A cost-of-living dynamic price index, with an application to indexing retirement accounts

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Abstract

If a consumer wishes to protect her retirement account from the risk of price changes in order to sustain a stable standard of living, then what price index should the account be indexed to? This paper constructs a dynamic price index (DPI) that answers this question. Unlike the existing theory on price indices (which is static and certain), the DPI measures the cost of living for a consumer who lives for many periods and faces uncertainty. The first contribution of this research is to define this price index and study its theoretical properties. The DPI: is homogeneous of degree 1 with respect to all prices, is forward-looking with respect to price shocks, responds more to permanent vis-à-vis transitory price changes, includes asset prices with a potentially large weight, and distinguishes between durable and non-durable goods prices. The second contribution of the paper is to construct a DPI for the United States from 1970 to 2004. It gives an account of the cost of living in the U.S. that is strikingly different from the one provided by the CPI. The DPI is less persistent, more volatile, and a large part of its movements are driven by changes in the prices of houses and bonds.

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1 Introduction

Price indices are among the most commonly used economic quantities. Not only are they monitored closely by the public and policymakers, but researchers use them almost daily as their object of study or to construct other variables of interest. There are three main uses of price indices: to guide monetary policy, to deflate nominal variables and obtain their real counterparts, and to measure the cost of living. For all of them, it is common practice to measure the price index as the change in expenditure required to buy a fixed basket of goods when facing a new set of prices, as Laspeyres or Paasche suggested.

The focus of this paper is on the measurement of the cost of living. Economists have long criticized the fixed-basket approach of aggregating prices into a measure of the cost of living.[1] They noted that, in general, consumers will substitute across goods in response to changes in prices, so keeping baskets fixed will lead to a substitution bias. Rather than keeping *baskets* fixed, instead one should keep *utility* fixed. The original definition of a cost-of-living price index was made by Konus (1924), who proposed measuring "the relative change occurring in the monetary cost of those consumers' goods which are necessary for the maintenance of a certain standard of living." If $V(W, \mathbf{p}^t)$ is the indirect utility function of a consumer mapping the wealth she has W and the prices she faces $\mathbf{p}^t$ to the standard of living she can achieve by acting optimally, then the Konus price index π_{t+1} is defined as the solution to:

$$V(\pi_{t+1}W, \mathbf{p}^{t+1}) = V(W, \mathbf{p}^t). \tag{1}$$

To proceed from this definition requires spelling out the model of behavior that is behind the indirect utility function.[2] Following the state of knowledge at his time, Konus (1924) considered a consumer who lives for one period in which she maximizes a utility function subject to a budget constraint and no uncertainty. Since his article, there has been a tremendous amount of progress using classical demand theory to construct price indices. Research characterized the circumstances under which the index is independent of wealth or utility, and those under which prices can be aggregated in stages through sub-indices. Researchers discovered the conditions for Laspeyres and Paasche price indices to provide

[1] This is not to say that there has not been an intense debate on which price indices should be used to guide monetary policy or measure real quantities. But this is not the topic of this paper.

[2] The price index is often defined using instead the expenditure function. Duality implies that it is equivalent to define it as in (1).

bounds on the cost-of-living index and proposed flexible functional forms to approximate the index for general utility functions. More recently, there has been a focus on incorporating changes in tastes and in the quality of the goods available to consumers. Some of these advances have influenced the way price indices are built today.[3]

However, the framework behind the economic price index is still the one suggested by Konus. Notably, it is still assumed that people maximize one-period utility with perfect certainty. The modern theory of consumption (see for instance Deaton, 1992) instead emphasizes dynamics and uncertainty by modeling people that maximize utility over many periods and face uncertainty. The aim of this paper is to bring this modern theory of consumption to shed light onto the old question of measuring the cost of living. The resulting price index is still defined by (1), but the underlying model of behavior is now dynamic and stochastic. For short, I call it the dynamic price index, or DPI.[4]

Why should we shift to using the DPI to measure the cost of living? There are at least three reasons. The first is simply, because we can. Konus (1924) did not have available at the time a model of consumption with dynamics and uncertainty. But over the past few decades, economists have developed a more realistic model of consumption that explicitly incorporates these features. The Konus insight was that given a model of consumption, one can construct a measure of the cost of living. Since we now have a better model of consumption, it is natural to consider the corresponding better measure of the cost of living.

The second reason is that taking dynamics into account unveils a new source of substitution bias that static price indices ignore. Consumers that live for more than one period react to higher prices today relative to the future by substituting away from present into future consumption. Only a dynamic model that takes intertemporal substitution into account can address this bias. The third reason is that, while price indices are typically used to compare two dates, it is theoretically awkward to do so when the implicit model of consumers' behavior assumes they live only for one period. These time comparisons follow naturally with the DPI.

[3] The International Labor Office (2004) outlines the procedures currently used in building consumer price indices, while the National Research Council (2002) is an excellent reference for the debates and progress in this literature. The Boskin commission (Boskin et al, 1997) makes a critical assessment of the CPI and a number of suggestions to make it closer to a cost-of-living index; see also Shapiro and Wilcox (1996) and the debate in the 1998 Winter issue of the Journal of Economic Perspectives.

[4] A more appropriate, though more lengthy, nomenclature might be DS-COL-CPI, for **D**ynamic **S**tochastic **C**ost-**o**f-**L**iving **C**onsumer **P**rice **I**ndex.

To make the DPI more concrete, this paper applies it to a specific question: consider retiring today with some amount in your retirement account that you will live off until you die. If instead you retire next year, when prices are different, how much must the account have so that you are indifferent as to when to retire? That is, by how much must the nominal amount in your account be adjusted so that you can afford the same standard of living after you retire regardless of what prices happen to be? While this question has been the focus of little research, its quantitative significance should not be understated. A large fraction of household wealth is held in retirement accounts. In the United States in 2001, 52% of the population had a retirement account, and 28% of the value of financial assets of families was in these accounts. Pension funds alone held $4.5 trillion in assets in 2004.[5] Current policy proposals for converting the U.S. social security system into a set of private retirement accounts would raise these numbers by even more.

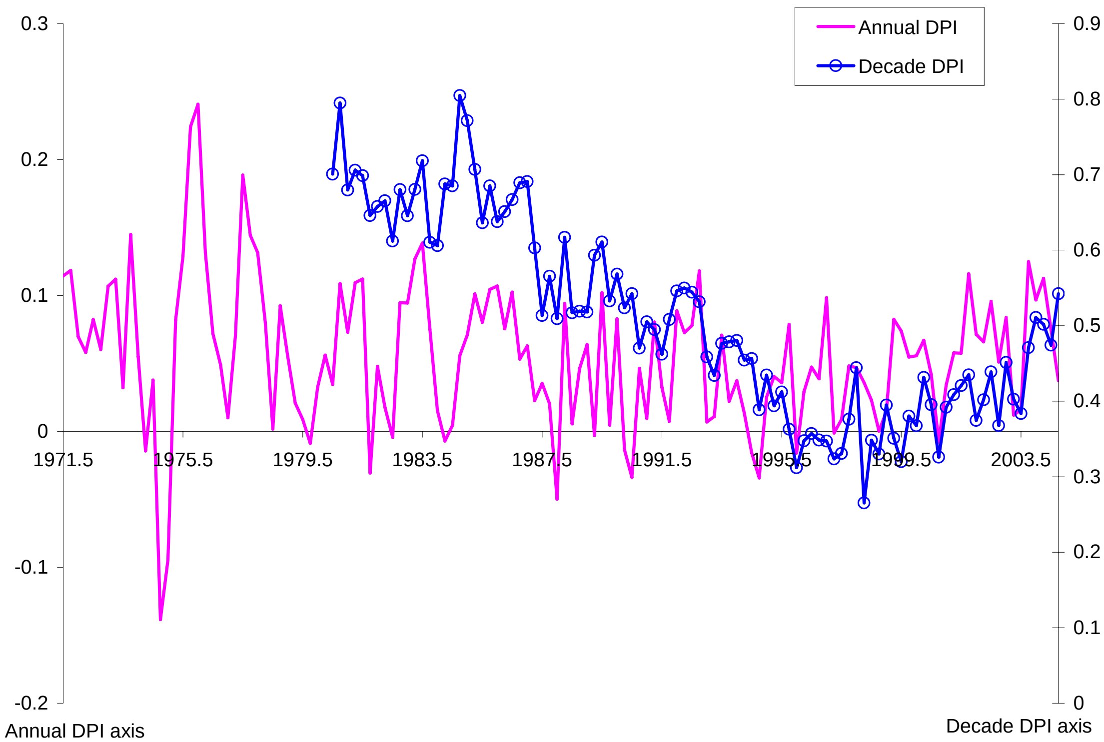
Moreover, a price index for retirement accounts could be given many other uses. A closely related one is to index the payments from disability insurance. Another one is to index the bequests given to children of different ages: if you have two children, one year apart, and choose to give each a lump-sum bequest when they turn 21 so as to leave both equally well off, then how much more or less should you give the younger one relative to the older? In the realm of public policy, there are scores of other applications in terms of indexing government fines and lump-sum transfers.[6]

Figure 1 gives away the main result of the paper in the form of a plot of the annual-DPI and the decade-DPI for retirement accounts in the U.S. between 1970 and 2004. The DPI gives a provocative new account of the cost of living during the last 30 years. According to it, the cost of living fell from the 1970s to the 1990s as in most conventional measures. However, this fall came with greater year-to-year volatility and less persistence than is commonly thought. Moreover, according to the DPI, during the 2000s, retirement accounts would have to grow at a considerably faster rate than during most of the 1990s in order to

[5] Sources: Survey of Consumer Finances, 2001, Tables 5.E and 4, and Flow of Funds, Tables L.119.b and L.119.c.

[6] How much can indexing matter? A curious incident provides an answer. When social security and disability benefits were first indexed to CPI-inflation in 1972, a mistake was made in the indexation formula. Among other factors, this contributed to make the social security system so generous that it quickly fell into disrepute. This led by the early 1980s to widespread reforms that scaled down the system, including an adjustment of the indexing formula (Bound and Burkhauser, 1999, pp. 3454-3456). Boskin and Jorgenson (1998) in turn calculate that if an estimated 1.1% upward-bias of the CPI due to quality change was corrected, the government savings on social programs indexed to the CPI would in a decade lower federal debt by $1,066 billion dollars.

Figure 1: The log annual and decade DPI for a household approaching retirement



leave the consumer's standard of living unchanged.

The remainder of this paper defines the DPI more carefully, explores its properties, and explains how figure 1 was constructed and what is behind its new account of the cost of living. There are many choices involved in constructing a price index, from which prices to consider to what preferences to assume, and this paper does not claim to deliver the definite price index. Its more modest aims are: to develop the theory for measuring the cost of living for a consumer that lives for many periods under uncertainty; to apply the theory to an important applied question that has been largely neglected; to take a first stab at building a dynamic price index; to identify the broad features of this index; and to give suggestions for future improvements.

Section 2 starts by describing the model of consumer behavior that I use and defining the DPI. Section 3 studies the theoretical properties of the price index. Section 4 describes the data that section 5 then uses to construct the DPI and interpret its movements. Section 6 looks at how the cost of living has changed for different groups of the population. Section 7 considers a few extensions to the basic model, and section 8 concludes.

A Brief Review of the Literature

There is a long and distinguished literature devoted to static Konus cost-of-living indices. Fisher and Shell (1972), Diewert and Montmarquette (1983), and Pollak (1989) offer book-length treatments, while Diewert (2001) is a comprehensive survey article. In the context of indexing retirement accounts, Jorgenson and Slesnick (1999) provide a thorough econometric application of the static approach.

The consideration of intertemporal trade-offs in the context of price indices was, to my knowledge, first articulated by Alchian and Klein (1973).[7] They proposed a definition of a price index that has many similarities with the one in this paper, but there are at least three key differences. First, they allow the consumer to trade a complete set of Arrow-Debreu securities whereas I consider the more realistic case where the consumer cannot fully insure against changes in the cost of living (section 7 discusses the role of insurance markets in the DPI). Second, their price index compares actual prices with an imaginary counterfactual while the DPI compares them with the previous period. Third, noting that the prices of the Arrow-Debreu futures contracts would enter the price index, Alchian and Klein (1973) proceeded to use several measures of asset prices in the estimation of money demand equations. This paper instead explores the theoretical properties of the price index and applies it to building an index for retirement accounts.

A closely related article by Pollak (1989, chapter 3) studies whether it is possible to form period sub-indices in an intertemporal context. Some of his results can be applied to the DPI; however, this paper focuses on a different set of questions. Shibuya (1992) and Wynne (1994) are the only studies that I am aware of that tried to build a price index taking dynamics and uncertainty into account. They used very restrictive assumptions though and can be seen as special cases of the more general results in this paper.

Other authors have informally defended the inclusion of asset prices in price indices. Goodhart (2001) persuasively argues that house prices should receive a special treatment. This paper provides a theoretical foundation to many of his comments. A recent literature instead assumes that the CPI is the correct welfare measure, but then suggests that asset prices may be used to forecast future CPI inflation. This paper instead suggests that regardless of their statistical relation with the CPI, asset prices enter directly the correct

[7] Irving Fisher (1906, 1910) was perhaps the first to propose price indices that include asset prices.

welfare measure, the DPI.[8]

Finally, note that this paper studies price indices that measure the cost of living. It does not address the impact of considering dynamics on the construction of adequate deflators or guides for monetary policy. Mankiw and Reis (2003) study the construction of inflation targets for central banks in economies with dynamic price rigidities. Using other dynamic models to inform the construction of price indices for purposes other than the cost of living is a likely fruitful avenue for future research.

2 The theoretical framework

2.1 The model of consumer behavior

Compactly written, the mathematical problem facing the consumer who retires at date t consists of choosing $\{C_{t+i}, S_{t+i}, B_{t+i}\}_{i=0}^{\infty}$ to maximize:

$$\mathbb{E}_t\left[\sum_{i=0}^{\infty}\beta^i\left(\sum_{j\epsilon ND}\alpha_j \ln\left(C_{j,t+i}\right)+\sum_{j\epsilon D}\alpha_j \ln\left(S_{j,t+i}\right)\right)\right] \tag{2}$$

subject to:

$$P_{t+i}^T C_{t+i} + R_{t+i}^T S_{t+i} + Q_{t+i}^T B_{t+i} \leq W_{t+i}, \tag{3}$$

$$W_{t+1+i} = D_{t+1+i}^T B_{t+i} + R_{t+1+i}^T \Delta S_{t+i}, \tag{4}$$

$$W_{t+1+i} \geq 0, \quad C_{t+i} \geq 0, \quad S_{t+i} \geq 0, \tag{5}$$

$$\text{for } i = 0, 1, 2, ..., \quad \text{ and } W_t = A_t. \tag{6}$$

In words, the consumer maximizes total utility which equals the expected discounted sum of period utilities. She faces a constant probability of dying, which combined with impatience, leads to a discount factor $\beta < 1$. Each period, she obtains utility from consuming non-durable goods, each denoted by $C_{j,t+i}$, and durable goods, $S_{j,t+i}$. Period utility takes a Cobb-Douglas form, with a set of taste weights α_j that sum to one across all goods.

The consumer allocates her wealth each period W_{t+i} to the uses in (3). She can acquire

[8] Cecchetti et al (2000) provide a good summary of the state of knowledge in this literature. Goodhart and Hofmann (2000) and Stock and Watson (2003) are useful references on the role of asset prices in forecasting inflation, while Bryan, Cecchetti and O'Sullivan (2001) use the prices of assets and other goods to statistically measure the common component in price increases.

non-durables, collected in the vector C_{t+i} at the price vector P_{t+i}, or invest in the durables in the vector S_{t+i} at the price vector R_{t+i}. She can also buy or sell two one-period financial assets: bonds which pay a certain amount next period, or stocks which have some random payoff next period. These assets trade at the price vector $Q_{t+i}^T = (Q_{B,t+i}\ Q_{E,t+i})$ and the portfolio holdings are collected in the vector B_{t+i}.

The consumer starts at the retirement date t with wealth equal to some exogenous amount in her retirement account A_t. After retirement, the sources of wealth are the payoffs from the financial assets plus the market value of the stock of durables after depreciation. The vector of payoffs is $D_{t+1+i}^T = (D_{B,t+i}\ Q_{E,t+1+i})$ where $D_{B,t+i}$ is the known payoff from bonds, so that the return from holding bonds and equity are respectively $D_{B,t+i}/Q_{B,t+i}$ and $Q_{E,t+1+i}/Q_{E,t+i}$. The diagonal matrix Δ has elements $1 - \delta_j$, where δ_j is the depreciation rate of durable j. Finally, the constraints in (5) impose that consumption cannot be negative and that the consumer cannot run Ponzi schemes, which in this case reduces to always having non-negative wealth.

The last component of the model to specify are the sources of uncertainty with respect to which expectations $\mathbb{E}_t[.]$ are formed. The aim of this paper is to calculate the appropriate index for the retirement account to insulate it from changes in prices. Therefore, I assume that prices are the only source of uncertainty: $\mathbf{p}_{t+i}$ is a random vector containing the three sets of positive prices, P_{t+i}, R_{t+i}, and Q_{t+i}, while tastes and the quality of goods (the α_j's and the δ_j's) are known. This assumption implies that the resulting index will be a *price* index, that responds to prices but nothing else.

I will further assume that $\mathbf{p}_{t+i}$ follows a Markov process so that if $\mathbf{p}^t = (\mathbf{p}_t, \mathbf{p}_{t-1}, \mathbf{p}_{t-2}, ...)$, it constitutes a sufficient statistic for forming expectations. This implies that at any date the consumer's wealth and this vector of past prices are all of the state variables that affect her plans. Evaluating total utility using the optimal behavior from $t+i$ onwards leads to a value function of the state variables: $V(W_{t+i}, \mathbf{p}^{t+i})$. It equals the consumer's expected life-long utility at date $t+i$ conditional on behaving optimally, and it defines what is commonly referred to as the standard of living.[9]

[9] For there to exist an optimal solution to the consumer problem leading to a finite value function requires some constraint on the stochastic process for prices. This is to ensure that, following a shock, prices do not go to zero too quickly driving consumption to infinity and so leading to unbounded utility. It is difficult to state these conditions for a general Markov process for all of the prices. Later, when I specialise to low-order Markov processes, I verify these conditions case by case.

2.2 Defining the appropriate dynamic price index

The dynamic price index is defined, following Konus (1924), as follows:

Definition: *The retirement account dynamic price index* π_{t+1} *is the scalar that solves:*

$$V(\pi_{t+1}A_t, \mathbf{p}^{t+1}) = V(A_t, \mathbf{p}^t). \tag{7}$$

That is, the price index measures how much more wealth the consumer needs if she retires at $t+1$ *rather than at* t*, so that she is equally well off given the prices at* $t+1$ *as she was facing the prices at* t*.*

Note that since the dynamic price index is defined taking as base the previous period, it corresponds to a measure of inflation. This is, of course, not essential. The base in the right-hand side of (7) could instead be with respect to a fixed date in time, leading to a measure of the price level.

Also, note that I consider the problem of a single consumer trying to compute an index that is important for her financial planning. All that is required therefore is a description of how the consumer behaves. The measures that I obtain may lead to other questions such as, who will provide this index in a market economy, or how might its adoption by many people affect the equilibrium of the economy. These further questions are not the emphasis of this paper, but section 7 offers some answers.

2.3 Basic properties of the DPI

One can establish some basic qualitative properties of the DPI.[10] To start:

Proposition 1 *If prices and wealth are positive and finite, the DPI exists and is unique.*

An important property of any price index is independence from the particular level of wealth at which it is computed. Since wealth differs widely across different people, without this property there is no hope of producing a price index that is broadly applicable. Samuelson and Swamy (1974) showed that for the static cost-of-living index, independence requires homotheticity of preferences. For the DPI, homotheticity of the period utility

[10] Proofs of this and other results are collected in the appendix.

function combined with time-separability ensures that total utility is homothetic. Since log preferences are homothetic, one obtains:

Proposition 2 *The DPI is independent of retirement wealth A_t.*

Another useful property of conventional price indices is that if all nominal prices and payoffs increase proportionally by the same amount, then so does the price index. The DPI shares this property since, letting M denote the common increase in all prices:

Proposition 3 *The DPI is proportional to M.*

A final question is: when does the DPI coincide with the static cost-of-living price index? Aside from the case $\beta \to 0$ in which the consumer cares only about present consumption, there is another case in which the two coincide:

Proposition 4 *If goods prices all follow random walks and financial asset returns are all i.i.d., up to a first-order approximation, the DPI equals the static cost-of-living price index.*

This particular case may justify looking at a price index such as the CPI as a rough indicator of the cost of living. The next section will show however that even slight departures from the conditions in this last proposition lead to stark differences between the static approach and the DPI in theory. And in practice, the CPI and the DPI are very different.

3 Theoretical properties of the price index

To better grasp the theoretical properties of the DPI, this section will start with a simpler version of the consumer problem and progressively build in ingredients towards the general problem stated in section 2. I start with a consumer that faces no uncertainty, only consumes non-durables, and has access to no financial assets aside from an annuity contract. I then proceed to allow for trade in financial assets and for uncertainty on prices. The next step is to add uncertainty on asset prices. The final step is to include durable goods.

3.1 Long lives and looking forward

To start off, consider a simpler version of the consumer problem in (2)-(5) in which the consumer faces no uncertainty and cannot trade resources over time. This will be the case if she has no access to durable goods or financial assets. In order for the consumer to have any resources after retirement, I assume that she can buy an annuity contract that converts her retirement account into a fixed stream of nominal income every period.

What is the impact of common trends in all prices on the price index? Letting P_{t+i} be the common component of prices, if $P_{t+i} \leq P_{t+i+1}$ for all i, then $\pi_{t+i+1} \geq 1$ for all i. That is, if prices are rising, the DPI will be above one. Moreover, if prices rise faster, then the DPI will be higher.[11] The DPI therefore shares with other familiar measures of inflation the property that it is high if prices are trending upwards.

However, even in this bare case in which the only link between dates comes from the consumer living for many periods, the DPI is not identical to a conventional static price index. Imagine that prices are the same at all dates with only one exception, h periods from now, when prices will be higher. The consumer today, realizing prices will rise, requires an increase in her nominal wealth to be able to afford these higher prices at the future date. The DPI will therefore be above one at date t, even though the increase in prices will actually only take place at date $t + h$. Because consumers are forward-looking, so is the price index. These results are collected in the following lesson:

Lesson 1: *The DPI is high if prices are trending upwards, and higher the steeper is this trend. The DPI is forward-looking reacting today to news about higher prices in the future.*

3.2 Non-durables goods prices and intertemporal trade

Next, I allow for intertemporal trade using bonds and equity, and investigate the impact of uncertainty on non-durable goods prices, in the case where each price equals the product of a common stochastic component and an idiosyncratic shock: $P_{j,t} = P_t \hat{P}_{j,t}$.

Starting with the common component, if it is i.i.d., then the only term involving P_{t+1} in the dynamic price index is: $\ln(\pi_{t+1}) = (1-\beta)\ln(P_{t+1}/P_t)$. A 1% increase in the goods price inflation raises the DPI by less than 1%. The reason is that, through financial assets, the consumer can smooth the temporarily high goods prices today by substituting consumption today for consumption tomorrow.

Empirically, goods prices are closer to a first-order autoregression in their log differences. If η is the non-negative coefficient on the autoregression, then: $\ln(\pi_{t+1}) = \ln(P_{t+1}/P_t)/(1-$

[11] Comparing two situations, A and B, distinguished by the fact that $(P_{t+i+1}/P_{t+i})^A \geq (P_{t+i+1}/P_{t+i})^B$ for all i, then $\pi^A_{t+i+1} > \pi^B_{t+i+1}$.

$\beta\eta)$. If $\eta = 0$, so prices follow a random walk, the DPI increases one-to-one with increases in prices. The reason is that with random walk prices, all shocks are permanent so there is no scope for the intertemporal substitution described in the previous paragraph. But if η is positive, a 1% increase in goods price inflation raises the DPI by more than 1%. The reason is that higher prices today now imply that the consumer should also expect that prices increase further by $(1+\eta)^h$ at each horizon h. The forward-looking consumer perceives this and thus requires a larger increase in wealth today in order not to be worse off. The larger is the persistence of shocks, the larger their impact on the DPI.

Considering next the idiosyncratic shocks, assume that each is i.i.d. Then, the relative marginal impact of shocks to two non-durable prices equals their relative weights in the utility function: $(\partial \ln(\pi_t)/\partial \ln(\hat{P}_{j,t}))/(\partial \ln(\pi_t)/\partial \ln(\hat{P}_{k,t})) = \alpha_j/\alpha_k$. It is also easy to show that this ratio equals the ratio of relative expenditures in the two goods. Intuitively, if the consumer cares more about a good and allocates a larger amount of spending to this good, then an increase in its price affects her cost of living by more.

Lesson 2: *Because consumers are forward-looking and intertemporally substitute consumption, the larger is the persistence of shocks to prices, the larger is their impact on the DPI. The relative weight of different goods depends on their relative shares in expenditure.*

3.3 Asset prices

One crucial difference between the DPI and a static price index concerns the role of asset prices. Once one takes an intertemporal perspective, it is evident that because asset prices are nothing more than the relative price of consumption in the future relative to the present, they must enter a dynamic price index. The relevant consumption bundle consists not only of consumption of two different goods today, but also of total consumption today and tomorrow. If today's prices of two goods enter the price index, then so must prices today and tomorrow. Asset prices give the relevant prices for the future.

To assess the quantitative significance of asset prices in the DPI, separate each asset price into a common and an idiosyncratic component: $Q_{B,t+i} = Q_{t+i}\hat{Q}_{B,t+i}$ and $Q_{E,t+i} = Q_{t+i}\hat{Q}_{E,t+i}$. Then, if the common component is i.i.d., the DPI depends on it through the term: $\ln(\pi_{t+1}) = \beta \ln(Q_{t+1}/Q_t)$. Higher asset prices raise the DPI because they make it more costly to transfer funds for future consumption. Moreover, recalling that a 1% increase in i.i.d. non-durables prices raised the DPI by $(1-\beta)$%, note that for plausible values of the discount factor, asset prices can receive a substantially larger weight than consumer prices in the DPI.

Still for the i.i.d. case, the impact of changes in the idiosyncratic components of the two asset prices is described by: $(\partial \ln(\pi_t)/\partial \ln(\hat{Q}_{E,t}))/(\partial \ln(\pi_t)/\partial ln(\hat{Q}_{B,t})) = \hat{Q}_{E,t}B_{E,t}/\hat{Q}_{B,t}B_{B,t}$. The relative marginal impact of a change in the price of each of the assets is proportional to their relative weights in the consumer's portfolio. The more of a specific asset the consumer chooses to hold, the more affected she will be by changes in its price.

As with non-durables though, the stochastic properties of asset prices play a key role. One important special case regards equity prices. If equity prices follow a random walk (which is a rough description of the actual data), then: $(\partial \ln(\pi_{t+1})/\partial \ln(\hat{Q}_{E,t+1})) = 0$. Changes in random walk equity prices have no impact on the DPI. The reason is that, in this case, higher equity prices today have no implication for current and expected future returns. Thus, no relative prices change and neither does welfare or the cost of living.

Lesson 3: *Asset prices generally enter the DPI and with a potentially large weight, with the exception of the special case when equity prices follow a random walk. The price of each asset receives a weight proportional to its portfolio share.*

3.4 Durable goods

Finally, I introduce durable goods. These goods are particularly interesting, because they combine features of both goods and assets: they yield utility and they also transfer wealth across time. For simplicity, I assume that the log of each price follows an AR(1) and use I^M_{t+1} to denote the return from holding the optimal portfolio of financial assets.

An important quantity is the cost of using a durable j between between t and $t+1$:

$$u_{j,t+1} = R_{j,t} - (1-\delta)R_{j,t+1}/I^M_{t+1}. \tag{8}$$

Holding the durable for one period requires paying $R_{j,t}$ for it at date t and then selling the remainder after depreciation for $R_{j,t+1}$ at date $t+1$, noting that the opportunity cost of investing a $t+1$ dollar in durables is $1/I^M_{t+1}$ dollars at date t. Thus, $u_{j,t+1}$ is the user cost of holding durable j between t and $t+1$.

The relative marginal impact of a change in durable j's price relative to a change in the price of non-durable i evaluated at the non-stochastic steady state equals:

$$\frac{\partial \ln(\pi_t)/\partial \ln(R_{j,t})}{\partial \ln(\pi_t)/\partial \ln(P_{i,t})} = \frac{\alpha_j}{\alpha_i} \times \left(\frac{1 - \beta \partial \ln(P_{i,t+1})/\partial \ln(P_{i,t})}{1 - \beta \partial \ln(R_{j,t+1})/\partial \ln(R_{j,t})} \right) \times \left(\frac{\partial u_{j,t+1}}{\partial R_{j,t}} \times \frac{R_{j,t}}{u_{j,t+1}} \right). \quad (9)$$

The first fraction in the expression captures the effect of tastes that was already discussed in section 3.2. The second fraction shows that the more persistent are shocks to the price of a good, the larger is its impact on the DPI. The reasons behind this were also discussed in section 3.2. If user costs are always proportional to prices, these are the only two effects. In this case, durability is irrelevant: whether the good is durable or not, it has the same weight on the DPI.

User costs will be proportional to prices if the log of the price of the durable follows a random walk. If, however, shocks are transitory, then user costs rise by more than 1% in response to a 1% rise in prices. A higher durable good's price then hurts the consumer in two ways: first because it raises the current price paid for the good, and second because the consumer expects a capital loss on holding it since the price is expected to fall. In this case, durable goods have a larger weight on the DPI, and one that increases with $1 - \delta$, the durability of the good.

In the extreme case in which the price of the durable is i.i.d., its impact on the DPI can be much larger than that of a non-durable. If the return on bonds and durables are approximately the same ($R_{j,t+1}/R_{j,t} \approx I^M_{t+1}$), the last fraction in (9) equals $1/\delta$. For a very durable good such as housing, for which $\delta \approx 0.01$, a temporary increase in its price raises the DPI by about 100 times more than a comparable increase in a non-durable price, and so can have a massive impact on the consumer's cost-of-living.

Alternatively, if shocks to durable prices are very permanent, in the sense that a 1% increase in $R_{j,t}$ comes with an expected increase in $R_{j,t+1}$ of more than 1%, then even though the consumer is hurt by paying more for the good, she benefits from the expected capital gains on it. The user cost of the durable falls, and so the change in its price has a smaller impact on the cost of living.[12]

Lesson 4: *The impact of durable prices on the DPI increases with their weight in period*

[12] One of the most important consumer durable goods is housing. Bajari, Benkard and Krainer (2004) study the impact of a change in house prices on welfare and provide some related results to the ones in this section. They did not consider uncertainty or the persistence of shocks however. Diewert (2003) provides a thorough review of the current state of knowledge on how to include house prices in price indices.

utility and with the persistence of shocks. Transitory shocks to a durable good's price have a larger impact on the DPI than shocks to a comparable non-durable because they raise current expenditures and also imply an anticipated capital loss. The more durable the good is, the larger its weight in the DPI. If instead an increase in the durable good's price leads to anticipating a capital gain on it, its impact on the DPI is smaller.

4 Data and methods for constructing the DPI

The empirical effort in this paper is limited by data constraints and the desire to provide a transparent first pass at the problem. I will consider only broad categories of goods and assets for which there are more reliable time-series.[13] This section explains the sources of these data. I will focus on the sub-set of households aged between 55 and 64, for whom indexing their retirement accounts is a more salient concern. Section 6 will consider different groups of the population.

4.1 Non-durables prices and consumption

The set of non-durable goods is the one included in the CPI: food, energy, services (without shelter or energy), and other non-durables. The price series come from the Bureau of Labor Statistics (BLS) CPI database. They cover the period from the start of 1960 to the end of 2004 at the quarterly frequency and are seasonally adjusted.[14] The growth rate of prices are plotted in figure 2.

For non-durables, the relative taste weights α_j equal the relative shares in the household's expenditure. I measure these using the relative importance of each good in the last revision of the CPI. These weights are allowed to vary deterministically over time to take into account the main trends in consumption in the last 50 years. For instance, the weight of shelter plus services in the consumer basket has risen from 39% in 1964 to 56% in 2004. The BLS has revised these weights six times since 1964. Table 1 shows these revisions.[15]

[13] There are many interesting details on goods prices that this paper will not consider. How to account for taxes is an important and difficult one. Triplett (1983) provides a lucid discussion of this and related issues.

[14] For the series for which the BLS does not report seasonally adjusted counterparts, I computed these using the Census Bureau's X-11 procedure. Comparing the available seasonally-adjusted series from the BLS with counterfactuals using this procedure produces correlation coefficients larger than 0.99.

[15] These expenditure shares refer to the average U.S. household, but the baseline DPI refers to the 55-64 age group. From 1998 onwards, the two are quite similar, and the differences have a negligible impact on the price index. Before 1998, it is difficult to find the expenditure shares for the 55-64 group.

Table 1. Relative weights of different components in the CPI

CPI weights	Food	Energy	Services	Other non-durables	Shelter	Other durables
1964-1977	0.224	0.067	0.185	0.210	0.202	0.115
1978-1982	0.177	0.086	0.190	0.131	0.291	0.124
1983-1986	0.190	0.124	0.213	0.131	0.213	0.128
1987-1997	0.162	0.074	0.229	0.139	0.277	0.120
1998-2001	0.153	0.070	0.238	0.125	0.298	0.116
2002-2003	0.147	0.062	0.237	0.118	0.315	0.121
2004	0.144	0.071	0.234	0.110	0.329	0.113

4.2 Durables prices and consumption

Durables comprise two goods: shelter and other durables. Combined with the four non-durables, these 6 categories include 100% of the goods in the CPI. The price series for other durables is that reported by the BLS.

For housing prices, the BLS shelter series suffers from many deficiencies. First, it includes both rents and house prices. There is ample evidence that these two prices are almost uncorrelated and that they are not even linked the way theory would predict.[16] Second, the shelter series in the BLS has an important break in 1983. Before that date, house prices were measured using the prices reported in recent sales. Afterwards, the BLS has been computing for homeowners a "rental-equivalent price," matching their house with similar rented ones and measuring the change in these rents as the relevant price variation. The shelter series before and after 1983 are, strictly speaking, non-comparable. Third and finally, the shelter series collected by the BLS seems to be downward biased at least over long horizons. Gordon and van Goethem (2004) note that according to this series, shelter is 5.1 times more expensive in 1999 than it was in 1925. Yet the asking price for single-family homes in Washington, DC is 22.5 times higher than it used to be.

In the baseline case, I will focus on home-owners. In the U.S. in 2001, 82.3% of households whose head was between 55 and 64 years old owned their primary residence.[17] To measure house prices, I will use the Conventional Mortgage House Price Index produced by

[16] See Gillingham (1983) or Verbrugge (2005).

[17] Source: Survey of Consumer Finances, 2001.

Freddie Mac. This contains quarterly data from 1970:1 to 2004:4 on house prices weighted and measured by repeat sales so as to control for the quality of the home.[18] An alternative index is the House Price Index produced by the Office of Federal Housing Enterprise Oversight. It is only available however from the first quarter of 1975. For the common sample, the two price series move very closely together: the correlation between them is 0.999 for their levels and 0.930 for their growth rates. I use the index from Freddie Mac because of its longer sample.[19]

To calibrate the four parameters, α_5, α_6, δ_5, and δ_6, which correspond to the tastes for the two durables and to their depreciation rates, I use four moments. The first two come from the Survey of Consumer Finances. They are the share of wealth held in durables relative to financial assets, and the amount of wealth in real estate relative to the wealth in other consumer durables. I use the averages from the 1989, 1992, 1995, 1998, and 2001 surveys for the households with a head aged between 55 and 64. The numbers are 0.86 for the ratio of durable to financial wealth, and 9.46 for the ratio of housing to other durables. The other two moments come from the last two columns of table 1, and they are the expenditure shares in housing and other durables. Using these four moments, the implied taste parameters are 2.2 times higher for housing and 1.5 times higher for other durables than the expenditure shares in table 1. The implied quarterly depreciation for housing is 0.6%. For other durables, it is 2.7%.[20]

4.3 Asset prices

I measure equity returns using a broad stock market fund: the value-weighted index of stocks in the NYSE, AMEX and NASDAQ compiled by the Center for Research in Security Prices. Bond returns refer to the yield on 3-month Treasury bills. Figure 3 plots both returns.

The portfolio shares of equity and bonds are set to match those of households in the

[18] See Case and Shiller (1989) for a discussion of the importance of this method in measuring house prices.

[19] The Census Bureau produces another house price series, with the advantage that it covers most of the post-war period. However, it refers only to new houses and it does not adjust perfectly for quality.

[20] For comparison, I computed these depreciations also using the Fixed Assets Table from the Bureau of Economic Analysis. They are 1.6% for housing and 21.1% for consumer durables, but at an annual rate. The wealth holdings in the Survey of Consumer Finances therefore imply that the depreciations for housing and other durables are about 3/2 and 1/2 respectively from those in the Fixed Assets table. I suspect that much of the discrepancy is due to the fact that some of the real estate declared in the Survey of Consumer Finances is used for non-residential purposes, and that the definition of durables in the two surveys differs.

Figure 2: Growth rates of goods prices 1970-2004

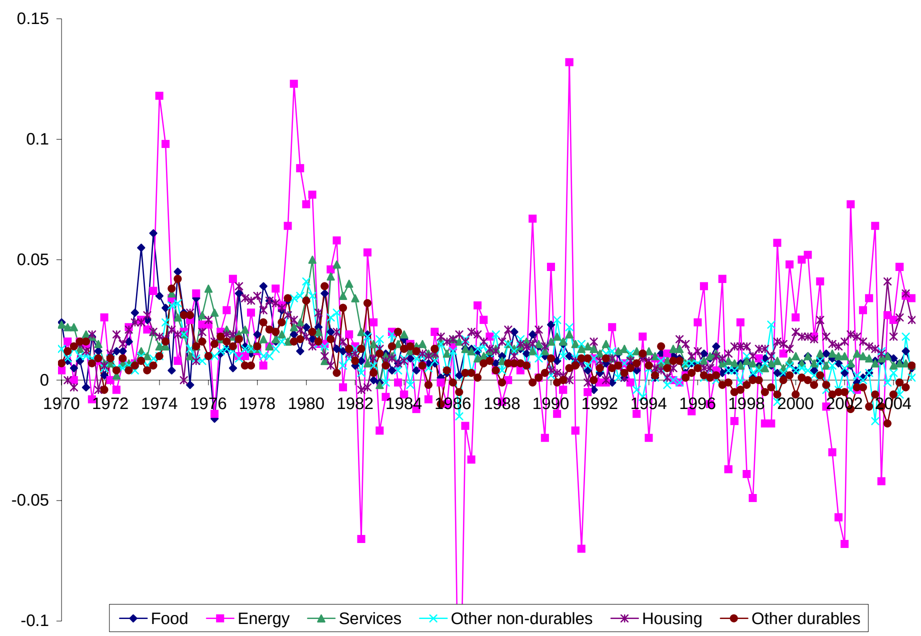
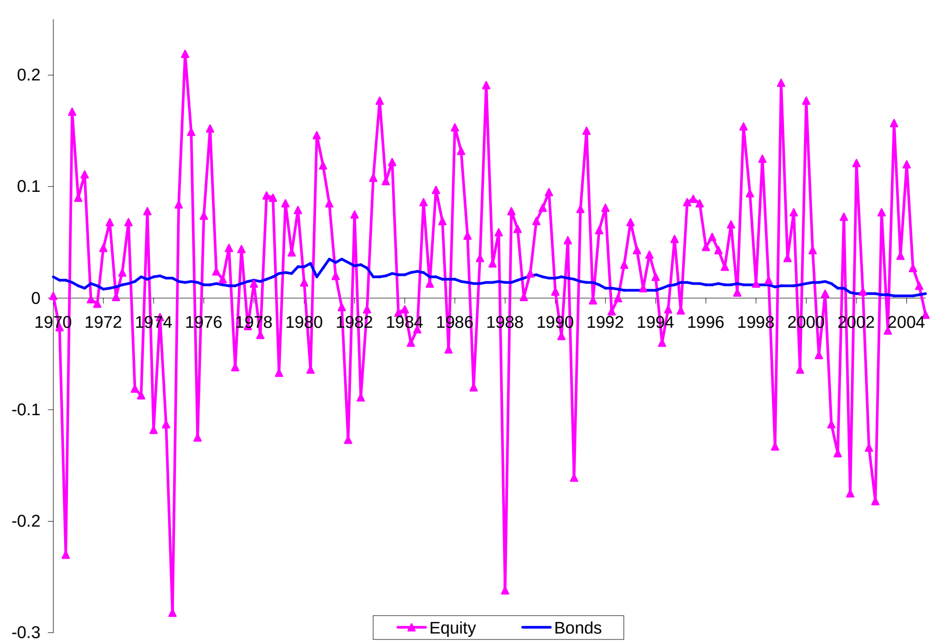


Figure 3: Asset returns 1970-2004



Survey of Consumer Finances. Subtracting liabilities from the holding of bonds (direct and indirect through pension funds) and adding certificates of deposits and transaction accounts, households with a head between 55 and 64 years old held on average 75% of their financial wealth in equity.

Finally, the discount factor is set at 0.99 to match the real return on the portfolio of financial assets so that steady state real wealth is stable.

4.4 Forecasting and solving the model

Forecasting the future of even only 8 prices using data from the first quarter of 1970 to the last quarter of 2004 is a daunting challenge. I go about doing this by using the assumption that the prices are finite-order Markov processes to estimate several vector autoregressive models (VARs) on the logs of the prices.

There is very strong statistical evidence that all price series are non-stationary according to standard unit root tests, so I first-difference the data. I estimate a VAR of order 2, since this leads to the lowest forecast prediction error and value of the Akaike information criteria. This is the most parsimonious statistical model that still performs reasonably at forecasting.[21] Section 6 will consider alternatives.

Computing the DPI in (7) requires solving the consumer problem in (2)-(5). If the Markov process followed by the vector of all prices is of order k, the size of the state vector for this problem is $1 + 8k$. Even for conservative choices of k, this number is too large to allow the use of numerical projection methods. One is left with perturbation methods, and the appendix takes this route to solve for a log-linear approximation for the DPI. One could of course also consider higher-order terms, but there are enough interesting first-order features of the DPI to study for now.

5 The U.S. DPI for households approaching retirement

Figure 4 plots the annual log price index using the parameters for households approaching retirement and the forecasting model described in the previous section. While the observations are quarterly, I focus on the annual DPI, which at date t equals $\sum_{i=0}^{3} \ln(\pi_{t-i})$, since this is easier to interpret. It corresponds to the cumulative percentage change in her

[21] To give an impression of the model's performance, the average adjusted R^2 is 0.45.

retirement account that the consumer would require during the preceding year to keep her standard of living unchanged.

In the figure is also plotted the annual CPI and the annual returns on bonds. Bond returns provide an interesting comparison because many financial institutions advise those approaching retirement to hold an account portfolio heavily composed of bonds, effectively indexing the amount in the account to their return.

The DPI is clearly strikingly different from either CPI inflation or bond returns. The correlation between the DPI and these two alternatives is -0.06 and -0.10 respectively. Part of the difference comes from the higher volatility of the DPI. While the standard deviation of CPI inflation is 2.9% and that of bond returns is 2.6%, the standard deviation of the DPI is 5.4%. Another part of the difference comes from the fact that while CPI inflation and bond returns are very persistent, the DPI exhibits many transitory movements with sharp rises followed by quick reversals. Consequently, while the serial correlation coefficients of CPI inflation and bond returns are respectively 0.97 and 0.99, the DPI's is only 0.43.

Despite these differences, the trends in all series are similar. While it may be difficult to see this in figure 4, it is clear in figure 5, which displays at each date the accumulated DPI, CPI-inflation, and bond returns over a decade. It is clear that the series now move closer together. For instance, the correlation between the DPI and CPI inflation is now 0.86. This rough similarity must not be taken too far however. There are important differences even over long horizons, the most striking of which during the 2000s. Whereas a retirement account indexed to the CPI between 1994:4 and 2004:4 would grow by 25%, one indexed to the DPI would grow by 54%. A retired household with an account indexed to the DPI would be substantially wealthier and better off than a household with an account indexed to the CPI.

Focussing instead on individual dates, there is an unusual fall in the DPI in 1974:4. The reason behind it is an unusual event: the largest stock market drop in a quarter in the sample. Equity prices fell by 28%, so consumers were able to buy future consumption with their retirement account at a discount price. Thus, the DPI fell. This incident was immediately followed by a run up in the DPI peaking in the first quarter of 1976. Not only were most goods prices rising (see figure 3) but also equity prices quickly recovered and were accompanied by also high bond prices. The DPI peaked again shortly after in late 1977. Behind this third episode was the second largest run up in house prices in the sample

Figure 4: The annual log DPI, CPI inflation and bond returns

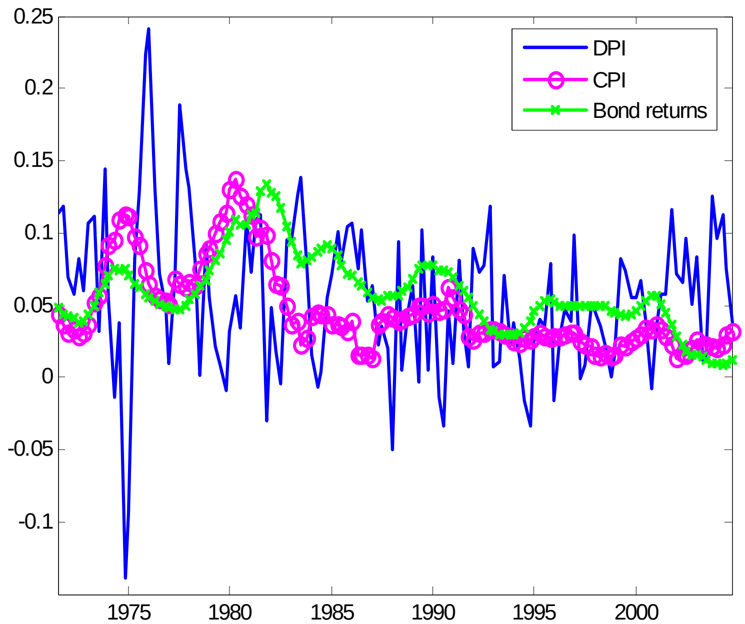
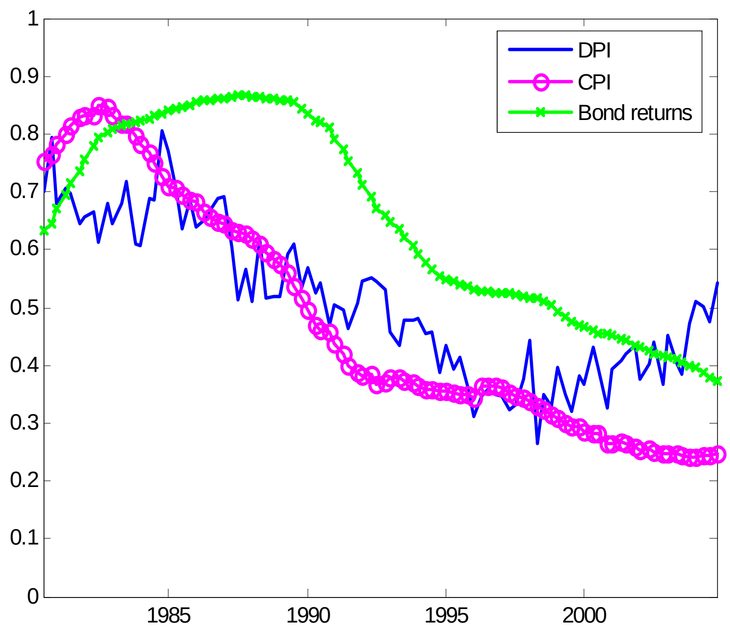


Figure 5: The decade log DPI, CPI inflation, and bond returns



period, which in 1977 alone increased by 12.5%. The final notable period is the high DPI since 2000 that was already mentioned. Behind this are two proximate causes: the largest increase in house prices in the sample and high bond prices.

Table 2 provides a more systematic analysis of the factors driving the DPI. The first row displays the standard deviation of changes in the log of each price. Two prices stand out for being more volatile than most others: energy prices and especially equity prices. Some of the volatility of the DPI is due to including these very volatile equity prices.

The next row in the table shows the impact on the DPI of changes in prices for the particular case in which each price follows an independent random walk. The assumption of independence allows one to associate changes in the price with shocks to that price. The assumption of a random walk leads to the case covered in Proposition 4, in which each good has an impact which matches its weight on consumer tastes.

Table 2. The volatility of the components of the DPI and their average impact

	Food	Energy	Services	Other non-durables
Volatility (standard deviation)	0.011	0.037	0.009	0.009
Marginal impact if random walk	0.132	0.057	0.155	0.111
Persistence (AR(1) coefficient)	0.360	0.258	0.636	0.437
Marginal impact if AR(1)	0.205	0.077	0.420	0.196
Marginal impact with VAR(2)	0.757	0.194	1.990	-0.418
	Housing	**Other durables**	**Equity**	**Bonds**
Volatility (standard deviation)	0.009	0.011	0.091	0.007
Marginal impact if random walk	0.412	0.132	0	0
Persistence (AR(1) coefficient)	0.608	0.663	0.020	0.954
Marginal impact if AR(1)	0.549	0.322	0.010	2.756
Marginal impact with VAR(2)	1.901	-0.823	-0.045	2.454

The third row reports the coefficient on an AR(1) regression for each price. Noticeably, equity returns are close to being serially uncorrelated, and energy prices are also little persistent. Consequently, despite being volatile, changes in these prices have a small impact on the DPI. Row four shows the marginal impact of shocks to each price using the independent

AR(1)'s as the forecasting model. Because most prices are quite persistent, the sum of their impacts on the DPI is well above one. This is the main reason behind the DPI's volatility. House price and bond price changes in particular are very serially correlated and thus have a large impact on the DPI.

Finally, the last row shows the impact of a 1% change in each price on the DPI with the baseline VAR(2) forecasting model. These numbers can no longer be interpreted as the response to shocks to individual prices, since they are not identified in the reduced-form VAR.[22] What they show is that if one observes in a period that house and bond prices have increased, then this comes on average with an increase in the DPI.

These different pieces of evidence paint the following characterization of the optimal price index for retirement accounts. It is volatile, because price changes are typically followed by further price increases.[23] Occasional sharp movements in equity prices may affect the DPI significantly, but on average equity prices have a small impact on the DPI because returns are very close to being serially uncorrelated.[24] Housing has an important role, with changes in its user cost accounting for much of the movements in the DPI. Finally, aside from housing, bond prices are the other main driving force in the DPI for two reasons: first, because changes in bond prices tend to be very persistent, and second because they affect the DPI both directly and indirectly through the user cost of housing.

6 Heterogeneity and the DPI

The price index in the previous section was intended for the average U.S. household approaching retirement. The prices used were those that this household would likely encounter and the parameters were calibrated to match its behavior. This section instead computes price indices for different segments of the population.

First, I consider how an average U.S. household differs from an average household ap-

[22] To calculate the impact of news on each component of the DPI on the index would require identifying structural shocks. In a VAR with 8 time-series, this would require 28 identifying restrictions. Since there aren't even a handful of possibly valid restrictions, this is virtually impossible.

[23] It is customary in building price indices to normalize the weights of each component so that they sum to one. Doing this for the DPI results in a price index that tracks very closely the movements in the DPI, but is more stable. However, this normalized price index can no longer be interpreted as the percentage adjustment in the retirement account.

[24] Goodhart (2001) and Bryan, Cecchetti and O'Sullivan (2001) worried that any price index that included equity prices would be very volatile. The DPI dispels this worry: it is more volatile than the CPI (though not to the point of being useless) but this is not mostly driven by equity prices, but rather by the persistence of other price shocks.

proaching retirement. The data from the Survey of Consumer Finances indicates that the average household holds more wealth in durables relative to financial assets (a ratio of 1.21) and that, among durables, housing is a smaller component relative to other durables (a ratio of 7.97). Moreover, the share of equity in their portfolio is higher, 0.92, mostly because they hold more debt. Using these three moments to calibrate the parameters produces the price index in the first panel of figure 6. It is almost identical to the price index for households approaching retirement.

Second, I measure goods prices using the components of the deflator for personal consumption expenditures (PCE), provided by the Bureau of Economic Analysis. The PCE deflator's expenditure shares are updated more frequently than those in the CPI and it is revised backwards with GDP revisions, so its series are consistent over time. The second panel of figure 6 displays the DPI using these goods prices and using also the expenditure shares for the PCE to calibrate the taste parameters. The resulting DPI is again closely associated with the baseline one.

Third, I consider different forecasting models. Unit root tests reveal some mixed evidence that price changes may also contain unit roots. If so, one might expect that these series share at least one common trend due to changes in the supply of money. Johansen's trace test of the number of cointegrating relations lends some support (at the 5% significance level) to this hypothesis, by finding that there are 7 cointegrating relations between the 8 price series. Following Stock and Watson (1988), this would imply that the series share one common trend. I measure this trend using CPI inflation, and idiosyncratic price changes using a VAR(2). This model of forecasting leads to the price index in the third panel of figure 6. A consumer that used this model to forecast future price changes would wish to index her retirement account to a price index that typically moves together with the baseline index.

Fourth, I consider another forecasting model: a Bayesian VAR with a Minnesota prior. Relative to a standard VAR, this procedure shrinks the coefficients that are imprecisely estimated towards a random walk for each price. Contrary to the previous models, and consistent with a Bayesian perspective, the VAR is estimated directly on the levels rather than the first differences of the series. The results are in the fourth panel of figure 6 and again provide a similar account of the cost of living.

The next four cases that I consider lead to more substantial changes in the price index.

The fifth scenario involves comparing homeowners to renters. For the latter, housing is now a non-durable. To measure its price, I use the series on "rent of primary residence" produced by the BLS. Also, different moments describe these households. Because they do not own a home, the renters' ratio of housing to durable wealth is zero versus 9.41 for homeowners. Their durable wealth is smaller (0.26 of their financial wealth against 0.96 for owners) and because they hold less debt, their portfolio has a smaller equity share (0.69 versus 0.95 for owners). The price indices for these two groups are in the first panel of figure 7. The two are quite different and even negatively correlated. This confirms the conclusion in the previous section that changes in the user costs of homeowners accounted for a large portion of the movements in the price index. Note also that starting in 2000, the price index for owners has been above that for renters, since house prices have risen relative to rents.

Sixth, I compare those that hold some equity with those that do not. According to the 2001 Survey of Consumer Finances, only 52% of U.S. families hold any equity, either directly or indirectly (and 57% of the households with a head aged 55 to 64). The second panel of figure 7 shows the price index for those households that do not hold equity. It and the baseline DPI are positively, but only weakly, correlated. On the one hand, households that are not exposed to equity are not affected by the occasional sharp movements in stock market prices. On the other hand, changes in the price of bonds play an even stronger role in their price index.

The seventh scenario I consider is that of households that discount the future more heavily. Among those approaching retirement, there are likely great differences in their health and life expectancy, which will lead them to discount the future differently. It could also be that some households are more impatient or are less altruistic towards their offspring. I consider the case in which the consumer has a quarterly discount rate of 2.5% as an alternative to the baseline 1%. The third panel of figure 7 shows that the price index for high-discounters is only weakly correlated with that for low-discounters. The reason is that those that discount the future by more, save less, and thus are less exposed to changes in the prices of financial assets and durables, but are more sensitive to changes in non-durable goods prices.

Eighth and finally, the last panel in figure 7 presents a compensation series associated with the DPI. The DPI measures by how much people would want to see their retirement account change so that by retiring today they are equally well off as they were one year ago.

Figure 6: The DPI for different groups: Small differences

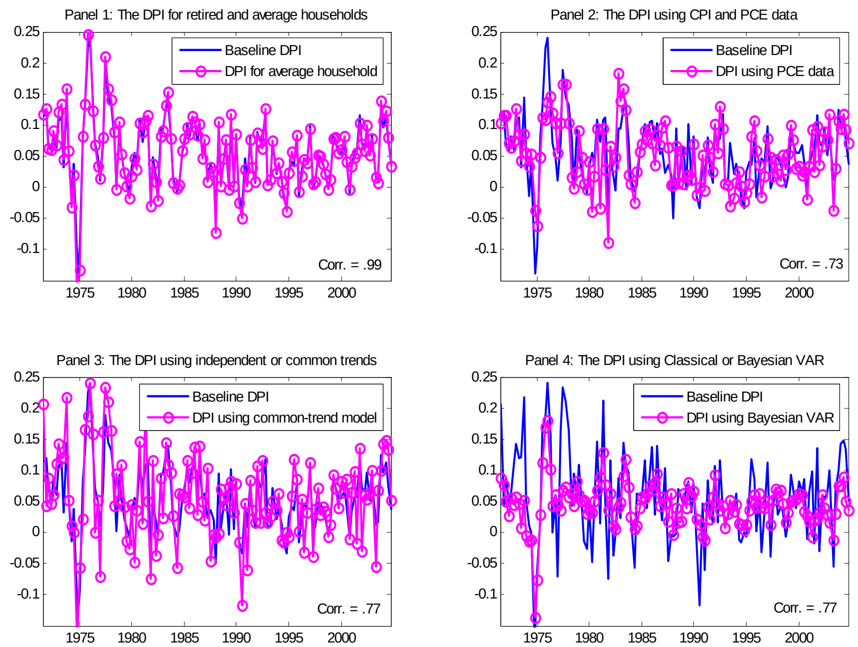
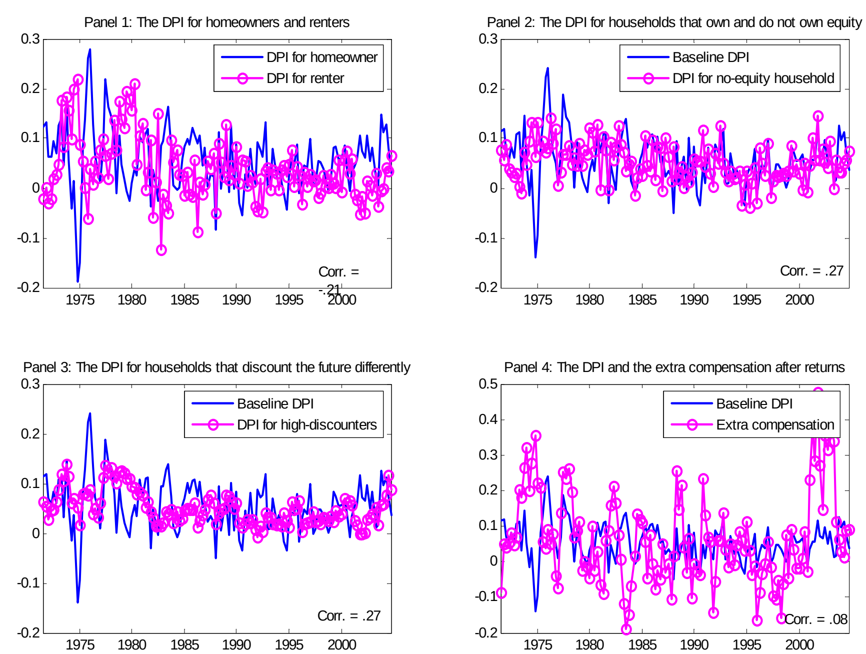


Figure 7: The DPI for different groups: Large differences



Part of that change in wealth might come through returns on the retirement account during this year, part from the wages earned from working an extra year, part perhaps through explicit insurance contracts, and a part probably never takes place. The first of these components can be measured directly using the baseline model. The last panel in figure 7 displays only the excess compensation beyond what is already earned on investments that the consumer requires to her retirement account. The movements in this measure are somewhat more extreme than those in the DPI. The reason is that high asset prices now not only raise the DPI but also come with lower expected returns.

7 Further questions about the DPI

7.1 The elasticity of intertemporal substitution and risk aversion

So far, the analysis has kept both the elasticity of intertemporal substitution and relative risk aversion fixed at 1. Given the key role that intertemporal substitution and uncertainty about the future play in the DPI, it is interesting to investigate alternative parameters.

For this purpose, I focus on the problem of a consumer of solely one non-durable good that can transfer funds over time using equity, which has an expected log return equal to the rate of time preference. I further assume that equity prices follow a random walk with innovations ε_t^Q, while the good's price follows an AR(1) in first differences: $\Delta \ln(P_t) = \eta \Delta \ln(P_{t-1}) + \varepsilon_t^P$, two rough descriptions of the U.S. data. Period preferences are now defined recursively as in Epstein and Zin (1991):

$$U_t = \left\{ (1-\beta) C_t^{1-1/\theta} + \beta \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-1/\theta}{1-\sigma}} \right\}^{1/(1-1/\theta)}, \tag{10}$$

where θ is the intertemporal elasticity of substitution and γ is the coefficient of relative risk aversion.

In this case, the DPI approximately equals:

$$\ln(\pi_t) \approx \frac{\ln(P_t/P_{t-1})}{1-\rho\eta} - \frac{\rho\eta \ln(P_{t-1}/P_{t-2})}{1-\rho\eta} \text{ with:} \tag{11}$$

$$\ln(\rho) = 0.5\,(1-\theta)\,\gamma Var(\varepsilon_t^Q + \varepsilon_t^P) + \ln(\beta). \tag{12}$$

If the intertemporal elasticity of substitution equals 1 (and regardless of risk aversion), we get the same solution as in section 3.2. That is, a 1% increase in goods prices raises the DPI by $1/(1-\beta\eta)$. Likewise, if goods prices follow a random walk (and regardless of θ and γ), proposition 4 still holds. Since in this case there is no scope for intertemporal substitution in response to shocks, a 1% increase in goods prices raises the DPI by exactly 1%.

Expression (12) reveals that the more willing the consumer is to substitute consumption across time, the smaller is the impact of price shocks on the DPI. Intuitively, this consumer will accommodate the shocks by more by changing the timing of her consumption so her effective cost of living changes by less. Almost all empirical studies have found that θ is smaller than one. In this case, an increase in γ raises the impact of price shocks on the DPI. A more risk-averse consumer is hurt by more when bad price shocks occur.

Most estimates of θ put it somewhere between 0.1 and 0.4, while reasonable estimates of γ are typically between 1 and 5. For the extreme cases of $\theta = 0$ and $\gamma = 10$, according to (11) the DPI in the previous sections that used the assumptions $\theta = \gamma = 1$ would have underestimated the impact of shocks on the DPI by only about 12%.[25]

7.2 Labor income and non-price uncertainty

This paper has focussed on constructing the DPI for indexing retirement accounts. By the nature of retiring, the consumer earned no labor income. However, one might want to use the DPI for other purposes for which considering labor income is important.

If the present value of labor income when discounted at the bond interest rates is deterministic, then the analysis is unchanged. As Merton (1971) showed, in this case we need only interpret initial wealth A_t as containing also the present value of labor income.

Otherwise, one must choose whether variations in labor income are seen as price (wage) variations that the price index should react to, or rather as a distinct set of shocks that the price index should not include. In the first case, interpreting one of the non-durable goods $C_{j,t}$ as leisure, one can adapt the previous analysis to accommodate wage variations. In the second case, we are in the more general problem of how to handle non-price uncertainty. If z_t is a vector of non-price shocks, which could include labor income but also changes in tastes or in the quality of goods, indirect utility is now given by: $V(W_t, p^t, z^t)$, where z^t is the sufficient statistic to forecast future z_t's. The DPI could then be defined as $V(\pi_{t+1}W_t, p^{t+1}, z^t) =$

[25] This calculation uses $\beta = 0.99$ and the estimated $\eta = 0.72$, $Var(\varepsilon_t^Q) = 0.091^2$ and $Var(\varepsilon_t^P) = 0.005^2$.

$V(W_t, p^t, z^t)$, or alternatively as $E_z[V(\pi_{t+1}W_t, p^{t+1}, z^{t+1})] = E_z\left[V(W_t, p^t, z^{t+1})\right]$.[26] In the first case, one would follow the traditional approach of keeping quality and tastes fixed at their base-period values. The second approach uses instead the consumer's expectations of what her welfare would be at the future tastes. Future research can consider these and other alternatives; the obstacles seem similar to those that face the static approach.[27]

7.3 Who would provide the DPI?

One interpretation of the DPI follows naturally under complete financial markets. If the prices of insurance contracts are fair so that the consumer is fully insured, and if consumption is stationary, then the DPI at each realization of prices measures how much of each Arrow-Debreu security the consumer purchases that pays in each of the states.

Seen from this perspective, there is no reason why private financial institutions could not provide the DPI. By pooling across households in different countries, they could diversify even the aggregate risks of price changes and sell households insurance policies in the form of indexed retirement accounts. As Shiller (2003) argued, there are few obstacles and many benefits from creating the markets for insuring against national inflation risks. Since there already exists a large competitive market for retirement accounts, the introduction of DPI-indexed contracts might even be easier.

According to the results in this paper, one important component of these financial products would be their variety. Section 6 demonstrated that different households will desire different indexed accounts. This provides an extra reason for these accounts to be provided by markets that can adapt their products to different groups in the population.

8 Conclusion

This paper has provided a dynamic price index for households approaching retirement to index their retirement accounts in order to secure a standard of living despite changing prices. While this price index is constructed to measure the cost of living (and not to deflate nominal variables or guide monetary policy), this use alone is sufficiently widespread and important for economic decisions to deserve a careful look.

[26] $E_z[.]$ denotes the expectations operator with respect to z^{t+1} conditional on information at t.

[27] See Pollak (1989) for an excellent discussion of conditional price indices, and Pakes (2005) for a recent survey of the potential of using hedonic models to take new goods and quality changes into account.

Some of the properties of the DPI may at first be a little startling. On second thought, perhaps they should not be so surprising. Macroeconomists have by now mostly abandoned static models that ignore uncertainty. In the study of consumption, Hall (1978) found that dynamics and uncertainty led to a new vision of reality. Consumption was no longer a stable function of current income, but instead it depended on news, interest rates, and on the structure of financial markets. Consumers responded differently to transitory and permanent shocks and future consumption growth was unpredictable. This paper brought this modern model of consumption to the study of cost-of-living price indices. Perhaps not surprisingly, it leads also to a new and different account of the cost of living. The DPI depends on news, on asset prices, on the persistence of price shocks, on the durability of goods, and it is less persistent than its static counterpart. As in Konus' (1928) original insight, the cost-of-living price index reflects the underlying theory of consumption.

While this paper moved a long way towards constructing such a price index, much remains to be done. Empirically, one could consider more disaggregated categories of goods and more involved utility functions that better fit the cross-sectional patterns of demand. A better understanding of the dynamics of prices and of how consumers form their expectations of future prices would be particularly useful. This could involve either better statistical models or perhaps economic models on which to base Muth-rational expectations. Another alternative would be to use direct survey measures of consumer expectations. The main obstacle to all of these extensions is obtaining the relevant data.

Note, however, that computing a DPI does not pose any insurmountable data problems. Relative to the static approach, one needs new information on consumer's preferences for trading over time (which is already routinely collected for portfolio and retirement advice) and on expectations of future price changes (for which we have massive amounts of data from markets and in the BLS records).[28] Given how many resources the government currently devotes to constructing price indices and given the data we already have, this seems feasible.

A second area for improvement is the model of consumer behavior. This paper has considered the model of consumption over time under uncertainty that is currently standard. However, there has been much progress in the last decade at understanding the implications for consumption of limitations on borrowing, habits, temptations, distorted expectations, and inattentiveness, among other realistic features of behavior. These could all be added

[28] The static approach does not need this information solely because it ignores the future.

to the model in this paper and alternative price indices could be constructed. This paper has only provided a first step in bringing modern models of consumption dynamics under uncertainty into the theory of building price indices. The importance of the questions involved and the tight link between the price index problem and theories of consumption should allow for fast progress in this dimension.

Appendix

A.1. Proof of the propositions in section 2

Proof of Proposition 1: For this standard consumption problem, it has been well-established that a solution exists leading to a bounded continuous value function (see e.g., Carroll, 2004). Moreover, it is clear that raising wealth relaxes the constraint of the maximization, so since the marginal utility of consumption is always positive, $V(W_t, .)$ increases with W_t. As π_t varies from 0 to infinity, the left-hand side therefore increases continuously from $-\infty$ to $+\infty$. Since for positive wealth and prices, the right-hand side is a finite number, a unique solution to the equation exists.

Proof of Proposition 2: Consider the transformation of the controls and states of the problem: $(C, S, B, W, \mathbf{p}) \rightarrow (\lambda C, \lambda S, \lambda B, \lambda W, \mathbf{p})$. It is clear that the feasibility set of the maximization problem is unchanged. As for the objective function, it goes through the transformation $V \rightarrow V + \ln(\lambda)/(1-\beta)$. Letting $\lambda = 1/W$, the symmetry theorem in Boyd (1990) implies that:

$$V(W_t, \mathbf{p}^t) = \ln(W_t)/(1-\beta) + V(1, \mathbf{p}^t). \tag{13}$$

Using this in the definition of the DPI in (7), it follows that:

$$\ln(\pi_t) = (1-\beta)(V(1, \mathbf{p}^{t-1}) - V(1, \mathbf{p}^t)), \tag{14}$$

which does not depend on wealth.

Proof of Proposition 3: Transform $(C, S, B, W, \mathbf{p}, D) \rightarrow (C, S, B, MW, M\mathbf{p}, MD)$. The feasibility set of the maximization problem is unchanged and so is the objective function. Thus, the transformation leaves the value function unchanged. From the definition of the DPI: $\pi = M$.

Proof of Proposition 4: To prove this proposition, one must first solve for a log-linear approximation to the DPI. This is done via three lemmas.

Lemma 1: *The solution to the consumption problem is:*

$$C_{j,t} = \alpha_j(1-\beta)W_t/P_{j,t} \textit{ for all } j\epsilon ND \tag{15}$$

and $S_{j,t} = \tilde{S}_{j,t}W_t/R_{j,t}$ *for all* $j\epsilon D$, $B_{E,t} = b_t(W_t - P_t^T C_t - R_t^T S_t)/Q_{E,t}$, $B_{B,t} = (1 -$

$b_t)(W_t - P_t^T C_t - R_t^T S_t)/Q_{B,t}$, and $W_{t+1} = \tilde{W}_{t+1} W_t$. Letting $\alpha_N = \sum_{j\epsilon ND} \alpha_j$, $I_{t+1}^M = (1-b_t)D_t/Q_{B,t} + b_t Q_{E,t+1}/Q_{E,t}$, and $\tilde{u}_{j,t+1} = 1-(1-\delta_j)R_{j,t+1}/R_{j,t} I_{t+1}^M$, then $\tilde{S}_{j,t}$, $\tilde{W}_{t+1}$ and b_t solve the system of equations:

$$\frac{(1-\beta)\alpha_j}{\beta \tilde{S}_{j,t}} = E_t \left[\frac{I_{t+1}^M \tilde{u}_{j,t+1}}{\tilde{W}_{t+1}} \right] \text{ for } j\epsilon D \tag{16}$$

$$\tilde{W}_{t+1} = I_{t+1}^M \left(W_t - (1-\beta)\alpha_N - \sum_{j\epsilon D} \tilde{u}_{j,t+1} \tilde{S}_{j,t} \right) \tag{17}$$

$$0 = E_t \left[\frac{Q_{E,t+1}/Q_{E,t} - D_{B,t}/Q_{B,t}}{I_{t+1}^M} \right] \tag{18}$$

Proof: Start by defining the new set of variables:

$$b_t = Q_{E,t} B_{E,t} / (W_t - P_t^T C_t - R_t^T S_t), \tag{19}$$

$$\tilde{C}_{j,t} = P_{j,t} C_{j,t} / W_t, \tag{20}$$

$$\tilde{S}_{j,t} = R_{j,t} S_{j,t} / W_t, \tag{21}$$

$$\tilde{W}_{t+1} = W_t / W_{t+1}, \tag{22}$$

and I_{t+1}^M and $\tilde{u}_{j,t+1}$ defined in the lemma. A change in variables then allows one to write the consumer problem in terms of the following Bellman equation:

$$V(W_t, \mathbf{p}^t) = \max_{\tilde{C}_{j,t}, \tilde{S}_{j,t}, b_t} \left\{ \begin{array}{c} \sum_{j\epsilon ND} \alpha_j \left(\ln(\tilde{C}_{j,t}) - \ln(P_{j,t}) \right) + \sum_{j\epsilon D} \alpha_j \left(\ln(\tilde{S}_{j,t}) - \ln(R_{j,t}) \right) + \ln(W_t) + \\ +\beta E_t \left[V\left(W_t I_{t+1}^M (1 - \sum_{j\epsilon ND} \tilde{C}_{j,t} - \sum_{j\epsilon D} \tilde{u}_{j,t+1} \tilde{S}_{j,t}), \mathbf{p}^{t+1} \right) \right] \end{array} \right\} \tag{23}$$

The optimality conditions for this problem are:

$$1 = \frac{\beta \tilde{C}_{j,t}}{(1-\beta)\alpha_j} \mathbb{E}_t \left[\frac{I_{t+1}^M}{\tilde{W}_{t+1}} \right], \tag{24}$$

$$1 = \frac{\beta \tilde{S}_{j,t}}{(1-\beta)\alpha_j} \mathbb{E}_t \left[\frac{I_{t+1}^M \tilde{u}_{j,t+1}}{\tilde{W}_{t+1}} \right], \tag{25}$$

$$0 = E_t \left[\frac{Q_{E,t+1}/Q_{E,t} - D_t/Q_{B,t}}{I_{t+1}^M} \right], \tag{26}$$

together with the law of motion for wealth. Guess now that $\tilde{C}_{j,t} = (1-\beta)\alpha_j$. You can verify that the Euler equations for non-durables are satisfied and that the law of motion for wealth becomes (17). This concludes the proof.

Lemma 2: *Letting a small letter denote the deviation of a variable with a tilde from its non-stochastic steady state and* $\chi_j = (1-\bar{u}_j)/\bar{u}_j$*, then:*

$$
\begin{aligned}
s_{j,t} &= \chi_j E_t\left[\Delta r_{j,t+1} - i^M_{t+1}\right] \textit{ for all } j\epsilon D & (27)\\
w_{t+1} &= i^M_{t+1} - (\beta^{-1}-1)\sum_{j\epsilon D}\alpha_j\chi_j\left[i^M_{t+1} - E_t[i^M_{t+1}] - (\Delta r_{j,t+1} - E_t[\Delta r_{j,t+1}])\right] & (28)
\end{aligned}
$$

Proof: At the non-stochastic steady state where $P_{j,t} = \bar{P}_j$, $R_{j,t} = \bar{R}_j$, and $I^M_{t+1} = \beta^{-1}$ we have: $\bar{C}_j = \alpha_j(1-\beta)W_t/\bar{P}_j$, $\bar{S}_j = \alpha_j(1-\beta)W_t/\bar{R}_j\bar{u}_j$. This follows from simply evaluating the optimality conditions in the previous lemma at the steady state prices.

Log-linearizing the optimality conditions in the previous lemma gives the system:

$$
\begin{aligned}
s_{j,t} &= E_t[w_{t+1} - i^M_{t+1} - u_{j,t+1}], & (29)\\
w_{t+1} &= i^M_{t+1} - (\beta^{-1}-1)\sum_{j\epsilon D}\alpha_j(s_{j,t} + u_{j,t+1}). & (30)
\end{aligned}
$$

Log-linearizing the definition of $\tilde{u}_{j,t+1}$:

$$
u_{j,t+1} = \chi_j\left(i^M_{t+1} - \Delta r_{j,t+1}\right). \quad (31)
$$

Rearranging these three equations gives the stated result.

Lemma 3: *The log-linearized value function is:*

$$
\begin{aligned}
V(.) \simeq\; & -\sum_{i=0}^{\infty}\beta^i\left(\sum_{j\epsilon ND}\alpha_j E_t\left[p_{j,t+i}\right] + \sum_{j\epsilon D}\alpha_j E_t\left[r_{j,t+i}\right]\right)\\
& + \sum_{i=0}^{\infty}\beta^i\sum_{j\epsilon D}\alpha_j\chi_j E_t\left[\Delta r_{j,t+1+i} - i^M_{t+1+i}\right]\\
& + \frac{\ln(W_t)}{1-\beta} + \sum_{i=1}^{\infty}\beta^i\sum_{k=1}^{i}E_t\left[i^M_{t+k}\right]. & (32)
\end{aligned}
$$

Proof: With the transformation of variables in lemma 1 and the value function in (23):

$$\begin{aligned} V(W_t, \mathbf{p}^t) &= -\sum_{i=0}^{\infty} \beta^i \left(\sum_{j\epsilon ND} \alpha_j E_t[\ln(P_{j,t+i})] + \sum_{j\epsilon D} \alpha_j E_t[\ln(R_{j,t+i})] \right) \\ &\quad + \sum_{j\epsilon ND} \alpha_j \ln(\tilde{C}_{j,t}) + \sum_{j\epsilon D} \alpha_j \ln(\tilde{S}_{j,t}) \\ &\quad + \sum_{i=0}^{\infty} \beta^i E_t[\ln(W_{t+i})] \end{aligned} \tag{33}$$

Log-linearizing the first line gives the first line in (32). Log-linearizing the second line, using the result for non-durable consumption in lemma 1 and the result for log-linearized durable consumption in lemma 2 gives the second line in (32). Finally, log-linearizing the term in the third line and using the result from lemma 2 that $E_{t+i}[w_{t+1+i}] = E_{t+i}[i^M_{t+1+i}]$, the third line in (32) follows.

We can now proceed to prove the proposition. If all prices follow a random walk in logs and returns are i.i.d. then the log-linearized value function in lemma 3 becomes:

$$V(.) \simeq \left(\ln(W_t) - \sum_{j\epsilon ND} \alpha_j p_{j,t} - \sum_{j\epsilon D} \alpha_j r_{j,t} \right) / (1-\beta). \tag{34}$$

Using the definition of the DPI it follows immediately that:

$$\ln \pi_{t+1} = \sum_{j\epsilon ND} \alpha_j (p_{j,t+1} - p_{j,t}) - \sum_{j\epsilon D} \alpha_j (r_{j,t+1} - r_{j,t}). \tag{35}$$

This is the static cost-of-living price index for this problem so this shows the proposition.

A.2. Proof of the lessons in section 3

Results for lesson 1: The assumptions in section 3.1 reduce the problem to:

$$V(A_t) = \max_{\{C_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \beta^i \sum_{j\epsilon ND} \alpha_j \ln(C_{j,t+i}) \tag{36}$$

$$\text{s.t.} \quad : \quad P^T_{t+i} C_{t+i} = Y = (1-\beta) A_t \tag{37}$$

The solution is clearly:

$$V(A_t) = const. + \frac{\ln(A_t)}{1-\beta} - \sum_{i=0}^{\infty} \beta^i \sum_{j \epsilon ND} \alpha_j \ln(P_{j,t+i}). \tag{38}$$

Solving for the DPI, it then follows that:

$$\ln(\pi_{t+1}) = (1-\beta) \sum_{i=0}^{\infty} \beta^i \sum_{j \epsilon ND} \alpha_j \ln(P_{j,t+1+i}/P_{j,t+i}). \tag{39}$$

Clearly, if $P_{j,t+1+i} \geq P_{j,t+i}$ for all i and j, then the log DPI is always non-negative. This proves the first result. Likewise, comparing two situation such that: $(P_{t+1+i}/P_{t+i})^A \geq (P_{t+1+i}/P_{t+i})^B$ for all i, clearly $\ln(\pi^A_{t+1}) > \ln(\pi^B_{t+1})$. Finally, if the price sequence is: $P_{j,t+i} = P$ for $i \neq h$, $P_{j,t+h} > P$ for all j, then clearly $\ln(\pi_{t+1}) > 0$.

Results for lessons 2 and 3: Letting b_t equal $Q_{E,t}B_{E,t}/(W_t - P_tC_t)$, the share of financial investments on equity, the consumer's problem is:

$$V(W_t,.) = \max_{C_{j,t},b_t} \left\{ \sum_{j\epsilon ND} \alpha_j \ln(C_{j,t}) + \beta \mathbb{E}_t \left[V\left(\left(\frac{(1-b_t)D_{B,t}}{Q_{B,t}} + \frac{b_t Q_{E,t+1}}{Q_{E,t}} \right) \left(W_t - \sum_{j\epsilon ND} P_{j,t}C_{j,t} \right), . \right) \right] \right\}, \tag{40}$$

where recall that $P_{j,t} = P_t \hat{P}_{j,t}$, and that $Q_{B,t} = Q_t \hat{Q}_{B,t}$ and $Q_{E,t} = Q_t \hat{Q}_{E,t}$ with Q_t i.i.d.

The Euler equations for consumption are:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{(1-b_t)D_{B,t}}{Q_{B,t}} + \frac{b_t Q_{E,t+1}}{Q_{E,t}} \right) \frac{P_{j,t}C_{j,t}}{P_{j,t+1}C_{j,t+1}} \right]. \tag{41}$$

The optimality condition for b_t is:

$$\mathbb{E}_t \left[\frac{W_{t+1}}{P_{j,t+1}C_{j,t+1}} \times \frac{D_{B,t}/\hat{Q}_{B,t} - \hat{Q}_{E,t+1}/\hat{Q}_{E,t}}{(1-b_t)\, D_{B,t}/\hat{Q}_{0,t} + b_t \hat{Q}_{E,t+1}/\hat{Q}_{E,t}} \right] = 0. \tag{42}$$

It is easy to see that the solution $P_{j,t}C_{j,t} = \alpha_j(1-\beta)W_t$ satisfies these two conditions, while the portfolio share solves:

$$\mathbb{E}_t \left[\frac{D_{B,t}/\hat{Q}_{B,t} - \hat{Q}_{E,t+1}/\hat{Q}_{E,t}}{(1-b_t)\, D_{B,t}/\hat{Q}_{B,t} + b_t \hat{Q}_{E,t+1}/\hat{Q}_{E,t}} \right] = 0, \tag{43}$$

and so depends only on the idiosyncratic shocks to asset prices.

The value function at date t equals the expected sum of discounted utility obtained by behaving optimally. Using the optimal consumption choices and the evolution of wealth implied by the budget constraint, evaluating expectations and summing over time, gives the value function:

$$\begin{aligned} V(.) &= const. + \frac{\ln(W_t)}{1-\beta} - \sum_{i=0}^{\infty}\beta^i \sum_{j\epsilon ND} \alpha_j \mathbb{E}_t\left[\ln(P_{j,t+i})\right] - \frac{\beta\ln(Q_t)}{1-\beta} \\ &\quad + \frac{\beta}{1-\beta}\mathbb{E}_t\left[\ln\left(\frac{(1-b_t)D_{B,t}}{\hat{Q}_{B,t}} + \frac{b_t\hat{Q}_{E,t+1}}{\hat{Q}_{E,t}}\right)\right]. \end{aligned} \tag{44}$$

Solving for the DPI then:

$$\begin{aligned} \ln(\pi_{t+1}) &= (1-\beta)\sum_{i=0}^{\infty}\beta^i\sum_{j\epsilon ND}\alpha_j\left(\mathbb{E}_{t+1}\left[\ln(P_{j,t+1+i})\right] - \mathbb{E}_t\left[\ln(P_{j,t+i})\right]\right) + \beta\ln(Q_{t+1}/Q_t) \\ &\quad -\beta\left(\mathbb{E}_{t+1}\left[\ln\left(\frac{(1-b_{t+1})D_{B,t+1}}{\hat{Q}_{B,t+1}} + \frac{b_{t+1}\hat{Q}_{E,t+2}}{\hat{Q}_{E,t+1}}\right)\right] - \mathbb{E}_t\left[\ln\left(\frac{(1-b_t)D_{B,t}}{\hat{Q}_{B,t}} + \frac{b_t\hat{Q}_{E,t+1}}{\hat{Q}_{E,t}}\right)\right]\right) \end{aligned} \tag{45}$$

Starting with the case when P_t is i.i.d., clearly, the term involving P_{t+1} in the log-DPI becomes $(1-\beta)\ln(P_{t+1}/P_t)$. The second case is when: $\Delta\ln(P_{t+1}) = \eta\Delta\ln(P_t) + \varepsilon^P_{t+1}$. Evaluating the expectations in the DPI, is it easy to see that the term involving P_{t+1} in the log-DPI now is: $\ln(P_{t+1}/P_t)/(1-\beta\eta)$. Third, if idiosyncratic goods prices are i.i.d., the terms involving them in the log-DPI are: $(1-\beta)\sum\alpha_j\ln(P_{j,t+1}/P_{j,t})$ from where the final result in section 3.2 follows.

Moving to section 3.3, the first result can be read directly from (45). The second result, on the relative impact of idiosyncratic asset price shocks, follows from taking derivatives:

$$\frac{\partial\ln(\pi_t)}{\partial\ln(\hat{Q}_{B,t})} = \beta(1-b_t)\mathbb{E}_t\left[\ln\left(\frac{D_{B,t}/\hat{Q}_{B,t}}{(1-b_t)D_{B,t}/\hat{Q}_{B,t} + b_t\hat{Q}_{E,t+1}/\hat{Q}_{E,t}}\right)\right], \tag{46}$$

$$\frac{\partial\ln(\pi_t)}{\partial\ln(\hat{Q}_{E,t})} = \beta b_t\mathbb{E}_t\left[\ln\left(\frac{\hat{Q}_{E,t+1}/\hat{Q}_{E,t}}{(1-b_t)D_{B,t}/\hat{Q}_{B,t} + b_t\hat{Q}_{E,t+1}/\hat{Q}_{E,t}}\right)\right]. \tag{47}$$

Using the optimality condition in (43), and taking the ratio of (47) and (46):

$$\frac{\partial\ln(\pi_t)/\partial\ln(\hat{Q}_{E,t})}{\partial\ln(\pi_t)/\partial\ln(\hat{Q}_{B,t})} = \frac{b_t}{1-b_t}. \tag{48}$$

Using the definition of b_t, the result claimed in the text follows. Finally, the third and last result is that if $\ln(\hat{Q}_{E,t}) = \ln(\hat{Q}_{E,t-1}) + \varepsilon_t^Q$ then $\ln(\pi_t)$ does not depend on $\hat{Q}_{E,t}$. This follows immediately from inspection of (45).

Results for lesson 4: Using the definition of the DPI and the implicit function theorem:

$$\begin{aligned}
\frac{\partial \ln(\pi_t)}{\partial \ln(P_{i,t})} &= -\frac{\partial \ln(V(.))/\partial \ln(P_{i,t})}{\partial \ln(V(.))/\partial \ln(W_t)}, && (49)\\
\frac{\partial \ln(\pi_t)}{\partial \ln(R_{j,t})} &= -\frac{\partial \ln(V(.))/\partial \ln(R_{j,t})}{\partial \ln(V(.))/\partial \ln(W_t)}. && (50)
\end{aligned}$$

Now, the problem in section 3.4 is:

$$V(W_t,.) = \max_{C_{j,t},S_{j,t},b_t} \left\{ \begin{array}{c} \sum_{i\epsilon ND} \alpha_i \ln(C_{i,t}) + \sum_{j\epsilon D} \alpha_j \ln(S_{j,t}) + \\ \beta \mathbb{E}_t \left[V \left(I_{t+1}^M \left(W_t - \sum P_{j,t} C_{j,t} - \sum u_{j,t+1} S_{j,t} \right), . \right) \right] \end{array} \right\}.$$

Where b_t and I_{t+1}^M are defined in lemma 1. The first-order conditions are:

$$\begin{aligned}
\alpha_i &= P_{i,t} C_{i,t} \beta \mathbb{E}_t [V_w(W_{t+1},.) I_{t+1}^M], && (51)\\
\alpha_j &= S_{j,t} \beta \mathbb{E}_t [V_w(W_{t+1},.) I_{t+1}^M u_{j,t+1}]. && (52)
\end{aligned}$$

Where $V_x(.)$ denotes the partial derivative of $V(.)$ with respect to x. The envelope theorem in turn implies that:

$$\begin{aligned}
V_w(W_t,.) &= \beta \mathbb{E}_t [V_w(W_{t+1},.) I_{t+1}^M] && (53)\\
V_j(W_t,.) &= -\beta S_{j,t} \mathbb{E}_t [V_w(W_{t+1},.) I_{t+1}^M \partial u_{j,t+1}/\partial R_{j,t}] + \beta \mathbb{E}_t [V_j(W_{t+1},.) \partial R_{j,t+1}/\partial R_{j,t}] && (54)\\
V_i(W_t,.) &= -\beta C_{j,t} \mathbb{E}_t [V_w(W_{t+1},.) I_{t+1}^M] + \beta \mathbb{E}_t [V_i(W_{t+1},.) \partial P_{i,t+1}/\partial P_{i,t}]. && (55)
\end{aligned}$$

Using the other optimality conditions to re-arrange the last two, and evaluating the derivatives at the non-stochastic steady state, one obtains:

$$\begin{aligned}
P_{i,t} V_i(.) &= -\alpha_i / (1 - \beta \partial \ln(P_{i,t+1})/\partial \ln(P_{i,t})), && (56)\\
R_{j,t} V_i(.) &= -\alpha_i / (1 - \beta R_{j,t} \partial u_{j,t+1}/\partial R_{j,t} u_{j,t+1})). && (57)
\end{aligned}$$

Using these to substitute in (54)-(55) gives the expression in the text.

Next, letting $\ln(R_{j,t}) = \eta \ln(R_{j,t-1}) + \varepsilon_t^R$, and recalling the definition of user costs:

$$\frac{\partial u_{j,t+1}}{\partial R_{j,t}} \times \frac{R_{j,t}}{u_{j,t+1}} = \frac{R_{j,t} - \eta(1-\delta)R_{j,t+1}/I_{t+1}^M}{R_{j,t} - (1-\delta)R_{j,t+1}/I_{t+1}^M}. \tag{58}$$

If the durable's price shocks are transitory ($\eta < 1$), then this is clearly larger than one. If however a 1% increase in the durable's price raises its future price by more than 1% ($\eta > 1$) then this is smaller than one. If $\eta = 0$ so shocks are i.i.d., and if $R_{j,t+1}/R_{j,t} \approx I_{t+1}^M$, then the right-hand side of (58) equals $1/\delta$.

A.3. Log-linearized DPI used in section 5

The starting point is the log-linearized value function in lemma 3. Using the definition of the DPI, the log-linearized DPI is:

$$\begin{aligned}
\frac{\ln(\pi_t)}{1-\beta} = & \sum_{i=0}^{\infty} \beta^i \left(\sum_{j\epsilon ND} \alpha_j \left(E_t \left[p_{j,t+i} \right] - E_{t-1} \left[p_{j,t-1+i} \right] \right) + \sum_{j\epsilon D} \alpha_j \left(E_t \left[r_{j,t+i} \right] - E_{t-1} \left[r_{j,t-1+i} \right] \right) \right) \\
& - \sum_{i=0}^{\infty} \beta^i \sum_{j\epsilon D} \alpha_j \chi_j \left(E_t \left[\Delta r_{j,t+1+i} - i_{t+1+i}^M \right] - E_{t-1} \left[\Delta r_{j,t+i} - i_{t+i}^M \right] \right) \\
& - \sum_{i=1}^{\infty} \beta^i \sum_{k=1}^{i} \left(E_t \left[i_{t+k}^M \right] - E_{t-1} \left[i_{t-1+k}^M \right] \right).
\end{aligned} \tag{59}$$

The models for the stochastic variables $p_t = (p_{1,t}, p_{2,t}, p_{3,t}, p_{4,t}, r_{5,t}, r_{6,t}, i_{E,t}, i_{B,t})$ all imply the following levels VAR representation:

$$p_t - \mu = \Phi_1(p_{t-1} - \mu) + ... + \Phi_k(p_{t-k} - \mu) + \varepsilon_t. \tag{60}$$

This can be written as a 1st order VAR on the expanded $8k$x1 vector: $p^t = [p_t - \mu, ..., p_{t-k+1} - \mu]^T$: $p^t = Fp^{t-1} + v_t$, where F is a $8p$x$8p$ matrix (see Hamilton, 1994, pp. 259). Moreover, it is easy to see that:

$$\begin{aligned}
\mathbb{E}_t[p_{t+s} - \mu] &= F_{11}^{(s)}(p_t - \mu) + ... + F_{1k}^{(s)}(p_{t-k+1} - \mu) && (61) \\
&= F_1^{(s)} p^t, && (62)
\end{aligned}$$

where $F_{1j}^{(s)}$ is the 8x8 matrix that includes rows 1 to 8 and columns $8(j-1)$ to $8j$ of F^s and $F_1^{(s)}$ is the 8x8k matrix from stacking all of these matrices horizontally.

Then, letting $A = \sum_{i=1}^{6} \alpha_i e_i$ where e_i is the 1x8 vector with a 1 in the i$^{\text{th}}$ column and zeros elsewhere, the term in the first line of (59) is:

$$\sum_{i=0}^{\infty} \beta^i A \left(\mathbb{E}_t(p_{t+i}) - \mathbb{E}_{t-1}(p_{t-1+i}) \right) = \sum_{i=0}^{\infty} \beta^i A F_1^{(i)} \Delta p^t. \tag{63}$$

Letting $B = \alpha_5 \chi_5 e_5 + \alpha_6 \chi_6 e_6 - (\alpha_5 \chi_5 + \alpha_6 \chi_6)(b e_7 + (1-b) e_8)$, the term in the second line likewise becomes:

$$-\sum_{i=0}^{\infty} \beta^i B \left(\mathbb{E}_t(\Delta p_{t+1+i}) - \mathbb{E}_{t-1}(\Delta p_{t+i}) \right) = -\sum_{i=0}^{\infty} \beta^i B \left(F_1^{(i+1)} - F_1^{(i)} \right) \Delta p^t. \tag{64}$$

Finally, letting $C = b e_7 + (1-b) e_8$, the term in the third line becomes:

$$-\sum_{i=1}^{\infty} \beta^i \sum_{k=1}^{i} C \left(E_t \left[\Delta p_{t+k} \right] - E_{t-1} \left[\Delta p_{t+k-1} \right] \right) = -\sum_{i=1}^{\infty} \beta^i \sum_{k=1}^{i} C \left(F_1^{(k)} - F_1^{(k-1)} \right) \Delta p^t. \tag{65}$$

Combining all of these terms gives the final expression for the log-linearized DPI:

$$\ln(\pi_t) = \sum_{i=0}^{\infty} \beta^i \left[(A+B) F_1^{(i)} - B F_1^{(i+1)} - \sum_{k=1}^{i} C \left(F_1^{(k)} - F_1^{(k-1)} \right) \right] \Delta p^t, \tag{66}$$

mapping the changes in the state vector p^t into the DPI.

A.4. Epstein-Zin utility case

The optimality conditions are:

$$\begin{aligned} W_{t+1} &= I_{t+1}^{E} (W_t - P_t C_t), && (67) \\ 1 &= \mathbb{E}_t \left[\left\{ \frac{\beta Q_{t+1}^{E} P_t}{Q_t^{E} P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-1/\theta} \right\}^{(1-\gamma)/(1-1/\theta)} \right]. && (68) \end{aligned}$$

These have a closed form solution only when $\eta = 0$. Then, $C_t = (1-\rho) W_t / P_t$, so the ratio of consumption expenditures to wealth is constant. The parameter ρ is defined in (12).

For the case $\eta \neq 0$, I follow the approach introduced by Campbell (1993) by log-linearizing the budget constraint around the point where the ratio of consumption expenditures to wealth is constant. Then, as Campbell (1993) shows (see his equation 20) the log-linear value function is:

$$\ln(V(W_t,.)) \approx w_t - p_t + \sum_{i=1}^{\infty} \rho^i \mathbb{E}_t[\Delta q_{t+1}^E - \Delta p_{t+1}]. \tag{69}$$

Using the definition of the DPI and the stochastic processes for equity and goods prices, the DPI in (11) follows immediately.

References

[1] Alchian, Armen and Benjamin Klein (1973) "On a Correct Measure of Inflation," *Journal of Money, Credit and Banking*, vol. 5 (1), pp. 173-191.

[2] Bajari, Patrick, C. Lanier Benkard, and John Krainer (2004) "House Prices and Consumer Welfare," Duke University.

[3] Boskin, Michael J., Ellen Dulberger, Robert Gordon, Zvi Griliches, and Dale Jorgenson (1997) "The CPI Commission: Findings and Recommendations," *American Economic Review Papers and Proceedings*, vol. 87, pp. 78-83.

[4] Boskin, Michael J. and Dale W. Jorgenson (1997) "Implications of Overstating Inflation for Indexing Government Programs and Understanding Economic Progress," *American Economic Review Papers and Proceedings*, vol. 87 (2), pp. 89-94.

[5] Bound, John and Richard V. Burkhauser (1999) "Economic Analysis of Transfer Programs Targeted on People with Disabilities," in *Handbook of Labor Economics*, Orley Ashenfelter and David Card eds., Amsterdam: North Holland.

[6] Boyd, John H. (1990) "Symmetries, Dynamic Equilibria, and the Value Function," in *Conservation Laws and Symmetry*, Ryuzo Sato and Rama Ramachandran eds., Kluwer.

[7] Bryan, Michael, Stephen Cecchetti and Roisin O'Sullivan (2001) "Asset Prices and the Measurement of Inflation," *De Economist*, vol. 149, 405-431.

[8] Campbell, John Y. (1993) "Intertemporal Asset Pricing without Consumption Data," *American Economic Review*, vol. 83 (3), pp. 487-512.

[9] Carroll, Christopher (2004) "Theoretical Results for Buffer-Stock Savings," Johns Hopkins University.

[10] Case, Karl and Robert Shiller (1989) "The Efficiency of the Market for Single-Family Homes," *American Economic Review*, vol. 79, pp. 125-137.

[11] Clark, Todd (1999) "A Comparison of the CPI and the PCE Price Index," FRB Kansas City Economic Review, vol. 3, pp. 15-29.

[12] Cecchetti, Stephen, H. Genburg, J. Lipsky, and S. Whadwani (2000) *Asset Prices and Central Bank Policy*, Geneva Reports on the World Economy, no. 2, International Center for Monetary and Banking Studies and Centre for Economic Policy Research.

[13] Deaton, Angus (1992) *Understanding Consumption*, Oxford: Oxford University Press.

[14] Diewert, W. Erwin (2001) "The Consumer Price Index and Index Number Theory," University of British Columbia Department of Economics discussion paper no. 01-02.

[15] Diewert, Erwin (2003) "The Treatment of Owner Occupied Housing and Other Durables in a Consumer Price Index," University of British Columbia discussion paper 03-08.

[16] Diewert, W. Erwin and Claude Montmarquette (1983) *Price Level Measurement*, Ottawa: Statistics Canada.

[17] Epstein, Lawrence and Stanley Zin (1991) "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis," *Journal of Political Economy*, vol. 99, pp. 263-286.

[18] Fisher, Irving (1906) *The Nature of Capital and Income*, New York: Macmillan Press.

[19] Fisher, Irving (1910) *The Purchasing Power of Money; its Determination and Relation to Credit, Interest and Crises*, New York: Macmillan Press.

[20] Fisher, Franklin and Karl Shell (1972) *The Economic Theory of Price Indices: Two Essays on the Effects of Taste, Quality, and Technological Change*, New York: Academic Press.

[21] Gillingham, Robert (1983) "Measuring the Cost of Shelter for Homeowners: Theoretical and Empirical Considerations," *Review of Economics and Statistics*, vol. 65 (2), pp. 254-265.

[22] Goodhart, Charles (2001) "What Weight Should Be Given to Asset Prices in the Measurement of Inflation?" *Economic Journal*, vol. 111, pp. F335-F356.

[23] Goodhart, Charles and Boris Hofmann (2000) "Do Asset Prices Help to Predict Consumer Price Inflation," *The Manchester School Supplement*, vol. 68, pp. 122-140.

[24] Gordon, Robert J. and Todd van Goethem (2004) "A Century of Downward Bias in the Most Important Component of the CPI: The Case of Rental Shelter, 1914-2003," Northwestern University.

[25] Hall, Robert E. (1978) "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, vol. 86, pp. 971-987.

[26] Hamilton, James (1994) *Time Series Analysis*, Princeton: Princeton University Press.

[27] International Labor Office (2004) *Consumer Price Index Manual: Theory and Practice*, International Labor Office: Geneva. (Available at http://www.ilo.org/public/english/bureau/stat/guides/cpi/)

[28] Jorgenson, Dale W. and Daniel T. Slesnick (1999) "Indexing Government Programs for Changes in the Cost of Living," *Journal of Business and Economic Statistics*, vol. 16 (2), pp. 170-181.

[29] Konus, A. (1924) "The Problem of the True Index of the Cost of Living," translated in *Econometrica* (1939), vol. 7, pp. 10-29.

[30] Mankiw, N. Gregory and Ricardo Reis (2003) "What Measure of Inflation Should a Central Bank Target?" *Journal of the European Economic Association*, vol. 1 (5), pp. 1058-1086.

[31] Merton, Robert C. (1971) "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, vol. 3., pp. 373-413.

[32] National Research Council (2002) *At What Price? Conceptualizing and Measuring Cost-of-Living and Price Indexes*, Panel on Conceptual, Measurement, and Other Statistical Issues in Developing Cost-of-Living Indexes, Charles Schultze and Christopher Mackie eds., Committee on National Statistics, Division of Behavioral and Social Sciences and Education, Washington, DC: National Academy Press.

[33] Pakes, Ariel (2005), "Hedonics and the Consumer Price Index," *Les Annales d'Economie et de Statistique*, forthcoming.

[34] Pollak, Robert (1989) *The Theory of the Cost-of-Living Index*, Oxford: Oxford University Press.

[35] Samuelson, Paul and Subramanian Swamy (1974) "Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis," *American Economic Review*, vol. 64, pp. 566-593.

[36] Shapiro, Matthew D. and David W. Wilcox (1996) "Mismeasurement in the Consumer Price Index: An Evaluation," *NBER Macroeconomics Annual 1996*, Cambridge MA: MIT Press.

[37] Shibuya, Hiroshi (1992) "Dynamic Equilibrium Price Index: Asset Prices and Inflation," *Monetary and Economic Studies*, vol. 10, pp. 95-109.

[38] Shiller, Robert (2003) *The New Financial Order: Risk in the 21st Century*, Princeton: Princeton University Press.

[39] Stock, James and Mark Watson (1988) "Testing For Common Trends," *Journal of the American Statistical Association*, vol. 83, pp. 1097-1107.

[40] Stock, James and Mark Watson (2003) "Forecasting Output and Inflation: The Role of Asset Prices," *Journal of Economic Literature*, vol. 41, pp. 788-829.

[41] Triplett, Jack (1983) "Escalation Measures: What is the Answer? What is the Question?" in *Price Level Measurement*, edited by W. Erwin Diewert and Claude Montmarquette, Ottawa: Statistics Canada.

[42] Verbrugge, Randal (2005) "The Puzzling Divergence of Rents and User Costs, 1980-2002," Division of Price and Index Number Research, Bureau of Labor Statistics.

[43] Wynne, Mark (1994) "An Intertemporal Cost-of-Living Index," FRB Dallas.