_d^l F\left(\sum \theta_i a + D^l, \sum e_i^l\right) - i^l D^u\right)(1-\tau) - D^l = 0. \tag{6}$$

We assume that all owners, prior to choosing their levels of effort, jointly agree on the firm's level of debt and on the share of assets θ, where $\theta_i = \theta$ for all i, that each owner should invest in the firm. Hence, in choosing effort each owner takes not only the firm's level of debt, but also her own and every other owner's investment in the firm as given.

Agent i's choice of effort is determined by

$$\max_{e_i} U\left(\left(1+r\right)\left(1-\theta\right)a, e_i\right) G\left(z_d^l\right) \tag{7}$$
$$+ \int_{z_d^l}^{\infty} U\left(\left(1+r\right)\left(1-\theta\right)a + \frac{\left(zF\left(N\theta a + D, \sum e_i\right) - \tilde{r}^l D\right)(1-\tau) - D}{N^l}, e_i\right) dG\,(z)$$

subject to (6). We obtain a symmetric NE $e_i = e\left(D^l, \tilde{r}^l, \theta\right)$.

Given this function, the owners jointly choose D^l and θ to maximize

$$\max_{D, \tilde{r}^l, \theta} U\left(\left(1+r\right)\left(1-\theta\right)a, e^l\right) G\left(z_d^l\right) \tag{8}$$
$$+ \int_{z_d^l}^{\infty} U\left(\left(1+r\right)\left(1-\theta\right)a + \frac{\left(zF\left(N^l\theta a + D^l, N^l e^l\right) - \tilde{r}^l D^l\right)(1-\tau) - D^l}{N^l}, e^l\right) dG\,(z)$$

subject to (5) and (6).

2.3 General Equilibrium

Suppose there is a continuum of two types of agents with low and high asset levels, a_L and a_H. And let the fraction of high asset agents be λ.

If high asset agents choose limited liability and low asset agents choose unlimited liability, market clearing in the bond market implies

$$(1-\lambda)\frac{D^u}{N^u}+\lambda\frac{D^l}{N^l}=\lambda(1-\theta)\,a_H. \tag{9}$$

This equation, in conjunction with the above-derived demand functions of debt, determines the equilibrium safe rate r. A corresponding equation holds if high asset agents choose unlimited liability and low asset agents choose limited liabilty.

An equilibrium of this form assumes that only firms with homogeneous owners come into formation. But mixed firms are possible. This is particularly the case if ownership rights can be sold at different prices to agents with different characteristics. Consider, in this respect, an equilibrium consisting of firms owned exclusively by low-asset agents and firms jointly owned by high- and low-asset agents in which the low asset agents have paid a price above par in purchasing their equity interests. Low-asset agents might be willing to pay an above-par price for joining a firm with high-asset owners because they realize that high-asset partners will providing more effort thanks to their larger share of the firm's eventual output. For an equilibrium consisting of firms with exclusively low-asset owners and firms with mixed ownership to exist, low-asset agents must be indifferent between joining one type firm or the other. The above-par price at which high asset agents sell equity in their firms to low-asset agents is the variable that would adjust to sustain such an equilbrium. The potential for mixed agent firms becomes greater once one entertains other differences across agents such as their degrees of risk aversion.

Pursuing these general equilibrium and asset pricing questions is high up on our short-term future work agenda, but our focus in this paper is exploring, in a partial equilibrium context, the choices of effort, debt, and number of owners for firms with unlimited and limited liability.

3 Solving the Unlimited Liability Case

The first order condition for effort in the case of unlimited liability is given by

$$
\begin{aligned}
& -U_2\left(0, e_i\right) G\left(z_d\right)-U\left(0, e_i\right) g\left(z_d\right) \frac{\partial z_d}{\partial e_i} && (10)\\
= & \int_{z_d}^{\infty} U_1\left(\frac{z F\left(N a+D, \sum e_i\right)-(1+\tilde{r}) D}{N}, e_i\right) \frac{z F_2\left(N a+D, \sum e_i\right)}{N} d G(z) && \\
& +\int_{z_d}^{\infty} U_2\left(0, e_i\right) d G(z)-U\left(0, e_i\right) g\left(z_d\right) \frac{\partial z_d}{\partial e_i}. && (11)
\end{aligned}
$$

The left-hand side represents the marginal cost of an increase in e_i. The right-hand side represents the marginal benefit. An increase in effort reduces the chances of bankruptcy by changing the firm's bankruptcy trigger z_d. But since it has no immediate effect on consumption, at the margin, this effect cancels out.

>From this first-order condition, we can derive a symmetric NE $e_i = e(D, \tilde{r})$ which satisfies

$$U_2(0, e)+\int_{z_d^u}^{\infty} U_1 \frac{z F_2(N a+D, N e)}{N} d G(z)=0. \tag{12}$$

We now turn to the owners' collective choice of debt. We use (2) to replace $\tilde{r}$ with z_d. Without risk of confusion, we still use the notation $e_i = e(D, z_d)$. The choice of $\tilde{r}$ is transformed to the choice of z_d. Then the firm chooses (D, z_d) to maximize total utility of all agents. Since all agents in the firm are identical, we have

$$\max_{D, z_d} U(0, e) G\left(z_d\right)+\int_{z_d}^{\infty} U\left(\frac{\left(z-z_d\right) F(N a+D, N e)}{N}, e\right) d G(z) \tag{13}$$

subject to

$$(1+r) D=F(N a+D, N e)\left\{\alpha \int_0^{z_d} z d G(z)+\left[1-G\left(z_d\right)\right] z_d\right\}, \tag{14}$$

where e is the solution to equation (12) as a function of (D, z_d).

Let λ be the Lagrange multiplier associated with (14). FOCs are

- For D:

$$\int_{z_d}^{\infty} U_1\left(\frac{(z-z_d)\,F\,(Na+D,Ne)}{N},e\right)\frac{(z-z_d)\,F_1\,(Na+D,Ne)}{N}dG\,(z)$$
$$+U_2G\,(z_d)\,\frac{\partial e}{\partial D}+\int_{z_d}^{\infty}\left[U_1\,(z-z_d)\,F_2+U_2\right]\frac{\partial e}{\partial D}dG\,(z)+\lambda\,(1+r)\,D\frac{F_2}{F}N\frac{\partial e}{\partial D}$$
$$-\lambda\left\{1+r-F_1\,(Na+D,Ne)\left(\alpha\int_0^{z_d} zdG\,(z)+\left[1-G\,(z_d)\right]z_d\right)\right\}=0, \tag{15}$$

- For z_d:

$$-\int_{z_d}^{\infty} U_1\left(\frac{(z-z_d)\,F\,(Na+D,Ne)}{N},e\right)\frac{F\,(Na+D,Ne)}{N}dG\,(z)$$
$$+U_2G\,(z_d)\,\frac{\partial e}{\partial z_d}+\int_{z_d}^{\infty}\left[U_1\,(z-z_d)\,F_2+U_2\right]\frac{\partial e}{\partial z_d}dG\,(z)+\lambda\,(1+r)\,D\frac{F_2}{F}N\frac{\partial e}{\partial z_d}$$
$$+\lambda F\,(Na+D,Ne)\left[\alpha z_d g\,(z_d)+1-G\,(z_d)-g\,(z_d)\,z_d\right]=0, \tag{16}$$

3.1 Parameterization

We suppose

$$F\,(K,L)=K^{\alpha_k}L^{\alpha_l}, \tag{17}$$

and

$$U\,(c,e)=\frac{c^{1-\gamma}}{1-\gamma}-H\frac{e^{1+\delta}}{1+\delta},\ \gamma\in(0,1)\,,\delta\geq 0 \tag{18}$$

We could also assume z is uniformly distributed over $[0,b]$ for some large b.

We solve equation (12) to obtain

$$-He^{\delta}+\frac{F_2\,(Na+D,Ne)\,F\,(Na+D,Ne)^{-\gamma}}{bN^{1-\gamma}}\int_{z_d}^{\infty}(z-z_d)^{-\gamma}\,zdz=0 \tag{19}$$

Thus,

$$e = e(D, z_d) = X^{1/(1-\alpha_l(1-\gamma)+\delta)}, \tag{20}$$

where

$$X = \frac{\alpha_l (Na + D)^{\alpha_k(1-\gamma)}}{HbN^{1+(1-\gamma)(1-\alpha_l)}} \left(\frac{(b - z_d)^{2-\gamma}}{2-\gamma} + z_d \frac{(b - z_d)^{1-\gamma}}{1-\gamma} \right). \tag{21}$$

We can derive some simple comparative statics.

$$\frac{\partial e}{\partial D} > 0, \frac{\partial e}{\partial z_d} > 0, \frac{\partial e}{\partial a} > 0 \tag{22}$$

since

$$\begin{aligned} \frac{\partial e}{\partial D} &= \frac{1}{1-\alpha_l(1-\gamma)+\delta} X^{1/(1-\alpha_l(1-\gamma)+\delta)-1} \\ &\quad \frac{\alpha_l \alpha_k (1-\gamma)(Na+D)^{\alpha_k(1-\gamma)-1}}{HbN^{1+(1-\gamma)(1-\alpha_l)}} \left(\frac{(b - z_d)^{2-\gamma}}{2-\gamma} + z_d \frac{(b - z_d)^{1-\gamma}}{1-\gamma} \right) \\ &= \frac{1}{1-\alpha_l(1-\gamma)+\delta} \frac{\alpha_k(1-\gamma)}{Na+D} X^{1/(1-\alpha_l(1-\gamma)+\delta)}. \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{\partial e}{\partial z_d} &= \frac{1}{1-\alpha_l(1-\gamma)+\delta} X^{1/(1-\alpha_l(1-\gamma)+\delta)-1} \\ &\quad \frac{\alpha_l (Na+D)^{\alpha_k(1-\gamma)}}{HbN^{1+(1-\gamma)(1-\alpha_l)}} \gamma \left(\frac{(b - z_d)^{1-\gamma}}{1-\gamma} + z_d \frac{(b - z_d)^{-\gamma}}{-\gamma} \right). \end{aligned} \tag{24}$$

The interpretations are as follows. If the debt is higher, or the risky interest rate is higher (higher z_d), the agent will provide more effort to avoid bankruptcy, ceteris paribus. If the agent puts in more asset in the firm, he has more to lose, and thus he will provide more effort to avoid bankruptcy. Finally, the effect of N on e is ambiguous because there are two opposing effects. First, if there are more agents (N is higher), the agent will provide less effort since he has a larger incentive to free ride. Second, when N is higher, the firm has more capital and offers a higher return to additional effort.

- For D :

$$\frac{F^{-\gamma}F_1}{bN^{1-\gamma}}\int_{z_d}^{\infty}(z-z_d)^{1-\gamma}\,dz$$
$$-H\frac{\partial e}{\partial D}+\frac{F^{-\gamma}}{bN^{-\gamma}}F_2\int_{z_d}^{\infty}(z-z_d)^{1-\gamma}\,dz\frac{\partial e}{\partial D}+\lambda(1+r)\,D\frac{F_2}{F}N\frac{\partial e}{\partial D}$$
$$-\lambda\left\{1+r-F_1(Na+D,Ne)\left(\alpha\int_0^{z_d}zdG(z)+[1-G(z_d)]\,z_d\right)\right\}=0, \tag{25}$$

- For z_d :

$$-\frac{F^{1-\gamma}}{bN^{1-\gamma}}\int_{z_d}^{\infty}(z-z_d)^{-\gamma}\,dz$$
$$-H\frac{\partial e}{\partial z_d}+\frac{F^{-\gamma}}{bN^{-\gamma}}F_2\int_{z_d}^{\infty}(z-z_d)^{1-\gamma}\,dz\frac{\partial e}{\partial z_d}+\lambda(1+r)\,D\frac{F_2}{F}N\frac{\partial e}{\partial z_d}$$
$$+\lambda F(Na+D,Ne)\left[\alpha z_d g(z_d)+1-G(z_d)-g(z_d)\,z_d\right]=0 \tag{26}$$

- We can solve for the three unknowns D, z_d, λ using three equations (25) (26) and (14).

- We will use the following expression for the integral

$$\int_{z_d}^{\infty}(z-z_d)^{1-\gamma}\,dz=\frac{(b-z_d)^{2-\gamma}}{2-\gamma} \tag{27}$$

4 Solving the Limited Liability Case

Again, we use the two stage formulations. In the last stage, we will solve a symmetric Nash equilibrium, taking D, $\tilde{r}$ and θ as given.

- FOC with e

$$-U_2((1+r)(1-\theta)a,e)\,G(z_d)-U((1+r)(1-\theta)a,e)\,g(z_d)\frac{\partial z_d}{\partial e} \tag{28}$$
$$=\int_{z_d}^{\infty}\left[U_1\frac{z(1-\tau)F_2(N\theta a+D,Ne)}{N}+U_2\right]dG(z)$$
$$-U((1+r)(1-\theta)a,e)\,g(z_d)\frac{\partial z_d}{\partial e}.$$

Or

$$U_2\left((1+r)(1-\theta)a,e\right)+\int_{z_d^u}^{\infty}U_1\frac{z(1-\tau)F_2(N\theta a+D,Ne)}{N}dG(z)=0. \tag{29}$$

We now move to the first stage. We can use (6) to substitute out for the borrowing rate $\widetilde{r}^l$ in (5) and (7). Let the NE be $e_i=e(D,z_d,\theta)$. We then have

$$\begin{aligned}&\max_{D,z_d,\theta}U\left((1+r)(1-\theta)a,e\right)G(z_d) \\ &+\int_{z_d^l}^{\infty}U\left((1+r)(1-\theta)a+\frac{(1-\tau)(z-z_d)F(N\theta a+D,Ne)}{N},e\right)dG(z)\end{aligned} \tag{30}$$

subject to

$$(1+r)D=(1-\tau)F(N\theta a+D,Ne)\left\{\alpha\int_0^{z_d}zdG(z)+\left[1-G(z_d)\right]z_d\right\}. \tag{31}$$

Let λ be the Lagrange multiplier associated with (31). FOCs are

- FOC with θ :

$$\begin{aligned}&-(1+r)aU_1\left((1+r)(1-\theta)a,e\right)G(z_d) \\ &+\int_{z_d^u}^{\infty}U_1\left((z-z_d)(1-\tau)aF_1(N\theta a+D,Ne)-(1+r)a\right)dG(z) \\ &+\lambda\frac{(1+r)D}{F}\left(NF_1a+NF_2\frac{\partial e}{\partial\theta}\right). \\ &+U_2\frac{\partial e}{\partial\theta}+\int_{z_d^u}^{\infty}U_1(z-z_d)(1-\tau)F_2dG(z)\frac{\partial e}{\partial\theta}=0\end{aligned} \tag{32}$$

- For D :

$$\begin{aligned}&\int_{z_d}^{\infty}U_1\frac{(1-\tau)(z-z_d)F_1(N\theta a+D,Ne)}{N}dG(z) \\ &-\lambda\left\{1+r-(1+r)DF_1(N\theta a+D,Ne)/F\right\} \\ &+U_2\frac{\partial e}{\partial D}+\int_{z_d}^{\infty}U_1(1-\tau)(z-z_d)F_2dG(z)\frac{\partial e}{\partial D} \\ &+\frac{\lambda(1+r)DN}{F}F_2\frac{\partial e}{\partial D}=0,\end{aligned} \tag{33}$$

- For z_d :

$$-\int_{z_d}^{\infty} U_1 \frac{(1-\tau) F(N\theta a + D, Ne)}{N} dG(z)$$

$$+\lambda (1-\tau) F(N\theta a + D, Ne) \{\alpha z_d g(z_d) + [1 - G(z_d) - g(z_d) z_d]\}$$

$$+U_2 \frac{\partial e}{\partial z_d} + \int_{z_d}^{\infty} U_1 (1-\tau)(z - z_d) F_2 dG(z) \frac{\partial e}{\partial z_d}$$

$$+\frac{\lambda (1+r) DN}{F} F_2 \frac{\partial e}{\partial z_d} = 0, \tag{34}$$

4.1 CRRA

- We now can solve the five unknowns $e, \theta, D, z_d, \lambda$ using the five equations

- For e :

$$-He^{\delta} + \frac{(1-\tau) F_2}{bN} \int_{z_d^u}^{\infty} \left((1+r)(1-\theta) a + \frac{(z - z_d) F(N\theta a + D, e)}{N} \right)^{-\gamma} z dz = 0. \tag{35}$$

- For θ :

$$-(1+r) a ((1+r)(1-\theta) a)^{-\gamma} z_d / b$$

$$+\frac{(1-\tau) a F_1}{b} \int_{z_d^u}^{\infty} U_1 (z - z_d) dz - (1+r) a \frac{1}{b} \int_{z_d^u}^{\infty} U_1 dz$$

$$+\lambda \frac{(1+r) D}{F} \left(F_1 a + N F_2 \frac{\partial e}{\partial \theta} \right) + U_2 \frac{\partial e}{\partial \theta} + (1-\tau) F_2 \int_{z_d^u}^{\infty} U_1 (z - z_d) dG(z) \frac{\partial e}{\partial \theta} = 0$$

- For D :

$$\frac{(1-\tau) F_1}{bN} \int_{z_d}^{\infty} U_1 (z - z_d) dz$$

$$-\lambda \left\{ 1 + r - \frac{(1+r) D F_1}{F} \right\}$$

$$+U_2 \frac{\partial e}{\partial D} + \frac{(1-\tau) F_2}{b} \int_{z_d}^{\infty} U_1 (z - z_d) dz \frac{\partial e}{\partial D}$$

$$+\frac{\lambda (1+r) DN}{F} F_2 \frac{\partial e}{\partial D} = 0, \tag{36}$$

- For z_d :

$$-\frac{(1-\tau)\,F\,(N\theta a+D,Ne)}{N}\int_{z_d}^{\infty}U_1dG\,(z)$$

$$+\lambda\,(1-\tau)\,F\,(N\theta a+D,Ne)\,\{\alpha z_d g\,(z_d)+[1-G\,(z_d)-g\,(z_d)\,z_d]\}$$

$$+U_2\frac{\partial e}{\partial z_d}+\frac{F_2\,(1-\tau)}{b}\int_{z_d}^{\infty}U_1\,(z-z_d)\,dz\frac{\partial e}{\partial z_d}$$

$$+\frac{\lambda\,(1+r)\,DN}{F}F_2\frac{\partial e}{\partial z_d}=0, \tag{37}$$

- We need the following explicit expressions of intergrals

$$\int_{z_d^u}^{\infty}U_1\,(z-z_d)\,dz=\int_{z_d}^{b}\left((1+r)\,(1-\theta)\,a+\frac{(1-\tau)\,(z-z_d)\,F}{N}\right)^{-\gamma}(z-z_d)\,dz \tag{38}$$

$$\begin{aligned}
&= \frac{N}{(1-\gamma)\,(1-\tau)\,F}\int_0^{b-z_d}xd\left((1+r)\,(1-\theta)\,a+\frac{x\,(1-\tau)\,F}{N}\right)^{1-\gamma}\\
&= \frac{N}{(1-\gamma)\,(1-\tau)\,F}\left[x\left((1+r)\,(1-\theta)\,a+\frac{xF\,(1-\tau)}{N}\right)^{1-\gamma}\Bigg|_0^{b-z_d}\right.\\
&\quad\left.-\int_0^{b-z_d}\left((1+r)\,(1-\theta)\,a+\frac{xF\,(1-\tau)}{N}\right)^{1-\gamma}dx\right]
\end{aligned}$$

$$=\frac{N}{(1-\gamma)\,(1-\tau)\,F}\left[(b-z_d)\left((1+r)\,(1-\theta)\,a+\frac{(b-z_d)\,F\,(1-\tau)}{N}\right)^{1-\gamma}\right]$$

$$-\frac{N^2}{(1-\tau)^2\,F^2\,(1-\gamma)\,(2-\gamma)}\left[\left((1+r)\,(1-\theta)\,a+\frac{(b-z_d)\,F\,(1-\tau)}{N}\right)^{2-\gamma}\right.$$

$$\left.-\left((1+r)\,(1-\theta)\,a\right)^{2-\gamma}\right]$$

- and

$$\int_{z_d^u}^{\infty}U_1dz=\int_{z_d}^{b}\left((1+r)\,(1-\theta)\,a+\frac{(z-z_d)\,F\,(1-\tau)}{N}\right)^{-\gamma}dz \tag{39}$$

$$=\frac{N}{F\,(1-\tau)\,(1-\gamma)}\left((1+r)\,(1-\theta)\,a+\frac{(z-z_d)\,F\,(1-\tau)}{N}\right)^{1-\gamma}\Bigg|_{z_d}^{b}$$

$$= \frac{N}{F(1-\tau)(1-\gamma)}\left[\left((1+r)(1-\theta)a + \frac{(b-z_d)F(1-\tau)}{N}\right)^{1-\gamma} - ((1+r)(1-\theta)a)^{1-\gamma}\right]$$

4.2 Results

Table 1 lays out our preliminary base case paramater values. In future work we will explore higher rates of risk aversion. Table 2 shows optimal choices of effort, debt, and firm size for the basecase paramaters and for alternative levels of assets per owner and degrees of risk aversion. The key point of interest is that these three decision variables as well as the borrowing rate and probability of default are highly sensitive to assumed asset levels and degrees of risk aversion.

Table 3 examines the optimal values of effort, debt, and investment share under limited liabilty, but assuming a zero corporate tax rate. To ease comparion with table 2 we hold firm size at the optimal values found in table 2. Table 4, whose values are not much different from those of table 3 also reports the optimal choice of firm size. The main finding here is that given the paramaters under consideration, the choice of limited liability makes very little difference to optimal effort or debt levels. Our sense is that we need to explore much higher levels of risk aversion to find meaningful differences between the two cases and to permit the possibility of limited liability being preferred over unlimited liabilty in the presence of a significant corporate tax rate. The final two tables – 5 and 6 – take firm size as exogenous and show that the effect of firm size on effort and debt levesl is what one would expect.

To conclude, this initial analysis yields qualitatively correct results and shows that the optimal choices of firm size, owner effort, and debt levels depends critically on asset levels and degrees of risk aversion. But the paramaterization we've explored so far does not produce meaningful differences in these choices between limited and unlimited liability enterprises. The key variable that we need to consider in future work is the degree of risk aversion. We intend to consider alternative specifications of utility, specifically constant absolute risk aversion, and constant relative risk aversion, but with a floor on consumption. Either approach should allow us to entertain much higher degrees of risk aversion. Doing so will lead to material differences in expected utilty between the two business forms and permit the possibility of some types of agents choosing limited liability even in the presence of a significant corporate income tax. Once we have captured such behavior, it will be a simple matter to change the corporate tax rate to see which agents are, in fact, paying the tax after all equilibrium adjustments have occurred.

Table 1. Baseline parameter values

	Parameter	Value
Asset exponent	α_k	0.52
Effort exponent	α_l	0.85
Risk aversion	γ	0.3
Effort curvature	δ	1
Effort cost	H	0.8
Assets	a	10
Maximum shock	b	5
Bankruptcy cost	$1-\alpha$	0.05
Riskfree rate	r	0.02

Table 2. Results for the unlimited liability case

	Baseline Model	$a = 5$ $\gamma = 0.1$	$a = 5$ $\gamma = 0.2$	$a = 5$ $\gamma = 0.3$	$a = 15$ $\gamma = 0.1$	$a = 15$ $\gamma = 0.2$	$a = 15$ $\gamma = 0.3$
N	6	3	4	5	3	5	6
Consumption	21.0	169	34.53	16.6	188.9	45.5	25.4
Effort	1.42	9.21	3.03	1.59	9.50	2.81	1.38
Debt	56.7	980	201	73.0	1008	223.6	26.3
Risky rate	0.12	0.31	0.26	0.19	0.30	0.21	0.06
Default prob	0.17	0.42	0.37	0.28	0.41	0.30	0.08
Utility	10.05	70.56	15.08	7.85	79.71	20.73	12.05

Table 3. Results for the limited liability case with N fixed at the unlimited liability level

	Baseline Model	$a = 5$ $\gamma = 0.1$	$a = 5$ $\gamma = 0.2$	$a = 5$ $\gamma = 0.3$	$a = 15$ $\gamma = 0.1$	$a = 15$ $\gamma = 0.2$	$a = 15$ $\gamma = 0.3$
Fixed N	6	3	4	5	3	5	6
Share	0.85	1	1	0.85	1	1	0.85
Consumption	20.45	169	34.53	16.0	188.9	45.5	25.2
Effort	1.40	9.21	3.03	1.56	9.50	2.81	1.38
Debt	67.34	980	201	75.97	1008	223.6	45.2
Risky rate	0.15	0.31	0.26	0.21	0.30	0.21	0.10
Default prob	0.21	0.42	0.37	0.30	0.41	0.30	0.14
Utility	10.09	70.56	15.08	7.90	79.71	20.73	12.09

Table 4. Results for the limited liability case with optimal N

	Baseline Model	$a = 5$ $\gamma = 0.1$	$a = 5$ $\gamma = 0.2$	$a = 5$ $\gamma = 0.3$	$a = 15$ $\gamma = 0.1$	$a = 15$ $\gamma = 0.2$	$a = 15$ $\gamma = 0.3$
N	5	3	4	5	3	5	6
Share	0.85	1	1	0.85	1	1	0.85
Consumption	20.45	169	34.53	16.0	188.9	45.5	25.2
Effort	1.55	9.21	3.03	1.56	9.50	2.81	1.38
Debt	57.34	980	201	75.97	1008	223.6	45.2
Risky rate	0.15	0.31	0.26	0.21	0.30	0.21	0.10
Default prob	0.21	0.42	0.37	0.30	0.41	0.30	0.14
Utility	10.10	70.56	15.08	7.90	79.71	20.73	12.09

Table 5. Baseline model with unlimited liability and different N

	$N = 4$	$N = 6$	$N = 8$
Consumption	21.94	21.0	20.2
Effort	1.76	1.42	1.21
Debt	37.8	56.7	71.5
Risky rate	0.12	0.12	0.12
Default prob	0.18	0.17	0.17
Utility	9.98	10.05	10.00

Table 6. Baseline model with limited liability and different N

	$N = 4$	$N = 6$	$N = 8$
Share	0.8	0.85	0.9
Consumption	21.3	20.4	19.9
Effort	1.73	1.40	1.20
Debt	47.7	67.3	81.7
Risky rate	0.15	0.15	0.14
Default prob	0.22	0.21	0.20
Utility	10.05	10.09	10.04