Divorce, Remarriage and Child Support

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Abstract

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1 Introduction

The last century has been characterized by changes in family structure, including a reduction in marriage and fertility and increased marital turnover. Divorce has been rising throughout the century and more men and women are now divorced and unmarried. However, the rise in divorce rates is associated with an increase in remarriage rates (relative to first marriage rates) see Figure B1 in Appendix B. Divorce and remarriage rates are substantially higher among recent cohorts (see Figures B2 to B5 in Appendix B).[1] About a quarter of men and women aged 40 to 49 in 1966 report that they have been married twice or more (see Table B1 in Appendix B). The remarriage rate among the young is similar to first marriage rate and exceeds the divorce rate suggesting that, despite the large turnover, marriage is the "natural" state.

One consequence of higher turnover is the large number of children who live in single parent and step parent households.[2] In this paper, we discuss the determination of expenditures on children and their welfare under various living arrangements. There is substantial evidence that children of divorced parents do not perform as well as comparable children in intact families.[3] It is more difficult to compare the welfare of children in high and low divorce environments and to identify the role of transfers.

[1] The especially quick rise in divorce during the seventies in many different countries was probably triggered by the oral contraceptive pill, and can be regarded as exogenous to a certain extent (see Michael, 1988, and Goldin-Katz, 2002).

[2] In the US, 2002, 69 percent of children less than 18 years old lived with two parents (including step parents), 23 percent lived only with their mother and 5 percent lived only with their father. The rest lived in households with neither parent present.

[3] The literature on this topic is large and covers psychological and economic outcomes. See, for instance, Argys et al.(1998), Lamb et al.(1999), Hetherington and Stanley-Hagan (1999), Gruber (2000) and Lerman (2002).

However, Picketty (2003) shows that the increase in the divorce rate in France have *reduced* the gap in school performance between children of divorced parents and children from intact families.[4] This paper explores a particular mechanism that may explain why the welfare loss of children from the separation of their parents can be *lower* when divorce and remarriage rates rise. We argue that a higher expectations for remarriage, associated with higher divorce rates, can serve as a coordination device that induces divorced parents to make more generous transfers that benefit the child and mother in the aftermath of divorce.

The production of children is a major reason for marriage, but an important aspect of the investment in children is the ex-post differences that are created between men and women who are otherwise identical. The basic reason for such differences is biological in nature. The mother is the one who gives birth and she is more capable of taking care of the child at least initially. This basic difference may have large economic consequences. If the couple produces children, the mother typically reduces her work in the market and, as a consequence, her future earning capacity is reduced. Thus, wage differences between men and women are created endogenously as a consequence of having children. The ex-post asymmetry between parents has strong implications for the divorce decision, the options for remarriage and the incentive to produce children. In this paper, we simplify the problem substantially by assuming that fertility is exogenous and all couples choose to have children even in the absence of any transfers. We further assume that the mother always has full custody of the child in the event of separation. We focus our attention on the *agency problems* that arise in caring for children and their relation to the aggregate conditions in the marriage market.

Children are a collective good for their natural parents and both care about their welfare. This remains true whether the parents are married or separated. However, marital status can affect the expenditures on children, and the welfare of parents and children. Separation may entail an inefficient level of expenditures on children for several reasons: 1) If the parents remarry, the presence of a new spouse who cares less about step children reduces the incentives to spend on children from previous marriages. 2) If the parents remain single then, in addition to the loss of the gains from joint consumption, the custodial parents may determine child expenditures without regard to the interest of their ex-spouse. 3) Parents that live apart from their children can contribute less time and goods to their children and may derive less satisfaction from them. These problems are amplified if the partners differ in income and cannot share custody to overcome the indivisibility of children. The custodial parent is

[4] Piketty shows that the school completion rates of children of divorced parents are lower than those of children in intact families. These gaps are similar whether the divorced parents remain single or remarry. However, the gaps in school completion declines as the proportion in the population of children who live in intact families declines over time and across regions. Piketty brings further evidence that shows that gaps are created in school completion even before the marriage breaks, suggesting that bad marriages (that end in divorce) also harm the children. A similar finding is reported by Bjorklund and Sundstrom (2002).

usually the mother who has some comparative advantage in caring for children but has lower income. The father has often limited access to the child and low incentive to provide for him. The outcome is that the level of child expenditures following separation is generally below the level that would be attained in an intact family, reducing the welfare of the children and possibly their parents.

To mitigate these problems, the partners have an incentive to sign *binding* contracts that will determine some transfers between the spouses. The purpose of the transfers is to induce an efficient level of child expenditures following divorce. We focus here on *contingent* contracts, in which the father commits to pay the mother some payment if and only if she remains single. Such a commitment increases the mother's bargaining power if she remarries and entails higher child expenditures in this case. The same payment may also induce higher child expenditures on the child if the mother remains single, provided that the income elasticity of child expenditures is positive. The incentive for such commitments is stronger when the prospects of remarriage are higher, because then the father is less likely to pay but more likely to benefit from his commitment. In addition, the incentive of each father to make such commitment depends on the commitments made by other father to their ex-wives, because such commitments also influence the bargaining outcome upon remarriage.

To explore these general issues we use a very stylized model in which the gains from marriage depend on economic considerations such as sharing consumption goods and on non monetary benefits such as companionship and love. Marriage is an "experience good" and the quality of match is discovered after some lag. Negative surprises about the quality of the match trigger divorce. However the probability of separation conditioned on a bad realization depends on the prospects of remarriage, the post divorce transfers made by the couple and also on the transfers made by potential mates to their ex-wives. In the absence of adequate transfers, remarriage may have a negative effect on the child because the new husband of the custodial mother may be less interested in the child's welfare. We may refer to this problem as the "Cinderella effect". This effect reduces the incentive of the non custodial father to support the child, because part of the transfer is "eaten" by the new husband. In addition, non custodial parents who are committed to their custodial ex-spouse are less attractive as potential mates for remarriage. Thus, the larger is the proportion of such individuals among the divorcees the less likely it is that a particular couple will divorce. In this way, we build into the model reinforcements that can create multiple equilibria.

We focus our attention on post divorce transfers that are signed "in the shadow of the law". In particular, we assume that child support payment are mandated at a minimal level that would allow the single mother to sustain the same level of child expenditure as under marriage. However, the non custodial father may augment the transfer if he wishes to influence the expenditures of the custodial mother on the child. Payments made to the custodial mother are fungible and the amount that actually reaches the child depends on whether the mother is single or remarries and on the commitments of prospective mates for remarriage to their-ex-wives. Thus, the commitments that a particular father wishes to make to his ex-wife upon separation depends on the commitments made by others and the prospects of remarriage. The model determines an equilibrium level of transfers and an equilibrium divorce and remarriage rates that are tied to each other. We identify two equilibria: A low divorce (remarriage) equilibrium in which all fathers transfer nothing to their ex-wives, above the minimal support mandated by law. In this equilibrium, the level of child expenditures falls short of the amount spent in an intact family. A high divorce (remarriage) equilibrium in which all fathers commit to transfer to their ex-wives a substantial amount if they remain single. This amount is sufficient to make the mother indifferent between remarriage and remaining single, so that the influence of the new husband on child expenditures is reduced and the level of child expenditures upon remarriage is the same as it would be if the parents did not separate.

A related paper is Aiyagari, Greenwood and Guner (2000) who construct and simulate a model of the marriage market which includes individual shocks, divorce and alimony payments among other things. They show that, at their chosen parameters, an increase in alimony raises welfare. Our model is substantially simpler than theirs, allowing us to discuss more explicitly the circumstances under which such an outcome is likely to occur. However, we achieve this added transparency at a substantial cost. In our model, individuals are assumed to be ex-ante identical so that all issues of assortative mating are set aside, and there is no role for ex ante redistribution. Similarly, there are no unexpected changes in earnings that can trigger divorce and create ex-post heterogeneity and we do not discuss wealth accumulation and the intergenerational implications of marriage and divorce.[5]

2 The basic ingredients

2.1 Incomes

All men are assumed to be identical and have a fixed income, y. Similarly, all women are identical and assumed to have the same fixed income z. However women earn less than men and $z < y$. The basic reason for this asymmetry is the presence of children, which, by assumption, requires that the mother who gives birth to the child and spends time caring for the child foregoes some of her earning capacity. Otherwise, we assume that labor supply is fixed and that incomes do not vary over time.

2.2 Preferences

A family spends its income on two goods an adult good a and a child good c. The adult good a is a public good for all members of the same household and the child

[5] Recent papers that touch on these issues are Burdett and Coles (1997 and 1999), Coles, Mailath and Postlewaite (1998) and Burdett and Wright (1998). .

good c is private to the child.

The utility of the child is

$$u_c = g(c), \tag{1}$$

where $g(c)$ is increasing and strictly concave.

Children are viewed as public good for their natural parents even if the children and parents live apart, with a correction for proximity by a discount factor, δ, that captures the idea that "far from sight is far from heart". In addition, each married couple derives utility from companionship that we denote by θ. The quality of match, θ, is an independent draw from a given symmetric distribution with a non negative mean, $\bar{\theta}$.

The utility of a single parent j is

$$u_j = a_j + u_c, \tag{2a}$$

if the parent and child live together and

$$u_j = a_j + \delta u_c, \tag{2b}$$

if the parent and child live apart, where $j = m$ indicates the mother and $j = f$ indicates the father. Similarly, the utility of a married parent i is

$$u_j = a_j + u_c + \theta_j, \tag{3a}$$

if the parent and child live together and

$$u_j = a_j + \delta u_c + \theta_j, \tag{3b}$$

if the parent and child live apart.

Adult consumption and the quality of match are viewed as household public goods. Any two married individuals who live in the same households share the same value of a and θ. Thus, parents who live together in an intact family have the same value of a and θ and enjoy equally the utility from their child u_c. However, if the parents divorce and live apart in different households, they will have different value of a and θ, and the custodial parent who lives with the child will have a higher utility from the child.[6]

2.3 Matching

There are equal numbers of males and females in each cohort. To keep things simple, we assume that, after separation, each partner can remarry only with a divorced person from the same cohort, provided that a "suitable match" who also wants to

[6] It is easy to generalize the model to allow the child to be affected (linearly) by the amount of the adult good consumed by the parents and by the quality of the match, θ.

remarry is found. However, the search process involves frictions and remarriage is neither immediate nor certain. Consequently, following divorce, agents may fail to meet an eligible new mate and hence remain single. A key ingredient of the model is that the probability of this event *decreases with the average divorce rate in the population*: remarriage is easier, the larger the number of singles around.[7]

There are several reasons why such increasing returns should be present in our context. One is that although the two sexes meet in a variety of occasions (work, sport, social life, etc.),[8] many of these meetings are "wasted", in the sense that one of the individuals is already attached and not willing to divorce. Obviously, non wasted meeting are more frequent when the proportion of divorcees in the population is larger. Another reason is that the establishment of more focused channels, where singles meet only singles, is costly and they will be created only if the "size of the market" is large enough. Thirdly, as noted by Mortensen (1988), the search intensity of the unattached decrease with the proportion of attached people in the population. The reason for that is that attached individuals are less likely to respond to an offer, which lowers the return for search. Empirical support for increasing returns is given by the geographic patterns of matching, which show that the degree of assortative mating into a given group tends to rise with the relative size of the group within the total population.[9] There is also a tendency of singles, of either sex, to congregate in large cities, especially if they have special marital needs.[10]

We do not fully specify the matching process and summarize it by a reduced form *matching function*, $m = \phi(d)$, where d is the common proportion of divorced men and women, and m is the probability that divorcees of opposite sex meet. In general, one expects each divorcee can meet with several potential mates - in which case $\phi(d)$ typically exceeds d. However, because the parties initiate contacts independently, it is also possible that some singles will be contacted by no one while others are contacted by more than one person - which might cause $\phi(d)$ to be smaller than d. In this paper, we assume that $\phi(d) > d$. The probability of remarriage is denoted by p, where $p \leq m$.

[7] This contrasts with most search models of the labor market that often assume constant returns, whereby the probability of meeting would depend on the *ratio* of single individuals of each sex.

[8] Lauman et al. (1994, Table 6.1) report that about half of the marriages arise from meeting in school, work, and private party and only 12 percent originate in specialized channels such as social clubs or bars.

[9] For instance, Bisin et al. (2001, Figure 2) report that individuals of a given religion are more likely to marry within their group than one would predict by the share of each religious group in the population, which suggests positive assortative marriage. They also report that the *difference* between within group marriage rates and population shares *rises* with the share of the religious group in the population, which suggests increasing returns. That is, a Jew, who presumably wants to marry a Jew, is more likely to do so if there are many Jews around.

[10] Costa and Kahn (2000) bring evidence that shows that singles, especially with high schooling, are more likely to reside in large metropolitan areas. Black et al.(2000) and Lauman et al. (1994, Table 8.1) report that gays are more likely to live in large cities.

2.4 Timing

Agents live two periods. In the beginning of each period, they can marry if they find a match. We assume that in the first period each agent finds a match with probability one. All matches end up in marriage, because individuals are identical and the expected gains from marriage are positive.

We think of marriage as a binding commitment to stay together for one period, with no search "on the job". The quality of the match θ is revealed with a lag at the end of each period, after having experienced the marriage. When the partners observe the common value of their match quality, θ, each partner chooses whether to continue the marriage or walk away and seek an alternative match.

All married couples produce a child at the beginning of the first period, at some cost, and receive the benefits from the child in the subsequent period. If the parents separate, the mother obtains the custody over the child and the father may make transfers to his ex-wife to induce her to maintain the welfare of the child, about whom he continues to care.

If two divorced man and woman meet at the beginning of the second period, they can choose whether to remarry. Hence, following separation, the parents can be in four different states, depending on the new marital status of the ex- spouses:

- Both parents are single, which we denote by ss.
- The father remains single while the mother is remarried, which we denote by sr.
- The mother is single but the father is remarried, which we denote by rs.
- Both parents are remarried, which we denote by rr.

2.5 Legal framework

We assume that the mother is always the custodial[11] and discuss two types of payment: a child support payment s that the wife receives if a separation occurs, independently of the subsequent marital status of the parents and an alimony payment σ that may depend on the mother's subsequent marital status. The child support payments s is determined by law but the partners can transfer an additional payment σ; σ has to be non negative, so that it cannot undo the child support payment mandated by law.[12] We further assume that the payments that the mother receives cannot be earmarked and hence can be freely reallocated in the new household which she forms.

[11] Although other custody arrangements are possible, this is still the prevalent arrangement. Mother custody can be justified by the economic comparative advantage of women in child care.

[12] While the amount of child support determined by law could (and should) in principle depend on the new marital status of both parents, in practice this feature is not observed.

The voluntary payment σ can be determined either *ex-post* (i.e., after the ex spouses' marital status has been determined), *interim* (i.e., after divorce but before remarriage) or *ex-ante* (i.e., at the time of marriage). The-ex ante and interim of agreements can be made contingent on future events. Moreover, they generally are binding contracts that require legal enforcement, which may raise renegotiation proofness issues. Ex- post payments, however, are voluntary and self-enforcing.

The focus of this paper is on transfers that are determined at the interim stage following divorce, which is the most common form of transfers. We shall therefore analyze the case in which, ex post, fathers have no incentive to make voluntary transfers to their ex-wives, yet, interim, they may *voluntarily commit* on contingent payments to the custodial mother that depend on whether she remarries or remains single. In principle, such contracts should also depend on the marital status of the father. However, observed divorce settlements are rarely contingent on the husband's marital situation, although they may depend on his income. As we have already simplified by assuming that incomes are constant, we shall further assume that σ is contingent *only* on the marital status of the mother.[13]

A common legal practice is to set child support at a level that would guarantee a standard of living similar to that obtained under marriage. In the framework presented here, this idea is captured by a payment to the custodial wife that is large enough to restore the same level of child expenditures as under marriage. However, because child support is fungible and child expenditures (especially time spent with the child) are not easily verifiable, it remains to determine what will be actually spent on the child.

3 The allocation of household resources

We begin by describing the allocation of household income between the adult and child goods under different household structures.

3.1 Intact family

If the parents remain married, they maximize their common utility

$$\max_{a,c} \; a + g(c) + \theta \tag{4}$$

s.t.

$$a + c = y + z,$$

[13] The limited scope of ex-ante marriage contracts and interim divorce contracts in modern societies is puzzling, especially in the light of the presence of such contracts in traditional societies. The rarity of ex-ante contracts can probably ascribed to a larger reliance, relative to the past, on emotional enforcement of commitments, and the presumption that too much contracting can "kill love". It is less clear why interim contracts, signed at the time of divorce, are not fully contingent.

implying that

$$g'(c) = 1. \tag{5}$$

We denote by c^* be the unique solution to (5) and assume that

$$z < c^* < y + z, \tag{6}$$

which means that the income of the mother, z, is not sufficient to support the optimal level of child expenditures, while the pooled income of the two parents $y + z$ is large enough to support the child and still leave some income for adult consumption.

3.2 Mother remains single

In this case, the mother solves

$$\max_{a,c} \; a + g(c) \tag{7}$$

s.t.

$$a + c = z + s + \sigma.$$

where σ denote the transfer that the mother receives from the father if she remains single in addition to the compulsory payment, s.

Given the quasi linear structure of preferences, the choice between adult consumption and child goods follows a very simple rule:

$$\begin{array}{cccc} a = 0 & and \;\; c = z + s + \sigma & if & z + s + \sigma \leq c^* \\ a = z + s + \sigma - c^* & and \;\; c = c^* & if & z + s + \sigma > c^* \end{array}. \tag{8}$$

That is, the mother spends all her income, $z + s + \sigma$, on the child if her income is lower than the child's "needs", as represented by c^*. If her total income exceeds c^* then the mother will spend c^* on the child and the rest on herself.

3.3 Mother remarries

If the custodial mother remarries, the problem becomes more complicated because of the involvement of a *new* agent, namely the new husband of the mother. The new husband receives little or no benefits from spending on the child good. To sharpen our results, we assume that the new husband derives no utility at all from the child, which means that the child good is a *private* good for the wife in the new household. In this case, if $c < c^*$, then an increase in the amount spent on child goods raises the utility of the mother, because she values this expenditure more than the forgone adult good. In this range, there is a *conflict* between the mother and her new husband. If $c > c^*$, there is no conflict and both partners prefer to spend the marginal dollar on the adult good.

We can distinguish two different mechanisms that determine the expenditures on the child in newly formed households, depending upon whether binding commitments on child expenditures can be made prior to remarriage. Without any commitment, the custodial mother will decide how much to spend on the child, taking as given the amount she receives from her former husband and the amount that her new husband gives to his former wife. Alternatively, the matched partners can bargain prior to remarriage on the division of the gains from remarriage and reach some binding agreement, (or an 'understanding') that will determine the expenditure on the child.

We may use a symmetric Nash-Bargaining solution to determine the bargaining outcome. The Nash axioms imply that the bargaining outcome must maximize the product of the gains from remarriage, relative to remaining single, of the two partners. The gain of the remarried mother depends on the transfers that she *expects* to receive from her ex-husband, when remarried or single. Similarly, the gain from marriage of the new husband depend on the *expected* payments that he is going to pay his ex-wife, when married or single. At the time of meeting between the two separated individuals, it is not known what is the marital status of their ex-spouses. However, because agents are assumed to be risk neutral, we can use the expected payments in calculating the gains from remarriage.

An important simplifying assumption of the model is that transfers made by the father do not depend on his *own* marital status. With this assumption we only need to keep track of whether or not the mother is remarried. We shall denote the payments by a given father to his ex-wife by σ and payment made by other men by σ^-.[14] With this notation, the voluntary payments of the *new* husband to his *ex-wife*, is $s + \sigma_s^-$ if his ex-wife remains single and $s + \sigma_r^-$ if she remarries. The realized value of the transfer is not known at the time of the bargaining and we shall denote its *expected* value by $\sigma_e^- = (1-p)\sigma_s^- + p\sigma_r^-$, where p is the probability of remarriage. We denote by y_e^- the expected net income that the new husband brings into the marriage, that is $y_e^- = y - s - \sigma_e^-$.

Since the new husband cares only about the adult good that he receives in the new household and because, by assumption, his payments to the ex-wife and thus the utility of his child are independent of his marital status, his gain from marriage depend only on the additional adult good and the value of companionship that he expects and given by

$$z + s + \sigma_r - c + \bar{\theta}. \tag{9}$$

The utility gain of the mother upon remarriage consists of the additional adult consumption and the change in her utility from child expenditures. For $s \geq c^* - z$, these amount to

$$\gamma(c) + y_e^- + \sigma_r - \sigma_s + \bar{\theta}, \tag{10}$$

[14] It is possible that identical agents will select different commitments to their identical ex-wives. However, because we are looking for symmetric equilibria, there is no loss of generality in assuming that all other fathers pay the same amounts to their ex-wives.

where

$$\gamma(c) \equiv g(c) - c - (g(c^*) - c^*). \tag{11}$$

Note that $\gamma(c)$ is non positive and concave with a maximum at c^*, where $\gamma(c^*) = \gamma'(c^*) = 0$.

The Nash bargaining solution can be written in the form

$$\gamma'(c) = \frac{\gamma(c) + y_e^- + \sigma_r - \sigma_s + \bar{\theta}}{z + s + \sigma_r - c + \bar{\theta}}, \tag{12}$$

where, $\gamma'(c)$ is the slope of the Pareto frontier (in absolute value) and $\frac{\gamma(c)+y_e^- +\sigma_r-\sigma_s+\bar{\theta}}{z+s+\sigma_r-c+\bar{\theta}}$ is the ratio of the utility gains of the two partners. Let $\hat{c}$ be the solution to (12) then, because remarriage occurs only if both partners have a non negative gain from marriage, $\gamma'(c) \geq 0$ and $\hat{c} \leq c^*$. That is, the step family generally spends less on child goods, which we may call the "Cinderella effect".

An increase in the payment to the wife as single, σ_s is more effective in influencing the child expenditures when the mother is remarried than a payment given to her directly when she is remarried in the form of σ_r. The basic reason is that any money given to the remarried mother in the form of fungible funds is partially "eaten" by the new husband, which is a form of an implicit tax on the father. The money given to the single mother is more effective because it raises the bargaining power of the mother if the wife remarries, without giving anything to the new husband. It is easily shown that an increase in σ_s unambiguously raises the amount spent on child goods, when the wife remarries. In contrast, an increase in σ_r raises the amount spent on the child if and only if $g\prime(\hat{c}) > 2$.[15] Moreover, under some conditions, the father can induce the efficient outcome $c = c^*$, which cannot be attained by any finite value of σ_r.[16]

An increase in the expected non monetary gain from marriage, $\bar{\theta}$, would reduce (increase) $\hat{c}$ if the monetary gain from marriage of the father is larger (smaller). That is, monetary transfers will partially offset the impact of $\bar{\theta}$ on the relative gains from marriage. Usually, we expect that the new husband will have a smaller monetary gain from marriage than the new wife. Thus, if both parties expect that the marriage will be successful, the new husband will agree to raise the expenditures on the child.

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[15] Rewrite (12) as

$$\gamma'(c)(z + s + \sigma_r - c + \bar{\theta}) - \gamma(c) = y_e^- + \sigma_r - \sigma_s + \bar{\theta}.$$

Then

$$[(z + s + \sigma_r - \hat{c} + \bar{\theta})\gamma''(\hat{c}) - 2\gamma'(\hat{c})]\frac{\partial \hat{c}}{\partial \sigma_r} = 1 - \gamma'(\hat{c})$$

and $\frac{\partial \hat{c}}{\partial \sigma_r} > 0$ iff $1 - \gamma'(\hat{c}) < 0$ or $g\prime(c) > 2$.

[16] For instance, if $\bar{\theta} > 0$, and $z + s = c^*$ then by setting $\sigma_r = 0$ and $\sigma_s = y_e^- + \bar{\theta}$, the father can induce c^*. In this solution, the mother is just indifferent between marriage and no marriage. The new husband receives no extra adult consumption from marriage, because the mother spends all her disposable income on the child, but he is still willing to marry the mother for her companionship.

An important feature of the Nash bargaining solution is that the amount of child goods in a remarried couple depends only on the difference $y_e^- - \sigma_s$. Thus, if the mother remarries a new husband with less commitments and, therefore, a higher bargaining power that induces lower child consumption, the father can fully offset this effect and maintain the level of child consumption by raising the payment that the mother receives as single.

4 Optimal Ex-Post Transfers

We begin our analysis with a case in which all transfers are voluntary. Such transfers are motivated by the father's continued interest in his child and he may give money to the custodial mother, aiming to influence her expenditures on the child. At the last stage of the game, when the marital status of *all* agents have been determined, each father may consider what is the transfer that he wishes to make, unilaterally. If prior commitment on the child expenditures, have been made, the father cannot influence the utility of the child, as all subsequent payments will be spent on the adult good. If, however, no commitments have been made then the father can influence the expenditure on the child.

If both parents are single, the father will *voluntarily* augment the mother's total income up to some minimal level c_δ, given by

$$\delta g'(c_\delta) = 1. \tag{13}$$

That is,

$$\sigma = \begin{cases} c_\delta - (z+s) & if \quad z+s \leq c_\delta \\ 0 & if \quad z+s > c_\delta \end{cases}. \tag{14}$$

Clearly, the father will never transfer more than c^*, because the mother will spend every marginal dollar beyond c^* on adult goods and not on the child, in which case the father gains nothing. However, if the mother is relatively poor and $z+s < c_\delta$, he will transfer up to c_δ. Clearly, $c_\delta \leq c^*$ if $\delta < 1$, implying a reduction in the child's welfare relative to continued marriage.

If the father or mother are remarried the father's incentive to transfer to the custodial mother is further reduced, because of the involvement of the new husband of the mother and the new wife of the father. Given these weakened incentives, there is a role for legal intervention, in the form of setting minimal child support standards and enforcing binding contracts. A common consideration in determining the size of the compulsory child support payments is the accustomed standard of living of the child, should the marriage continue. For this reason, we shall assume that s is set at the level which can support the level of child expenditures c^*, that is, $s = c^* - z$. Such a policy clearly "crowds out" all voluntary ex-post payments. However, it is generally insufficient to guarantee that the child receives c^* if the mother remarries.

4.1 Optimal interim Contracts

Our previous analysis reveals an interesting dilemma; if the custodial mother remains single, the father is unwilling to give her any transfer beyond the minimum set by law. However, if she remarries, he would like her to have more money as single. This situation calls for *voluntary binding contract*, whereby the father commits to pay a certain amount if his ex-wife if she remains single. Our focus in this paper is on such payments and to simplify our analysis of such contingent contracts we shall assume that the father is unwilling to give the custodial mother any additional payment if she remarries, given that the mother is already protected by a compulsory payment $s = z - c^*$. A sufficient condition for such an outcome is $g'(\hat{c}) < 2$. Clearly, the father will refrain from promising a transfer in the contingency that the wife remarries, if such a transfer in fact reduces the expenditure on the child. However, he may still want to commit *interim* to a payment conditioned on her being single, because such a commitment will influence her bargaining position (hence child expenditures) if she remarries. We can then omit the subscript and will refer to σ as an alimony payment that the wife receives as single and is stopped if she remarries. Correspondingly, we define the expected net income of the father as $y_e = y - s - (1-p)\sigma$ and of the new husband as $y_e^- = y - s - (1-p)\sigma_e^-$.

Setting $s = z - c^*$ and letting $x = \sigma - y_e^-$, condition (12) can be rewritten as

$$\gamma'(c) = \frac{\gamma(c) - x + \bar{\theta}}{c^* - c + \bar{\theta}}, \tag{15}$$

or equivalently as

$$\gamma'(c)(c^* - c + \bar{\theta}) - \gamma(c) = -x + \bar{\theta} \tag{15'}$$

and solved for $\hat{c}$ as a function of x, for a given parameter $\bar{\theta} \geq 0$. We denote by $\hat{c} = h(x;\bar{\theta})$, the unique solution of (15), where $0 \leq h(x;\bar{\theta}) \leq c^*$.[17] The level of child expenditure upon remarriage cannot exceeds c^* because the remarried mother and the new husband have a common interest not to exceed this level of expenditure. The function $h(x;\bar{\theta})$ is defined only for values of x in the range $-(y-s) \leq x \leq \bar{\theta}$. The lower bound is a consequence of the requirement that σ and σ^- cannot be negative and the upper bound arises from the requirement that both partners must have non negative gains from marriage, otherwise a remarriage would not occur. From

[17] Let

$$f(c) = \gamma'(c)(c^* - c + \bar{\theta}) - \gamma(c).$$

Because of the assumed properties of $\gamma(c)$, $f'(c) < 0$. Therefore, if a solution of (15') exists in the region $0 \leq c \leq c^*$ then it must be unique. Because $f(c^*) = 0$, the solution for $\hat{c}$ is positive if

$$\gamma'(0)(c^* + \bar{\theta}) - \gamma(0) > \bar{\theta} - x$$

and equals zero otherwise. Sufficient conditions for the solution to be positive are $g\prime(0) = 2$ and $g(c^*) > y + z - c^*$. That is, the utility from the child must be sufficiently large relative to the utility from the adult good.

(15) and the properties of $\gamma(c)$, we see that if $x = \bar{\theta}$ then $h(x;\bar{\theta}) = c^*$, because $\gamma(c^*) = \gamma'(c^*) = 0$. At this point, the mother is just indifferent between remarriage and remaining single, while the new husband has a positive gain if $\bar{\theta} > 0$ and is indifferent towards remarriage if $\bar{\theta} = 0$.[18]

When $0 < h(x;\bar{\theta}) < c^*$,

$$\begin{aligned} h'(x;\bar{\theta}) &= \frac{1}{2\gamma'(\hat{c}) - (c^* - \hat{c} + \bar{\theta})\gamma''(\hat{c})} > 0, \\ h''(x;\bar{\theta}) &= [h'(x;\bar{\theta})]^3((c^* - \hat{c} + \bar{\theta})\gamma'''(\hat{c}) - 3\gamma''(\hat{c})). \end{aligned} \tag{16}$$

An interesting feature of the Nash bargaining solution is that the bargaining outcome $h(x;\bar{\theta})$ can be *convex* in x. As seen in (16), a sufficient condition for that is that the marginal utility from child expenditures $g\prime(x)$ is convex, implying $\gamma''(c) = g''(\hat{c}) < 0$ and $\gamma'''(c) = g'''(\hat{c}) \geq 0$. That is, the mother gains less from a marginal increase in c at higher levels of child expenditures while the marginal cost for her new husband (in terms of adult good) remains the same. The remarried mother is, therefore, more willing to give up child consumption and the bargaining outcome becomes more responsive (in terms of the child good) to transfers from the father or the new husband. This situation is illustrated in Figure 1.

Recalling that $x = \sigma - y_e^- = \sigma + (1-p)\sigma^- + s - y$, we see that the convexity or concavity of $h(x;\bar{\theta})$ creates *strategic interactions* among different agents, in the sense that the *marginal* impact of the commitment made by the father to his ex-wife, σ, is affected by the commitments made by others, σ^-. These interactions have different consequences at different marital states. If $h(x;\bar{\theta})$ is convex (concave) then, if the mother remarries, a larger σ^-will increase (decrease) the marginal impact of σ. However, if the father remarries, a higher commitment by others raises the bargaining power of the new wife and the marginal cost of the commitment made by the father will be higher (lower). Thus, to fully describe the strategic interactions we need to look at the effects of commitments on the *expected* utilities of the fathers.

The expected utility of a particular father upon separation consists of several parts and can be written as $E(u_f) = \delta E(u_c) + E(a_f) + p\bar{\theta}$. Where,

$$E(a_f) = y_e + p(c^* - h(\sigma^- - y_e;\bar{\theta})),$$

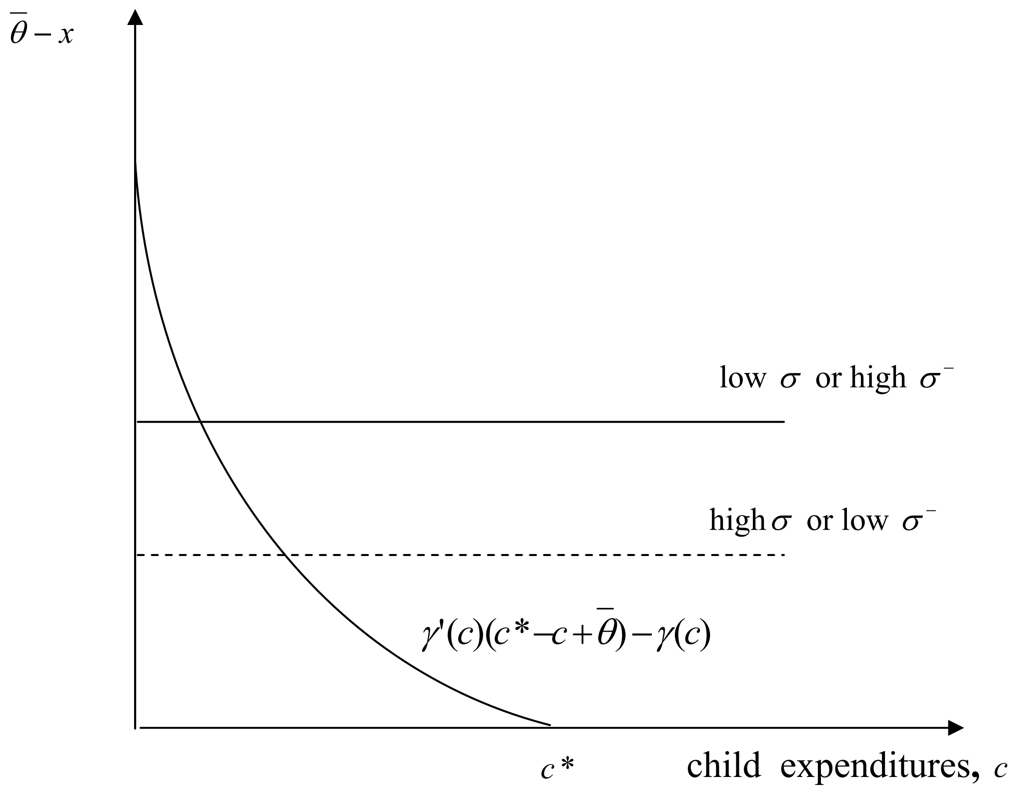
is the father's expected adult consumption,

$$E(u_c) = pg(h(\sigma - y_e^-;\bar{\theta})) + (1-p)g(c^*),$$

is the expected utility of the child upon separation, and $\bar{\theta}$ is the father's expected value of companionship upon remarriage.

[18] Note that $h(x;\bar{\theta})$ is usually not differentiable at $x = \bar{\theta}$, because behavior changes if the boundary is crossed and the mother prefers to stay single, but the left derivative exists.

Figure 1 : Determination of child expenditures when the mother remarries[1]



[1] c^* is the efficient level of child expenditures, $\overline{\theta}$ is the expected quality of match and x is the difference between the father's commitment, σ, and the new husband's expected income, $(y-s)-\sigma^-(1-p)$. The function $\gamma(c)$ equals $g(c)-c-(g(c^*)-c^*)$. The graph is drawn for the case in which $g'(c)$ is convex.

Taking the derivative of $E(u_f)$ with respect to σ, holding σ^- constant, we obtain

$$\begin{aligned} \frac{\partial E(u_f)}{\partial \sigma} &= -(1-p) - p(1-p)h'(\sigma^- - y_e;\bar{\theta}) \\ &\quad + p\delta g'(h(\sigma - y_e^-;\bar{\theta}))h'(\sigma - y_e^-;\bar{\theta}). \end{aligned} \tag{19}$$

The father pays σ only if the mother remains single, which occurs with probability $(1-p)$. At the margin, this commitment would cost him 1 dollar if he remains single and $h'(\sigma^- - y_e;\bar{\theta}) + 1$ dollars if he remarries, where the added term $h'(\sigma^- - y_e;\bar{\theta})$ represents the additional expenditures on the child of the *new* wife, resulting from the decline in the father's bargaining power when he increases the commitment to his ex-wife. The father gets benefits from σ only if the mother remarries, which occurs with probability p. In this case, the payment raises the mother expenditures on the child because her bargaining power is stronger. The increase in child expenditures is $h'(\sigma - y_e^-;\bar{\theta})$ and the father gain from this increase is $\delta g'(h(\sigma - y_e^-;\bar{\theta}))h'(\sigma - y_e^-;\bar{\theta})$.

The expression in (19) is valid only in the range in which the commitments are consistent with remarriage of the mother and the new wife of the father, that is

$$\begin{aligned} \sigma &\leq \bar{\theta} + y_e^- = \bar{\theta} + y - s - \sigma^-(1-p), \\ \sigma^- &\leq \bar{\theta} + y_e = \bar{\theta} + y - s - \sigma(1-p). \end{aligned} \tag{20}$$

We shall refer to condition (20) as the incentive compatibility constraint. In addition, the commitments must be feasible and satisfy

$$\begin{aligned} 0 &\leq \sigma \leq y - s, \\ 0 &\leq \sigma^- \leq y - s. \end{aligned} \tag{21}$$

An interior optimal solution for σ given σ^- must satisfy these two constraints and the necessary condition for individual optimum $\frac{\partial E(u_f)}{\partial \sigma} = 0$ and $\frac{\partial^2 E(u_f)}{\partial \sigma^2} < 0$.[19] However, as we shall show shortly, *corner solutions* in which agents select either $\sigma = 0$ or the maximal level permitted by constraints (20) and (21) will play an important role in the analysis.

A salient feature of the model is that the probability of remarriage, p, has a systematic influence on the willingness of each father to commit. The reason is quite simple. The father commits to pay only if the mother remains single and gets the benefits only if she remarries. Thus, if p is low he is more likely to pay and less likely to benefit. Conversely, if p is high, the father less likely to pay and more likely to benefit.

---

[19] The second order derivative is

$$\begin{aligned} \frac{\partial^2 E(u_f)}{\partial \sigma^2} &= p\delta g''(h(\sigma - y_e^-;\bar{\theta}))[h'(\sigma - y_e^-;\bar{\theta})]^2 + p\delta g'(h(\sigma - y_e^-;\bar{\theta})h''(\sigma - y_e^-;\bar{\theta}) \\ &\quad - p(1-p)^2 h''(\sigma^- - y_e;\bar{\theta}). \end{aligned}$$

We can now see how the commitments of others affect the expected utility of each father and his incentives to commit. These interactions between agents are summarized by

$$\begin{aligned}\frac{\partial E(u_f)}{\partial \sigma^-} &= -ph'(\sigma^- - y_e;\bar{\theta}) \\ &\quad +p(1-p)\delta g'(h(\sigma - y_e^-;\bar{\theta}))h'(\sigma - y_e^-;\bar{\theta}),\end{aligned} \tag{22}$$

and

$$\begin{aligned}\frac{\partial^2 E(u_f)}{\partial \sigma \partial \sigma^-} &= p(1-p)[\delta g''(h(\sigma - y_e^-;\bar{\theta}))[h'(\sigma - y_e^-;\bar{\theta})]^2 \\ &\quad +\delta g'(h(\sigma - y_e^-;\bar{\theta})h''(\sigma - y_e^-;\bar{\theta}) - h''(\sigma^- - y_e;\bar{\theta})].\end{aligned} \tag{23}$$

We see in (22) that an increase in σ^- reduces the gains of the father if he remarries by $h'(\sigma^- - y_e;\bar{\theta})$, because his new wife will have a higher bargaining power. On the other hand, if the mother remarries she will have a higher bargaining power if her prospective new husband has higher commitments to his ex-wife, which raises the utility of the father by $(1-p)\delta g'(h(\sigma - y_e^-;\bar{\theta}))h'(\sigma - y_e^-;\bar{\theta})$. Generally, it is not clear which of these two effects is stronger. However, it is seen from (22) and (23) that both $\frac{\partial E(u_f)}{\partial \sigma^-}$ and $\frac{\partial^2 E(u_f)}{\partial \sigma \partial \sigma^-}$ are negative if $\sigma - y_e^- = \bar{\theta}$. This happens because the father already sets σ at a high level and the father's gain from further increase in child consumption is small (or non existent) while an increases in σ^- can still reduce his own consumption upon remarriage. In contrast, when σ is small and child expenditures upon remarriage are set at a low level, then an increase in σ^- can be beneficial to the father and increase his incentive to commit. The impact of others also depends on the probability of remarriage, either directly or because σ depends on p. The upshot is that although we can easily determine the impact of others ex-post, this impact is generally ambiguous when marital status is still unknown.

4.2 Partial Equilibrium

A symmetric partial (or conditional) equilibrium exists when, given the probability of remarriage p, all agents choose the same level of σ, taking the choices of others as given. The term partial is used here because the remarriage rate is endogenous in our model and must be determined too.

We first consider an interior equilibrium with a common σ that satisfies the feasibility and incentive compatibility constraints as strict inequalities. For such an equilibrium to exist, it is necessary that the first order condition,

$$\frac{\partial E(u_f)}{\partial \sigma} = -(1-p)(1+ph'(h(\sigma - y_e;\bar{\theta})) + p\delta g'(h(\sigma - y_e;\bar{\theta}))h'(\sigma - y_e;\bar{\theta}) = 0, \tag{24}$$

and the second second order condition,

$$\frac{\partial^2 E(u_f)}{\partial\sigma^2} = p\delta g''(h(\sigma - y_e;\bar{\theta}))[h'(\sigma - y_e;\bar{\theta})]^2 + p\delta g'(h(\sigma - y_e;\bar{\theta})h''(\sigma - y_e;\bar{\theta}) \\ -p(1-p)^2 h''(\sigma - y_e;\bar{\theta}) < 0, \quad (25)$$

be satisfied, together with the incentive compatibility constraint

$$0 < \sigma < \frac{y - s + \bar{\theta}}{2 - p}, \quad (26)$$

and feasibility constraint

$$0 < \sigma < y - s. \quad (27)$$

In the appendix we prove that

Proposition 1 *If the marginal utility from child expenditures is convex (i.e., $g'''(c) \geq 0$), $\frac{\partial E(u_f)}{\partial\sigma} = 0$ entails $\frac{\partial^2 E(u_f)}{\partial\sigma^2} > 0$ for all $p \in [0, \frac{2}{3}]$, implying that an interior symmetric equilibrium* does not exist *in this range.*

The reason for non existence can be traced back to the convexity of the Nash bargaining outcome in the commitment made by each father to the custodial mother. This convexity implies that each father can individually gain from a unilateral departure from the interior equilibrium. However, a symmetric equilibrium can still occur at the *boundaries* of either the feasibility constraints or the incentive compatibility constraint, whichever is binding.

To simplify the analysis of equilibria at the boundary, we shall now assume that $\bar{\theta} = 0$ and $\delta = 1$. The restriction on the non monetary factor, $\bar{\theta}$, guarantees that the incentive compatibility constraint is the binding constraint for all p. The assumption that $\delta = 1$ guarantees that the father cares sufficiently about the child to support an equilibrium in which everyone is willing to commit. Finally, we shall assume that $g(c)$ is quadratic, which is the borderline case for the class of functions for which $g'''(c) \geq 0$ and also the easiest one to calculate.

We can provide only a partial characterization, for the case in which the best response functions are always at the boundary. That is, if all individuals set $\sigma^- = 0$ then each father who deviates makes the maximal feasible transfer $\sigma = y - s$, and if all other individual selects $\sigma^- = \frac{y-s}{2-p}$ then any father who deviates chooses the minimal transfer, $\sigma = 0$. This extreme behavior can be supported by sufficient convexity of $h(x;0)$. In the appendix we prove that such a pattern must hold when others set $\sigma^- = 0$. For the case in which others set $\sigma^- = \frac{y-s}{2-p}$, we cannot prove that the optimal deviation is necessarily to 0, but shall assume that this is the case.[20]

[20]In the numerical examples that follow, we do verify that this property holds.

We can then characterize the strategic interaction globally and say that σ and σ^- are complements if, for all p

$$V_f(\frac{y-s}{2-p},\frac{y-s}{2-p}) - V_f(0,\frac{y-s}{2-p}) > V_f(y-s,0) - V_f(0,0) \tag{28}$$

and substitutes if the inequality is reversed, where $V_f(\sigma,\sigma^-)$ stands for the expected utility of the father at given σ and σ^-. We can also define critical points for the probability of remarriage, p_0 and p_1 such that each father is indifferent between deviating and conforming, given that others choose, $\sigma^- = \frac{y-s}{2-p}$ and $\sigma^- = 0$, respectively.

We can then prove (see Appendix)

Proposition 2 *Suppose that the utility of the child, $g(c)$, is quadratic with $g'(c) < 2$ and that the implied best response functions are always at the boundary. Assume no expected gain from companionship, $\bar{\theta} = 0$, and no discounting if the parent and child live apart, $\delta = 1$. Then p_0 and p_1 are uniquely determined. If σ and σ^- are complements then $p_1 > p_0$ and for $p < p_0$ all fathers set $\sigma = 0$, while for $p > p_1$ all fathers voluntarily commit to pay their ex-wife the maximal σ that is incentive compatible $\frac{y-s}{2-p}$. For $p_1 > p > p_0$ both types of equilibrium can exist. If σ and σ^- are substitutes then $p_0 > p_1$ and for $p < p_1$ all fathers set $\sigma = 0$, while for $p > p_0$ all fathers voluntarily commit to pay their ex-wife the maximal σ that is incentive compatible $\frac{y-s}{2-p}$. If $p_0 > p > p_1$ there is no symmetric equilibrium. In either case, equilibria at the upper, incentive compatible, boundary imply that child expenditures are set at the efficient level c^*, while equilibria at the lower boundary imply that child expenditures are set at an inefficient level $\hat{c} < c^*$.*

The pattern described in the Proposition 2 is illustrated in Figure 2. We see that, irrespective of the strategic interactions, a higher probability of remarriage is conducive to equilibria in which fathers are willing to commit on a payment that is conditioned on the event that the mother remains single, because such promises are carried out less often and are more likely to yield benefits. In this regard, the probability of remarriage serves a *coordination device* that induces fathers to behave similarly in terms of their commitments. The strategic interactions can reinforce or mitigate this pattern depending on complementarity or substitution. If σ and σ^- are complements each father is more willing to contribute if others do, and thus the critical value at which all fathers contributes occurs at a lower p than it would if others do not contribute. Conversely, if σ and σ^- are substitutes.

The basic characterization of the equilibrium of Proposition 2 holds also if $\bar{\theta} > 0$. However, in this case, the feasibility constraint will bind at high remarriage rates, and it is then impossible to maintain efficiency of child expenditures. The modified statement is then that equilibria at the upper feasible level, $\sigma = y - s$, imply higher levels of child expenditures than equilibria in the lower feasible boundary with $\sigma = 0$.

Figure 2a: Incentives to commit $V_f(\sigma^-,\sigma^-)$-$V_f(\sigma,\sigma^-)$ in relation to the probability of remarriage, p, and commitments of others, $\sigma^- = \frac{y-s+\overline{\theta}}{2-p}$ or $\sigma^- = 0$, when σ^- and σ are strategic complements

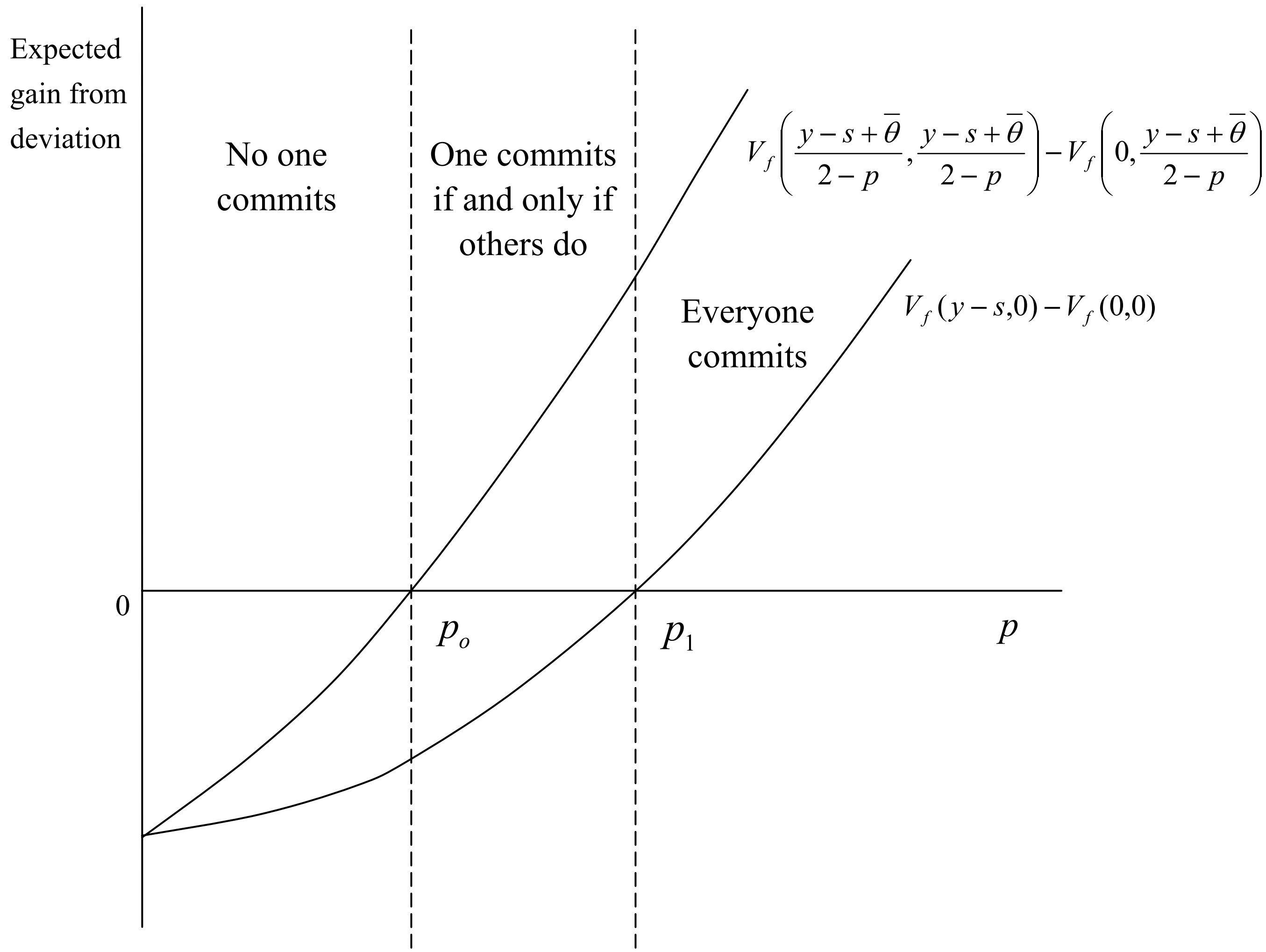
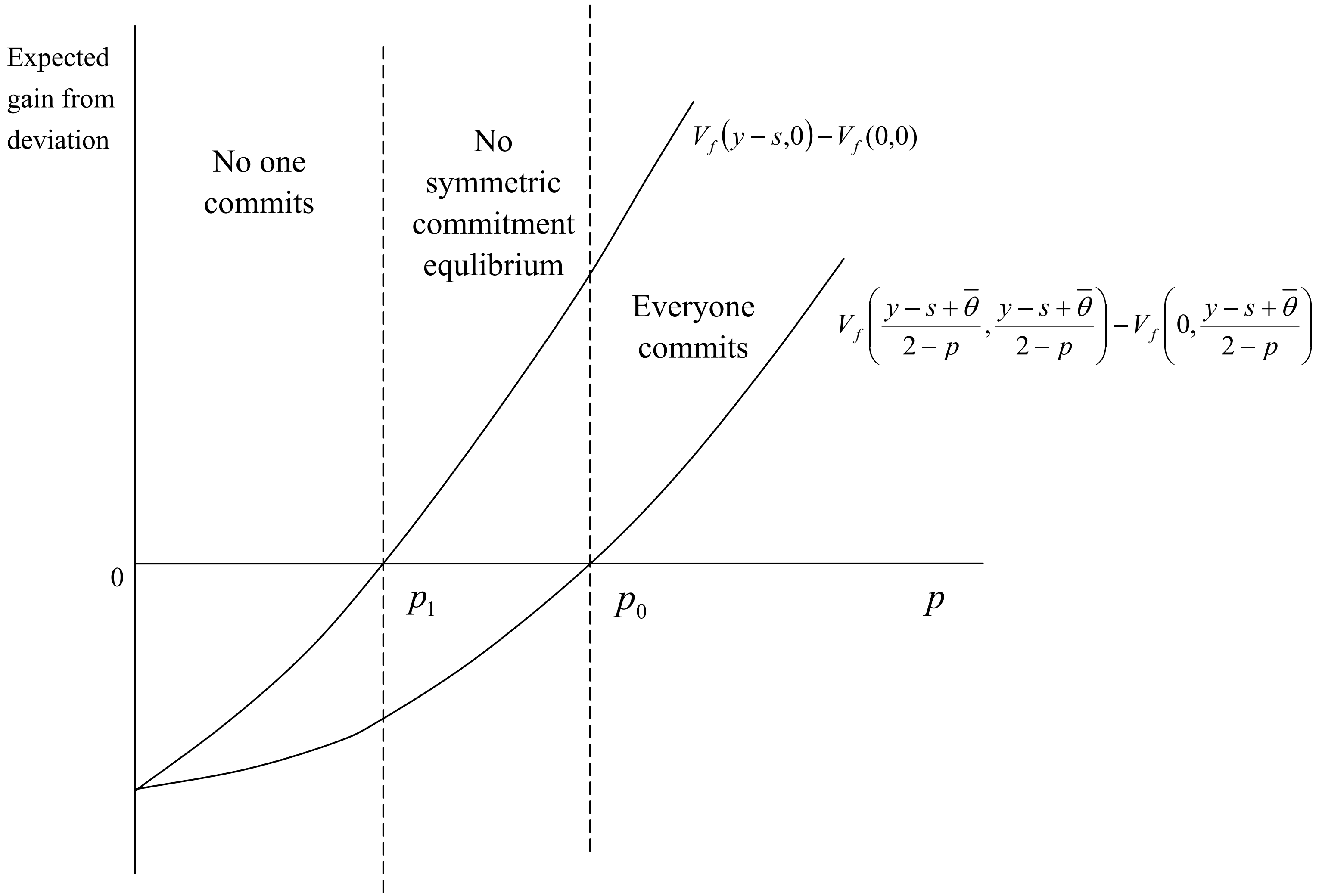


Figure 2b: Incentives to commit $V_f(\sigma^-,\sigma^-)$-$V_f(\sigma,\sigma^-)$ in relation to the probability of remarriage, p, and commitments of others, $\sigma^- = \frac{y-s+\bar{\theta}}{2-p}$ or $\sigma^- = 0$, when σ^- and σ are strategic substitutes



5 Divorce

Having observed the realized quality of the current match, each spouse may consider whether or not to continue the marriage. A parent will agree to continue the marriage if, given the observed θ, the utility in marriage exceeds his/her expected gains from divorce. Under divorce at will, the marriage breaks if

$$u^* + \theta < max\{E(u_m), E(u_f)\}, \tag{29}$$

where $E(u_m)$ and $E(u_f)$ are the expected utility of the mother and father at divorce, and

$$u^* = y + z + g(c^*) - c^* \tag{30}$$

is the common utility of the husband and wife if the marriage continues, not incorporating the quality of the match.

Let us define the critical value of θ that triggers divorce as

$$\theta^* = max\{E(u_m), E(u_f)\} - u^*. \tag{31}$$

Excluding the quality of match θ_i, the utility of each parent following separation cannot exceed the common utility that the parents attain if marriage continues, because the allocation between adult goods within marriage is efficient and all the opportunities of sharing consumption are exploited. Therefore, the critical values θ^* must be lower than the expected quality of match following remarriage.

The probability that a couple will divorce is

$$\Pr\{\theta \leq \theta^*\} = F(\theta^*), \tag{32}$$

where $F(.)$ is the cumulative distribution of θ. Assuming independence of the marital shocks across couples and a large population, the proportion of couples that will choose to divorce is the same as the probability that a particular couple divorces. Symmetry implies that $F(\bar{\theta}) = \frac{1}{2}$ and, therefore, the fact that divorce is costly from an economic point of view implies that less than half of the marriages will end up in divorce as a consequence of "bad" realizations for the quality of match.

An important feature of the model is that the decision of each couple to divorce depends on the probability of remarriage, that in turn depends on the decision of others to divorce, because a remarriage is possible only with a divorcee. In addition, the decision to divorce, depends on the nature of the commitments that the couples makes, as well as the commitment made by others. Post divorce transfers between the parents can reduce their cost of separation in the event of a bad quality of match. However commitments made by others imply that prospective matches are less attractive for remarriage, which can increase the cost of divorce.

To analyze these complex issues, we limit our attention to the commitment equilibria that occur at the boundary. We shall also maintain our simplifying assumptions that $\bar{\theta} = 0$, $\delta = 1$ and that $g(c)$ is quadratic with $g'(c) < 2$. Based on Proposition 2, we can now examine the expected utilities of the husband and wife, evaluated at the time of divorce.

In equilibria without commitment, $\sigma = 0$,

$$\begin{aligned} E(u_c) &= pg(\hat{c}) + (1-p)g(c^*), \\ E(u_f) &= y + z - c^* + p(c^* - \hat{c}) + E(u_c), \\ E(u_m) &= p(y + z - \hat{c}) + E(u_c), \end{aligned} \tag{33}$$

where the Nash Bargaining outcome $\hat{c}$ is given by the solution to

$$\gamma'(\hat{c}) = \frac{\gamma(\hat{c}) + y + z - c^*}{c^* - \hat{c}}. \tag{34}$$

The child's expected utility declines with the probability of remarriage, p, because child expenditure if the mother remarries, $\hat{c}$, is lower than if the mother remains single, c^*. The father pays his ex-wife the compulsory payment $s = c^* - z$ and his net income is therefore $y + z - c^*$. If he remarries, he obtains additional adult good from his new wife, $c^* - \hat{c}$. Therefore, his expected adult consumption rises with the probability of remarriage, p. However, taking into account the utility of the child, the expected utility of the father declines in p, because $g(c^*) - c^* > g(\hat{c}) - \hat{c}$. The mother, has no adult good if she remains single because she spends all the compulsory payment s on the child. If she remarries, the amount received from her ex-husband is exactly offset but the commitment of the new husband so that the adult consumption in the new household is $y + z - \hat{c}$. It is clear that she gains more from remarriage than her ex-husband, because he meets "poor" women and she meets "rich" men. If her gain from adult consumption upon remarriage is more important than the loss of child good, she gains from an increase in the probability of remarriage.[21] Nevertheless, the expected utility of the father exceeds that of the mother by $(1-p)(y + z - c^*)$, because of his higher consumption of adult goods as a single man.

In equilibria in which all fathers commit to $\sigma = \frac{y-s}{2-p}$,

$$\begin{aligned} E(u_c) &= g(c^*), \\ E(u_m) &= \frac{y + z - c^*}{2-p} + g(c^*) \\ E(u_f) &= \frac{y + z - c^*}{2-p} + g(c^*) \end{aligned} \tag{35}$$

[21] The effect of p on the mother's expected utility is

$$\frac{\partial E(u_m)}{\partial p} = (y + z - \hat{c}) - (g(c^*) - g(\hat{c})).$$

Given our assumption that the marginal utility from child expenditures is bounded by 2, $g(c^*) - g(\hat{c}) < 2(c^* - \hat{c})$. Hence, $\frac{\partial E(u_m)}{\partial p} > 0$ if $\hat{c} + y + z - 2c^* \geq 0$.

That is, the efficient level of child expenditures is attained whether or not the mother remarries. Both the father and the mother are indifferent between remarriage and remaining single. The expected utility of the mother upon divorce equals that of the father and both rise with the probability of remarriage, p.

We conclude

Proposition 3 *Suppose that proximity does not matter, $\delta = 1$, and all gains from remarriage are monetary, $\bar{\theta} = 0$. Then, the expected utility of the father upon divorce is at least as large as that of the mother and he determines whether or not the marriage will continue. If no father commits, $\sigma = 0$, then $E(u_f) > E(u_m)$ and the father will break the marriage for all θ such that $\theta < E(u_f) - u^*$. Inefficient separations occur when the father wants to leave but and the mother wants to maintain the marriage, $E(u_f) - u^* > \theta > E(u_m) - u^*$. If all fathers commit to $\sigma = \frac{y-s}{2-p}$ then $E(u_f) = E(u_m)$ and separations are efficient.*

The assumptions that $\delta = 1$ and $\bar{\theta} = 0$ are crucial for the result that the father and mother have the same expected utility. Clearly, the father is at a disadvantage if proximity is valuable and the mother gains custody. The level of $\bar{\theta}$ matters because it affects the payment that is required to maintain the mother's indifference between marriage and non marriage. The result that $E(u_f) = E(u_m)$ when $\bar{\theta} = 0$ and the associated efficiency of child expenditures reflect a *knife edge* situation in which neither men nor women gain from remarriage. When $\bar{\theta}$ is raised then, as long as $\frac{y-s+\bar{\theta}}{2-p} < y + z - c^*$, it is possible to maintain the level of child expenditure at c^*, by raising the commitment σ and keeping the mother indifferent between marriage and non marriage. In this case, the mother's expected utility is $\frac{y-s+\bar{\theta}}{2-p} + g(c^*)$ but the expected utility of the father is now smaller, $\frac{y-s+\bar{\theta}}{2-p} + g(c^*) - (1-p)\bar{\theta}$, because a larger transfer is needed to maintain indifference. In this case, the *mother* will determine whether the couple divorces. Finally, if $\bar{\theta}$ is such that $\frac{y-s+\bar{\theta}}{2-p} > y - s$ then *both* men and women gain from remarriage, but it is impossible to maintain the child expenditures at the efficient level c^*, which means that the child suffers from the mother's remarriage.

It is also clear that equilibrium outcome in the aftermath of divorce is inferior to the utility that an average couple obtains in marriage, because $\frac{y-s+\bar{\theta}}{2-p} \leq y - s + \bar{\theta}$. This difference reflects the lack of companionship and the inability to share in the adult good when one of the partners remains single. It is only when remarriage is certain, that one can expect to recover the average utility in the first marriage.

6 Full equilibrium

We can now close the model and determine the equilibrium levels of divorce and remarriage. Equilibrium requires that all agents in the marriage market act optimally, given their expectations, and that expectations are realized. The decision of each couple to divorce depends on the expected remarriage rate, p. Given a matching function $m = \phi(d)$, and that all meetings end up in marriage,[22]we must have

$$p = m = \phi[F(\theta^*(p))]. \tag{36}$$

In addition, the contracting choices of all participants in the marriage market must be optimal, given p. Based on our previous analysis, we define $\theta_0^*(p)$ as the trigger if all fathers set $\sigma = 0$ and $\theta_1^*(p)$ as the trigger if all fathers set $\sigma = \frac{y-s+\bar{\theta}}{2-p}$. Then, the equilibrium divorce and remarriage rates are determined by $p = \phi[F(\theta_0^*(p))]$ or $\phi[F(\theta_1^*(p))]$, depending upon whether the induced commitment is $\sigma = 0$ or $\sigma = \frac{y-s+\bar{\theta}}{2-p}$.[23]

To separate the economic considerations embedded in $\theta^*(p)$ from the exogenous distribution of match quality $F(\theta)$ and matching function $\phi(d)$, it is useful to rewrite condition (36) in the form

$$F^{-1}[\phi^{-1}(p)] = \theta^*(p). \tag{36'}$$

The function $F^{-1}[\phi^{-1}(p)]$ is always increasing in p, while $\theta^*(p)$ depends on the commitments that individual fathers wish to make, given their evaluation of the remarriage prospects of their ex-wife. The linearly declining line in Figure 3 represents $\theta_0^*(p)$, which is the critical remarriage rate that would trigger divorce if no father pays his ex-wife beyond the minimum required by law. The increasing convex line in the figure represents $\theta_1^*(p)$ which is the critical value of remarriage rate that would trigger divorce if all father commit to an additional payment $\sigma = \frac{y-s}{2-p}$ that the mother receives if she remains single. The vertical lines at $p = p_1 = .53$ indicates that for all lower values of the expected remarriage rate, no father is willing to commit if others do not. The vertical line at $p = p_0 = .59$ indicates that for all higher values of the expected remarriage rates each father is willing to commit if others do. The result that $p_0 > p_1$ shows that, for the chosen parameters, σ and σ^- are strategic substitutes. The two steep convex lines represent the function $F^{-1}[\phi^{-1}(p)]$, where $F(\theta)$ is uniform over $[-u, u]$ and the matching function is given by $\phi(d) = 1 - (1-u)^2$. The curve to the right represents a higher variability in θ than the curve to the left. The increase in the variance holding the mean constant (i.e., an increase in u) shifts the equilibrium from a low divorce- remarriage equilibrium with $p = .5$ and $d = .29$ to high divorce-remarriage equilibrium with $p = .71$ and $d = .46$. The higher equilibrium occurs above p_1 and is associated with more generous commitments by fathers who pay $\sigma = \frac{y-s}{2-p}$ to their ex-wives if they remain single. The lower equilibrium is

[22]Strictly speaking, all agents are indifferent toward marriage if. $\bar{\theta} = 0$. However, for any positive $\bar{\theta}$, the father gains and the mother is either indifferent or gains too. Thus, we interpret the case with $\bar{\theta} = 0$ as a limit in which the expected gains from companionship approach zero.

[23]These equilibrium requirements implicitly assume symmetric equilibria in which all agents behave in the same manner. Such equilibria are a natural choice given that all agents are initially identical, but other equilibrium may exist. In a more general analysis, one can incorporate also mixed equilibria such that some couples choose to have a child, some choose to remain childless and all couples are *indifferent* between having and not having a child.

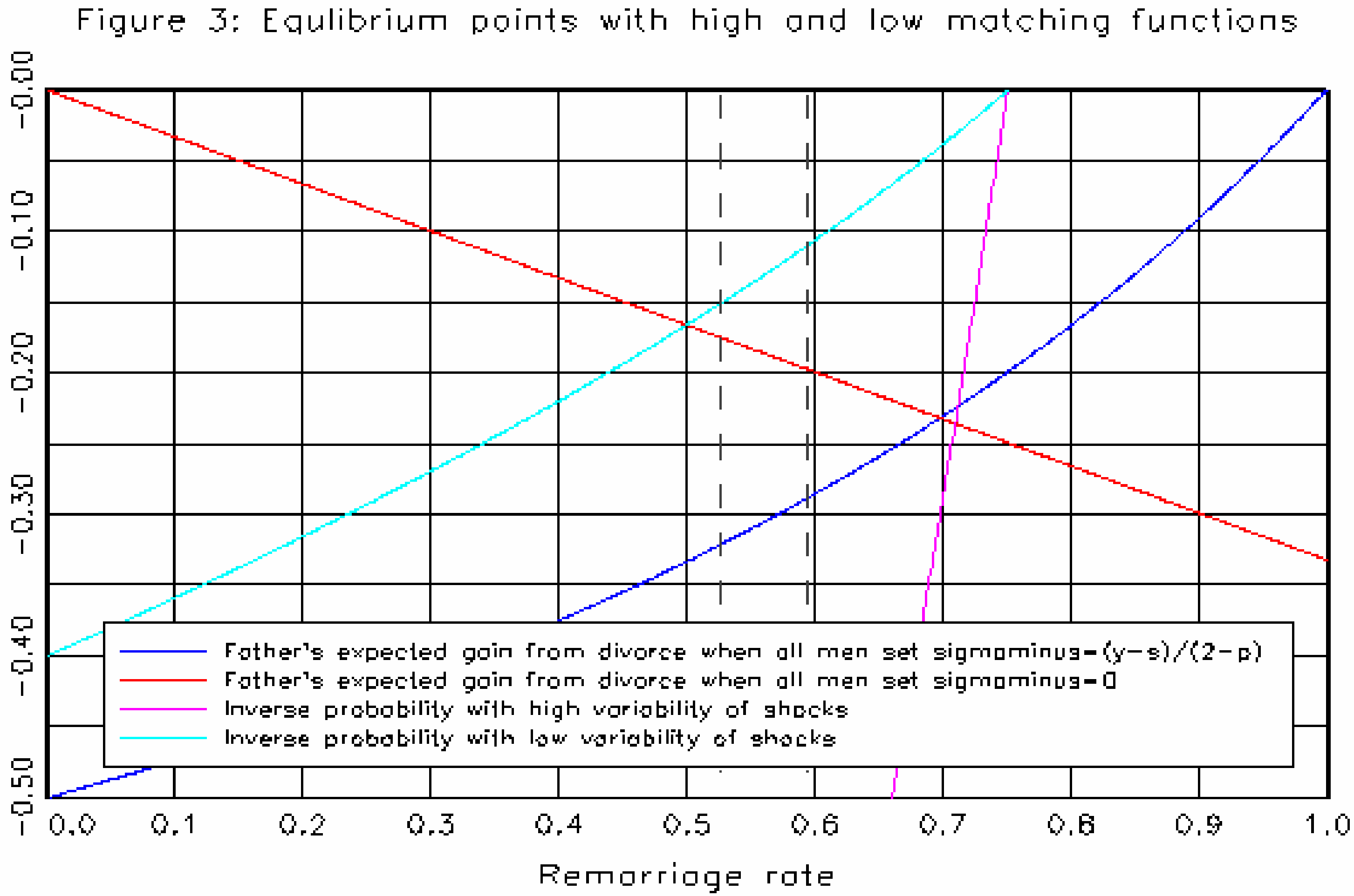


Figure 3: Equlibrium points with high and low matching functions
-0.00
-0.10
-0.20
-0.30
-0.40
-0.50
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Remarriage rate
Father's expected gain from divorce when all men set sigmaminus=(y-s)/(2-p)
Father's expected gain from divorce when all men set sigmaminus=0
Inverse probability with high variability of shocks
Inverse probability with low variability of shocks

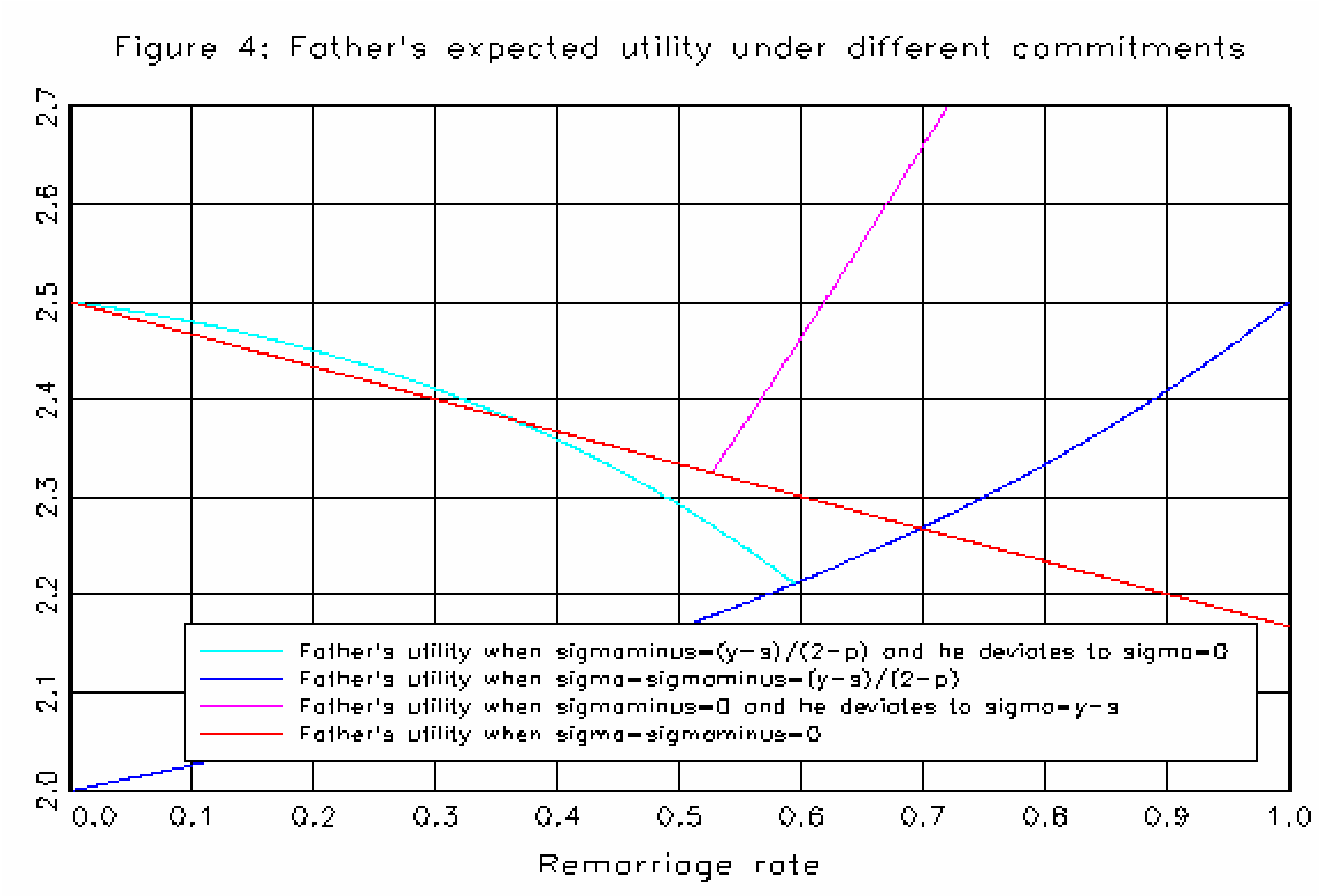


Figure 4: Father's expected utility under different commitments
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Remarriage rate
Father's utility when sigmaminus=(y-s)/(2-p) and he deviates to sigma=0
Father's utility when sigma=sigmaminus=(y-s)/(2-p)
Father's utility when sigmaminus=0 and he deviates to sigma=y-s
Father's utility when sigma=sigmaminus=0

below p_0 and in this case fathers pay nothing to their ex-wives ($\sigma = 0$), beyond the amount stipulated by law, $s = c^* - z$. This example illustrates the coordination role of aggregate divorce in inducing stronger commitments.

The model is capable of generating multiple equilibria that arise with the *same* basic parameters. This requires, however, that σ and σ^- be strategic complements, which creates a reenforcement mechanism whereby each father is more likely to commit if others do.[24]

7 Welfare

The expected utility of a member j in a particular couple, evaluated at the time of marriage, is

$$W_j(p) = u^0 + \bar{\theta} + \int_{\theta^*(p)}^{\infty} (u^* + \theta) f(\theta) d\theta + F(\theta^*(p)) V_j(p), \tag{37}$$

where, $V_j(p)$ denote the expected utility upon divorce, $j = f$ for the father and $j = m$ for the mother. The term u^0 represent the parents average utility in the first period and is given by $y + z$, while $u^* = y + z + g(c^*) - c^*$ represents the utility in an intact family (excluding the impact of the non monetary return θ).[25] The expected utility following divorce, $V_j(p)$, may be different for the two parents, depending upon the agreement they make about transfers and on the agreement made by others, which determines their value as potential mates for remarriage. As a consequence, the expected life time utility evaluated at the time of marriage, $W_j(p)$, may differ for males and females.

The expected life time utility is higher for the partner with the higher gains from divorce, who determines the divorce decision. In fact, the expected life time utility can be rewritten as

$$W_j(p) = \begin{cases} u^0 + u^* + 2\bar{\theta} + \int_{-\infty}^{\theta^*(p)} (\theta^*(p) - \theta) f(\theta) d\theta & if \quad V_j(p) \geq V_i(p) \\ u^0 + u^* + 2\bar{\theta} + \int_{-\infty}^{\theta^*(p)} (\theta^*(p) - \theta) f(\theta) d\theta & \\ \quad -F(\theta^*(p))(V_i(p) - V_j(p)) & if \quad V_j(p) < V_i(p) \end{cases} \tag{38}$$

[24] Given the specification $g(c) = 2c - \frac{1}{2}c^2$, that implies $c^* = 1$, σ and σ^- are strategic complements when $z + y$ is close to 1. However, there are no multiple equilibria at such parameters. It seems that a strict convexity for $g'(c)$ is needed to generate "realistic" examples with multiple equilibria.

[25] The economic costs of bearing and raising children are reflected by the assumption that $z < y$. Because these cost are largely borne by the mother, she may refrain from having children unless the father make further commitments at the time of marriage about post divorce settlements. To avoid these further complications, we assume here that the total gains from having children (including possible benefits in the first period) exceed these economic costs.

Table 1 : Equlibrium divorce and remarriege rate for different parameters values

Changes in the variance of the shocks, u. Matching parameter, $\alpha = 2$

Commitment	Variability parmeter	Divorce rate	Remar. rate	Child c	Child $E(u_c)$	Mother $E(u_m)$	Father $E(u_f)$
$\sigma = \frac{y-s}{2-p}$	3.0	.46	.71	1.0	1.5	2.28	2.28
$\sigma = 0$	.4	.29	.50	.18	.93	1.83	2.33

Changes in the matching parameter, u. Variability parameter, $u = 1.5$

Commitment	Matching parmeter	Divorce rate	Remar. rate	Child c	Child $E(u_c)$	Mother $E(u_m)$	Father $E(u_f)$
$\sigma = \frac{y-s}{2-p}$	3.0	.45	.84	1.0	1.5	2.36	2.36
$\sigma = 0$	1.33	.44	.54	.4	.88	1.86	2.32

The fixed parameters are : Proximity, $\delta = 1$. Expected match quality, $\bar{\theta} = .001$. Efficient level of child expenditure, $c^* = 1$. Family income, $y + z = 2$. Critical value when other fathers commit, $p_0 = .594$. Critical value when other fathers do not commit, $p_1 = .526$.

where the term $u_\delta^0 + u_\delta^* + 2\bar{\theta}$ is the expected value of the marriage if it never breaks and the term $\int_{-\infty}^{\theta_\delta^*(p)} (\theta_\delta^*(p) - \theta) f(\theta) d\theta$ is the *option value* of breaking the marriage if it turns sour because of a bad draw of θ. The option to sample from the distribution of θ is a motivation for marriage that exists even if marriage provides no other benefits. However, this option is available only to the person with higher gains from divorce, who determines the divorce. When the marriage breaks, an event that happens with probability $F(\theta^*(p))$, the spouse who does not initiate the divorce and is left behind suffers a capital loss given by $V_i(p) - V_j(p)$. The value of the option for the spouse who determines the divorce, increases with the variability in the quality of match, because then the ability to avoid negative shocks becomes more valuable.

Using the expressions in (37) and (38), we can calculate the welfare of each agent in equilibrium. The main result is that exogenous shocks that raise the divorce rates can increase the welfare of the child and the mother, because they provide incentive to fathers to raise their commitments. An important feature of the model is

Proposition 4 *The welfare of the child declines with the probability of remarriage, p, if fathers make no commitments, $\sigma = 0$, and is unaffected by p if fathers make full commitments, $\sigma = \frac{y-s+\bar{\theta}}{2-p}$. Starting at a low p, a* large *change in p is needed to induce fathers to commit to a level of transfer that entails an improvement in the child's welfare.*

In Table 1, we show the outcomes for some particular parameter values. The upper panel correspond to the outcomes shown in Figure 3 and, as seen, the expected utility of the father is lower in the equilibrium with higher divorce and remarriage. In the lower panel, we illustrate the impact of improved matching, which might be an outcome of increased use of the internet in looking for mates. Such exogenous shift raises the welfare of all family members, father mother and child. Notice that, despite the large increase in the remarriage rate, the divorce rate is hardly affected. This happens because in the range without commitment the gains from marriage decline in the probability of remarriage so that easier remarriage *reduces* divorce. It is only when transfers are operative that the gains from divorce rise with the probability of remarriage and the two move together.

8 Conclusion

Broadly viewed, divorce is a corrective mechanism that enables the replacement of bad matches by better ones. The problem, however, is that private decisions may lead to suboptimal social outcomes because of the various externalities that infest search markets. These externalities exists at the level of a single couple and the market at large. At the level of a couple, the spouse who initiates the divorce fails to internalize the interest of the other spouse in continued marriage and the parent that remarries fails to internalize the impact on the child and consequently of the ex-spouse who continues to care about the child. At the market level, a person who chooses to divorce fails to take into account the impact on the remarriage prospects of others, and if commitments are made, on the quality of prospective mates. We have shown that the problems at the couple's level can be resolved by voluntary commitments that entail efficient level of child expenditures and efficient separation. However such commitments are made only if the expected remarriage rate is sufficiently high.

The willingness to commit at high divorce levels is a consequence of the social interaction between participants. In the marriage market, as in other "search markets", finding a mate takes time and meetings are random; the decision of each couple to terminate its marriage depends not only on the realized quality of the particular match, but also on the prospects of remarriage and, therefore, on the decisions of others to divorce and remarry (see Mortensen, 1988, and Becker et al., 1977). If, in addition, search in the marriage market is characterized by 'increasing returns', in the sense that having more singles around makes it easier for each one to marry, then one obtains positive feedbacks that can lead to "social multiplier" effects, implying large aggregate responses to small exogenous changes (see the recent survey by Scheinkman and Glaeser 2000). We have shown that such feedbacks may also improve the welfare of children, because fathers may be more willing to commit on post divorce transfers to their ex-wives in high divorce environments, in order to influence their bargaining power upon remarriage. Of course, a higher divorce rate can induce other mechanisms that affect the welfare of children, in addition to the impact on voluntary commitments discussed here. In fact, the rise in divorce was associated with new guidelines for the courts that facilitate child support agreements and with increased enforcement of child support awards (see Del Boca, 2002, and Lerman and Sorensen, 2003).

The analysis of this paper can be extended to include endogenous fertility. As we have shown, the ex-post asymmetry between parents caused by having children can create problems in caring for them if the marriage breaks up and contracts are incomplete. Because men often have higher expected gains from divorce, they initiate the divorce, at some situations in which the mother would like to maintain the marriage. Such inefficient separations imply that the gains from having children are smaller to the mother than to the father. Because the production of children requires both parents, the mother may avoid birth in some situations in which the husband would like to have a child. The consequences is an inefficient production of children. This suggests another role for contracts, to regulate fertility, which may require some ex-ante contracting at the time of marriage. However, contracts that couples are willing to sign at the time of marriage may be inconsistent with contracts that the partners are willing to sign in the interim stage, after divorce has occurred and the impact of the contract on the divorce and fertility decisions is not relevant any more. With such time inconsistency, the partners may wish to renegotiate, thereby creating a lower level of welfare for both of them from an ex-ante point of view. Assuming that renegotiation takes place, the contracts will be similar to the interim contracts discussed here but they would apply for a broader range of remarriage probabilities. It can then be shown that fertility choice creates further feedbacks that can generate multiple equilibria, with and without children. These are important issues for further research.

9 Appendix A

9.1 Non Existence of a symmetric equilibrium with an interior solution

Suppose that $\sigma = \sigma^-$ and the solution is interior. Then the first order condition

$$\frac{\partial E(u_f)}{\partial \sigma} = -(1-p)(1+ph'(h(\sigma - y_e;\bar{\theta})) + p\delta g'(h(\sigma - y_e;\bar{\theta}))h'(\sigma - y_e;\bar{\theta}) = 0, \quad \text{(A1)}$$

and the second second order condition,

$$\begin{aligned} \frac{\partial^2 E(u_f)}{\partial \sigma^2} &= p\delta g''(h(\sigma - y_e;\bar{\theta}))[h'(\sigma - y_e;\bar{\theta})]^2 + p\delta g'(h(\sigma - y_e;\bar{\theta})h''(\sigma - y_e;\bar{\theta}) \\ &\quad -p(1-p)^2 h''(\sigma - y_e;\bar{\theta}) < 0, \end{aligned} \quad \text{(A2)}$$

are satisfied, together with the incentive compatibility constraint

$$\sigma \leq \frac{y - s + \bar{\theta}}{2-p}, \quad \text{(A3)}$$

and feasibility constraint

$$0 \leq \sigma \leq y - s. \quad \text{(A4)}$$

Rewriting

$$\frac{\partial^2 E(u_f)}{\partial \sigma^2} = p[h'(\sigma - y_e;\bar{\theta})]^2\{\delta g''(\hat{c}) + (\delta g'(\hat{c}) - (1-p)^2)h'(\sigma - y_e;\bar{\theta})\frac{h''(\sigma - y_e;\bar{\theta})}{[h'(\sigma - y_e;\bar{\theta})]^3}\} \quad \text{(A5)}$$

and using equations (16) and (A1) we obtain

$$\begin{aligned} \frac{\partial^2 E(u_f)}{\partial \sigma^2} &= p[h'(\sigma - y_e;\bar{\theta})]^2\{\delta g''(\hat{c}) - \\ &\quad \frac{\delta g'(\hat{c}) - (1-p)^2}{\delta g'(\hat{c}) - (1-p)}\frac{1-p}{p}(3g''(\hat{c}) - (c^* - \hat{c} + \bar{\theta})g'''(\hat{c}))\} \\ &= p[h'(\sigma - y_e;\bar{\theta})]^2\{g''(\hat{c})[\delta - 3\frac{\delta g'(\hat{c}) - (1-p)^2}{\delta g'(\hat{c}) - (1-p)}\frac{1-p}{p}] + \\ &\quad \frac{\delta g'(\hat{c}) - (1-p)^2}{\delta g'(\hat{c}) - (1-p)}\frac{1-p}{p}(c^* - \hat{c} + \bar{\theta})g'''(\hat{c})\}. \end{aligned} \quad \text{(A6)}$$

Now because $\delta \leq 1$ and $\frac{\delta g'(\hat{c}) - (1-p)^2}{\delta g'(\hat{c}) - (1-p)} \geq 1$, the first term must be positive if $3\frac{1-p}{p} > 1$ or $p < \frac{3}{4}$. The second term is non negative if $g'''(\hat{c})) \geq 0$. Therefore, if $g'''(\hat{c})) \geq 0$ and $p < \frac{3}{4}$, $\frac{\partial E(u_f)}{\partial \sigma} = 0 \Rightarrow \frac{\partial^2 E(u_f)}{\partial \sigma^2} > 0$ and a symmetric equilibrium with an interior solution for the commitment σ does not exist.

9.2 Equilibria at the boundary

The possible symmetric commitment equilibria at the boundary are at $\sigma = 0$, at $\sigma = y - s$ and at $\sigma = \frac{y-s+\bar{\theta}}{2-p}$.

To analyze these potential equilibria we shall assume that $g(c) = g_1 c - \frac{g_2}{2}c^2$ is quadratic, where g_1 and g_2 are fixed parameters such that $2 \geq g_1 > 0$ and $g_2 > 0$. The restriction that $g_1 \leq 2$ ensures that the father never wants to transfer money to the custodial mother if she remarries. The restriction that $g'''(c) = 0$ implies that

$$h''(x;\bar{\theta}) = 3g_2[h'(x;\bar{\theta})]^3 > 0.$$

We can now prove the following

Lemma 5 *For a quadratic function $g(c)$, there is no interior solution for σ such that $\sigma \geq \sigma^-$ in the region where $p < \frac{3}{4}$. In particular, if all agents set $\sigma^- = 0$, then any agent that considers deviation must choose between the lower boundary, i.e., $\sigma = 0$ and the upper boundary given by $\sigma = y - s$.*

Proof. Suppose that

$$\begin{aligned}\frac{\partial E(u_f)}{\partial \sigma} &= -(1-p) - p(1-p)h'(\sigma^- - y_e;\bar{\theta}) \\ &\quad + p\delta g'(h(\sigma - y_e^-;\bar{\theta}))h'(\sigma - y_e^-;\bar{\theta}) \geq 0.\end{aligned}$$

At such a point, the second order derivative is

$$\begin{aligned}\frac{\partial^2 E(u_f)}{\partial \sigma^2} &= p\delta g''(h(\sigma - y_e^-;\bar{\theta}))[h'(\sigma - y_e^-;\bar{\theta})]^2 + p\delta g'(h(\sigma - y_e^-;\bar{\theta})h''(\sigma - y_e^-;\bar{\theta}) \\ &\quad -p(1-p)^2 h''(\sigma^- - y_e;\bar{\theta}) \\ &= -pg_2[h'(\sigma - y_e^-;\bar{\theta})]^2\{\delta - 3\delta h'(\sigma - y_e^-;\bar{\theta})g'(h(\sigma - y_e^-;\bar{\theta}) \\ &\quad +3(1-p)^2 h'(\sigma^- - y_e;\bar{\theta})\frac{[h'(\sigma^- - y_e;\bar{\theta})]^2}{[h'(\sigma - y_e^-;\bar{\theta})]^2}\} \\ &\geq -pg_2[h'(\sigma - y_e^-;\bar{\theta})]^2\{\delta - 3(\frac{1-p}{p} + (1-p)h'(\sigma^- - y_e;\bar{\theta})) \\ &\quad +3(1-p)^2 h'(\sigma^- - y_e;\bar{\theta})\frac{[h'(\sigma^- - y_e;\bar{\theta})]^2}{[h'(\sigma - y_e^-;\bar{\theta})]^2}\} \\ &= -pg_2[h'(\sigma - y_e^-;\bar{\theta})]^2\{(\delta - 3\frac{1-p}{p}) \\ &\quad +3(1-p)h'(\sigma^- - y_e;\bar{\theta})((1-p)\frac{[h'(\sigma^- - y_e;\bar{\theta})]^2}{[h'(\sigma - y_e^-;\bar{\theta})]^2} - 1)\}.\end{aligned}$$

Now

$$\begin{aligned}\sigma^{-} - y_e &= \sigma^{-} - [(y-s) - (1-p)\sigma], \\ \sigma - y_e^{-} &= \sigma - [(y-s) - (1-p)\sigma^{-}],\end{aligned}$$

and

$$\sigma^{-} - y_e - (\sigma - y_e^{-}) = \sigma^{-} - \sigma + (1-p)(\sigma - \sigma^{-}) = p(\sigma^{-} - \sigma).$$

Because for a quadratic $g(.)$, $h''(.;\bar{\theta}) > 0$, we have

$$\sigma^{-} \leq \sigma \Rightarrow \frac{[h'(\sigma^{-} - y_e; \bar{\theta})]^2}{[h'(\sigma - y_e^{-}; \bar{\theta})]^2} \leq 1.$$

Thus, for $\sigma^{-} \leq \sigma$ and $1 - 3\frac{1-p}{p} < 0$ (or $p < \frac{3}{4}$) we see that $\frac{\partial E(u_f)}{\partial \sigma} \geq 0 \Rightarrow \frac{\partial^2 E(u_f)}{\partial \sigma^2} > 0$. That is, there is no interior solution. ■

We shall now consider the choice between these two alternatives.

9.2.1 Equilibrium at the lower boundary

If all men set $\sigma = 0$ then

$$E(u_c) = pg(c_0) + (1-p)g(c^*), \tag{A7}$$

where c_0 solves

$$\gamma'(c) = \frac{\gamma(c) + y - s + \bar{\theta}}{c^* - c + \bar{\theta}}. \tag{A8}$$

The expected utility of each father if no one commits and $\sigma = 0$ is

$$\begin{aligned}E(u_f) &= (1-p)(y-s) + p(y+z-c_0) + \delta[pg(c_0) + (1-p)g(c^*)] + p\bar{\theta} \\ &= y - s + p(c^* - c_0) + \delta[pg(c_0) + (1-p)g(c^*)] + p\bar{\theta}. \qquad \text{(A9)}\end{aligned}$$

If one father deviates and promises his wife $\sigma = y - s$, the amount spent on his child if the mother remarries will be c_1 where c_1 solves

$$\gamma'(c) = \frac{\gamma(c) + \bar{\theta}}{c^* - c + \bar{\theta}}. \tag{A10}$$

If the father remarries, the amount spent on the child of his new wife will be c_2, where c_2 solves

$$\gamma'(c) = \frac{\gamma(c) + p(y-s) + \bar{\theta}}{c^* - c + \bar{\theta}} \tag{A11}$$

and $c_0 < c_2 < c_1 < c^*$. The expected utility of the father is then

$$\begin{aligned}E_d(u_f) &= (1-p)[p(y-s) + (1-p)0] + p[p(y+z-c_2) + (1-p)(s+z-c_2)] \\ &\quad \delta[pg(c_1) + (1-p)g(c^*)] + p\bar{\theta} \qquad \text{(A12)} \\ &= p(y-s) + p(c^* - c_2) + \delta[pg(c_1) + (1-p)g(c^*)] + p\bar{\theta}.\end{aligned}$$

Taking differences, and setting $\delta = 1$, we have

$$E(u_f) - E_d(u_f) = (1-p)(y-s) + p[c_2 - c_0 + g(c_0) - g(c_1)]] \equiv D_1(p). \tag{A13}$$

We see that $D_1'(p) < 0$, because $\frac{\partial c_2}{\partial p} < 0$, $c_2 - c_0 + g(c_0) - g(c_1) < c_1 - c_0 + g(c_0) - g(c_1) < 0$, and c_0 , c_1 are independent of p. That is, the father is more likely to deviate the higher is p. Now $D_1(p) = y - s > 0$ and $D_1(1)$ and there must be p_1 such that $D_1(p_1) = 0$ and the father is indifferent between deviating and conforming. Note that

$$D_1(\frac{1}{2}) = \frac{1}{2}[(y-s) + (c_2 - c_0) + g(c_0) - g(c_1)] \tag{A14}$$

is negative for c^* close to $y + z$, implying that, in this case, $p_1 < \frac{1}{2}$.

9.2.2 Equilibrium at the upper boundary

A symmetric equilibrium at the upper boundary is either at $\sigma = y - s$, or at $\sigma = \frac{y-s+\bar{\theta}}{2-p}$, whichever is larger. The incentive compatibility constraint is the binding one if $\bar{\theta} < (y-s)(1-p)$ and the feasibility constraint binds otherwise. We shall assume here that $\bar{\theta}$ is sufficiently small to guarantee that incentive compatibility is the binding one in the "relevant range". We shall later set $\bar{\theta} = 0$ to ensure that this always holds.

If all fathers commit to pay $\sigma = \frac{y-s+\bar{\theta}}{2-p}$, implying that $c = c^*$ upon remarriage, then the expected utility of each father is

$$\begin{aligned} E(u_f) &= (1-p)[p(y-s) + (1-p)(y-s-\frac{y-s+\bar{\theta}}{2-p})] + \\ &\quad p[p(y+z-c^*) + (1-p)(y+z-c^*-\frac{y-s+\bar{\theta}}{2-p})] \\ &\quad +\delta g(c^*) + p\bar{\theta} \\ &= y - s - \frac{y-s+\bar{\theta}}{2-p}(1-p) + \delta g(c^*) + p\bar{\theta} \\ &= \frac{y-s+\bar{\theta}}{2-p} + \delta g(c^*) - (1-p)\bar{\theta}. \end{aligned} \tag{A15}$$

We first note that no father wants to deviate up from this pattern and select $\sigma > \frac{y-s+\bar{\theta}}{2-p}$. Such deviation would mean that the father cannot remarry, because $\sigma^- = \frac{y-s+\bar{\theta}}{2-p} > \bar{\theta} + y - s - \sigma(1-p)$ and also that the mother does not remarry because $\sigma > \frac{y-s+\bar{\theta}}{2-p} = \bar{\theta} + y - s - (1-p)\frac{y-s+\bar{\theta}}{2-p}$. In this case, the gain from not deviating becomes

$$\begin{aligned} E(u_f) - E_d(u_f) &= \frac{y-s+\bar{\theta}}{2-p} - (1-p)\bar{\theta} - (y-s-\sigma) \\ &> \frac{y-s+\bar{\theta}}{2-p} - (1-p)\bar{\theta} - (y-s) + \frac{y-s+\bar{\theta}}{2-p} \qquad \text{(A16)} \\ &= p\frac{y-s+\bar{\theta}}{2-p} + p\bar{\theta} > 0. \end{aligned}$$

Now suppose that a father that deviates selects some $\sigma < \frac{y-s+\bar{\theta}}{2-p}$. Then, if the mother remarries, $c = \hat{c} = h(\sigma - y_e^-, \bar{\theta})$. That is, $\hat{c}$ is determined by

$$\begin{aligned} \gamma'(c) &= \frac{\gamma(c) + y - s - (1-p)\frac{y-s+\bar{\theta}}{2-p} - \sigma + \bar{\theta}}{c^* - c + \bar{\theta}} \\ &= \frac{\gamma(c) + \frac{y-s+\bar{\theta}}{2-p} - \sigma}{c^* - c + \bar{\theta}}. \end{aligned} \tag{A17}$$

If the father remarries $\tilde{c} = h(\sigma^- - y_e, \bar{\theta})$. That is $\tilde{c}$ is determined by

$$\begin{aligned} \gamma'(c) &= \frac{\gamma(c) + y - s - (1-p)\sigma - \frac{y-s+\bar{\theta}}{2-p} + \bar{\theta}}{c^* - c + \bar{\theta}} \\ &= \frac{\gamma(c) + (\frac{y-s+\bar{\theta}}{2-p} - \sigma)(1-p)}{c^* - c + \bar{\theta}}. \end{aligned} \tag{A18}$$

Thus, $\hat{c} \leq \tilde{c} < c^*$ and, for any fixed σ, $\hat{c}$ declines p, and $\tilde{c}$ rises in p if $\sigma < \frac{y-s+\bar{\theta}}{(2-p)^2}$ and declines in p if $\frac{y-s+\bar{\theta}}{(2-p)^2} < \sigma < \frac{y-s+\bar{\theta}}{2-p}$. The father expected utility upon deviation is now

$$\begin{aligned} E_d(u_f) &= (1-p)[p(y-s) + (1-p)(y-s-\sigma] + \\ &\quad p[p(y+z-\tilde{c}) + (1-p)(y+z-\tilde{c}-\sigma)] \\ &\quad +\delta[(1-p)g(c^*) + pg(\hat{c})] + p\bar{\theta} \\ &= (1-p)(y-s-\sigma) + p(y+z-\tilde{c}) \\ &\quad +\delta[(1-p)g(c^*) + pg(\hat{c})] + p\bar{\theta}. \end{aligned} \tag{A19}$$

Let $\sigma(p)$ be the optimal deviation given that others choose $\sigma = \frac{y-s+\bar{\theta}}{2-p} \equiv \sigma^-(p)$, then using the envelope theorem,

$$\begin{aligned} \frac{dE_d(u_f)}{dp} &= c^* - \tilde{c} + \delta(g(\hat{c}) - g(c^*)) + p\delta g'(\hat{c})h'(\sigma - y_e^-, \bar{\theta})(\frac{d\sigma^-}{dp}(1-p) - \sigma^-(p)) \\ &\quad -ph'(\sigma^- - y_e, \bar{\theta})(\frac{d\sigma^-}{dp} - \sigma(p)) + \bar{\theta}. \end{aligned} \tag{A20}$$

Setting $\delta = 1$, and defining

$$D_0(p) \equiv E(u_f) - E_d(u_f) = \frac{y-s+\bar{\theta}}{2-p} - (y-s) - p(c^* - \tilde{c}) + (1-p)\sigma(p) + p[(g(c^*) - g(\hat{c})], \tag{A21}$$

we see that $D_0(0) = -(y-s) + \frac{y-s}{2}$ and $D_0(1) = g(c^*) - c^* - (g(\hat{c}) - \tilde{c}) > g(c^*) - c^* - (g(\hat{c}) - \tilde{c}) > 0$. Therefore, there must exist p_0 such that $D_0(p_0) = 0$ and the father is indifferent between conforming to $\frac{y-s+\bar{\theta}}{2-p}$ and deviating to $\sigma(p)$.

It is seen from (A20) that if $\sigma(p) < \frac{d\sigma^-}{dp} = \frac{y-s+\bar{\theta}}{(2-p)2}$then $\frac{dE_d(u_f)}{dp} - \bar{\theta} < 0$, because $\frac{d\sigma^-}{dp}(1-p) - \sigma^-(p) = -\frac{y-s+\bar{\theta}}{2-p}$ and $c^* - \tilde{c} + g(\hat{c}) - g(c^*)$ are both negative. In contrast, $\frac{dE(u_f)}{dp} - \bar{\theta} > 0$, because $\frac{y-s+\bar{\theta}}{2-p}$ rises in p. It follows that $D_0'(p) > 0$ if the best response against $\sigma^- = \frac{y-s+\bar{\theta}}{2-p}$ is to set $\sigma(p) < \frac{y-s+\bar{\theta}}{(2-p)2}$ and, as a special case, if $\sigma(p) = 0$. Such an outcome arises, if $E_d(u_f)$ is convex globally or locally, in the sense that $\frac{\partial E_d(u_f)}{\partial \sigma} = 0 \Rightarrow \frac{\partial^2 E_d(u_f)}{\partial \sigma^e} > 0$, so that there is no interior solution for σ in the range $(0, \frac{y-s+\bar{\theta}}{2-p})$. In this case, $D_0'(p) > 0$ and there is a unique p_0 such that $D_0(p_0) = 0$ and the father is indifferent between conforming to $\frac{y-s+\bar{\theta}}{2-p}$ and deviating to $\sigma = 0$.

Assuming that either $\sigma(p) = 0$ or $\sigma(p) = \frac{y-s+\bar{\theta}}{2-p}$ are the only possible best responses against $\frac{y-s+\bar{\theta}}{2-p}$, definition (28) implies that $D_0(p) > -D_1(p)$ if σ and σ^- are strategic complements and $D_0(p) < -D_1(p)$ if σ and σ^- are strategic substitutes, as defined in (28). From the results that $D_0'(p) > 0$ and $D_1'(p) < 0$, it then follows that $p_1 > p_0$ if σ and σ^- are strategic complements $p_1 > p_0$ if σ and σ^- are strategic complements and $p_1 < p_0$ if σ and σ^- are strategic substitutes.

9.2.3 An example

The functional forms used in the examples are as follows. The utility of the child is

$$g(c) = 2c - \frac{c^2}{2}.$$

This specification satisfies $g'(\hat{c}) \leq 2$ so that the father does not want to transfer if the mother remarries. In this case,

$$\begin{aligned} c^* &= 1, \\ g(c^*) &= \frac{3}{2}. \end{aligned}$$

The implied function used in the solution of the Nash Bargaining model is

$$\begin{aligned} \gamma(c) &= g(c) - c - (g(c^*) - c^*) = c - \frac{c^2}{2} - \frac{1}{2} \\ \gamma'(c) &= 1 - c. \end{aligned}$$

Solving

$$\gamma'(c) = \frac{\gamma(c) - x + \bar{\theta}}{c^* - c + \bar{\theta}},$$

we get

$$\hat{c} = 1 + \frac{\bar{\theta}}{3} - \frac{[\bar{\theta}(\bar{\theta} + 6) + 6(y_e^- - \sigma)]^{\frac{1}{2}}}{3}$$

which is reduced to

$$\hat{c} = 1 - (\frac{2}{3}(y_e^- - \sigma))^{\frac{1}{2}},$$

if $\bar{\theta} = 0$. Also if $\bar{\theta} = x$ then $\hat{c} = c^* = 1$.

We can use this specification to verify that the Nash bargaining problems defined above have interior solutions, as we have implicitly assumed. Rewriting the Nash requirement as

$$\gamma'(c)(c^* - c + \bar{\theta}) - \gamma(c) = y + z - c^* + \bar{\theta}$$

and noting that $\gamma'(0) = 1$, we require that

$$\begin{aligned} c^* + \bar{\theta} + g(c^*) - c^* &> y + z - c^* + \bar{\theta} \\ g(c^*) + c^* &> y + z, \end{aligned}$$

which would guarantee that $0 < c_0 < c^*$. Because c_0 is the lowest solution, the other cases values $c_{1,} c_{2,}$ $\hat{c}$, and $\tilde{c}$ are also positive.

A convenient choice for the matching function $p = \phi(d)$ is

$$\phi(x) = 1 - (1 - d)^{\alpha},$$

where $\alpha \geq 1$. This function maps from $[0, 1]$ to $[0, 1]$ and satisfies $\phi(0) = 0$, $\phi(1) = 1$, $\phi(d) > x$, $\phi'(d) > 0$.

A higher α corresponds to a better matching function. With a uniform distribution for θ on $[-u, u]$, the equilibrium condition becomes

$$\begin{aligned} p &= 1 - (1 - F(\theta^*(p)))^{\alpha} \\ &= 1 - (1 - \frac{\theta^*(p) + u}{2u})^{\alpha} \end{aligned}$$

or,

$$\theta^*(p) = u(1 - 2(1 - p)^{\frac{1}{\alpha}}).$$

Figure A1 : The feasibility and incentive compatibility constraints on the commitment by the father, σ, and the commitments of others σ^{-}

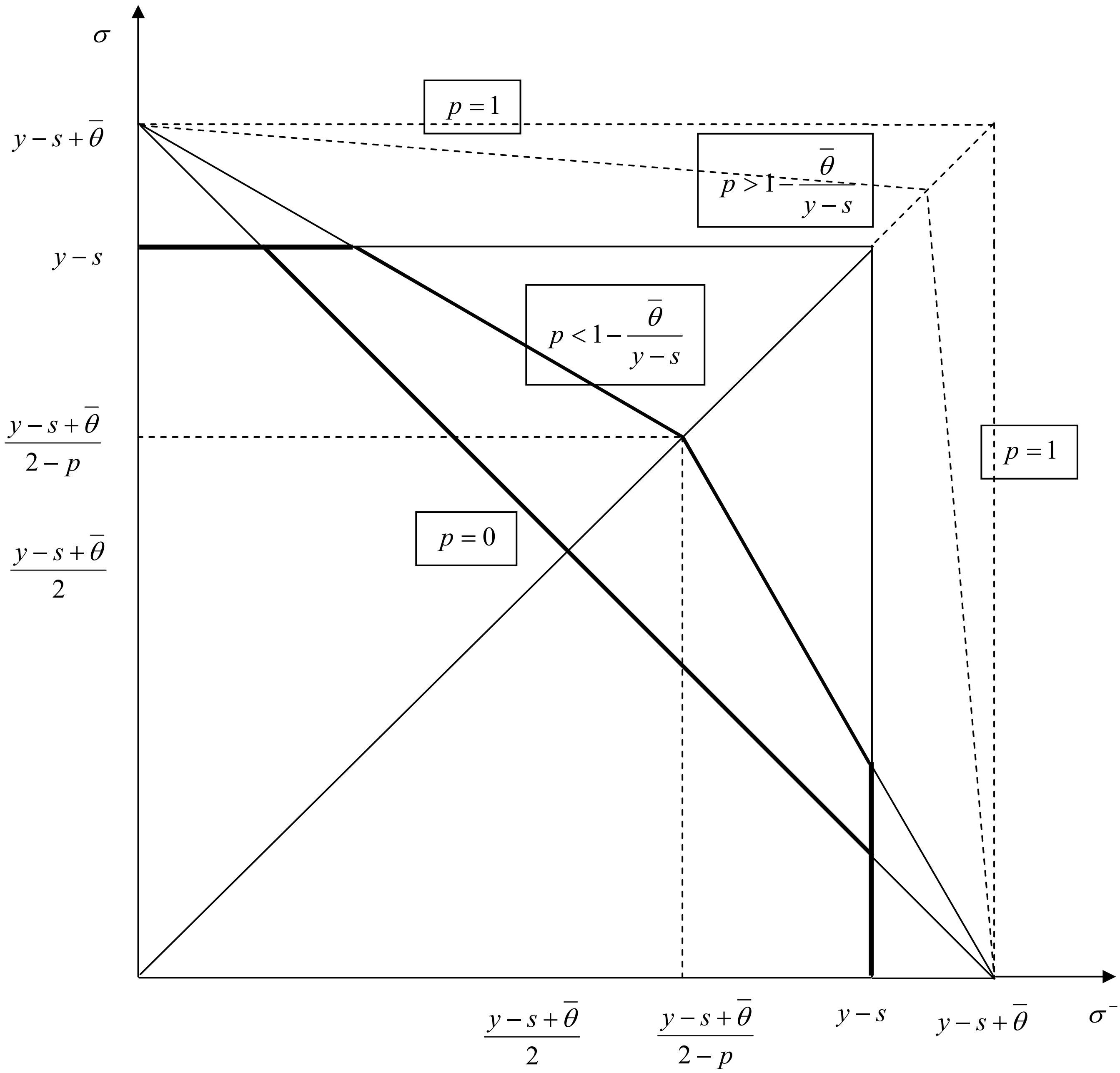


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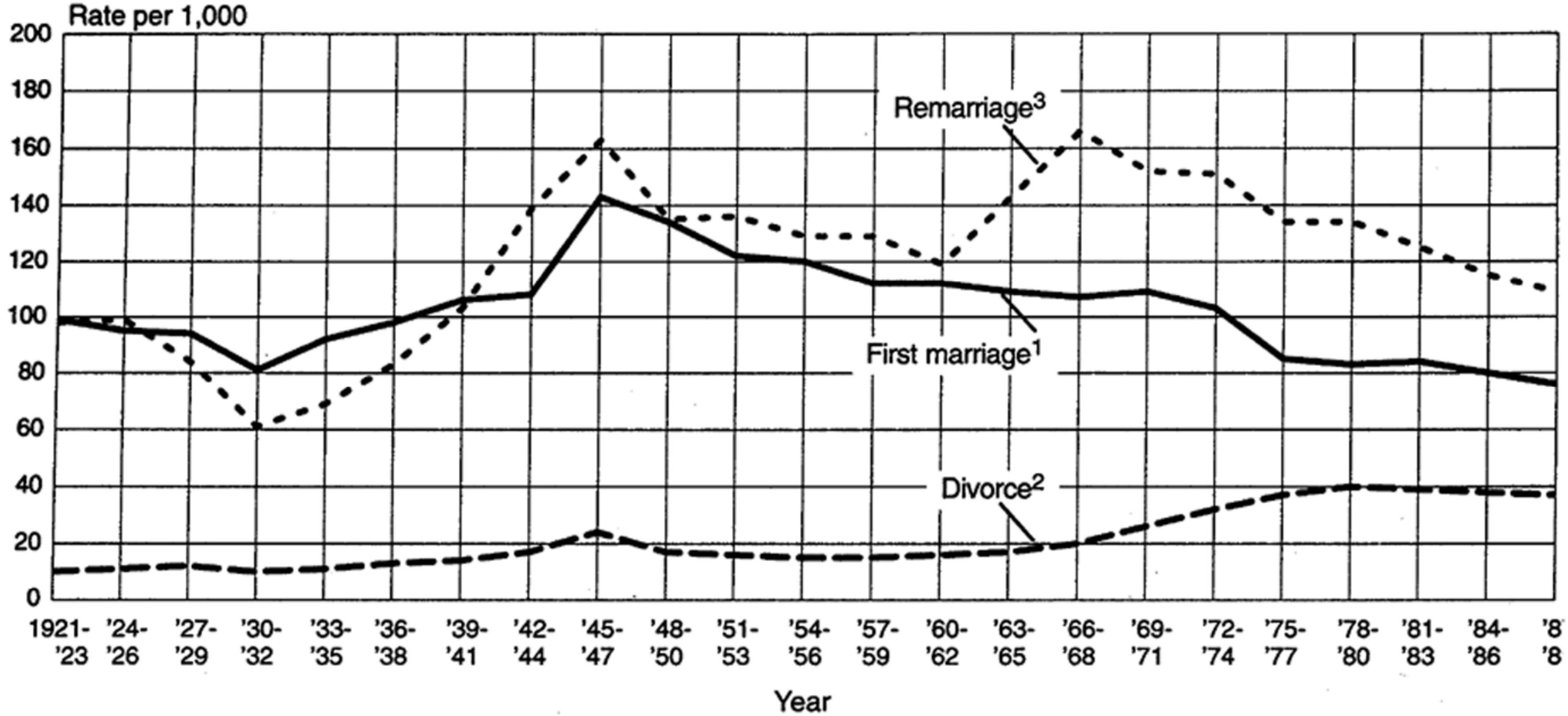
Table B1: Marital history by age and sex US, 1996[1]

Age:	30-34	35-39	40-49	50-59	60-69
Men (White, non-Hispanic)					
Never married	25.5	16.9	10.5	5.1	4.6
Married Once	64.1	66.0	62.9	64.4	70.4
Married Twice	9.7	14.6	21.2	22.8	19.2
Married 3 or more times	0.7	2.5	5.4	7.8	5.9
Women (White, non-Hispanic)					
Never married	14.3	10.9	6.8	4.2	3.2
Married Once	70.5	67.8	65.3	68.7	74.6
Married Twice	13.5	17.4	21.9	20.0	17.7
Married 3 or more times	1.7	3.8	5.9	7.2	4.4

[1] Source: U.S. Census Bureau, Survey of Income and Program Participation (SIPP), 1996 wave.

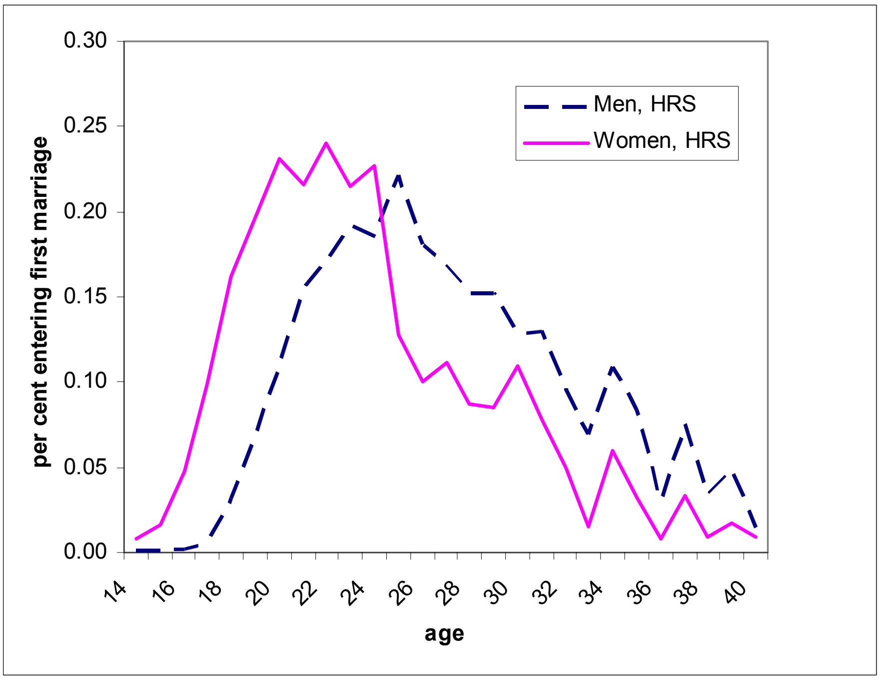
Figure 1. **Rates of First Marriage, Divorce, and Remarriage: 1921 to 1989**
(3-Year Averages)



Source: National Center for Health Statistics.

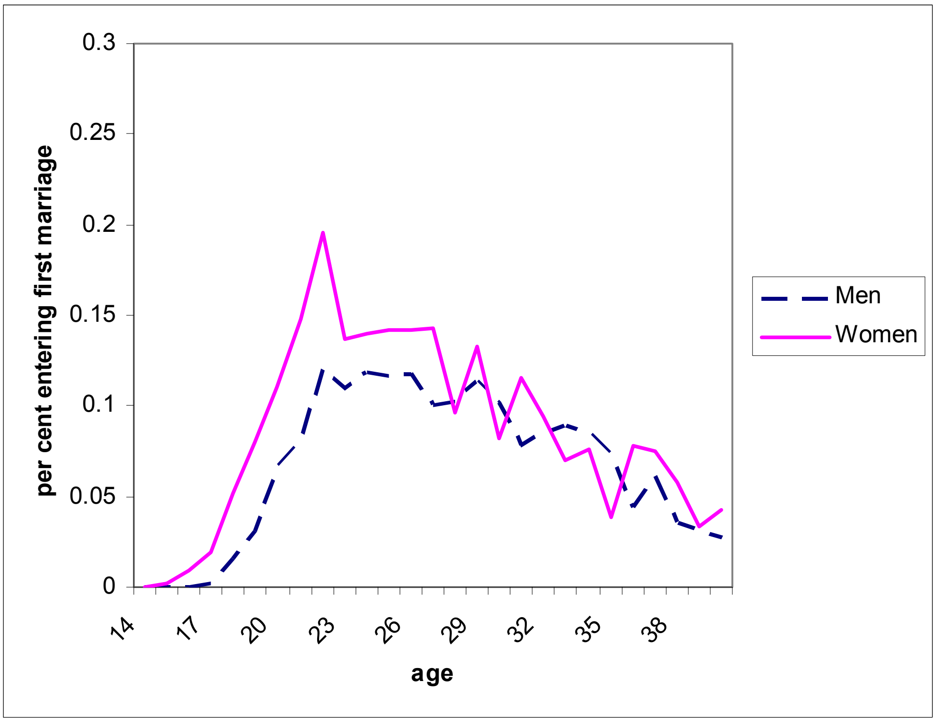
[1]First marriages per 1,000 single women, 15 to 44 years old.
[2]Divorces per 1,000 married women, 15 to 44 years old.
[3]Remarriages per 1,000 widowed and divorced women, 15 to 54 years old.

Figure B2 : Entry into first marriage, US., HRS



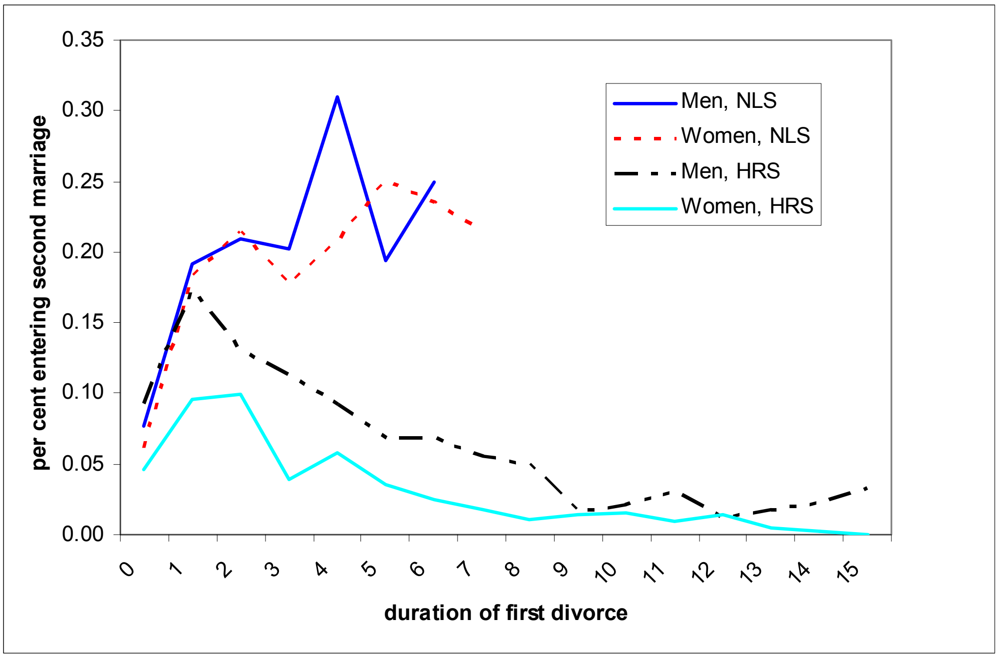
Source : HRS panel data , 1992 wave

Figure B3: Entry into first marriage, US., NLS



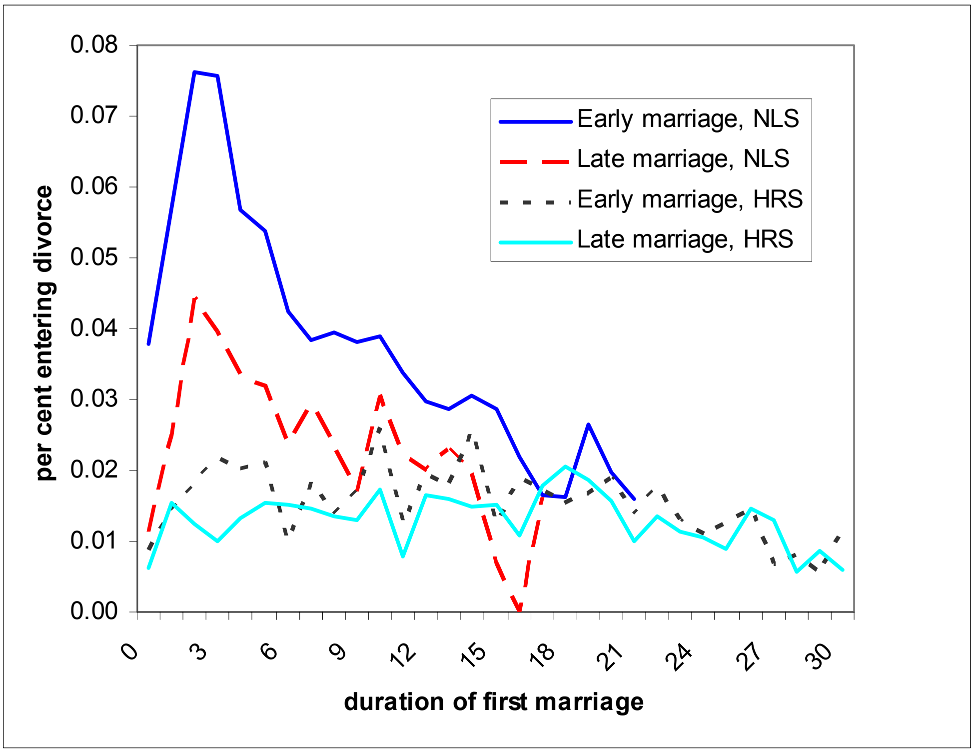
Source : NLS panel data, Youth 1979

Figure B4: Entry in to sencond marriage



Source: NLS panel data, Youth 1979, HRS panel data, 1992 wave

Figure B5: Entry into divorce



Source: NLS panel data, Youth 1979, HRS panel data, 1992 wave