Longevity and Pay-as-you-Go Pensions*

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May 16, 2006

Abstract

This paper aims at investigating whether or not a utilitarian social planner should subsidize longevity-enhancing expenditures in an economy with a PAYG pension system. For that purpose, a simple two-period OLG model is developed, in which the length of the second period of life can be raised by private health spendings. Focussing on the steady-state, it is shown that the sign of the optimal subsidy on health expenditures tends to be negative when the replacement ratio is sufficiently large. Moreover, the optimal health subsidy is also shown to depend significantly on the longevity production process and on the production technology.

Keywords: longevity, health care, PAYG social security.

JEL classification: E13, E21.

1 Introduction

Whereas a particular attention has been recently paid to the positive study of interactions between the economic growth process and the lengthening of life, the normative question of the optimal public intervention in a growing economy with endogenous longevity has remained so far most often unexplored.[1]

That question can be formulated as follows. Suppose that each individual can influence the length of his life through some health expenditures. Should a benevolent utilitarian government let individuals invest in health as much as they want, or, on the contrary, should the State intervene, and subsidize - or tax - private health expenditures?

Intuitively, the answer to that question depends on how imperfect the market economy is. Several factors - either at the individual or social level - may cause a departure from a perfect economy, justifying some form of public intervention.

*The authors are grateful to F. Schroyen for very helpful suggestions. The first two authors achnowledge support from FNRS and ARC.

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[1] On growth models with endogenous longevity, see Chakraborty (2004), Aisa and Pueyo (2004), Leung and Wang (2005), and Bhattacharya and Qiao (2005). See Zhang *et al* (2006) for a more normative perspective.

At the individual level, it might be the case, for instance, that a person, when deciding his investment in health, does not fully internalize the consequences of his decision, and, hence, does not choose what is the best for him, justifying some correction by the government.[2] Moreover, even if the economy is composed of well-informed, fully rational agents, it may be the case that particular institutions, inherited from the past, prevent the economy from being at the social optimum. For instance, a Pay-as-you-Go pension system may exist, and, by discouraging savings, may lead to an under-accumulation of capital, which invites some governmental intervention.

The present paper aims at examining the definition of the optimal long-run public policy in an economy composed of rational agents influencing their longevity by health expenditures, but where there exists a PAYG pension system with a fixed replacement ratio.[3] The question to be addressed in this paper is twofold. First, would a benevolent utilitarian planner choose, in the presence of a PAYG pension system, to subsidize or to tax individual health expenditures? Second, to what extent is the optimal (second-best) policy sensitive to the size of the replacement ratio?

This paper is organized as follows. Section 2 presents a two-period OLG model aimed at identifying the determinants of the optimal long-run health subsidy. Then, Section 3 develops the analytical example of an economy with CES utility, a Cobb-Douglas technology and a longevity production function with constant elasticity. The sensitivity of the optimal (second-best) policy to the replacement ratio is studied numerically in Section 4. Section 5 concludes.

2 The basic model

2.1 A competitive economy with endogenous longevity

2.1.1 Environment

Let us consider a two-period OLG model with identical individuals within each generation (each generation being denoted by the time of its birth). The first period is of unitary length, while the length of the second period of life for a cohort born at time t is given by h_{t+1} ($0 < h_{t+1} < 1$), which depends on health investments H_t made during the first period of life:

$$h_{t+1} = h(H_t) \tag{1}$$

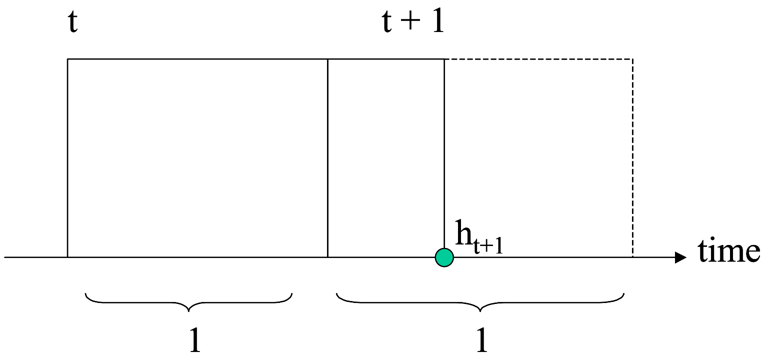
Whereas the first period is a period of labour, the second period, of length h, is a period of retirement. Given that h is not the probability of survival in

[2] For instance, Becker and Philipson (1998) show that individuals may tend to overinvest in longevity when they benefit from schemes whose generosity depends on longevity (e.g. annuities). See also Davies and Kuhn (1992).

[3] However, fertility is here supposed to be exogenous and constant over time, which might be regarded as resulting from some form of imperfect rationality at the individual level. For a study on the design of PAYG systems under endogenous fertility, see Cremer *et al* (2006).

the second period, but the length of the second period, there are no accidental bequests: individuals know exactly how long they live in their second period.[4]

Figure 1



If one supposes additive lifetime welfare, the welfare of a generation t is:

$$u(c_t) + \beta h(H_t)u(d_{t+1}) \tag{2}$$

where c_t and d_{t+1} denote consumption at the first and the second period of life, while β is the time preference factor.

2.1.2 Production

The production technology involves labour and capital:

$$Y_t = F(K_t, L_t) \tag{3}$$

where Y_t denotes the output, while K_t is the stock of capital and L_t is the labour force, which is supposed to be constant. In intensive terms, the production process, which is supposed to be homogenous of degree one, can be written as:

$$y_t = F(k_t, 1) = f(k_t) \tag{4}$$

where y_t and k_t denote respectively the output and capital per worker.

Factors are paid at their marginal productivities:

$$R_t = f'(k_t) \tag{5}$$

$$w_t = f(k_t) - k_t f'(k_t) \tag{6}$$

where R_t is one plus the interest rate, while w_t is the wage.

[4] Note here that Eeckhoudt and Pestieau (2006) introduced a two-parameter longevity, with a survival probability π of living the whole second period, and the length h of that second period. Hence, the average longevity is then $1 + \pi h$. See also Ponthiere (2006) on this.

2.1.3 Government

The government taxes labour income during the first period at a rate τ, and, then, uses the resulting tax revenue to fund the PAYG pension system (with pension scheme p), and to subsidize health spendings at a rate θ. Note that we assume that this pension scheme is given: this means that either τ or the replacement rate p/w is given.[5]

It is supposed that the government has no budget deficit, so that:

$$\tau w_t = h_t p_t + \theta H_t \tag{7}$$

or,

$$\tau = h_t \varphi + \theta H_t / w_t \tag{8}$$

where φ denotes the replacement ratio granted by the PAYG scheme.

2.1.4 Consumption, saving and health spending

Consumption during the first and the second period can be written as:

$$c_t = (1-\tau)w_t - s_t - (1-\theta)H_t \tag{9}$$

$$d_{t+1} = \frac{1}{h_{t+1}} R_{t+1} s_t + p_{t+1} \tag{10}$$

where s_t denotes savings.

The problem of each individual is to choose the savings s_t and health expenditures H_t that maximize lifetime welfare:

$$u\left((1-\tau)w_t - s_t - (1-\theta)H_t\right) + \beta h_{t+1} u\left(\frac{R_{t+1}s_t}{h_{t+1}} + p_{t+1}\right) \tag{11}$$

Optimal savings imply:

$$u'(c_t) = \beta u'(d_{t+1}) R_{t+1} \tag{12}$$

while optimal investment in health yields:

$$(1-\theta)u'(c_t) = \beta h'(H_t) u(d_{t+1}) - \beta u'(d_{t+1}) \frac{R_{t+1}s_t}{h_{t+1}} h'(H_t) \tag{13}$$

or

$$u'(c_t)\left(1 - \theta + \frac{h'(H_t)}{h_{t+1}} s_t\right) = \beta h'(H_t) u(d_{t+1}) \tag{14}$$

From these FOCs, one can derive the indirect utility function:

$$\tilde{V}(w_t, R_{t+1}, \tau, \theta) \tag{15}$$

[5] A certain amount of public debt can be given as initial condition.

with partial utilities: $\tilde{V}_\tau = -u'(c_t)w_t$; $\tilde{V}_\theta = u'(c_t)H_t$; $\tilde{V}_{w_t} = u'(c_t)\left(1-\tau+\frac{H_t}{R_{t+1}}\varphi\right)$ and $\tilde{V}_{R_{t+1}} = \frac{u'(c_t)s_t}{R_{t+1}}$.

2.1.5 Steady-state

Let us now suppose that a steady-state equilibrium exists. Within the present model, the (intertemporal) capital market equilibrium condition is:

$$k_{t+1} = s_t \tag{16}$$

Hence, if one focuses on the steady-state (and thus drops time indexes), and supposes that factors are paid at their marginal productivities, the above FOCs can be rewritten as:

$$u'(c) = \beta u'(d) f'(k) \tag{17}$$

$$u'(c)\left(1-\theta+\frac{h'(H)}{h}k\right) = \beta h'(H) u(d) \tag{18}$$

From this, one can define a new indirect utility function that depends exclusively on the public policy parameters τ and θ:

$$V = V(\tau, \theta) \tag{19}$$

with

$$V_\tau = -u'(c)w + u'(c)\zeta\left[(1-\tau)(R-1)-(\tau-h\varphi)\right]\frac{\partial k}{\partial \tau} \tag{20}$$

$$V_\theta = u'(c)H + u'(c)\zeta\left[(1-\tau)(R-1)-(\tau-h\varphi)\right]\frac{\partial k}{\partial \theta} \tag{21}$$

where $\zeta \equiv -f''(k)k/f'(k)$. Those two expressions summarize how public policy parameters τ and θ affect individual utility at the steady-state.

2.2 The planner's problem

Having characterized individual indirect utility at the steady-state as a function of the two policy parameters τ and θ, we can now present the problem of the benevolent utilitarian social planner. The planner wants to choose, for a given replacement ratio φ, the pair (τ^*, θ^*) maximizing steady-state lifetime utility.

If one supposes that the budget constraint must be satisfied, the planner's problem consists of maximizing the following Lagrangean:

$$\mathcal{L} = V(\tau, \theta) + \mu\left[(\tau - \varphi h(H))\, w(k) - \theta H\right] \tag{22}$$

where both H and k depend on τ and θ, while μ is the Lagrange multiplier.

The FOCs for τ and θ are:

$$\frac{\partial \mathcal{L}}{\partial \tau} = V_\tau + \mu \left[w'(k) \frac{\partial k}{\partial \tau} (\tau - \varphi h) + w \left(1 - \varphi h' \frac{\partial H}{\partial \tau} \right) - \theta \frac{\partial H}{\partial \tau} \right] = 0 \quad (23)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = V_\theta + \mu \left[w'(k) \frac{\partial k}{\partial \theta} (\tau - \varphi h) - w \varphi h' \frac{\partial H}{\partial \theta} - \theta \frac{\partial H}{\partial \theta} - H \right] = 0 \quad (24)$$

Combining those FOCs yields:

$$\frac{\partial \mathcal{L}^c}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial \theta} + \frac{\partial \mathcal{L}}{\partial \tau} \frac{H}{w} = u'(c) \zeta \left[(1-\tau)(R-1) - (\tau - h\varphi) \right] \frac{\partial k^c}{\partial \theta}$$
$$+\mu \left[w'(k) (\tau - \varphi h) \frac{\partial k^c}{\partial \theta} - (w\varphi h' + \theta) \frac{\partial H^c}{\partial \theta} \right] = 0$$

where the superscript c denotes that we deal with compensated demands.[6]

Hence,

$$\frac{\partial \mathcal{L}^c}{\partial \theta} = \left[u'(c)\zeta (1-\tau)(R-1) + (\tau - \varphi h) \left[\mu w'(k) - u'(c)\zeta \right] \right] \frac{\partial k^c}{\partial \theta}$$
$$-\mu \left[\theta + w\varphi h' \right] \frac{\partial H^c}{\partial \theta}$$

Let us now suppose that θ is equal to 0. Hence, given the budget constraint, $\tau - h\varphi = \frac{\theta H}{w} = 0$, so that:

$$\left. \frac{\partial \mathcal{L}}{\partial \theta} \right|_{\theta=0} = u'(c)\zeta (1-\tau)(R-1) \frac{\partial k^c}{\partial \theta} - \mu \left[w\varphi h' \right] \frac{\partial H^c}{\partial \theta} \quad (25)$$

At the Golden Rule (i.e. $R = 1$), the first term of the above expression vanishes, so that, given that $\partial H^c / \partial \theta$ is positive, a tax on H is desirable. The size of that tax depends positively on the replacement ratio φ.

If some under-accumulation prevails (i.e. $R > 1$), then a subsidy on health investment becomes desirable if $\partial k^c / \partial \theta$ is positive and if φ is small. Otherwise, it is optimal to tax health spendings.

One should notice here the economic significance of the two terms of expression (25). The first term of the RHS reflects the desirable effect of θ on capital accumulation when $R > 1$. Naturally, if the compensated effect of θ on s is negative, a negative θ occurs. The second term of the RHS reflects the fiscal burden of PAYG when longevity increases. It pushes for a negative θ given that normally $\partial H^c / \partial \theta$ is positive.

Assuming an interior solution for θ, the optimal health subsidy is:

$$\theta^* = \frac{u'(c)\zeta (1-\tau)(R-1) \frac{\partial k^c}{\partial \theta} - \mu \varphi w h' \frac{\partial H^c}{\partial \theta}}{\mu \frac{\partial H^c}{\partial \theta} - \frac{\zeta H}{w} (R\mu - u'(c)) \frac{\partial k^c}{\partial \theta}} \quad (26)$$

[6] More precisely, $\frac{\partial k^c}{\partial \theta} = \frac{\partial k}{\partial \theta} + \frac{\partial k}{\partial \tau} \frac{H}{w}$, while $\frac{\partial H^c}{\partial \theta} = \frac{\partial H}{\partial \theta} + \frac{\partial H}{\partial \tau} \frac{H}{w}$.

To interpret it, let us consider some polar cases. First, if R equals 1 and φ equals 0, θ^* is equal to 0. This is not surprising: in the absence of imperfections in the economy, *laissez-faire* is optimal. Let us now consider the case of under-accumulation of capital (i.e. $R > 1$). If we assume that $\partial k^c/\partial\theta$ is close to 0, meaning that the tax-subsidy combination has no effect on capital accumulation, the optimal subsidy θ^* becomes a tax that increases with φ. If, on the contrary, we suppose that $\partial H^c/\partial\theta$ is close to 0, the sign of θ^* is, in the case of under-accumulation, negative ($\mu > u'(c)/R$, i.e. the cost of public funds is larger than 1): the health-subsidy must be turned into a tax to foster capital accumulation.

Expression (26), although general, suffices to identify the determinants of the optimal (second-best) health subsidy under a PAYG pension system. However, this does not allow us to say much about the sign and size of θ^*. For that purpose, it is necessary to assume specific functional forms for individual utility, and for the production of output and longevity. This is the task of Section 3.

3 An analytical example

Let us now consider a particular analytical example of our basic framework.

3.1 A standard economy

3.1.1 Preferences

For convenience, individual lifetime welfare is supposed to be additive over time, whereas temporal utility takes a CES form:

$$U_t = \frac{1}{1-\delta}c_t^{1-\delta} + \beta\left[h_{t+1}\frac{d_{t+1}^{1-\delta}}{1-\delta}\right] \tag{27}$$

where $\delta < 1$ is the inverse of the elasticity of intertemporal substitution, while β is the time preference factor.

3.1.2 Production

The production technology is specified as a Cobb-Douglas function:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha} \tag{28}$$

where $A > 0$ and $0 < \alpha < 1$. Hence, in intensive terms:

$$y_t = Ak_t^{\alpha} \tag{29}$$

3.1.3 Longevity

Longevity is supposed to be determined by private health spendings as follows:

$$h_t = \eta H_t^{\varepsilon} \tag{30}$$

where η and ε are positive constants. The parameter ε corresponds to the elasticity of longevity with respect to health spendings. When health expenditures are completely unproductive (i.e. $\varepsilon = 0$), longevity is constant and equal to η.

3.1.4 Consumption, savings and health spendings

From the individual's utility maximization, one can obtain the following FOCs for the optimal s_t and H_t:

$$s_t^* = \frac{(\beta R_{t+1})^{1/\delta} h_{t+1} [(1-\tau)w_t - (1-\theta)H_t^*] - h_{t+1}p_{t+1}}{R_{t+1} + (\beta R_{t+1})^{1/\delta} h_{t+1}} \quad (31)$$

$$H_t^* = \frac{\varepsilon}{(1-\delta)(1-\theta)} \left[\delta s_t^* + h_{t+1}\frac{p_{t+1}}{R_{t+1}}\right] \quad (32)$$

The first expression suggests that, not surprisingly, the net wage has a positive impact on savings, whereas the pension benefit p_{t+1} has a negative impact on s_t^*. However, the influence of h_{t+1} on savings is unknown. The second expression suggests that, without surprise, H_t^* tends to 0 when ε tends to 0.

3.1.5 Steady-state

Whereas de la Croix and Michel (2002) show that there exists a unique and stable steady-state equilibrium with a log linear utility and a constant h, the present assumptions - CES utility and endogenous longevity - do not allow us to prove the existence of an equilibrium. Hence, the present analysis will be carried out under the postulate of existence of such a steady-state.[7]

If one substitutes the two FOCs in the capital market equilibrium condition $s_t = k_{t+1}$, some manipulations yield, at the steady-state:

$$(1-\theta)H = (1-\theta)\left(\frac{h}{\eta}\right)^{\frac{1}{\varepsilon}} = \frac{\varepsilon}{1-\delta}k\left(\delta + h\varphi\frac{1-\alpha}{\alpha}\right) \quad (33)$$

$$\left(1 + h\varphi\frac{1-\alpha}{\alpha}\right)Ak^{\left(\frac{1}{\delta}-1\right)(1-\alpha)} - BA^{\frac{1}{\delta}}h\left[(1-\tau)(1-\alpha)Ak^{\alpha-1} - \left(1 + \frac{\varepsilon\left(\delta + h\varphi\frac{1-\alpha}{\alpha}\right)}{1-\delta}\right)\right] = 0 \quad (34)$$

where $B = \beta^{\frac{1}{\delta}}\alpha^{\frac{1-\delta}{\delta}}$.

Moreover, if one supposes that the budget constraint $\tau w = h\varphi w + \theta H$ holds, the latter expression can be rewritten as:

$$\Delta \equiv (\alpha + h\varphi(1-\alpha))\,Ak^{\alpha+\frac{1-\alpha}{\delta}} - (\alpha\beta A)^{\frac{1}{\delta}}h\left((1-h\varphi)(1-\alpha)Ak^{\alpha} - k - \left(\frac{h}{\eta}\right)^{\frac{1}{\varepsilon}}\right) = 0 \quad (35)$$

[7] The possibility of stability of the equilibrium is discussed in the Appendix.

This expression, which characterizes the steady-state of the economy, can be used as a basis for investigating, in this analytical example, the conditions under which the optimal long-run health subsidy θ is positive or negative.

For that purpose, we shall proceed in two stages. Firstly, we shall carry out a comparative statics exercise, which will allow us to identify the different *kinds* of equilibrium that can occur in our economy. Then, in a second stage, we shall, in the light of our findings, consider the planner's problem, and derive the optimal (second-best) health subsidy in the economy under study.

3.1.6 Comparative statics

The expression Δ yields an implicit function of k, h and φ. Writing this as:

$$\Delta(k, h, \varphi) = 0 \tag{36}$$

we easily check that Δ_k and Δ_φ are positive. Regarding the sign of Δ_h, it appears, after some manipulations, that, at the steady-state (i.e. at $\Delta = 0$):

$$\Delta_h \gtreqless 0 \Leftrightarrow -\frac{\alpha}{h} A k^{\alpha + \frac{1-\alpha}{\delta}} + (\alpha\beta A)^{\frac{1}{\delta}} h \left(\varphi(1-\alpha) A k^{\alpha} + \frac{k\left(\delta + h\varphi\frac{1-\alpha}{\alpha}\right)}{h(1-\delta)(1-\theta)} \right) \gtreqless 0 \tag{37}$$

That condition suggests that there exist two distinct kinds of equilibrium in this economy, depending on whether Δ_h is positive or negative at the steady-state. In other words, two types of steady-state can occur, depending on whether the two curves (33) and (34) intersect where $\partial k^* / \partial h^*$ is positive or negative.

One should notice that the higher the replacement ratio φ is, the more likely is the occurrence of an equilibrium at which Δ_h is positive, that is, an equilibrium such that a rise in h would have a negative impact on steady-state capital per worker. On the contrary, when φ is low, an equilibrium with $\Delta_h < 0$ is more likely.[8]

Another way to identify the condition of occurence of each kind of equilibrium is to isolate h in the above inequality, which yields:

$$\Delta_h \gtreqless 0 \Leftrightarrow h \gtreqless \left[A^{\frac{1}{\delta}-1} B \frac{\varepsilon}{1-\delta} \frac{1-\alpha}{\alpha} \varphi \right]^{-\frac{1}{2}} k^{\frac{1-\alpha}{2}\frac{1-\delta}{\delta}} \tag{38}$$

Thus, a steady-state at which $\Delta_h > 0$ is, *ceteris paribus*, more likely in economies with a high steady-state longevity.

Let us now consider these two kinds of equilibrium, which shall be named case (a) ($\Delta_h < 0$) and case (b) ($\Delta_h > 0$).

Under case (a), it follows from the signs of Δ_k, Δ_h and Δ_φ that:

[8] It is straightforward to see that, if there is no pensions scheme ($\varphi = 0$), the above condition collapses to $R \gtreqless \beta^{-\frac{1}{1-\delta}} \left(\frac{1-\delta}{\delta} \frac{1-\theta}{h} \right)^{\frac{\delta}{1-\delta}}$.

$$k^* = \underset{+\ \ -}{k(h, \varphi)} \tag{39}$$

that is, h has here a positive effect on k^*, whereas φ has, without surprise, a negative impact.

Using the SOCs [i.e. $\frac{\partial}{\partial h}\left(\frac{h}{\eta}\right)^{\frac{1}{\varepsilon}} > \frac{\varepsilon k\left(\delta + h\varphi\frac{1-\alpha}{\alpha}\right)}{(1-\delta)(1-\theta)}$], it appears that:

$$h^* = \underset{+\ +\ +}{h(k, \varphi, \theta)} \tag{40}$$

so that h^* depends positively on k, and on the parameters φ and θ.

By combining the above expressions, one can identify the sign of the impact of φ and θ on steady-state k and h. Actually, given that θ has a positive effect on h^*, it must also have a positive impact on k^*. Hence, if the steady-state is of kind (a), raising θ has a positive influence on both steady-state capital and longevity. However, the above expressions do not allow us to deduce the impact of changing φ on h^*: indeed, whereas a higher replacement ratio has a positive direct effect on h^*, it has also a negative, indirect influence on h^*, due to the negative effect of φ on k^*, so that the final impact on h^* is unknown. A similar indeterminacy holds as far as the impact of φ on k^* is concerned.

Turning now to equilibrium of kind (b), at which $\Delta_h > 0$, the signs of Δ_k, Δ_h and Δ_φ imply:

$$k^* = \underset{-\ \ -}{k(h, \varphi)} \tag{41}$$

that is, h has here, unlike in case (a), a negative effect on k^*.

Moreover,

$$h^* = \underset{+\ +\ +}{h(k, \varphi, \theta)} \tag{42}$$

Hence, while the impact of a higher subsidy on steady-state h is, here again, positive, its impact on steady-state capital is now negative, unlike what prevailed in panel (a). Furthermore, whereas the effect of a higher φ on h^* remains unknown, its impact on k^* is here unambiguously negative [unlike in case (a)].

It follows from all this that the impact of public policy parameters θ and φ on steady-state capital and longevity - the budget constraint being satisfied - depends on which kind of equilibrium prevails, as summarized by Table 1.

Table 1: Sign of comparative statics

	Panel (a)		Panel (b)	
	k^*	h^*	k^*	h^*
θ	+	+	-	+
φ	?	?	-	?

Having emphasized, for the analytical example under study, the possible existence of two kinds of equilibrium, as well as the - distinct - impact of public policy under each of these, we can now consider the planner's problem of selecting the optimal health subsidy θ for a fixed replacement ratio φ.

3.2 The planner's problem

The benevolent planner's problem consists of choosing the health-subsidy θ and the tax rate τ in such a way as to maximize individual steady-state lifetime utility under the budget constraint. The Lagrangean associated to that optimization problem can be written as:

$$\mathcal{L} = V(k^*, \tau, \theta, \varphi) + \mu \left[\tau w^* - h^* \varphi w^* - \theta \hat{H}\right] \tag{43}$$

where V denotes the individual's lifetime utility, k^* and w^* denote the capital per worker and the wage at the steady-state, whereas $\hat{H}$ is given to the planner as a solution to $h^* = \eta \hat{H}^{\varepsilon}$, where h^* denotes steady-state longevity. μ is the Lagrange multiplier.

The FOCs are:[9]

$$\frac{\partial \mathcal{L}}{\partial \tau} = V_\tau + V_k \frac{\partial k}{\partial \tau} + \mu \left(w + (\tau - \varphi h) w'(k) \frac{\partial k}{\partial \tau} - \frac{\partial h}{\partial \tau} \varphi w - \theta \frac{\partial \hat{H}}{\partial h} \frac{\partial h}{\partial \tau} \right) \tag{44}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = V_\theta + V_k \frac{\partial k}{\partial \theta} + \mu \left((\tau - h\varphi) w'(k) \frac{\partial k}{\partial \theta} - \frac{\partial h}{\partial \theta} \varphi w - \hat{H} - \theta \frac{\partial \hat{H}}{\partial h} \frac{\partial h}{\partial \theta} \right) \tag{45}$$

where

$$V_k = \frac{(1-\alpha)\left[(R-1)(1-\tau) - (\tau - \varphi h)\right]}{c^\delta}; V_\tau = -\frac{(1-\alpha) A k^\alpha}{c^\delta}; V_\theta = \frac{H}{c^\delta}$$

where $c = \left[(1 - \varphi h)(1-\alpha) A k^\alpha - k - H\right]$.

Hence, combining the FOCs yields:

[9] For convenience, the superscript * is here omitted, as it is clear that the analysis is carried out at the steady-state.

$$\frac{\partial \mathcal{L}}{\partial \theta}+\frac{\partial \mathcal{L}}{\partial \tau}\frac{H}{w} = (1-\alpha)\left[\frac{(R-1)(1-\tau)}{c^{\delta}}+(\mu R-\frac{1}{c^{\delta}})(\tau-h\varphi)\right]\left(\frac{\partial k}{\partial \theta}+\frac{\partial k}{\partial \tau}\frac{H}{w}\right)$$
$$-\mu\varphi w\left(\frac{\partial h}{\partial \theta}+\frac{\partial h}{\partial \tau}\frac{H}{w}\right)-\theta\frac{\partial \hat{H}}{\partial h}\left(\frac{\partial h}{\partial \theta}+\frac{\partial h}{\partial \tau}\frac{H}{w}\right) \quad (46)$$

Thus, the derivative of the compensated Lagrangean is:[10]

$$\frac{\partial \mathcal{L}^c}{\partial \theta}=(1-\alpha)\left[\frac{(R-1)(1-\tau)}{c^{\delta}}+(\mu R-c^{-\delta})\frac{\theta H}{w}\right]\frac{\partial k^c}{\partial \theta}-\mu\varphi w\frac{\partial h^c}{\partial \theta}-\theta\frac{\partial \hat{H}}{\partial h}\frac{\partial h^c}{\partial \theta} \quad (47)$$

Assuming an interior solution for optimal θ yields:

$$\theta^*=\frac{c^{-\delta}(1-\alpha)(1-\tau)(R-1)\frac{\partial k^c}{\partial \theta}-\mu\varphi w\frac{\partial h^c}{\partial \theta}}{\mu\frac{\partial \hat{H}}{\partial h}\frac{\partial h^c}{\partial \theta}-\frac{(1-\alpha)H}{w}(R\mu-c^{-\delta})\frac{\partial k^c}{\partial \theta}} \quad (48)$$

It is straightforward to see that this expression corresponds to a particular case of the general solution derived in Section 2, for the case of CES utility (implying $u'(c)=c^{-\delta}$) and Cobb-Douglas technology (implying $\zeta=1-\alpha$).

To interpret the above expression, let us first notice that, if the economy is at the Golden Rule (i.e. $R=1$) and if there is no PAYG pension (i.e. $\varphi=0$), the optimal θ is zero, which only reflects that, in the absence of any market imperfection, the optimal policy is, without surprise, the *laissez-faire*.

However, in the other - more plausible - cases, the optimal θ cannot be easily signed. In the case of under-accumulation of capital ($R>1$), the sign of θ^* becomes dependent on the sign of $\partial k^c/\partial\theta$. If, for instance, $\partial k^c/\partial\theta$ is negative, it follows, given that $\partial h^c/\partial\theta$ is positive, that the numerator of the above expression is negative, whereas the denominator is positive [under $\mu>c^{-\delta}/R$], so that θ^* is necessarily negative. On the contrary, if $\partial k^c/\partial\theta$ is positive, then, the sign of θ^* is no longer necessarily negative: it depends on the size of the replacement ratio φ, as well as on the longevity production (*via* the size of $\partial h^c/\partial\theta$). Clearly, if $\partial k^c/\partial\theta$ is positive, it is likely, in the light of what was said in the previous subsection, that h is low, so that η and ε are likely to be small, implying a small $\partial h^c/\partial\theta$. Hence, in that case, θ^* is likely to be positive.

All this emphasizes the importance of the above discussion concerning the kind of equilibrium at which the economy lies. Indeed, the sign of $\partial k^c/\partial\theta$ depends crucially on the sign of $\partial k/\partial\theta$, which is positive under an equilibrium of kind (a), but negative under an equilibrium of kind (b). Hence, if the equilibrium is of type (b), which is more likely if φ is large, the optimal health subsidy θ^* is a tax, whereas, if the equilibrium is of type (a), the sign of θ^*, which depends on both φ and longevity technology, is more likely to be positive.

To summarize, while the model developed in Section 2 allowed us to emphasize, in general terms, the dependency of the optimal health subsidy on the

[10]Note that $\frac{\partial k^c}{\partial \theta}\equiv\frac{\partial k}{\partial \theta}+\frac{\partial k}{\partial \tau}\frac{H}{w}$ and $\frac{\partial h^c}{\partial \theta}\equiv\frac{\partial h}{\partial \theta}+\frac{\partial h}{\partial \tau}\frac{H}{w}$.

size of the PAYG system, and on the distance of the economy from the Golden Rule, the present analytical example allows us to say a bit more about the determinants of the optimal policy. Actually, in an economy with CES utility and Cobb-Douglas technology, a central determinant of the sign of the optimal health subsidy consists of the relationship between longevity and capital at the steady-state, relationship that is influenced by the size of the PAYG scheme. The above solution also underlines the crucial role played by demographic parameters, which affect each of the four terms in the expression defining θ^*.

However, in order to have a more precise idea of the sensitivity of the optimal health subsidy to the replacement ratio, a numerical application is required.

4 A numerical example

Let us now study, in the light of a numerical application, the sensitivity of the optimal long-run 'second-best' policy (τ^*, θ^*) to the size of the PAYG system. By optimal 'second-best' policy, we mean the pair (τ^*, θ^*) maximizing the steady-state lifetime utility of a typical agent under a given replacement ratio.

While this numerical application can have, as we shall see, significant policy implications, it should be stressed here that our focus on the steady-state constitutes, from a normative perspective, a non-negligible simplification, because it ignores generations living during the transition towards the steady-state.[11]

One should also notice that we shall, throughout this Section, impose a strict budget constraint: the sum of pensions and subsidies on health expenditures is supposed to be equal to fiscal revenues from income taxation.

The present numerical application is made under the assumptions of α equal to 1/3 and A equal to 10, while δ is fixed to 0.8 (which corresponds to an elasticity of intertemporal substitution of 1.25).[12] The discount factor β is fixed to 0.99. That low value is based on recent work by Bommier (2005), suggesting that most of observed discounting may reflect uncertainty about future survival rather than pure preference for the present. Given that there is no uncertainty in our model, it makes sense to suppose low discounting. Finally, demographic parameters ε and η are supposed to be equal to 0.10 and 0.16.[13]

As suggested by Table 2, the optimal public policy is significantly sensitive to the size of the PAYG system. Whereas the optimal subsidy on health expenditures H_t is positive and equal to 6 percents in the absence of a PAYG scheme, it becomes negative for higher values of the replacement ratio: under φ equal to 0.25, the optimal health subsidy becomes a tax of 2 percents, while θ^* is equal to -0.22 under φ equal to 0.75. Hence, the optimal subsidy on health expenditures is unambiguously decreasing with the size of the PAYG system. This illustrates the conclusions drawn from the planner's problem in Section 3:

[11] One should notice that all steady-states presented in this numerical exercise satisfy the stability condition stated in the Appendix.

[12] This is conform with Browning *et al* (1999)'s survey concluding that the IES should be close to 1.

[13] Those values are obtained by regressing life expectancy at age 65 on health expenditures per capita (data source: OECD, 2005).

a rise in φ increases the absolute value of the second term of the numerator of expression (48), which is negative, implying a lower θ^*. Thus, the optimal subsidy depends significantly on the prevailing replacement ratio: under a low φ, the optimal subsidy is positive, whereas, when the PAYG system is larger, this becomes a tax that grows with the size of φ.

Table 2: Optimal second-best policy:

ε	η	φ	τ^*	θ^*	h	k	U
0.10	0.16	0	0.004	0.06	0.147	1.030	8.334
0.10	0.16	0.25	0.035	-0.02	0.146	0.906	8.235
0.10	0.16	0.50	0.066	-0.11	0.144	0.801	8.136
0.10	0.16	0.75	0.095	-0.22	0.142	0.711	8.037

Table 2 also shows that, without surprise, a higher PAYG system leads to a lower steady-state capital per worker and, also, to a lower steady-state longevity. As a consequence, a higher φ implies a fall in steady-state lifetime utility, as shown by the last column of Table 2.

While Table 2 illustrates the substantial sensitivity of the optimal public policy to the prevailing PAYG pension scheme, it is important to notice that the replacement ratio is not the unique determinant of θ^*. Besides the influence of φ, the optimal health subsidy is also affected by how the production process is calibrated, that is, by the values of the parameters A and α in the Cobb-Douglas production function. To illustrate this, Figure 2 contrasts, for different values of φ, the optimal health subsidy under A equal to 10 and α equal to 1/3, with the one under A equal to 20 and α equal to 1/2. Under each scenario, demographic parameters ε and η remain equal to 0.10 and 0.16.

Figure 2: Sensitivity of optimal health subsidy to the production process

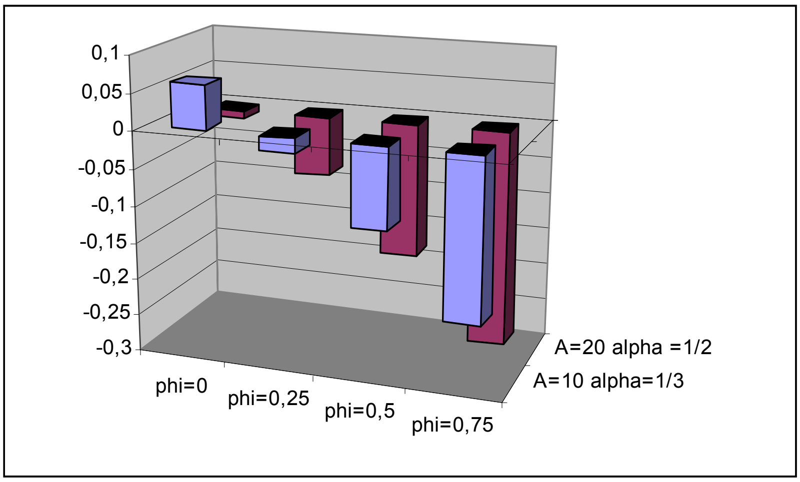


Figure 2 shows that, while the *pattern* of $\theta^*$with respect to φ is, in each case, decreasing, the *level* of θ^*differs significantly according to how the production function is calibrated. For instance, in the absence of a PAYG scheme, the optimal subsidy is, under α equal to 1/3 and A equal to 10, positive and equal to 6 percents, whereas, under α equal to 1/2 and A equal to 20, it becomes a tax of 1 percent. Thus, under a more capital-intensive production process, it becomes optimal for the government to tax private health expenditures whatever the level of the replacement ratio is. This conclusion differs from the one under a less capital-intensive production, under which it may be optimal to subsidize health expenditures if the replacement ratio is sufficiently low.

To explain this non-negligible sensitivity of the optimal second-best public intervention to the prevailing production process, let us turn back to the solution of the planner's problem in Section 3. If φ is zero, the numerator of expression (48) consists of a single term, which is likely to be positive (given that $\partial k^c/\partial\theta$ is likely to be positive under φ equal to 0). However, the denominator may be either positive, implying a positive θ^*, or negative, implying a negative θ^*, depending on the size of the second term (in which $\partial k^c/\partial\theta > 0$) with respect to the first term. That second term is, *ceteris paribus*, likely to be lower under α equal to 1/3 and A is equal to 10, than under α equal to 1/2 and A equal to 20, explaining why θ^* is, at φ equal to zero, positive in the former case, but negative in the latter case. When φ becomes larger, the numerator is likely to be negative (because $\partial k^c/\partial\theta$ is likely to be negative in the first term), whereas a second negative term is added to the numerator. Moreover, under a larger φ, the second term of the denominator becomes positive (because $\partial k^c/\partial\theta < 0$ is then more likely), which explains, in each case, the negative θ^*.

While the optimal subsidy on health expenditures is sensitive to the underlying production process, it is also strongly dependent on the longevity production function. To illustrate this, Figure 3 compares the optimal health subsidy (under $A = 10$ and $\alpha = 1/3$) for different values of the parameter η, which can be interpreted as the hypothetical longevity that would prevail if health expenditures were totally unproductive.

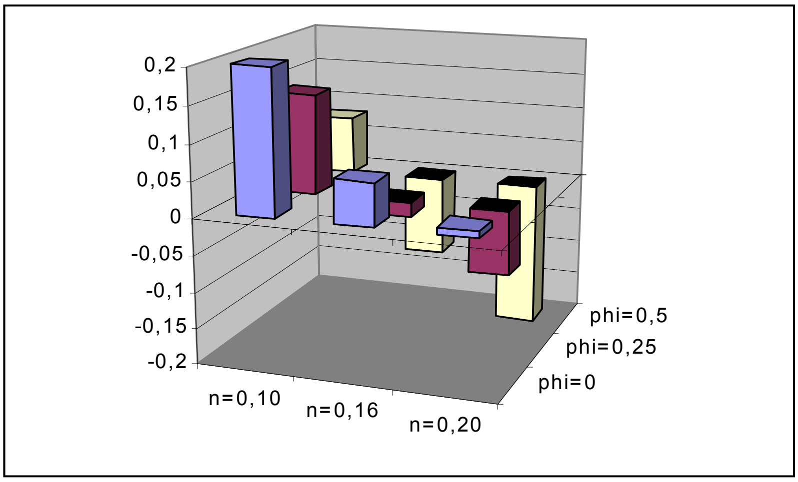
Figure 3 shows that θ^* is significantly sensitive to the postulated η. For instance, in the absence of a PAYG system, the optimal θ is as high as 20 percents under η equal to 0.10, but equal to only 1 percent under η equal to 0.20. Moreover, whereas Figure 3 confirms that θ^* remains, *ceteris paribus*, decreasing with φ, it suggests that the level of the optimal subsidy can be positive even under a high replacement ratio φ, provided η is sufficiently low. For instance, under η equal to 0.1, the optimal health subsidy remains positive (i.e. equal to 8 percents), even under a replacement ratio equal to 0.5. This result contrasts sharply with the optimal subsidy under η equal to 0.16 and 0.20, equal respectively to -11 and -20 percents.

To explain the dependency of θ^*on η, one can once again refer to the planner's problem in Section 3: given that η affects directly the impact of health expenditures on longevity - and, hence, via the effect of h on savings, on capital - it is not surprising that the optimal second-best policy is sensitive to η.

This result supports the view that the optimal public intervention under a

PAYG system cannot be uniform across countries, but should reflect the specificities of the prevailing demography. For instance, in poor countries, exhibiting a low η, the optimal second-best policy is to subsidize health expenditures, because such a subsidization improves both longevity *and* capital accumulation. However, in a country with a higher η, the extra-longevity resulting from subsidizing health expenditures would not suffice to compensate, in utility terms, the fall in capital accumulation, so that taxing health expenditures is the second-best policy. Naturally, taxing H_t leads to a lower longevity, but the corresponding gain in capital - and thus in consumption - does more than compensate the lower longevity, making the taxation of health expenditures socially desirable.

Figure 3: Sensitivity of optimal health subsidy to η

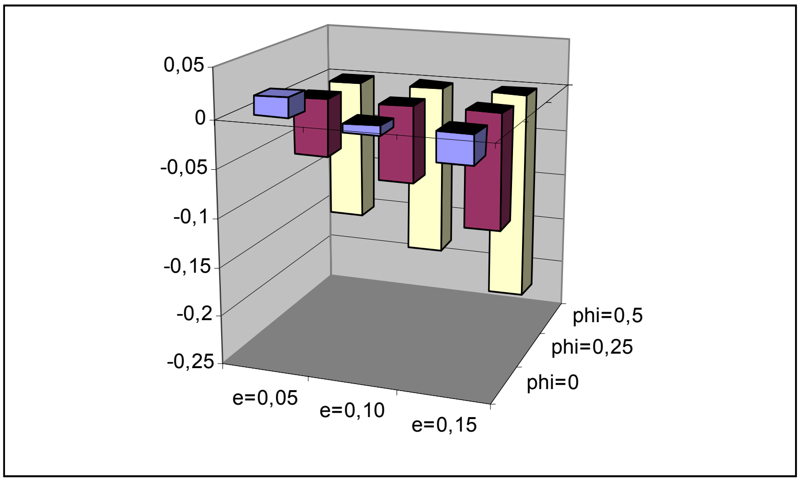


Finally, one should notice that the optimal health subsidy is also significantly sensitive to the other demographic parameter, ε, which is the elasticity of longevity with respect to health expenditures. That sensitivity is illustrated by Figure 4, which compares the optimal subsidy θ^* for ε equal to 0.05, 0.10 and 0.15, under different values of φ (under $A = 20$ and $\alpha = 1/2$).

As shown by Figure 4, the higher ε is, the lower the optimal subsidy θ^* is *ceteris paribus*. This means that the more productive private expenditures H_t are in terms of longevity, the higher is the optimal 'second-best' tax rate on these. In the light of the first-order conditions derived in Section 3, this is not a surprise: a higher ε implies that individuals tend to spend more, *ceteris paribus*, on health expenditures. Hence, if the goal of the planner is to correct an under-accumulation of capital [i.e. first term of expression (48)], a higher ε must lead, *ceteris paribus*, to a higher tax rate on health expenditures.

To summarize, this numerical exercise illustrates the significant sensitivity of optimal public policy to the size of the PAYG system, but, also, its dependency on the material production process and the longevity production process.

Figure 4: Sensitivity of optimal health subsidy to ε



5 Conclusions

In conclusion, let us turn back to the question raised in the introductionary section of this paper: should a benevolent utilitarian social planner subsidize private health expenditures in the presence of a PAYG pension scheme?

To answer that question, we firstly set up, in Section 2, a two-period OLG model with endogenous longevity, in which individuals can affect the length of their life by means of some health expenditures. Solving the utilitarian planner's problem allowed us to identify, in general terms, the factors affecting the sign and the size of the optimal subsidy on health spendings. Basically, the optimal (second-best) health subsidy was shown to depend negatively on the replacement ratio, whereas its level is also affected by whether the economy is in a situation of under- or over-accumulation of capital.

An analytical example was then developed in Section 3, to study more concretely the determinants of the optimal health subsidy. Under a Cobb-Douglas technology, CES utility and a longevity production function with constant elasticity, it was shown, by means of a comparative statics exercise, that the optimal public intervention depends crucially on the kind of steady-state equilibrium prevailing in the economy, that is, on how capital and longevity are related to each other at the steady-state. Moreover, solving the planner's problem allowed us to show how the production technology and the longevity technology interact in the definition of the optimal public policy.

Finally, the analytical example of Section 3 was calibrated to be able to discuss the sensitivity of the optimal second-best policy to the various parameters of the model (Section 4). This brief numerical application confirmed that the optimal health subsidy is decreasing with the size of the replacement ratio: whereas it is optimal to subsidize health expenditures under a low φ, it becomes socially desirable to tax health expenditures once φ is larger. While this result tends to highlight the importance of institutions for the design of optimal public intervention, it was also shown that the level of the optimal subsidy is significantly influenced - for a given replacement ratio - by the longevity production process and the prevailing production technology.

As a consequence, the answer to the question raised in this paper depends not only on the size of the PAYG system, but, also, on how the economy produces goods and life-years. Given the large international heterogeneity on those dimensions, the main normative conclusion of this paper might be that the optimal public policy cannot be uniform across countries, but should rather take into account the specificities of the economies considered, at the institutional, economic and demographic levels.

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7 Appendix

Stability of the steady-state equilibrium

From (39) we have:

$$\frac{dh}{dk} = \frac{\varepsilon}{k} h, \tag{A.1}$$

and from (40) we obtain:

$$\Delta_k = \frac{BA^{\frac{1}{\delta}}}{k^2} h (1-\alpha) \left[\left(\frac{1}{\delta} - 1 \right) c + (1-\tau) w \right] \tag{A.2}$$

and

$$\Delta_h = \frac{BA^{\frac{1}{\delta}}}{k} \left[h \frac{\varepsilon}{1-\delta} \varphi \frac{1-\alpha}{\alpha} k - \frac{c}{1 + h\varphi \frac{1-\alpha}{\alpha}} \right] \tag{A.3}$$

where $c \equiv (1-\tau)(1-\alpha) A k^{\alpha} - \left(1 + \frac{\varepsilon}{1-\delta} \left(\delta + h\varphi \frac{1-\alpha}{\alpha} \right) \right) k.$

Combining (A.2) and (A.3) yields:

$$\frac{dk}{dh} = \frac{\Delta h}{\Delta k} = \frac{k}{h} \frac{1}{1+h\varphi} \frac{1-\alpha}{\alpha} \frac{c - \left(1 + h\varphi \frac{1-\alpha}{\alpha} \right) h \frac{\varepsilon}{1-\delta} \varphi \frac{1-\alpha}{\alpha} k}{(1-\alpha) \left[\left(\frac{1}{\delta} - 1 \right) c + (1-\tau) w \right]}. \tag{A.4}$$

Using (A.1) and (A.4), we write the stability condition:

$$\frac{dh}{dk} \cdot \left| \frac{dk}{dh} \right| < 1$$

that is equivalent to:

$$\begin{aligned} & \varepsilon \left| c - \left(1 + h\varphi \frac{1-\alpha}{\alpha} \right) h \frac{\varepsilon}{1-\delta} \varphi \frac{1-\alpha}{\alpha} k \right| \\ < \quad & \left(1 + h\varphi \frac{1-\alpha}{\alpha} \right) (1-\alpha) \left[\left(\frac{1}{\delta} - 1 \right) c + (1-\tau) w \right] \end{aligned}$$