Abortions, Inequality and Family Formation

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Abstract

In the last three decades over a million abortions were performed annually in the United States. Empirical studies such as Gruber, Levine and Staiger (1999) assess the impact of legalization of abortions on living conditions of children. They argue that legalization of abortions provides better living conditions and human capital endowments to surviving children. This paper takes seriously the hypothesis that legalized abortion can improve the living conditions of children and hence alter their future labor market outcomes. The main question of the paper is what are the implications of abortions for income inequality, intergenerational transmission of income and family formation. A model of fertility, human capital transmission, contraception and abortion decisions is built to answer this question quantitatively.

JEL Classifications: E24, D31, J13, J17, J62.
Key Words: Fertility, Abortions, Contraception, Income Inequality, Family Formation, Intergenerational Mobility.

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1 Introduction

Unintended pregnancies accounted for around half of the 6.4 million pregnancies in the United States in 2001. Half of these unintended pregnancies resulted in abortion (Finer and Henshaw, 2006).[1] Several recent papers have studied the consequences of abortion access empirically. Gruber, Levine and Staiger (1999) ask the following question: Would children who were not born because of abortion live in different circumstances than the average child in their cohort? The answer depends on the magnitude of two opposing effects, (i) positive selection: women use abortion to avoid bearing children in adverse circumstances and the marginal child has worse living conditions than the average child of the cohort or/and; (ii) negative selection: if the most disadvantaged women are constrained in their abortion access (geographically or financially), the marginal child has better living conditions than the average child of the cohort. They discover sizable positive selection:

> "[...]the average living circumstances of cohorts of children born immediately after abortion became legalized improved substantially relative to preceding cohorts, and relative to places where the legal status of abortion was not changing. Our results suggest that the marginal children who were not born as a result of abortion legalization would have systematically been born into less favorable circumstances if the pregnancies had not been terminated: they would have been 60 percent more likely to live in a single-parent household, 50 percent more likely to live in poverty, 45 percent more likely to be in a household collecting welfare, and 40 percent more likely to die during the first year of life." (p. 265)

In a similar study, Donohue and Levitt (2001) analyze the impact of legalized abortion on crime. Their analysis suggests that legalized abortion reduced crime rates with a twenty-year lag and find that an increase of 100 abortions per 1000 live births reduces a cohort's crime by 10%. If one uses their estimates to create a counterfactual, it turns out that crime would be 15% to 25%

[1]The debate on abortion legalization in the U.S. is dominated by two ideological positions. Pro-life supporters consider the fetus a living being and therefore view abortion as taking life. On the other extreme, the pro-choice stand views abortion as an essential woman's right to control her own body. Although these two groups often use well-rehearsed arguments for the economic consequences of legalizing abortion, the discussion between them is of moral nature. This paper views abortion access as an economic policy, and its only goal is to gleam some light on the economic consequences of the legalization of abortion for the agrregates of the economy. For more details, see Levine (2004).

higher in 1997 if abortions were not legal. They claim that using this counterfactual and previously estimated cost of crime, the social benefit of reduced crime due to legalization is of order of 30 billion U.S. dollars (about 113 dollars per capita) annually. The explanations for this strong effect of abortion on crime are either due to reduced cohort size or lowered offending rates per capita. The effect of abortions on crime comes predominantly from the lower offending rates per capita. The intuitive explanation for the lower average rates goes through two channels: (i) women who have abortions are those who are more likely to give birth to children who engage in crime or/and; (ii) women may use abortion to optimize the timing of childbearing, and consequently children can have better living environment and better human capital endowments.

The empirical studies on abortion access are not able to assess the long-run aggregate implications of the change in the average living standards of children due to abortion. The reduced form estimates of the cited works are inappropriate when computing aggregate changes due to abortions.[2] A more suitable framework would be that of a general equilibrium model of fertility and abortion decision which maps the level of abortion access into a particular intergenerational mobility pattern and ultimately, into different labor market outcomes. In such a framework, the main mechanism will work through initial human capital endowments given to children by their parents. Parents' decisions will be determined by preferences, income levels and availability of methods (contraception and abortion) to regulate fertility.

The goal of this paper is to examine the quantitative importance of access to abortion for the income inequality. A dynamic equilibrium model of marital matching, abortions, contraception and fertility choice is built to match the fertility and abortions behavior in the US economy for the late 1990s and early 2000s. The model economy is populated by heterogenous agents and this gives rise to an income distribution as an equilibrium outcome. First, the benchmark income distribution is derived in the estimated economy where abortions are available. Then, a counterfactual income distribution is derived for the case in which abortions are not allowed. The resulting difference in terms of inequality is interpreted as an evaluation of the role of abortions.

The model economy is populated by males and females who live for 3 periods, one as a child (teenager), and two as adults. Children are born with certain ability level which is correlated with the ability level of their mothers.. Parents spend resources on children which determine their

[2] Another strand of the emprical literature utilizes structural dynamic models of dicrete choice with stochastic fertility to estimate key structural paramters of the models. See, for example, Wolpin (1984), Hotz and Miller (1993) and Carro and Mira (2006).

education (skill) level when they are adults. Children (teenagers) are also engaged in premarital sex. Female teenagers can put effort to use contraceptives. They can also abort their pregnancy. Hence, some females start their adult lives with a child that was conceived out-of-wedlock in their teenage years.

Each adult (male or female) is characterized by an education level and their ability, which together determine their earnings. Adult females also differ by the presence of premarital children from their teenage years. At the start of their adult lives, males and females mate and decide to form married households or remain single. A male receives disutility from the presence of out-of-wedlock children in the household. After households are formed, married households and single females decide how many children to have and what amount of resources to spend on them. Fertility is stochastic, i.e., the quantity of children they desire is not realized with probability of one. They can use contraception and abortion as instruments to mitigate the risk stemming from this uncertainty. Contraception is an instrument that reduces the fertility risk before the realization of the fertility process, while abortion is a tool that can correct the fertility outcome after the final realization of the process. Households use a particular mix of these two instruments depending on their costs and the preferences over quantity and quality of children. Human capital endowments given to children (quality), thus, depend on the cost of abortion. The future income of children is positively correlated with their human capital endowments, and therefore is conditional on the cost of abortion as well. Using this link, the model can assess changes in the cost of abortion and their influence on the future income distribution.

After the first period of adult life, married household members may decide to dissolve the household unit. Divorced people remain single in their second period of adult life. At the start of this second period, single males and females in the first period mate with each other again. Households in this period face the same decisions as they do in the first period.

Children stay with their parents only for a period and then they become adults. Their initial conditions as adults are determined by the decisions of their parents, providing a natural framework to study intergenerational mobility.

The general equilibrium modeling approach is essential in this exercise, because any change of the cost of abortion will alter the way these costs affect future incomes. This is so because households can reconsider the way they use contraception, the number of their children, or the way they invest in their children.

This research is related to the work of Choi (2011). He embeds stochastic fertility in a life-cycle model and finds that fertility risk is more spread among less educated people and thus can be a powerful source of life-time inequality. The present paper incorporates endogenous channels of investment in children in order to evaluate the importance of abortion access for inequality. This is done by assuming a quantity-quality trade-off in the preferences for children and allowing educational achievements to be a function of the resources parents invest in their children. Models of children's quality-quantity trade-offs relating intergenerational mobility and income inequality date back to Becker and Tomes (1979). Aiyagari, Greenwood and Guner (2000) and Greenwood, Guner and Knowles (2003) use this approach in a search equilibrium framework to analyze the interaction between the marriage market and investment in children. **?** studies the link between rising income inequality and delay in fertility in a Huggett framework with a quantity-quality fertility choice. Restuccia and Urrutia (2004) emphasize the role of early children's education in the intergenerational persistence of income.[3] The most important channel for inequality in the framework presented here is the difference between individuals in terms of their initial ability and parental investments. There is a large literature that emphasize the role of initial conditions for inequality. Recently, this approach is taken by Huggett, Ventura and Yaron (2006). They use the human capital accumulation model of Ben-Porath (1967) to reproduce the dynamics of the U.S. earnings distribution. Huggett, Ventura and Yaron (2010) discuss the role of the initial conditions in explaining lifetime inequality.

2 Facts

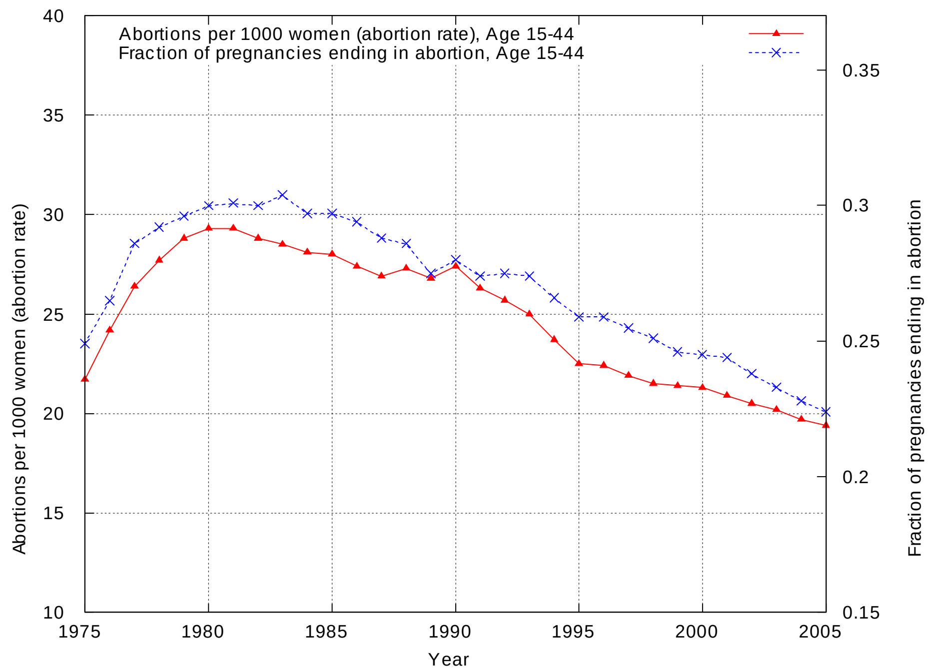
History. Roe v. Wade (1973) made abortion legal across the United States. The Supreme Court held that the constitutional right to privacy extends to a woman's decision to have an abortion. The decision also stated that this right should be balanced against the health status of the pregnant woman and "the potentiality of human life" (Page 410 U. S. 114). These two interests were considered at their weakest in the first trimester of the pregnancy and the decision of abortion is left to the pregnant woman (and the attending physician).

The number of induced abortions was consistently increasing in the decades following the Court's decision. From around 745 000 procedures performed in 1973, the abortion number

[3] Cuhna and Heckman (2007) present a formal model of child development that also emphasize the improtance of early child investments.

skyrocketed to 1.6 million procedures in 1990.[4] In the last twenty years, the total number of induced abortions decreased but was always above 1.2 million.

Figure 1: Evolution of Abortions, 1975-2005



When population changes are accounted for, the abortion practices follow a similar pattern. The abortion rate defined as the number of induced abortions per 1000 women of age 15-44 is depicted in Figure 1. It reaches a historical peak of 29 abortions per 1000 women in 1980 and declines in the 90s and 00s. This diminishing trend, however, never reaches levels of less than 19 abortions.

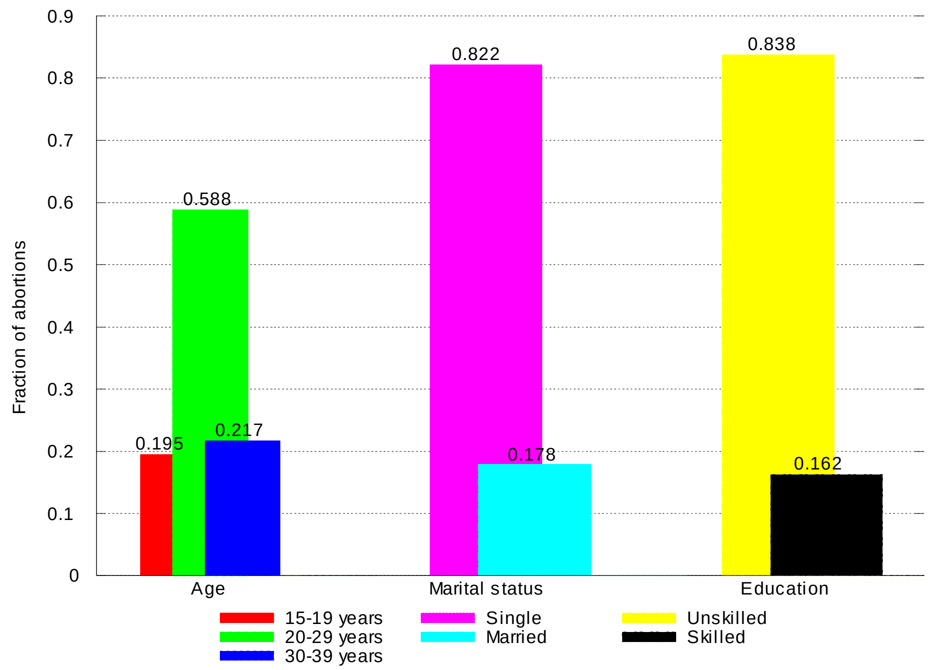
To understand the real extent to which abortions are used to control women's fertility, consider the number of interventions as a fraction of all pregnancies occurring to women of age 15-44. This measure (also shown in Figure 1) peaks in the early 80s when one third of all pregnancies are aborted. In recent years the utilization of abortion declines but the fraction of the aborted pregnancies is never less than 20%.

The historical evolution of abortion practices reveals a major boom of their usage in the years after the legalization and a minor decline in recent time. The numbers show that the abortions nowadays are used as much as in the 70s and continue to be a major way of correcting for unfavor-

[4] All data sources for the figures below are presented in Appendix A.

able fertility outcomes.

Figure 2: Fraction of Abortions by Age, Marital Status, and Education, 2000



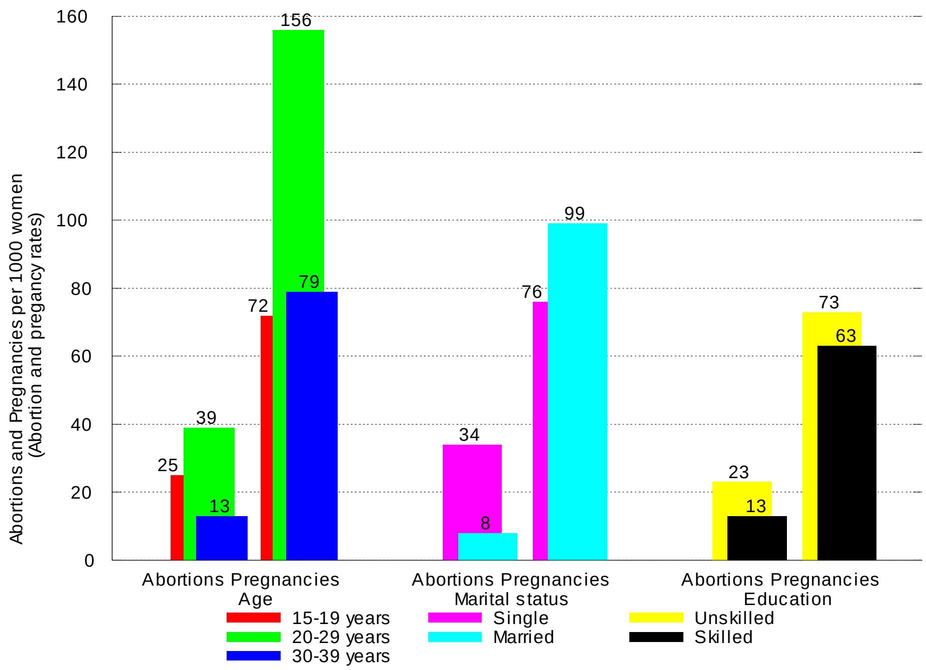
Who has abortions? A possible channel that links abortion numbers and income inequality can be the living environment and initial human capital endowments of children. In order to assess the feasibility of such a channel, one should look at the cross-sectional facts on women who abort: The typical abortion patient is young, single and unskilled (defined as non-college educated). Figure 2 plots the fractions of abortions to women by their age, marital status and education. Specifically, one out of every five women (19.5%) having abortions in 2000 were adolescents. Women in their 20s accounted for more than half of all abortions (58.8%), and 21.7% of the abortions occurred to women in their 30s.[5] On the other hand, the faction of abortions performed by single women (82.2%) is almost 5 times higher than the abortions of married (17.8%). Finally, the educational division of abortion patients reveal that unskilled women account for 83.8% of all abortions in 2000, while skilled recipients of the medical procedure are only 16.2%.

The high proportion of abortions occurring to young, single and unskilled women may be a result of high fertility risk (unintended pregnancies) for these populations. High acceptance and willingness to use the abortion procedure might have contributed as well this differential use of

[5]The fraction of abortions to women 40 or older is very small and is neglected in this analysis.

abortion. Figure 3 depicts the number of abortions and pregnancies per 1000 women (abortion and pregnancy rates) in subgroups of the female population by age, marital status and education. The pregnancy rate for women in their 20s is much higher than for any other age subgroup of the population. On the other hand, the abortion rate for women of age 20-29 is also the highest.

Figure 3: Abortion and Pregnancy Rates by Age, Marital Status, and Education, 2000



3 Economic Environment

Consider an overlapping generations economy with a continuum of individuals, each of which lives for three periods. Individuals are either males or females, and these groups are of equal size. In the first period of their lives agents are children (teenagers). They are born with an ability level and live with their parents who invest in the human capital development of the teenagers. The ability of teenagers is correlated with the ability of their mother. This emphasizes the intergenerational transmission of talent. Female teenagers may encounter a sexual contact with their male peers and face a probability of getting pregnant. They exert effort in avoiding the pregnancy through contraception. This effort comes at a cost and reduces the probability of getting pregnant. They also have the option to terminate a realized pregnancy through an abortion at a given cost. If they do not terminate it, a premarital teenage birth is realized.

At the end of their teenage period, teenagers become skilled (college educated) with certain probability or else stay unskilled (non-college educated). The odds of becoming skilled for a teenager are increasing in the human capital investment of the parents, and in the case of the females, is decreasing if a premarital teenage birth occurs. The educational level of teenagers, jointly with their ability, determine their starting positions in the labor market, that is, they are the inputs for building the initial human capital stock with which adults operate.

After the teenage period, individuals become adults. They are fertile for the next two periods (young and old adults) and are also active workers. During the two periods they have a unit time endowment per period which they supply inelastically to the labor market in exchange for a wage per human capital unit. People accumulate human capital between periods. In the case of females the evolution of human capital is influenced by the number of children present in the household.[6]

Young adult females and males match in a marriage market at the start of the period and form household units for joint consumption, and making and raising children. These units can be married, single female, or single male households. Married and single female households can have children and make decisions about consumption, fertility, and resources spent on their children's development while male-headed households cannot have children and care only about their consumption.

When young married adults turn old, they can break out of their household units at the start of the period. In this case they spend their time when old as single. The household units that do not experience a divorce continue their lives as old married households. Young single adults turned old, on the other hand, match again with other never-married old and can form married, single female, or single male households. Old households make the same decisions as the young ones.

Fertility decision of a household consists of the number of desired children, contraception and abortion choice. Fertility is stochastic, i.e., the number of desired children is not realized with probability one. Before making the fertility choice households may decide to purchase contraception treatment which reduces the chances of having more pregnancies than desired. After the realization of the stochastic process for fertility, households may decide to use abortion to terminate some of the pregnancies. Hence, contraception and abortion are instruments which households utilize in coping with the stochastic nature of fertility.

[6]This formulation of female human capital accumulation is well-established in the literature. See, for instance Attanasio, Low and Sanchez-Marcos (2008) and Miller (2011).

3.1 Adults

Human Capital and its Evolution. The level of human capital at the start of the adult life is determined by the ability level, λ, and whether the person is skilled, $e = 1$, or not, $e = 0$. The starting values of human capital are given by

$$h_1 = \lambda \chi_e$$

where the subscript 1 denotes that individuals are in the first period of their adult lives.[7] The parameters χ_0 and χ_1 indicate the dependence of human capital levels on educational achievements. Skilled and unskilled workers of the same gender and of same ability levels have different human capital levels. In particular, the skilled workers operate with higher stock of human capital, i.e., $\chi_1 > \chi_0$.

Human capital evolution for females over the life cycle is described by

$$h_2^f = \begin{cases} \xi h_1^f \text{ if } n = 0 \\ (1-\tau)\xi h_1^f \text{ if } n > 0 \end{cases}$$

where n denotes the number of children of the female in the period.

The parameter $\xi > 1$ summarizes the rate at which human capital evolves. In the presence of children in the family, the growth of human capital for the female is taxed at a rate $0 < \tau < 1$. This formulation reflects the fact that human capital accumulation process for females is disrupted in the presence of children.

Human capital for males evolves according to

$$h_2^m = \xi h_1^m.$$

Income. Male workers receive wage w per efficiency unit of human capital. Female workers, however, receive just $\varkappa w$ per unit of human capital. The parameter $0 < \varkappa < 1$ reflects the gender gap in income. A married household income equals $\varkappa w h^f + w h^m$, while a single female household income is $\varkappa w h^f$. Finally, a single male household's income is given by $w h^m$.

Household Formation and Dissolution. Young adult individuals form households in a marriage market where they meet other young adults of the opposite gender. Individuals match randomly.

[7]The life period of the individuals are indexed for the remainder of the paper as 0 for the teenage period, 1 for the yound adult period, and 2 for the old adult period.

A potential match appears together with a match-specific quality parameter γ drawn from a distribution $\mathbf{\Gamma}(\gamma)$. In addition, all things equal males prefer females without a teenage premarital birth (see section on household preferences). Young adults compare the present discounted utilities of becoming married and staying single and based on that choose to marry or not.

As adults age and become old, the married households formed when adults were young draw a new match-specific quality from a conditional distribution $\mathbf{\Phi}(\gamma|\gamma_{-1})$. Based on that old married adults may decide to break out the married household they have formed (divorce) and stay single for the last period of their lives. On the other hand, old adults who were single as young match again and may form married households.

Fertility. Married and single female households can have children. Females in the household choose the number of children which they desire to have $k \in \{0, 1, 2, \ldots, N\}$ in a period. In the first adult period of her life, the female of the household might already have had a birth as a teenager. This potential child is considered to be delivered in the first period of adulthood and its presence is known when k is chosen. The probability of having male or female children is the same for all pregnancies. Due to the stochastic nature of human fertility, the number of realized children $\widetilde{n} \in \{0, 1, 2, \ldots, N\}$ is described by a $N+1$-by-$N+1$ matrix $\Pi_j^{r,s}$. Each row of this matrix represents the probabilities that $\widetilde{n}$ children will be realized given the choice k. Therefore,

$$\Pi_j^{r,s} = \begin{bmatrix} \Pr(\widetilde{n}=0|k=0) & \cdots & \Pr(\widetilde{n}=N|k=0) \\ \vdots & \ddots & \vdots \\ \Pr(\widetilde{n}=0|k=N) & \cdots & \Pr(\widetilde{n}=N|k=N) \end{bmatrix}$$

where $j \in \{1, 2\}$ is the age of the fertile household, $r \in \{f, fm\}$ indexes whether the household is single female (f) or married (fm), and $s \in \{0, 1\}$ is an indicator whether the household has purchased contraception treatment or not (discussed shortly). For example, $\Pi_1^{fm,0}$is the transition matrix for young, married household which does not use contraception. The first row of this matrix represents the probabilities with which 0, 1,..., or N births might occur given that the household has decided not to have children. The first row, first column element, $\pi_{1,00}^{fm,0} = \Pr(\widetilde{n}=0|k=0)$ is the probability that no birth occurs given that the household has decided on being childless. The matrix index of each element π follows the number of desired children k, and the number of realized births $\widetilde{n}$. This implies that this particular element is situated on the $k+1$ row and on the $\widetilde{n}+1$ column of the matrix. Note that each row sums up to one: $\sum_{i=0}^{N} \Pr(\widetilde{n}=i|k=i') = 1$, for every $i' \in \{0, 1, 2, \ldots, N\}$.

Contraception and abortion. If contraception treatment is purchased by a household ($s = 1$), then the $N + 1$-by-$N + 1$ transition matrix from k to $\widetilde{n}$ is $\Pi_j^{r,1}$, otherwise ($s = 0$) it is $\Pi_j^{r,0}$. The purchase of contraception treatment strengthens the diagonal of the transition matrix. In the first period of adulthood the fertility matrices $\Pi_1^{r,s}$ assign relatively higher probabilities to fertility outcomes higher than the desired, while in the second period, probabilities are relatively higher for lower fertility outcomes than the desired. This reflects the fact that human ability of reproduction decreases with age. The fertility matrices are also contingent on the household type. This reflects the fact that single females experience higher unintended pregnancy rates than married females.

Abortion within the household, on the other hand, is defined as a medical procedure that can be performed after the realization of stochastic fertility process. It can bring back the realization of the number of children to the original choice k or to $k+1, \ldots, \widetilde{n}$ if $\widetilde{n} > k$ with $\widetilde{n}-k$ or $\widetilde{n}-k-1, \ldots, 0$ abortions performed. If $\widetilde{n} \leq k$, no abortions are performed. The final number of children born in a household per period is $n = \widetilde{n} - a + y$ for young adults, and $n = \widetilde{n} - a$ for old adults, where y is the number of teenage premarital births (see next subsection) and a is the number of performed abortions per period ($\widetilde{n} \geq a \geq 0$).

Parents' spending on their children. Once the final number of children in a household per period, n, is realized, parents choose b, the resources spent on the human capital formation of each of their children.

Consumption. Within the household, consumption is a public good subject to congestion. Its level is decided by the adult members of the household.

Preferences. Adult individuals derive utility from consumption. Within the household, consumption is a public good subject to congestion. Its level is decided by the adult members of the household. They are altruistic towards their children, and therefore if children are present in the household, they derive additional utility from the number of children and the resources they spend jointly on each child for human capital development. If adults live in a married household they receive a match-specific quality γ and have a negative preference towards children coming from a premarital teenage births (in the case of young adults).

3.2 Teenagers

Teenage premarital fertility, contraception and abortion. Female teenagers may have sex with male teenagers. The probability of getting pregnant in the process is $p(z)$, where $z \in \{0, 1\}$ is a discrete choice whether the female teenager puts effort into contraception. If effort is exercised $(z = 1)$, it brings a utility cost to the female teenager, κ_z.

If a pregnancy occurs, the female teenager has the option to abort the pregnancy at a utility cost κ_a. The variable $y_p \in \{0, 1\}$ takes the value of one if the female teenager gets pregnant, and zero otherwise. If she does not perform an abortion the resulting birth is called a teenage premarital birth. The variables $y \in \{0, 1\}$ takes the value of one if a female teenager has a teenage premarital birth, and zero otherwise.

Educational Achievements. Teenagers can be educated, $e = 1$, or not, $e = 0$. Female teenagers can obtain a college education and become skilled with probability $q^f(b, y)$. This probability is increasing in the investment given by the parents, b, and it is lower if a teenage premarital birth is realized, i.e., $q^f(b, 0) > q^f(b, 1)$.

Male teenagers face a corresponding probability function of attaining high educational degree and becoming skilled given by $q^m(b)$. It is again increasing in the investment received by the parents, b.

Ability. Teenagers are assumed to obtain a labor market ability which they can utilize when entering the labor market as adults. Denote this labor ability by λ. Labor abilities are drawn from a probability distribution function $\mathbf{\Lambda}(\lambda|\lambda^f_{-1})$ which is conditional on the ability of the mother, λ^f_{-1}.

4 Decision Making

The economic environment poses several decision problems for the individuals. Teenagers do not have a say when determining consumption levels within their parents' households. They are given access to the consumption good which is determined by the parent(s). The female teenagers, however, have to make a choice in respect to whether to exert a contraception level or not, and whether to perform an abortion if a teenage premarital birth occurs. These decisions are made so that the present discounted value of the their future utility streams is maximized.

Young adults have to decide whether they marry or not their potential partners for marriage. They do so by comparing the expected values of utility streams of single and married life. Adults derive utility from consumption, the number of children they have, and the resources they invest in them. Households need to make decisions concerning their contraception treatment, the number of children they desire to have, the abortions they perform (if needed), consumption levels and the resources they invest in each of their children. When making their decision choices young adults maximize the current utility levels they may obtain and the present discounted values of their utility as old adults. Consumption and investments in children are public goods within the household. Moreover, their choices do not affect the future utility stream of adults in the household. Therefore, the wife and the husband in a married household choose the same levels of consumption and investments in children given the realized number of children in the household. Children are also a public good but their presence affect the human capital accumulation process of the young female. At the end of the period young married couples may split. The number of children of these young couples may affect differently the future prospectives of males and females within the young married households. Based on that, the current decisions for the contraception treatment, the desired number of children and the number of performed abortions (if needed) may differ for husbands and wives. Therefore, it is assumed that these decisions are made solely by the female in a young married household.

Old adults also make a decision of whether to marry or stay single if they were single as young, and whether to divorce or stay married if they were married as young. Further on, they make the same decisions as in their young age with the only difference is that here they maximize just their current utility levels in respect to the decision variables since this is the terminal period in their lives.

Young and old single males do not have any children attached to them and consume their endowments.

In each period, the sequence of events within single female and married households goes like this:

1. Households choose k and whether to purchase contraception treatment ($s = 1$ or $s = 0$).

2. k is realized as $\widetilde{n}$ and households decide whether to perform abortions (if $\widetilde{n} > k$) and how many to perform (a). n is determined.

3. Households choose the human capital spending per teenager, b, and household consumption, c.

Given the structure of the decision making, it is convenient to start describing the decision problems in the terminal period of life, and move back to the start of life when individuals are teenagers.

4.1 Old Adults' Problems

4.1.1 Old Single Female Adults

Consider the problem of an old single female after the fertility and contraception choices are made, and fertility outcome $\widetilde{n}$ is realized. The single female is about to decide how many abortions to perform given the realization $\widetilde{n}$ and the contraception decision s. Let the value function associated with this problem be $\widetilde{V}_2^f(h_2^f, e^f, \widetilde{n}, s)$. Denote by $V_2^f(h_2^f, e^f)$, the value of the problem that a household faces before fertility and contraception choices are made, and the realization of the fertility outcome $\widetilde{n}$ is not yet known. The subscript 2 signifies the second period of adult life, while the superscript f stands for a single female.

The problem after the fertility outcome is realized is given by

$$\widetilde{V}_2^f(h_2^f, e^f, \widetilde{n}, s) = \max_{a,c,b}\{u_2^f(c, b, n) - C_2^{f,a}(a, e^f)\} \tag{1}$$

subject to

$$c + bn \leq wh_2^f,$$

and

$$n = \widetilde{n} - a,$$

where a is the number of performed abortions. The per period utility function for old single females $u_2^f(c, b, n)$ has as arguments the consumption level c, the investment in children b, and the number of children n. The utility cost related to the number of performed abortions is represented by $C_2^{f,a}(a, e^f)$. It is a function of the number of abortions and the education of the female.[8] The per period utility net of the abortion cost is maximized with respect to the number of abortions, consumption, and the resources spent on each teenager in the household. The budget constraint of the problem states that the sum of consumption, spending on children should be feasible given the household's income. The second constraint of the problem states that the final number of children

[8]The utility cost of abortions might depend on the age of the female (the subscript 2), and the fact that the female is single (the superscript f). The second superscript, a, stands for abortions. This is needed, so that the utility cost of abortions $C_j^{r,a}$ is distinguishable from the utility cost of contraception $C_j^{r,s}$, and the optimal decision rule for consumption C_j^r, where $r \in \{f, fm\}$ stands for the type of household, and $j \in \{1, 2\}$ is the age.

within the family, n, is derived by subtracting the number of abortions performed (if any) from the fertility realization in terms of pregnancies, $\widetilde{n}$. The decision rules associated with problem (1) are $A_2^f(h_2^f, e^f, \widetilde{n}, s)$, $C_2^f(h_2^f, e^f, \widetilde{n}, s)$ and $B_2^f(h_2^f, e^f, \widetilde{n}, s)$.

Then, the problem before the realization of the fertility process is

$$V_2^f(h_2^f, e^f) = \max_{k,s}\{\sum_{\widetilde{n}=0}^{N} \pi_{2,k\widetilde{n}}^{f,s} \widetilde{V}_2^f(h_2^f, e^f, \widetilde{n}, s) - C_2^{f,s}(s, e^f)\}, \tag{2}$$

where $\pi_{2,k\widetilde{n}}^{f,s}$ is the $(k+1, \widetilde{n}+1)$-th element of $\Pi_2^{f,s}$ and $C_2^{f,s}(s, e^f)$ denotes the utility cost of contraception. Here the objective function is maximized with respect to the desired number of children and contraception. When making these decisions, individuals take into account the value function $\widetilde{V}_2^f(h_2^f, e^f, \widetilde{n}, s)$ and the corresponding decision rules for abortions, consumption and investments in children. The decisions for problem (2) are given by $K_2^f(h_2^f, e^f)$ and $S_2^f(h_2^f, e^f)$.

There is an important trade-off between contraception and abortion in the two-step decision procedure utilized by people in this economy to determine their consumption, the number of children and the investments in these children. At the start of the period, single females (and married people, whose decisions are described later) uncertainty about the realization of their fertility. They choose how many children they would like to have (k) but also decide whether to reduce the fertility uncertainty by using contraception (s). The usage of contraception comes at a cost ($C_2^{f,s}(s, e^f)$). An alternative (and complementary) way to cope with fertility risk is to abort some of the realized pregnancies at a utility cost $C_2^{f,a}(a, e^f)$. Depending on the underlying fertility uncertainty, the cost structure, and the individual state variables, some females may put their effort into contraception, others may rely solely on abortions. Moreover, some individuals may both utilize contraception and abortions to achieve their desired number of children, while others may use neither of these two insurance techniques.

4.1.2 Old Single Male Adults

Old single males do not have any children attached to them and consume everything they have. Let the value function associated with the problem of the single males be

$$V_2^m(h^m) = u_2^m(wh_2^m). \tag{3}$$

4.1.3 Old Married Adults

Old married female adults (fm) solve

$$\widetilde{V}_2^{fm}(h_2^f, e^f, h_2^m, \gamma, \widetilde{n}, s) = \max_{a,c,b}\{u_2^{fm}(c, b, n) + \gamma - C_2^{fm,a}(a, e^f)\} \tag{4}$$

subject to

$$c + bn \leq wh_2^f + wh_2^m,$$

and

$$n = \widetilde{n} - a$$

before the realization of the fertility process. The per period utility function here is $u_2^{fm}(c, b, n)$ and the cost of abortions is $C_2^{fm,a}(a, e^f)$. The corresponding decision rules are: $A_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s)$ for the number of performed abortions, $C_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s)$ for the household's consumption, and $B_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s)$ for the resources invested in each of the teenagers in the household. The major difference of the old married single people problem compared to the problem (1) is that the income level of the household is now given by the joint income of the wife and the husband, $wh_2^f + wh_2^m$. The problem before the fertility uncertainty is resolved is

$$V_2^{fm}(h_2^f, e^f, h_2^m, \gamma) = \max_{k,s}\{\sum_{\widetilde{n}=0}^{N} \pi_{2,k\widetilde{n}}^{fm,s} \widetilde{V}_2^{fm}(h_2^f, e^f, h_2^m, \gamma, \widetilde{n}, s) - C_2^{fm,s}(s, e^f)\}. \tag{5}$$

where $\pi_{2,k\widetilde{n}}^{fm,s}$ is the $(k+1, \widetilde{n}+1)$-th element of $\Pi_2^{fm,s}$ and $C_2^{fm,s}(s, e^f)$ is the utility cost of contraception. The corresponding decision rules for the desired number of children and contraception are $K_2^{fm}(h_2^f, e^f, h_2^m)$ and $S_2^{fm}(h_2^f, e^f, h_2^m)$.

What about old married males ? The superscript mf is used for them in the value functions below. This is the terminal period of the lives of adults. The fertility decisions here do not have any dynamic effects, that is, there is no differential influence of the fertility choices on the future of males and females because there are no future values. All goods are public within the family, and males and females have identical preferences over them. Therefore, the value function for old married males, after the realization of the fertility shock $\widetilde{n}$, is given by

$$\widetilde{V}_2^{mf}(h_2^m, h_2^f, e^f, \gamma, \widetilde{n}, s) = \max_{c,b}\{u_2^{mf}(c, b, n) + \gamma - C_2^{fm,a}(A_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s), e^f)\} \tag{6}$$

subject to

$$c + bn \leq wh_2^f + wh_2^m,$$

and

$$n = \widetilde{n} - A_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s).$$

Here the males take as given the decision rule for abortions of the females, $A_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s)$. Also,

$$u_2^{mf}(c, b, n) = u_2^{fm}(c, b, n),$$

as preferences over consumption and children are identical for husbands and wives. Then, it can be shown that the choices over consumption and investments in children are identical for both spouses,

$$C_2^{mf}(h_2^m, e^f, h_2^f, \widetilde{n}, s) = C_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s)$$
$$B_2^{mf}(h_2^f, e^f, h_2^m, \widetilde{n}, s) = B_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s).$$

Therefore, it is easy to conclude that

$$\widetilde{V}_2^{mf}(h_2^m, h_2^f, e^f, \gamma, \widetilde{n}, s) = \widetilde{V}_2^{fm}(h_2^f, e^f, h_2^m, \gamma, \widetilde{n}, s).$$

Then, at the start of the period, and before the fertility uncertainty is resolved, the value function of old married males is

$$V_2^{mf}(h_2^m, e^f, h_2^f, \gamma) = \sum_{\widetilde{n}=0}^{N} \pi_{2,K_2^{fm}(h_2^f, e^f, h_2^m)\widetilde{n}}^{fm,S_2^{fm}(h_2^f, e^f, h_2^m)} \widetilde{V}_2^{mf}(h_2^f, e^f, h_2^m, \gamma, \widetilde{n}, S_2^{fm}(h_2^f, e^f, h_2^m))$$
$$- C_2^{fm,s}(S_2^{fm}(h_2^f, e^f, h_2^m), e^f)\}.$$

The male takes as given the decisions of his wife about the desired number of children $K_2^{fm}(h_2^f, e^f, h_2^m)$ and contraception $S_2^{fm}(h_2^f, e^f, h_2^m)$. The probability of getting $\widetilde{n}$ pregnancies conditional on these decisions is denoted by $\pi_{2,K_2^{fm}(h_2^f, e^f, h_2^m)\widetilde{n}}^{fm,S_2^{fm}(h_2^f, e^f, h_2^m)}$. This is the $(K_2^{fm}(h_2^f, e^f, h_2^m) + 1, \widetilde{n} + 1)$-th element of the fertility matrix $\Pi_2^{fm,S_2^{fm}(h_2^f, e^f, h_2^m)}$. Finally, it follows that the value functions at the start of the period are identical for husbands and wives,

$$V_2^{mf}(h_2^m, e^f, h_2^f, \gamma) = V_2^{fm}(h_2^f, e^f, h_2^m, \gamma).$$

4.1.4 Old Adults Matching Decisions

At the start of their old age period, adults make decisions with respect to household formation and dissolution. Individuals who enter the old age period as singles match randomly and each matched couple draws a match quality γ from a distribution $\mathbf{\Gamma}(\gamma)$. Females and males compare the expected utilities of staying single $V_2^f(h_2^f, e^f)$ and $V_2^m(h^m)$, respectively, and the expected utility associated with a marriage, $V_2^{fm}(h_2^f, e^f, h_2^m, \gamma)$.

Individuals who enter old age as married draw a new match-specific quality γ, conditional on their initial match quality γ_{-1} from a distribution $\mathbf{\Phi}(\gamma|\gamma_{-1})$. They also compare the value of single life versus the value of continuing being married and make a decision whether to get divorced and stay single for the rest of their lives, or alternatively stay married to their current match.

Consider a particular match pair of previously single people $(h_2^f, e^f, h_2^m, \gamma)$. This couple will get married if and only if

$$V_2^{fm}(h_2^f, e^f, h_2^m, \gamma) \geq V_2^f(h_2^f, e^f) \text{ and } V_2^{mf}(h_2^m, e^f, h_2^f, \gamma) \geq V_2^m(h^m), \tag{7}$$

that is, both parties agree on a marriage by comparing the expected utilities of marriage and single life. Let the indicator function $\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma)$ take the value of one if both people in the match agree to marry and the value of zero otherwise.

Take instead an existing married household from the young age whose members turn old. In the process they draw a new match quality and contemplate on whether to keep their unit or get a divorce. The couple's marriage will survive if and only if (7) holds. In this case both parties find keeping their match profitable compared to single life as a divorcee. Therefore, the indicator function $\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma)$ describes the divorce behavior of old people who were married as young. If this function takes a value of zero, the married household under consideration dissolves, otherwise it stays intact.

4.2 Young Adults' Problems

Before starting the description of the young adults' decision making, a note about the probability of meeting a partner at the start of the old age is in order. Here the description concerns individuals who lived as single young adults and passed to the old age period as such.

An old female who was single as young faces a distribution of male partners $\mathbf{P}_2^m(h_2^m)$ at the start of the old age period. The number of these partners is not necessarily summing up to one. The normalized version of the male partners distribution is given by

$$\widehat{\mathbf{P}}_2^m(h_2^m) = \frac{\mathbf{P}_2^m(h_2^m)}{\int_{\mathcal{H}^m} \mathbf{P}_2^m(h_2^m) d(h_2^m)}, \tag{8}$$

where $\mathcal{H}^m$ is the set of all possible values for the human capital levels of the old single males in the marriage market. Under a suitable law of large numbers assumption, $\widehat{\mathbf{P}}_2^m(h_2^m)$ gives the individual probabilities that an old single female will meet a particular type of old single male in the marriage market.

Likewise, an old male who was single as young faces a pool of female candidates summarized by the distribution $\mathbf{P}_2^f(h_2^f, e^f)$.[9] The normalized version of this distribution which also gives the individual probabilities that an old single male will meet a particular female match is given by

$$\widehat{\mathbf{P}}_2^f(h_2^f, e^f) = \frac{\mathbf{P}_2^f(h_2^f, e^f)}{\sum_{e^f=0}^{1} \int_{\mathcal{H}^f} \mathbf{P}_2^f(h_2^f, e^f) d(h_2^f)}, \tag{9}$$

where $\mathcal{H}^f$ is the set of all possible values for the human capital levels of the old single females in the marriage market. This detour is needed because the probabilities described in (8) and (9) will play a crucial role in defining the present discounted expected utility of the young single adults for the next period when they are old.

Having at hand the value functions defined in problems (2), (3), (5),(**??**), the indicator function for marriage and divorce derived from (7), and the normalized distributions of partners in the old age marriage market (8) and (9), one can formulate the problems of the young adult people in the economy. The decision process of single females and married people is the same as in the case of old age problems but the objective functions have a different structure. Young individuals maximize their current per period utility and the present discounted expected value of their future streams of utilities.

4.2.1 Young Single Female Adults

Young single females face the following problem when the fertility realization is already known:

$$\widetilde{V}_1^f(h_1^f, e^f, y, \widetilde{n}, s) = \max_{a,c,b} \{ u_1^f(c, b, n) - C_1^{f,a}(a, e^f) \tag{10}$$

$$+ \beta \int_{\mathcal{G}\times\mathcal{H}^m} [(1 - \mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma))V_2^f(h_2^f, e^f)$$
$$+ \mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma)V_2(h_2^f, e^f, h_2^m, \gamma)]\widehat{\mathbf{P}}_2^m(h_2^m)\mathbf{\Gamma}(\gamma)d(h_2^m, \gamma)\}$$

[9]The distribution of the old single female partners has education level as an argument because this information is a state variable in the problems of the young single people (see next).

subject to

$$c + bn \leq wh_1^f,$$
$$n = y + \widetilde{n} - a,$$

and

$$h_2^f = \begin{cases} \xi h_1^f \text{ if } n = 0 \\ (1-\tau)\xi h_1^f \text{ if } n > 0 \end{cases}.$$

The young single females maximize their current utility plus the discounted expected value of the their future utility (the continuation value). The subjective discount factor β reflects the fact that the utility stream is discounted to the present day. The continuation value is part of the objective function because the current decisions of the single female may affect the way she accumulates human capital (whether or not there are children in the household) and therefore, its evolution. This part of the objective function is in expected terms because of the random match with a partner that occurs at the start of the next period, and the random quality γ associated with this match. Therefore, the expectation of the continuation value is with respect to whom she might meet and what would be the quality of the proposed marriage. The double integral in the second line of the objective function of problem (10) defines formally this expectation.

The young single female will meet a male with human capital stock h_2^m at the start of the old age period and the match quality of the pair will be γ. The probability of meeting such a male is given by $\widehat{\mathbf{P}}_2^m(h_2^m)$, the probability that a particular match quality γ is associated with this match is $\mathbf{\Gamma}(\gamma)$. The support of the distribution $\mathbf{\Gamma}$ is given by $\mathcal{G}$. Given the characteristics of the match, a marriage might occur if the marriage indicator function $\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma)$ is one. If this indicator is one, then both parties of the match find that marrying their mate is better than staying single. The expression within the double integral of (10) consists of two terms. The first term summarizes the cases in which the future mate h_2^m and the match quality γ are such that marriage does not occur: $\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma) = 0$. The utility of the female in the old age period is given by the value of single life, $V_2^f(h_2^f, e^f)$. The second term brings the future utility of the female in the cases in which she marries a male h_2^m with a quality γ, that is, $\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma) = 1$.

The first constraint of problem (10) states that consumption c, and the total investment in children bn should be less or equal to the budget of the household wh_1^f. The second constraint summarizes the fact that the final number of children within the household n, is equal to a possible teenage birth y, plus the pregnancies conceived by the female, $\widetilde{n}$, minus the number of abortions, a. finally the third constraint deals with the evolution of the human capital of the female. If children are not present, the human capital stock of the female grows up to ξh_1^f. If the female has children, then her human capital evolves to $(1-\tau)\xi h_1^f$.

The decision rules of problem (10) in terms of number of performed abortions, household consumption level, and investment per child are $A_1^f(h_1^f, e^f, y, \widetilde{n}, s)$, $C_1^f(h_1^f, e^f, y, \widetilde{n}, s)$ and $B_1^f(h_1^f, e^f, y, \widetilde{n}, s)$, respectively.

The problem before the realization of the fertility outcome when the number of pregnancies is not yet revealed is summarized by

$$V_1^f(h_1^f, e^f, y) = \max_{k,s}\{\sum_{\widetilde{n}=0}^{N} \pi_{1,k\widetilde{n}}^{f,s} \widetilde{V}_1^f(h_1^f, e^f, y, \widetilde{n}, s) - C_1^{f,s}(s,, e^f)\}. \tag{11}$$

with decision rules $K_1^f(h_1^f, e^f, y)$ and $S_1^f(h_1^f, e^f, y)$ for desired number of pregnancies and contraception.

4.2.2 Young Single Male Adults

Young single males consume all their income but the present value of their lifetime utility includes a continuation term which reflects who they might meet at the next round of the marriage market.

$$V_1^m(h_1^m) = u_1^m(c) + \tag{12}$$

$$+\beta \sum_{e^f=0}^{1} \int_{\mathcal{G}\times\mathcal{H}^f} [(1-\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma))V_2^m(h_2^m)$$
$$+\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma)V_2^{mf}(h_2^m, e^f, h_2^f, \gamma)]\widehat{\mathbf{P}}_2^f(h_2^f, e^f)\mathbf{\Gamma}(\gamma)d(h_2^f, \gamma)\}$$

subject to

$$c = wh^m,$$

and

$$h_2^m = \xi h_1^m.$$

To understand the continuation value of equation (12), consider a young single male who matches with a particular single female at the start of the old age period. Who this female might be? Is he going to marry her? This depends on the characteristics of the female, that is her human capital stock h_2^f and her education level e^f. The realization of the potential marriage is also influenced by the match-specific quality γ. The second line of equation (12) describes the two possible outcomes for the match. If the marriage indicator functions has a value of zero, then the match is not realized and the male into consideration continues to live as single receiving a utility stream $V_2^m(h_2^m)$. In the case when marriage is realized ($\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma) = 1$), the same male obtains a utility stream $V_2^{mf}(h_2^m, e^f, h_2^f, \gamma)$. The integral in respect of the human capital and the summation over the possible educational levels of the potential female mate transform the for future benefits for the male into expected terms. The discount factor β brings back the whole expression in present terms.

4.2.3 Young Married Adults

Start solving the problem of the young married female adults (fm) towards the end of the young age period when the fertility outcome is already realized. The problem looks like this,

$$\widetilde{V}_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s) = \max_{a,c,b}\{u_1^{fm}(c, b, n, y) + \gamma - C_1^{fm,a}(a, e^f) \tag{13}$$

$$+\beta\int_{\mathcal{G}}[(1-\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma'))V_2^f(h_2^f, e^f) + \mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma')V_2^{fm}(h_2^f, e^f, h_2^m, \gamma')]\mathbf{\Phi}(\gamma'|\gamma)d\gamma'\}$$

subject to

$$c + bn \leq wh_1^f + wh_1^m,$$

$$n = y + \widetilde{n} - a,$$

$$h_2^f = \begin{cases} \xi h_1^f \text{ if } n = 0 \\ (1-\tau)\xi h^f \text{ if } n > 0 \end{cases},$$

and

$$h_2^m = \xi h_1^m.$$

The continuation value of the problem again describes what would happen in the next period when the young adults become old and might stay married or get divorced. In particular given a new quality draw for the couple in old age, γ', the female and the male may contemplate whether to stay married or file for divorce. If the marriage indicator has the value of zero, at least one of the partners finds single life as a divorcee more attractive than married life. If however, both partners find it profitable to continue their common marital life ($\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma') = 1$), then the female gets the utility stream $V_2^{fm}(h_2^f, e^f, h_2^m, \gamma')$. The associated decision rules for the number of performed abortions, consumption, and investments in children are $A_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s)$, $C_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s)$ and $B_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s)$, respectively.

The problem before the realization of the fertility shock is given by

$$V_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma) = \max_{k,s}\{\sum_{\widetilde{n}=0}^{N} \pi_{1,k\widetilde{n}}^{fm,s} \widetilde{V}_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s) - C_1^{fm,s}(s, e^f)\} \tag{14}$$

with decision rules $K_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)$ and $S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)$ for the desired number of children and contraception, respectively.

How about young married males (mf)? After the realization of the fertility shock, the abortion decisions are made by the females in the family. The problem of a male h_1^m, who is currently married to a young female (h_1^f, e^f, y) with match quality γ, fertility shock $\widetilde{n}$, and contraception level s, is

$$\begin{aligned} \widetilde{V}_1^{mf}(h_1^m, h_1^f, e^f, y, \gamma, \widetilde{n}, s) = \max_{c,b}\{ & u_1^{mf}(c, b, n, y) + \gamma \\ & - C_1^{fm,a}(A_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s), e^f) \end{aligned} \tag{15}$$

$$+\beta \int_{\mathcal{G}} [(1 - \mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma'))V_2^m(h^m) + \mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma')V_2^{mf}(h_2^m, h_2^f, e^f, \gamma')]\mathbf{\Phi}(\gamma'|\gamma)d\gamma'\}$$

subject to

$$c + bn \leq wh_1^f + wh_1^m,$$

$$n = y + \widetilde{n} - a,$$

$$h_2^f = \begin{cases} \xi h_1^f \text{ if } n = 0 \\ (1-\tau)\xi h^f \text{ if } n > 0 \end{cases},$$

and

$$h_2^m = \xi h_1^m.$$

Here the decision rule for abortions of the female is taken as given. Therefore, the decision rules for consumption and investments in children of husbands and wives in problems (13) and (15) are identical within the couple,

$$C_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s) = C_1^{mf}(h_1^m, h_1^f, e^f, y, \gamma, \widetilde{n}, s)$$
$$B_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s) = B_1^{mf}(h_1^m, h_1^f, e^f, y, \gamma, \widetilde{n}, s).$$

The per period indirect utilities are identical for the members of the couple. Nevertheless, the value function of young husbands and wives, $\widetilde{V}_1^{mf}(h_1^m, h_1^f, e^f, y, \gamma, \widetilde{n}, s)$ and $\widetilde{V}_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s)$, are in general different. This result comes from the fact that upon a future divorce decision, young males and females within the couple face different future streams of utility.

At the start of the period, before the realization of the fertility uncertainty, the value function of the married male is given by

$$V_1^{mf}(h_1^m, h_1^f, e^f, y, \gamma) = \sum_{\widetilde{n}=0}^{N} \pi_{1,K_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)\widetilde{n}}^{fm, S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)} \widetilde{V}_1^{mf}(h_1^m, h_1^f, e^f, y, \gamma, \widetilde{n}, S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma))$$
$$- C_1^{fm,s}(S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma), e^f)\}.$$

The young male takes as given the decisions of his wife about the desired number of children $K_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)$ and contraception $S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)$. The probability of getting $\widetilde{n}$ pregnancies conditional on these decisions is denoted by $\pi_{1,K_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)\widetilde{n}}^{fm, S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)}$ for this young couple. This is the $(K_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)+1, \widetilde{n}+1)$-th element of the fertility matrix $\Pi_1^{fm, S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)}$.

4.2.4 The Young Adults Matching

In the first instance of their adult lives, young individuals have to make a marriage decision. They randomly mate with a young partner from the opposite gender and draw a quality specific to this match, γ, from distribution $\mathbf{\Gamma}(\gamma)$. Clearly, a marriage will be realized if both parties find it profitable relative to single life. That is, a marriage with a match quality γ between a female with human capital stock h_1^f, education achievement e^f and a possible teenage premarital birth y, and a male with h_1^m will be realized if and only if

$$V_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma) \geq V_1^f(h_1^f, e^f, y) \text{ and } V_1^{mf}(h_1^m, h_1^f, e^f, y, \gamma) \geq V_1^m(h_1^m). \tag{16}$$

Let the marriage decision be summarized in an indicator function $\mathbf{I}_1(h_1^f, e^f, y, h_1^m, \gamma)$ which takes the value of one if condition (16) is met, and zero otherwise.

4.3 Teenagers' Problems

Before moving on to the decisions made by the female teenagers, a detour on the probabilities with which young adults meet each other for marriage is needed. These probabilities are essential for defining the problems of the teenagers and the equilibrium of the economic model.

Young female adults searching for a spouse are confronted by a distribution of potential husbands $\mathbf{P}_1^m(h_1^m)$. The probabilities of meeting each of those husbands are summarized in the normalized distribution

$$\widehat{\mathbf{P}}_1^m(h_1^m) = \frac{\mathbf{P}_1^m(h_1^m)}{\int_{\mathcal{H}^m} \mathbf{P}_1^m(h_1^m) d(h_1^m)}. \tag{17}$$

Young male adult individuals face a distribution of potential female partners $\mathbf{P}_1^f(h_1^f, e^f, y)$. The normalized version of this distribution, which can be interpreted as the probabilities of meeting a particular female partner is

$$\widehat{\mathbf{P}}_1^f(h_1^f, e^f, y) = \frac{\mathbf{P}_1^f(h_1^f, e^f, y)}{\sum_{y=0}^{1} \sum_{e^f=0}^{1} \int_{\mathcal{H}^f} \mathbf{P}_1^f(h_1^f, e^f, y) d(h_1^f)}. \tag{18}$$

Equipped with the probabilities (17), we can proceed in defining the female teenagers' problems. The sequence of events in the teenage period is as follows. At the start of the period female teenagers have to decide whether to exert contraception effort. Based on that, with certain probability they might become pregnant. If this is the case, they either carry the pregnancy to term, or abort it. At the last instance of the teenage period, the education level of the teenager is revealed. The probability of getting skilled is increasing in the amount of parental investments in the child. The presence of a teenage birth decreases the odds of obtaining higher education for females. Start solving the problem of the female teenager from the last instance of teenagehood.

At the end of the teenage period when teen pregnancies, births and education levels are revealed, the female teenager is just about to enter adulthood. She has an ability level (λ), an education level (e), and a possible out-of-wedlock birth ($y = 1$ if birth is present, and $y = 0$ if not). Her life-time utility at that point of time is given by

$$\begin{aligned}\widetilde{\widetilde{V}}_0^f(\lambda, e^f, y) = \beta \int_{\mathcal{G}\times\mathcal{H}^m} &[\mathbf{I}_1(h_1^f, e^f, y, h_1^m, \gamma) V_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma) \\ &+ (1 - \mathbf{I}_1(h_1^f, e^f, y, h_1^m, \gamma)) V_1^f(h_1^f, e^f, y)] \widehat{\mathbf{P}}_1^m(h_1^m) \mathbf{\Gamma}(\gamma) d(h_1^m, \gamma)\end{aligned} \tag{19}$$

subject to

$$h_1^f = \varkappa \lambda \chi_e.$$

Here the teenager considers the expected discounted stream of utility from her future adult life. Once the teenager becomes an adult she will pick a partner from the male distribution $\widehat{\mathbf{P}}_1^m(h_1^m)$ with a match quality γ. Based on the the marriage decision function $\mathbf{I}_1(h_1^f, e^f, y, h_1^m, \gamma)$, she will either marry him, or stay single. In the case she marries, the utility she gets is measured by

$V_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)$, while if she stays single the appropriate value is $V_1^f(h_1^f, e^f, y)$. The expression within the double integral of (19) summarizes this. The integral is needed, because the appropriate probabilities of whom the female might meet (and what is the level of γ) should be assigned when calculating the expected value of the future stream of utility. The discounted factor β is there because the the expected utility should be converted in present terms for the teenager. The constraint $h_1^f = \varkappa\lambda\chi_e$ states how the ability λ and the education level e map into the stock of human capital of the young female.

Now move back in the teenage period when education levels are still not revealed but the female teenager knows whether she is pregnant or not. The expected value of $\widetilde{\widetilde{V}}_0^f(\lambda, e, y)$ at this stage is denoted by $\widetilde{V}_0^f(\lambda, b, y_p)$, where b is the amount of the parental investment to the teenager, and y_p is a variable that takes the value of 1 if the teenager is pregnant, and 0 otherwise. If conception is present $(y_p = 1)$, she has to make a decision whether to abort the pregnancy or keep it:

$$\widetilde{V}_0^f(\lambda, b, y_p = 1) = \max_{a\in\{0,1\}}\{q(b, y)\widetilde{\widetilde{V}}_0^f(\lambda, e = 1, y) + (1 - q(b, y))\widetilde{\widetilde{V}}_0^f(\lambda, e = 0, y) - \kappa_a a\} \quad (20)$$

subject to

$$y = y_p - a.$$

If the female teenager performs an abortion $(a = 1)$, then the probability of getting educated, $q(b, y)$, increases since $q(b, 0) > q(b, 1)$. The value of educated life, $\widetilde{\widetilde{V}}_0^f(\lambda, e = 1, y)$, is strictly better than the value when uneducated, $\widetilde{\widetilde{V}}_0^f(\lambda, e = 0, y)$. This creates incentives for teenagers to abort their pregnancies. However, the abortion procedure comes at cost κ_a. The female teenager weights the benefits of abortion in terms of increased chances of higher education versus this abortion cost. The abortion decision rule for problem (20) is $A_0(\lambda, b, y_p)$.

If no pregnancy occurs, the expected value of $\widetilde{\widetilde{V}}_0^f(\lambda, e^f, y = 0)$ is just its weighted sum,

$$\widetilde{V}_0^f(\lambda, b, 0) = q(b, 0)\widetilde{\widetilde{V}}_0^f(\lambda, 1, 0) + (1 - q(b, 0))\widetilde{\widetilde{V}}_0^f(\lambda, 0, 0)$$

Further, consider the problem before the realization of the teenage pregnancy y_p. Here, the female teenager has to make a decision whether to use contraception $(z = 1)$ or not $(z = 0)$. This decision determines the probability of a teen pregnancy $p(z)$, where $p(1) < p(0)$. There is a cost associated to contraception, κ_z. The teenager weights the benefits of contraception in reducing the odds of pregnancy versus this cost:

$$V_0^f(\lambda, b) = \max_{z\in\{0,1\}}\{p(z)\widetilde{V}_0^f(\lambda, b, 1) + (1 - p(z)\widetilde{V}_0^f(\lambda, b, 0)) - \kappa_z z\}. \quad (21)$$

The contraception decision rule is denoted by $Z(\lambda, b)$.

4.4 Definition of Equilibrium

The households in the economy at hand make an explicit fertility choice, therefore the economy can grow or shrink in terms of population size. In such an environment, a steady-state equilibrium requires the distributions of households of different ages and teenagers, normalized to the measure of each generation, to be identical over time. This is true if and only if the normalized distributions of females ($\widehat{\mathbf{P}}_1^f(h_2^f, e^f, y)$) and males ($\widehat{\mathbf{P}}_1^m(h_1^m)$) entering adulthood are constant over time. Then we are ready to define the equilibrium.

Definition. A steady-state equilibrium is a set of decision rules:

(i) for teenagers: $Z(\lambda, b)$ and $A_0(\lambda, b, y_p)$;

(ii) for young adults: $\mathbf{I}_1(h_1^f, e^f, y, h_1^m, \gamma)$, $K_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)$, $S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)$, $A_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s)$, $C_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s)$, $B_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s)$, $K_1^f(h_1^f, e^f, y)$, $S_1^f(h_1^f, e^f, y)$, $A_1^f(h_1^f, e^f, y, \widetilde{n}, s)$, $C_1^f(h_1^f, e^f, y, \widetilde{n}, s)$, and $B_1^f(h_1^f, e^f, y, \widetilde{n}, s)$;

(iii) for old adults: $\mathbf{I}_2(h_2^f, h_2^m, \gamma)$, $K_2^{fm}(h_2^f, e^f, h_2^m)$, $S_2^{fm}(h_2^f, e^f, h_2^m)$, $A_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s)$, $C_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s)$, $B_2^{fm}(h_2^f, e^f, h_2^m, \widetilde{n}, s)$, $K_2^f(h_2^f, e^f)$, $S_2^f(h_2^f, e^f)$, $A_2^f(h_2^f, e^f, \widetilde{n}, s)$, $C_2^f(h_2^f, e^f, \widetilde{n}, s)$, and $B_2^f(h_2^f, e^f, \widetilde{n}, s)$;

and a set of distributions $\widehat{\mathbf{P}}_1^f(h_1^f, e^f, y)$, $\widehat{\mathbf{P}}_1^m(h_1^m)$ such that:

(i) The decision rules solve problems (1)-(6), (10)-(15), and (20)-(21).

(ii) The distributions are consistent with individual decisions.

5 Parametrization

5.1 Functional Forms

The economic environment cannot be solved analytically. Instead it is simulated numerically. In order to do that, specific functional forms should be assumed.

Preferences. The utility functions of people in single female households are specified as

$$u_j^f(c,b,n) = \alpha_c \frac{(c/(1+\zeta_2 n))^{1-\xi_c}}{1-\xi_c} + \alpha_n \frac{(\varepsilon+n)^{1-\xi_n}}{1-\xi_n} + (1-\alpha_c-\alpha_n)\frac{I(n>0)b^{1-\xi_b}}{1-\xi_b} \tag{22}$$

for both young and old aged adult individuals, i.e. $j \in \{1,2\}$. On the other hand, the utility functions of people in young married households is specified as

$$u_1^{fm}(c,b,n,y) = \alpha_c \frac{(c/(1+\zeta_1+\zeta_2 n))^{1-\xi_c}}{1-\xi_c} + \alpha_n \frac{(\varepsilon+n)^{1-\xi_n}}{1-\xi_n} + (1-\alpha_c-\alpha_n)\frac{I(n>0)b^{1-\xi_b}}{1-\xi_b} - \theta y \tag{23}$$

while the old married individuals derive utility

$$u_2^{fm}(c,b,n) = \alpha_c \frac{(c/(1+\zeta_1+\zeta_2 n))^{1-\xi_c}}{1-\xi_c} + \alpha_n \frac{(\varepsilon+n)^{1-\xi_n}}{1-\xi_n} + (1-\alpha_c-\alpha_n)\frac{I(n>0)b^{1-\xi_b}}{1-\xi_b}. \tag{24}$$

Finally, the utility for the single male households is given by

$$u_j^m(c) = \alpha_c \frac{c^{1-\xi_c}}{1-\xi_c} + \alpha_n \frac{\varepsilon^{1-\xi_n}}{1-\xi_n}, j \in \{1,2\}. \tag{25}$$

Several comments are in order on the choice of these functional forms. First, the utility derived from consumption in the household, which is a public good, is subject to congestion. The relevant parameters here are ζ_1 and ζ_2. They represent the equivalence scales due to the presence of a second adult member (ζ_1) or a child in a household (ζ_2). Second, the altruistic nature of people toward their children is summarized by the second and third terms in (22), (23), and in (24). Married people and single females derive utility from the number of children they have, n, and from the investments in each of these children, b. If there are no children present, people cannot derive utility from children's investments. Therefore, the indicator function $I(n > 0)$ multiplies the utility term for investments in (22), (23), and in (24). This indicator function takes the value of one if the number of children, n, is positive, and zero otherwise. The utility terms for consumption, children, and investments in each child, are separable. This specification can generate a negative income-fertility relationship[10] with discrete number of children and without any cross-terms in the utility or time costs of children in terms of income. For this purpose, (i) the marginal utilities of extra units of consumption and investment should be large and decreasing very slowly, that is, ξ_c and ξ_b should be small, and (ii) the marginal utility of an extra child should be sufficiently small. This implies that ε and ξ_n should be sufficiently large. If these conditions are met, a discrete jump

[10] An early discussion of the negative relationship between fertility and income appears in Becker (1960). See Jones, Schoonbroodt and Tertilt (2008) for more details on this stylized fact and economic models which generate it.

in the number of children from n to $n+1$ decreases the utility coming from consumption and investment in children. This decrease is larger for higher levels of income, and can dominate the extra utility from the increase in the number of children. Formal analysis of the conditions under which the negative income-fertility relationship holds in this model are presented in Appendix C. Finally, the parameter θ captures the stigma associated with having teenage premarital children in a married couples.

Utility costs. The utility cost of abortions for young and old single female and married households is given by

$$C_1^{fm,a}(a,\cdot) = C_1^{f,a}(a,\cdot) = C_2^{fm,a}(a,\cdot) = C_2^{f,a}(a,\cdot) = \varphi a. \tag{26}$$

The utility cost of contraception for young and old single female and married households is similarly described by

$$C_1^{fm,s}(s,\cdot) = C_1^{f,s}(s,\cdot) = C_2^{fm,s}(s,\cdot) = C_2^{f,s}(s,\cdot) = \phi s. \tag{27}$$

The specifications in (26) and (27) pose utility costs of abortion and contraception that are independent of age and education.

Distributions for types and match qualities. The ability types λ are assumed to be distributed log-normally , i.e. $\ln\lambda \sim N(0,\sigma_\lambda^2)$. Teenagers may pick randomly ability of this distribution with probability $1-\rho_\lambda$, and may inherit the ability of their mother with probability ρ_λ. On the other hand, the match specific quality of the prospective marriages is drawn from

$$\gamma \sim N(0,\sigma_\gamma^2), \tag{28}$$

while the future quality for married couple stays the same with probability $1-\rho_\gamma$, or may be drawn again from distribution (28) with probability ρ_γ. For now, set $\rho_\gamma = 1$, that is, all young couples draw a new quality level γ at the start of their old age period.

Probabilities of becoming skilled. The probability functions of getting educated are given by

$$q^f(b,y) = 1 - \exp(\psi_1 y - \psi_2 b) \tag{29}$$

for female teenagers and

$$q^m(b) = 1 - \exp(-\psi_3 b)$$

for male teenagers. These functional forms have all the desirable properties. The odds of education increase with the investment b and decrease with a teen birth.[11]

Fertility matrices. The maximum number of children a household can have within a period is set to 2 $(N = 2)$. Some further assumptions should be put in place in order to parametrize the fertility matrices $\Pi_j^{s,r}$. Recall that $j \in \{1, 2\}$ is the index for the age of the household members, $s \in \{0, 1\}$ signifies whether contraception is used, and $r \in \{f, fm\}$ indexes whether the household is single female or married. Suppose that the probability of making a fertility mistake upwards, i.e. having one more child than desired is $\upsilon_{j,s}$. In addition, assume that the fertility mistakes are independent of each other. Therefore, the probability of making two consecutive mistakes upwards is $\upsilon_{j,s}^2$. Now assume that the probability of being sterile is ω_j. Note that this probability does not depend on whether or not the households use contraception. This is so because here the source of uncertainty is the possibility of multiple miscarriages, ectopic pregnancies, and different forms of sterility. With these assumptions at hand and keeping in mind that each row of the fertility matrices should sum up to one, one can derive the particular shape of the matrices:

$$\Pi_j^{s,r} = \begin{bmatrix} 1 - \upsilon_{j,s,r} - \upsilon_{j,s,r}^2 & \upsilon_{j,s,r} & \upsilon_{j,s,r}^2 \\ \omega_j & 1 - \omega_j - \upsilon_{j,s,r} & \upsilon_{j,s,r} \\ \omega_j^2 & \omega_j & 1 - \omega_j - \omega_j^2 \end{bmatrix}.$$

5.2 Estimation

The distributions for abilities and marital match quality are discretized and the model is simulated numerically.[12] The model period is set to 10 years. Teenagers leave their parents house at the age of 20 and enter adulthood. They obtain their education before they enter into adulthood. This might sound a bit unrealistic since the university education continues till mid 20s, however, the period labor income in the model is equivalent to pooled income of individuals between their twenty-first year and their thirtieth year and it accounts for the fact that educated people (university graduates) spent some time in their 20s at school instead of working. The period in which each person in the model is young adult corresponds to real-life age between 20 and 29. The terminal period in the lives of the model people is equivalent to age 30-39.

The benchmark economy is parametrized so that it represents closely certain features of the United States economy circa 2000. A few parameters are set directly to their empirical coun-

[11]The parameters ψ_1, ψ_2 and ψ_3 are restricted to positive values which allow for upper bounds of q^f and q^m consistent with their probabilistic interpretation.

[12]The outline of the numerical solution algorithm is presented in Appendix D.

terparts. The rest of the parameters are chosen so that the benchmark economy fits the demographic structure of the United States economy in respect to: (i) pregnancy and abortion behavior by age, marital status, and education; (ii) marriage and divorce patterns, and the proportion of single/married young mothers with premarital teenage births, and (iii) educational achievements for females and males.

Parameters set from data. The parameters $\upsilon_{j,s}$ and ω_j are estimated using data from the National Survey of Family Growth (NSFG), Cycle VI , which was conducted by the National Center for Health Statistics (NCHS) in 2002. The survey consists of interviews conducted with females 15-44 years of age. A female pregnancy file is also compiled, containing a record for each pregnancy for all female respondents. This pregnancy file contains basic demographic information for the females to whom the pregnancy occurs and whether it is intended or not. The discrete choice formulation of contraception in the model requires the identification of the parameters of the fertility matrices for the people who use full contraception and the ones who never use. The pregnancy cases to females who have never used contraception are known in the survey. Take these pregnancies and divide them according to the age of the females (20-29 and 30-39), and whether they are single or married the For each of these groups of pregnancies, the probability of having an unintended pregnancy conditional on not using contraception $(\upsilon_{j,0,r})$ is estimated is as the ratio of unintended pregnancies to all pregnancies within the group. Furthermore, following Greenwood and Guner (2010) and Fernández-Villaverde, Greenwood and Guner (2010), assume that the failure rate of contraceptives in 2000 is 28%. Therefore, the parameters $\upsilon_{j,1,r}$ are obtained by augmenting $\upsilon_{j,0,r}$ by this failure rate. The parameter ω_j is set to the ratio of female respondents who report sterility to all female respondents within the appropriate age group.

Table 1: Paramters Set from Data I - Fertility

Teenagers
$p(0) = 0.813, p(1) = 0.228$
Young single adults
$\upsilon_{1,0,f} = 0.586, \upsilon_{1,1,f} = 0.192, \omega_1 = 0.073$
Young married adults
$\upsilon_{1,0,fm} = 0.451, \upsilon_{1,1,fm} = 0.126, \omega_1 = 0.073$
Old single adults
$\upsilon_{2,0,f} = 0.575, \upsilon_{2,1,f} = 0.189, \omega_2 = 0.277$
Old married adults
$\upsilon_{2,0,fm} = 0.240, \upsilon_{2,1,fm} = 0.067, \omega_2 = 0.277$

The probability of unintended pregnancy of teenagers when contraception is not used $(p(0))$ is calculated in a similar fashion. Its counterpart when contraception is employed $(p(1))$ is calculated as 28% of $p(0)$.

The ratio $\frac{\chi_{e_1}}{\chi_{e_0}}$ represents the educational premium in terms of income for agents with identical ability levels in the first period of their adult life. The parameter χ_{e_0} is normalized to 1, while χ_{e_1} takes the value of of the average educational premium for male workers of age 20-29 in the 2000 1% census data sample of the Integrated Public Use Microdata Series (IPUMS).[13]The gender gap parameter $\varkappa$ is set to the average gender difference in income among agents of age 20-29. The parameter ξ, which represents the rate of accumulation of human capital is set to the average growth rate of income for men between age periods 20-29 and 30-39.

The equivalence scale parameters are in accord with the OECD scale which assigns a value of 1 to the first adult household member, 0.7 to the second, and 0.5 to each child. Following Knowles (1999) the coefficient which determines the intergenerational persistence of ability is set to 0.7.

[13]The 2000 1% census data sample of the IPUMS is used for setting all parameters and targets unless specified otherwise.

Table 2: Parameters Set from Data II

Parameter	Explanations/Source
$\beta = 0.665$	Annual discount rate of 0.96
$\chi_{e_1} = 1.713$	$\frac{\chi_{e_1}}{\chi_{e_0}}$, premium of education for males 20-29
$\varkappa = 0.799$	Gender gap for workers 20-29
$\xi = 1.619$	Income growth for men from 20-29 to 30-39
$\zeta_1 = 0.7$	OECD equivalence scale for a second adult
$\zeta_2 = 0.5$	OECD equivalence scale for a child
$\rho_\lambda = 0.7$	Knowles (1999)

Parameters set in equilibrium. The rest of the parameters are set in equilibrium. Several data targets are chosen and the distance between them and the equivalent statistics produced in the benchmark model economy is minimized in respect to those parameters. The estimation technique is a simplified minimum distance estimator in which the squared sum of the difference between the data and the model moments is minimized.

The parameters left to be set in equilibrium are:

- Preferences: $[\alpha_c, \alpha_n, \varepsilon, \xi_c, \xi_n, \xi_b, \theta]$ **7** parameters;
- Utility costs: $[\varphi, \phi, \kappa_a, \kappa_z]$ **4** parameters;
- Probabilities of becoming skilled: $[\psi_1, \psi_2, \psi_3]$ **3** parameters;
- Distributions for types and match qualities: $[\sigma_\lambda, \sigma_\gamma]$ **2** parameters
- Tax rate of human capital accumulation due to the presence of children: $[\tau]$ **1** parameters

All in all, the vector of estimated parameters consists of **17** parameters Let's turn our attention to the chosen targets to be matched by the benchmark economy:

(i) Pregnancy and abortion behavior by age, marital status, and education (**16** targets)

1. Proportions of pregnancies and abortions among teenagers, young adults, and old adults: **6** targets
2. Proportions of pregnancies and abortions by marital status: **4** targets
3. Proportions of pregnancies and abortions by education: **4** targets
4. Fraction of pregnancies ending in abortions: **1** target
5. Total fertility rate (TFR): **1** target

(ii) Marriage and divorce patterns and the proportion of single young mothers with premarital teenage births (**4** targets):

1. Proportion of never married singles: **1** target
2. Proportion of divorced singles: **1** target
3. Proportion of young single females with births as teenagers: **1** target
4. Proportion of young married females with births as teenagers: **1** target

(iii) Educational achievements for females and males (**4** targets):

1. Education rates for females and males: **2** targets
2. Proportion of young unskilled females with births as teenagers: **1** target
3. Proportion of young skilled females with births as teenagers: **1** target

The values of the estimated parameters are summarized in Table 3.

Table 3: Parameters Set in Equilibrium

Preferences
$\alpha_c = 0.5, \alpha_n = 0.2, \varepsilon = 10.25$
$\xi_c = 0.1, \xi_n = 0.4, \xi_b = 0.2, \theta = 0.21$
Utility costs
$\varphi = 0.02, \phi = 0.50, \kappa_a = 0.0495, \kappa_z = 0.000016$
Probabilities of becoming skilled
$\psi_1 = 0.02, \psi_2 = 0.98, \psi_3 = 0.85$
Distributions for types and match qualities
$\sigma_\lambda = 0.97, \sigma_\gamma = 0.75$
Tax rate on human capital accumulation (children)
$\tau = 0.15$

The model moments and the corresponding data moments are presented in Tables 4, 5, and 6.

Table 4: Pregnancy and Abortion Proportions - Model and Data

Proportions of Pregnancies			**Proportions of Abortions**		
	Model	Data		Model	Data
By Age of Parents			**By Age of Parents**		
Teen (15-19)	0.130	0.132	Teen (15-19)	0.254	0.195
Young (20-29)	0.508	0.550	Young (20-29)	0.445	0.588
Old (30-39)	0.362	0.318	Old (30-39)	0.301	0.217
By Marital Status of Household			**By Marital Status of Household**		
Single	0.597	0.542	Single	0.703	0.523
Married	0.403	0.458	Married	0.297	0.477
By Education of Mother			**By Education of Mother**		
Unskilled	0.809	0.771	Unskilled	0.725	0.838
Skilled	0.191	0.229	Skilled	0.275	0.162

Table 5: Overall Fertility - Model and Data

	Model	Data
Pregnancies ending in abortion	0.297	0.270
TFR	2.122	2.10

Table 6: Marital Status and Education - Model and Data

Marital Status			**Education**		
	Model	Data		Model	Data
Never married	0.352	0.391	Females	0.230	0.247
Divorced	0.112	0.100	Males	0.214	0.218
Young single females with teen birth out of all young females	0.166	0.155	Young unskilled females with teen birth out of all young females	0.183	0.207
Young married females with teen birth out of all young females	0.129	0.148	Young skilled females with teen birth out of all young females	0.022	0.027

The economic environment presented and parametrized above generates an economy which closely follows certain demographic features of the United States economy in the 2000s. This benchmark model economy fits well the pregnancy and abortions behavior for the U.S. and matches the marital and education statistics observed in the data. The economy is constructed so that it mimics teenage births occurring to young women by their marital status and education. This is needed for the identification of the parameters ψ_1 and θ which are responsible for the reduction of the probability of getting educated because of a teenage birth and the marriage market stigma associated with a teenage birth.

6 Computational Experiments

6.1 The Importance of Legal Abortions

What is the importance of legal abortions for the U.S. economy in the 2000s? What if the abortions policy is reversed and no female can use this medical procedure? The model economy is simulated in the case in which abortions are not legal. This is done by setting the cost of abortions for teenagers and adults prohibitively high so that no abortions occur. The estimated benchmark economy is denoted as "Benchmark" in the tables that follow, while the counterfactual one (with no availability of abortions) is called "No abortions".

The benchmark economy and the counterfactual economy in which abortions are not available differ in terms of average individual and family income. In particular, individual (family) income declines from the benchmark economy to the counterfactual economy by 1.9% (1.5%). The main reason for that are the different education rates in the two economies due to different levels of parental investments to children and the changing mean of the ability distribution. The fraction of skilled females (males) out of all females (males) reduces from 0.23 (0.214) to 0.218 (0.205). Since the mean of the income distribution changes in the experiment, an appropriate measure of inequality should correct for the mean of the income distribution. The measure used here is the coefficient of variation which is defined as the standard deviation over the mean of the distribution.

The lack of abortions increases long-term inequality. The coefficient of variation of individual income rises from 0.987 to 1.227. Furthermore, the coefficient of variation of family income goes up from 0.760 to 0.951. Finally, the intergenerational persistence[14] of family income decreases from 0.476 to 0.379. This poses an interesting situation in which cross-sectional inequality increases along with mobility between generations. More discussion on that is to follow in a bit when changes in the investments in children are discussed.

[14]The intergenerational persistence of family income is the coefficient β_1 in the regression

$$\log(H') = \beta_0 + \beta_1 \log(H) + \varepsilon,$$

where H' is the family income of individuals and H is the family income of their parents. High persistence, i.e. β_1 relatively close to 1 indicates low levels of intergenerational mobility, and lower persistence, i.e. β_1 relatively close to 0, implies high intergenerational mobility.

Table 7: Inequality,Benchmark and No Abortions

	Benchmark	No Abortions
Inequality		
CV individual income	0.987	1.227
CV family income	0.760	0.951
Intergenerational persistence of family income	0.476	0.379

There are several factors that might contribute to the increase of inequality due to intensified fertility risk when abortions are not available. Start with the changes to family formation patterns that occur when we move from the benchmark to the counterfactual economy. The fraction of married people increased by 4% (from 54% to 58%). Divorce reduced by 1%(from 11% to 10%). These two changes may work against the increasing individual income inequality because the women that are now married instead of single or divorced face a reduced amount of fertility risk within the marriage. At the same time correlation went down from 0.141 to 0.102. Thus, now marriages are a bit more random in the counterfactual world which could potentially work towards increasing family income inequality. Overall, changes in family formation patterns are small. This is a first indication that family formation might not play a significant role in explaining the rise in inequality.

Table 8: Family Formation, Benchmark and No Abortions

	Benchmark	No Abortions
Family Formation		
Fraction of married	0.536	0.577
Fraction of divorced	0.112	0.098
Fraction of never married	0.352	0.325
Correlation of education between spouses	0.141	0.102

Another mechanism which can amplify inequality in the presence of more fertility risk due to the lack of abortions are the changing parental investments to children. These investments are lower and more variable. Table 9 reports investments in children as a fraction of household income and the corresponding coefficient of variation of this fraction in brackets. Moving to a world with no abortions makes everybody in the economy to invest less in their children. However, the single households reduce these investments by around 3% of family income (from 19% to 16%) compared to 20% for the married households (from 53.60% to 33.40% of family income). At the same time, the coefficient of variation of investments of married stays the same (0.239), while the one of singles increases from 0.367 to 0.433. Bear in mind that single female households have on average lower income than married households because the former consist of just one wage earner. The reduction of investments is of similar magnitude for young and old, and unskilled and skilled but again the young and unskilled families are on average poorer than the old and the skilled ones. The coefficient of variation is again larger at the bottom of the distribution. The overall effects of this differential change in the amounts invested in children is that the intergenerational persistence of family income drops from 0.476 in the benchmark economy to 0.379 in the new equilibrium with no abortions (Table 7). This change can be explained with the fact that children raised at the bottom of the income distribution receive lesser cut in their investments than the children raised at the top of the distribution. Therefore, the children that were likely to grow as unskilled (and poor) adults receive a minor decrease in their human capital investments, while the children who were likely to grow as skilled (and rich) adults in the benchmark get a major decrease in their parents' investments. Thus, the new economy is much more mobile between generations. Children born to poor parents have on average similar (but more volatile) chances of climbing up the income distribution as before, while children born to rich parents have on average lower chances (but less volatile compared to the prospects of the poor) of education when abortions are banned.

Table 9: Investments in Children, Benchmark and No Abortions

	Benchmark	No Abortions
Investments in Children as a Fraction of Income		
(Coefficient of Variation)		
Age of Parents		
Young	0.334	0.252
	(0.251)	(0.322)
Old	0.420	0.279
	(0.243)	(0.264)
Education of Mother		
Unskilled	0.389	0.274
	(0.244)	(0.332)
Skilled	0.330	0.230
	(0.244)	(0.315)
Marital Status of Household		
Single	0.190	0.160
	(0.367)	(0.433)
Married	0.536	0.334
	(0.239)	(0.239)

The final channel which might contribute to rising inequality is the changing fertility patterns. Final number of children n is reported for the benchmark and the counterfactual economy in Table 10. In parenthesis are the coefficients of variation. The average number of children per household rises for all types of households. This increase is more pronounced for the young versus the old (average increase of 0.305 versus 0.035 children), the unskilled versus the skilled (0.156 versus 0.156), and the single versus the married (0.438 versus 0.030). This is consistent with the higher fertility risk faced by these groups. In terms of variability of children per household, the young, the unskilled, and the single have again a lead over the old, the skilled, and the married judging by the magnitudes of the increasing coefficients of variation.

Table 10: Number of Children, Benchmark and No Abortions

	Benchmark	No Abortions
Number of Children		
(Coefficient of Variation)		
Age of Parents		
Young	1.294	1.599
	(0.263)	(0.288)
Old	0.880	0.915
	(0.250)	(0.262)
Education of Mother		
Unskilled	1.193	1.334
	(0.268)	(0.277)
Skilled	0.757	1.114
	(0.172)	(0.135)
Marital Status of Household		
Single	1.215	1.653
	(0.263)	(0.290)
Married	0.987	1.017
	(0.253)	(0.265)

6.2 Decomposing the Rise in Inequality due to the Lack of Abortions

Three mechanisms which might contribute to the rise in income inequality were proposed in the previous section: (i) changes in family formation patterns between the benchmark and the counterfactual economy, (ii) more volatile investments in children due to the higher fertility risk, and (iii) increasing and more volatile fertility especially for the young, the unskilled, and the single.

Here is a thought experiment. Suppose abortions are not available but marriages are formed and dissolved according to the marriage decision rule for the young, $\mathbf{I}_1(h_1^f, e^f, y, h_1^m, \gamma)$, and the marriage and divorce decision rule for the old, $\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma)$ from the benchmark economy. That is, exogenously insert the rules as defined by the value functions inequalities of the benchmark economy in (16) and (7) into an economy where abortions are not allowed. What is the resulting level of

inequality now? Is it lower than in the No Abortions economy with endogenous marriage decision rules? If yes, the amount of reduced inequality towards the benchmark levels must be due to the fact that the family formation channel is shut down. Table 11 (Column 2: No Abortions, Fix Marriages) shows that the family formation can account partially for the rise in inequality of individual and

Table 11: Decomposing the Rise in Inequality

	1	2	3	4	5
	No Abortions	No Abortions Fix Marriages	No Abortions Fix Parental Investments	No Abortions Fix Marriages and Parental Investments	Benchmark
Inequality					
CV individual income	1.227	1.191	1.145	1.116	0.987
CV family income	0.951	0.910	0.883	0.846	0.760

family income. Without it, the inequality in the absence of abortions increases from the benchmark coefficient of variation 0.987 to 1.191 instead to 1.227 as in the proper No Abortions economy. In the case of family income, the coefficient of variation increases from 0.760 to 0.910, which is also slightly less than the number in Column 1, 0.951. Clearly, the family formation has only a marginal contribution for the rise of family income, and almost none to the increase of individual income disparity.

Now try a different experiment. Take an economy in which abortions are not allowed but fix the parental investments to those in the benchmark. That is, make parents in this No Abortions economy invest in their children the amounts they would have invested in the benchmark economy. Fertility decisions are endogenous. In this case, moving from the benchmark economy to the No Abortions economy with the parental investments channel shut down produces an increase of individual (family) inequality from 0.987 (0.760) to 1.91 (0.910) (Table 11, Column 3). The parental investments channel can account for larger fraction of the rise of inequality due to the lack of abortions compared to the family formation channel. With parental investments channel shut down, inequality rises to 1.145, while with family formation shut down, it goes up to 1.191.

Further, shut down both of these channels in the No Abortions economy. The marriages and divorce decisions and the parental investments decisions are fixed to the benchmark. Inequality rises only for 0.987 to 1.116 for the individual income coefficient of variation, and from 0.760 to 0.846 for the family income inequality measure. This shows that there are certain complementarities between the family formation channel and the parental investment channel for creating inequality in the lack of abortions.

The residual inequality (from Column 4 to Column 5 in Table 11) is due to the fertility channel. Comparing the magnitudes of the coefficients of variation from Column 1 to Column 5, one can conclude that increasing and more volatile fertility in the absence of abortions accounts for around a half of the total rise in inequality.

7 Conclusions

The enormous amount of induced abortions performed in the US today stirs intense social discussions about the moral grounds of the medical procedure. This study takes a more pragmatic stand on the issue and explores the availability/lack of abortions as a source of changes in the dispersion of long-term income in the US economy. The economic environment built here incorporates abortions and contraception decisions in a quality-quantity fertility model with overlapping generations. The model is estimated to fit pregnancy and abortion behavior by age, education and marital status of the population.

The role of abortions for the economic outcomes in the environment is assessed by simulating the economy under the alternative policy regime of no abortions. The results show that inequality of income rises in the absence of abortions. There are three mechanisms that may account for this rise. The first candidate is the changing pattern of family formation. It is shown that the contribution of this channel to the rise in income disparity is marginal. The second channel is more powerful. This is the changing pattern of parental investments when abortions are not available. These investments become lower relative to household income (especially at the bottom of the distribution) and more volatile (especially at the bottom of the distribution). An interesting implication of this channel is that the intergenerational persistence of family income falls as the children born at the top of the parental income distribution now have lower chances of getting education (and higher income), while the children at the bottom of the distribution are as deprived of education (and income) as before. Finally, when abortions are not available, household fertility increases and becomes more volatile. This effect is more pronounced for the young, the unskilled, and the single. The changing fertility can account for about a half of the total increase in inequality.

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Appendices

A Data Sources

Figure 1. The annual time series for the abortion rate and the fraction of pregnancies ending in abortions for 1973-2005 are taken from Jones et al. (2008).

Figure 2 and 3. The statistics are taken from Jones, Darroch and Henshaw (2002) and are adjusted to the age groups used in the analysis.

B Distributions of Teenagers, and Single and Married Households

B.1 Old Adults

At the start of the old age, there are single females and males and married couples from the previous period. These are the potential entrants in the secondary marriage market that is to be conducted. Singles search for spouses and may choose get married to their matches or stay singles. The married couples from the previous period may decide to separate.

Denote the distribution of single females at the start of the period by $\mathbf{P}_2^f(h_2^f, e^f)$, and the distribution of single males by $\mathbf{P}_2^m(h_2^m)$. The married couples from the previous period (that may separate) is described by $\mathbf{P}_2(h_2^f, h_2^m, \gamma_{-1})$ where γ_{-1} is the match specific quality they have experienced in the last period.

In the next instance of this period, previously single people meet potential spouses in the marriage market, while married people from the last period find out what the new match specific quality of their units will be and decide whether to stay married or become single.

The distributions of the old single females and males and the married households after this stage can be derived from the distributions of the potential wives and husbands in the marriage market, $\mathbf{P}_2^f(h_2^f, e^f)$ and $\widehat{\mathbf{P}}_2^m(h_2^m)$ and the married $\mathbf{P}_2(h_2^f, e^f, h_2^m, \gamma_{-1})$ from last period. The other necessary object for the derivation of these new distributions is the decision rule for marriage and divorce in the old period $\mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma)$ derived from condition (7) The old single females distribution is given by

$$\mathbf{S}_2^f(h_2^f, e^f) = \int_{\mathcal{G}\times\mathcal{H}^m} (1 - \mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma))\mathbf{P}_2^f(h_2^f, e^f)\widehat{\mathbf{P}}_2^m(h_2^m)\mathbf{\Gamma}(\gamma)d(h_2^m, \gamma) \tag{B-1}$$
$$+ \int_{\mathcal{G}\times\mathcal{G}\times\mathcal{H}^m} (1 - \mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma))\mathbf{P}_2(h_2^f, e^f, h_2^m, \gamma_{-1})\Phi(\gamma|\gamma_{-1})d(h_2^m, \gamma, \gamma_{-1}).$$

The first term of equation (B-1) sums all previously single females who decided to stay single after the old age marriage market, and the second term describes all previously married females who chose to separate from their husbands and become old singles.

The old single males distribution (B-2) is also composed of two terms, the first of which describes the previously single males who stayed single in the old age period as well, and the second is for the previously married males who chose divorce in the old age.

$$\mathbf{S}_2^m(h_2^m) = \sum_{e^f=0}^{1}[\int_{\mathcal{G}\times\mathcal{H}^f} (1 - \mathbf{I}_2(h_2^f, e^f, h_2^m, \gamma))\mathbf{P}_2^m(h_2^m)\widehat{\mathbf{P}}_2^f(h_2^f)\mathbf{\Gamma}(\gamma)d(h_2^f, \gamma) \tag{B-2}$$
$$+ \int_{\mathcal{G}\times\mathcal{G}\times\mathcal{H}^f} (1 - \mathbf{I}_2(h_2^f, h_2^m, \gamma))\mathbf{P}_2(h_2^f, h_2^m, \gamma_{-1})d(h_2^f, \gamma, \gamma_{-1})]$$

The old married couples distribution (B-3) is also composed of people who were previously single, met in the marriage market and decided to get married (first term), and people who were previously married, observed their new match specific quality and chose to stay married (second term).

$$\mathbf{M}_2(h_2^f, e^f, h_2^m) = \int_{\mathcal{G}} \mathbf{I}_2(h_2^f, h_2^m, \gamma)\mathbf{P}_2^f(h_2^f)\widehat{\mathbf{P}}_2^m(h_2^m)\mathbf{\Gamma}(\gamma)d(\gamma) \tag{B-3}$$
$$+ \int_{\mathcal{G}\times\mathcal{G}} (1 - \mathbf{I}_2(h_2^f, h_2^m, \gamma))\mathbf{P}_2(h_2^f, h_2^m, \gamma_{-1})\Phi(\gamma|\gamma_{-1})d(\gamma, \gamma_{-1})$$

B.2 Young Adults

At the start of the young age, all adult people are single and are categorized in distributions of potential mates ($\mathbf{P}_1^f(h_1^f, e^f, y)$ and $\mathbf{P}_1^m(h_1^m)m$) in the forthcoming marriage market. Then the distributions of the single females and males after the marriage market are given by

$$\mathbf{S}_1^f(h_1^f, e^f, y) = \int_{\mathcal{G}\times\mathcal{H}^m} (1 - \mathbf{I}_1(h_1^f, e^f, y, h_1^m, \gamma))\mathbf{P}_1^f(h_1^f, e^f, y)\widehat{\mathbf{P}}_1^m(h_1^m)\mathbf{\Gamma}(\gamma)d(h_1^m, \gamma) \tag{B-4}$$

for females, and

$$\mathbf{S}_1^m(h_1^m) = \sum_{y=0}^{1}\sum_{e^f=0}^{1}\int_{\mathcal{G}\times\mathcal{H}^f}(1-\mathbf{I}_1(h_1^f,e^f,y,h_1^m,\gamma))\mathbf{P}_1^m(h_1^m)\widehat{\mathbf{P}}_1^f(h_1^f,e^f,y)\mathbf{\Gamma}(\gamma)d(h_1^f,\gamma) \quad \text{(B-5)}$$

for males.

$$\mathbf{M}_1(h_1^f,y,h_1^m,\gamma) = \mathbf{I}_1(h_1^f,y,h_1^m,\gamma)\mathbf{P}_1^f(h_1^f,y)\widehat{\mathbf{P}}_1^m(h_1^m)\mathbf{\Gamma}(\gamma). \quad \text{(B-6)}$$

With these distributions at hand we can express the distributions of these partners when they get to the start of the old age, just before the secondary marriage market starts. The pool of old potential husbands is given by

$$\mathbf{P}_2^m(\xi h_1^m) = \mathbf{S}_1^m(h_1^m). \quad \text{(B-7)}$$

The expression (B-7) is derived by the the distribution of the young single males taking into account that the human capital of the young adults, h_1^m, grows over time to ξh_1^m in their old age. The pool of young single females who become old and have the chance to find husbands in the secondary marriage market at the start of the old age is summarized by

$$\mathbf{P}_2^f(\xi h_1^f,e^f) = \sum_{y=0}^{1}\sum_{\widetilde{n}=0}^{N}\pi_{1,K_1^f(h_1^f,e^f,y)\widetilde{n}}^{f,S_1^f(h_1^f,e^f,y)}\mathbf{1}(N_1^f(h_1^f,e^f,y,\widetilde{n},S_1^f(h_1^f,e^f,y))=0)\mathbf{S}_1^f(h_1^f,e^f,y) \quad \text{(B-8)}$$

for the females who do not have children in their households as young, and by

$$\mathbf{P}_2^f((1-\tau)\xi h_1^f,e^f) = \sum_{y=0}^{1}\sum_{\widetilde{n}=0}^{N}\pi_{1,K_1^f(h_1^f,e^f,y)\widetilde{n}}^{f,S_1^f(h_1^f,e^f,y)} \quad \text{(B-9)}$$

$$\times\mathbf{1}(N_1^f(h_1^f,e^f,y,\widetilde{n},S_1^f(h_1^f,e^f,y))>0)\mathbf{S}_1^f(h_1^f,e^f,y)$$

for the females who do have children as young adults. The left-hand side of equations (B-8) and (B-9) take the mass of young females of type (h_1^f,e^f,y) who might have children in the household $(N_1^f(h_1^f,e^f,y,\widetilde{n},S_1^f(h_1^f,e^f,y)) = 0)$ [15] or not $(N_1^f(h_1^f,e^f,y,\widetilde{n},S_1^f(h_1^f,e^f,y)) > 0)$ and multiply it by the probability that given a desired number of children $K_1^f(h_1^f,e^f,y)$ and a contraception treatment $S_1^f(h_1^f,e^f,y)$, these females had exactly $\widetilde{n}$ pregnancies. Note that this probability $\pi_{1,K_1^f(h_1^f,e^f,y)\widetilde{n}}^{f,S_1^f(h_1^f,e^f,y)}$ is the $(K_1^f(h_1^f,e^f,y),\widetilde{n})$-th element of the fertility matrix $\Pi_1^{f,S_1^f(h_1^f,e^f,y)}$. The evolution of distribution $\mathbf{S}_1^f(h_1^f,e^f,y)$ to distribution $\mathbf{P}_2^f(h_2^f,e^f)$ is described in two separate equations

[15] The number of children in a young single female household is

$$N_1^f(h_1^f,e^f,y,\widetilde{n},s) = y+\widetilde{n}-A_1^f(h_1^f,e^f,y,\widetilde{n},s).$$

because the human capital of females grows differently for the ones who had children and the ones with no children in their households.

The young married couples who now enter into old age and are about to choose whether to get divorced or not, are summarized in a similar way by the distribution

$$\mathbf{P}_2(\xi h_1^f, e^f, \xi h_1^m, \gamma) = \sum_{y=0}^{1} \sum_{\widetilde{n}=0}^{N} \pi_{1,K_1^{fm}(h_1^f,e^f,y,h_1^m,\gamma)\widetilde{n}}^{fm,S_1^{fm}(h_1^f,e^f,y,h_1^m,\gamma)} \tag{B-10}$$

$$\times \mathbf{1}(N_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)) = 0)\mathbf{M}_1(h_1^f, e^f, y, h_1^m, \gamma),$$

$$\mathbf{P}_2((1-\tau)\xi h_1^f, e^f, \xi h_1^m, \gamma) = \sum_{y=0}^{1} \sum_{\widetilde{n}=0}^{N} \pi_{1,K_1^{fm}(h_1^f,e^f,y,h_1^m,\gamma)\widetilde{n}}^{fm,S_1^{fm}(h_1^f,e^f,y,h_1^m,\gamma)} \tag{B-11}$$

$$\times \mathbf{1}(N_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)) > 0)\mathbf{M}_1(h_1^f, e^f, y, h_1^m, \gamma).$$ [16]

B.3 Teenagers

Teenagers are the children of the living adults in the economy. Their distribution is indexed by the ability level they own, λ, and the amount of resources invested in their human capital development, b. Denote this distribution by $T(\lambda, b)$ and note that the mass of teenagers of type (λ, b) is just the sum of all children of this type born to young single female, young married, old single female, and old married households. Thus,

$$T(\lambda, b) = \tag{B-12}$$

$$\int_{\mathcal{L}} \mathbf{\Lambda}(\lambda|\lambda_{-1}^f) d(\lambda_{-1}^f) \sum_{e^f=0}^{1} [\int_{\mathcal{H}^f} \sum_{y=0}^{1} \sum_{\widetilde{n}=0}^{N} \pi_{1,K_1^f(h_1^f,e^f,y)\widetilde{n}}^{f,S_1^f(h_1^f,e^f,y)} N_1^f(h_1^f, e^f, y, \widetilde{n}, S_1^f(h_1^f, e^f, y)) \mathbf{S}_1^f(h_1^f, e^f, y) d(h_1^f)$$

$$+ \int_{\mathcal{G}\times\mathcal{H}^m\times\mathcal{H}^f} \sum_{y=0}^{1} \sum_{\widetilde{n}=0}^{N} \pi_{1,K_1^{fm}(h_1^f,e^f,y,h_1^m,\gamma)\widetilde{n}}^{fm,S_1^{fm}(h_1^f,e^f,y,h_1^m,\gamma)} N_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma))$$

$$\times \mathbf{M}_1(h_1^f, e^f, y, h_1^m, \gamma) d(h_1^f, h_1^m, \gamma)$$

$$+ \int_{\mathcal{H}^f} \sum_{\widetilde{n}=0}^{N} \pi_{2,K_2^f(h_2^f,e^f)\widetilde{n}}^{f,S_2^f(h_2^f,e^f)} N_2^f(h_2^f, e^f, \widetilde{n}, S_2^f(h_2^f, e^f)) \mathbf{S}_2^f(h_2^f, e^f) d(h_2^f)$$

$$+ \int_{\mathcal{H}^m\times\mathcal{H}^f} \sum_{\widetilde{n}=0}^{N} \pi_{2,K_2^{fm}(h_2^f,e^f,h_2^m)\widetilde{n}}^{fm,S_2^{fm}(h_2^f,e^f,h_2^m)} N_2^{fm}(h_2^f, h_2^m, \widetilde{n}, S_2^{fm}(h_2^f, e^f, h_2^m)) M_2(h_2^f, e^f, h_2^m) d(h_2^f, h_2^m)].$$

[16] The number of children in a young married household is

$$N_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, S_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)) = y + \widetilde{n} - A_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s).$$

Then, the gender specific distributions for teenagers are given by

$$T^f(\lambda, b) = T^m(\lambda, b) = \frac{1}{2}T(\lambda, b).$$

This is so because the probability of having female or male children is equal. Take the distribution for female teenagers. It evolves to

$$T_z^f(\lambda, b, Z(\lambda, b))$$

after the contraception decision is made by the female. Furthermore, after the pregnancy outcome, y_p is revealed and the abortion decision, $A_0(\lambda, b, y_p)$ is made, the distribution changes to

$$T_y^f(\lambda, b, y)$$

having in mind the a premarital birth occurs ($y = 1$) if and only if there is a teen pregnancy ($y_p = 1$) and it is not aborted ($A_0(\lambda, b, 1) = 0$). Finally, the education outcomes are revealed for both female and male teenagers. The relevant distributions for female teenagers who are now ready to step into adult life are

$$\mathbf{P}_1^f(h_1^f, y) = T_e^f(\lambda, y, e)$$

taking into account that $h_1^f = \varkappa\lambda\chi_e$. Similarly for male teenagers,

$$\mathbf{P}_1^m(h_1^m) = T_e^m(\lambda, e)$$

with $h_1^m = \lambda\chi_e$.

C Preferences and the Negative Relationship between Income and Fertility

The goal of this section is to explain why the preference specification in Section 5.1 for consumption, number of children, and investments in them can generate a negative relationship between income and fertility.

Think of a simple static model of a representative female who spends her income I on consumption goods c, chooses whether to have one child $(n = 1)$ or two children $(n = 2)$, and how much to invest in each child she has (b). The problem to be solved is

$$\max_{c,b,n\in\{0,1\}} \alpha_c \frac{c^{1-\xi_c}}{1-\xi_c} + \alpha_n \frac{(\varepsilon + n)^{1-\xi_n}}{1-\xi_n} + (1 - \alpha_c - \alpha_n)\frac{b^{1-\xi_b}}{1-\xi_b}$$

subject to

$$c + bn \le I. \tag{C-1}$$

The indirect utility for a fixed number of children n is

$$v(n, I) = \max_{c,b} \alpha_c \frac{(I - bn)^{1-\xi_c}}{1 - \xi_c} + \alpha_n \frac{(\varepsilon + n)^{1-\xi_n}}{1 - \xi_n} + (1 - \alpha_c - \alpha_n) \frac{b^{1-\xi_b}}{1 - \xi_b}.$$

Then, the problem of choosing fertility is

$$\max_{n \in \{1,2\}} v(n, I). \tag{C-2}$$

The fertility solution to problem (C-2) is denoted by

$$N(I) = \arg \max_{n \in \{1,2\}} v(n, I).$$

To make the analysis more tractable, assume that $\xi_c = \xi_b$. In practice, the results that follow will hold for ξ_c close to ξ_b.

The offer curve for consumption and investment is given by

$$c = \left(\frac{\alpha_c}{1 - \alpha_c - \alpha_n} \right)^{\frac{1}{\xi_c}} n^{\frac{1}{\xi_c}} b. \tag{C-3}$$

Substitute (C-3) in the budget constraint (C-1) to get the demand functions for consumption and investment,

$$C(n, I) = \frac{\left(\frac{\alpha_c}{1-\alpha_c-\alpha_n} \right)^{\frac{1}{\xi_c}} n^{\frac{1}{\xi_c}}}{\left(\frac{\alpha_c}{1-\alpha_c-\alpha_n} \right)^{\frac{1}{\xi_c}} n^{\frac{1}{\xi_c}} + n} I,$$

and

$$B(n, I) = \frac{1}{\left(\frac{\alpha_c}{1-\alpha_c-\alpha_n} \right)^{\frac{1}{\xi_c}} n^{\frac{1}{\xi_c}} + n} I.$$

Then, the indirect utility as a function of n and I can be expressed as

$$v(n, I) = \frac{1}{1 - \xi_c} \frac{\alpha_c \left(\frac{\alpha_c}{1-\alpha_c-\alpha_n} \right)^{\frac{1}{\xi_c}} n^{\frac{1}{\xi_c}} + (1 - \alpha_c - \alpha_n)}{\left[\left(\frac{\alpha_c}{1-\alpha_c-\alpha_n} \right)^{\frac{1}{\xi_c}} n^{\frac{1}{\xi_c}} + n \right]^{1-\xi_c}} I^{1-\xi_c} + \alpha_n \frac{(\varepsilon + n)^{1-\xi_n}}{1 - \xi_n}. \tag{C-4}$$

When ξ_c is sufficiently small (close to zero), the fraction

$$\frac{\alpha_c \left(\frac{\alpha_c}{1-\alpha_c-\alpha_n} \right)^{\frac{1}{\xi_c}} n^{\frac{1}{\xi_c}} + (1 - \alpha_c - \alpha_n)}{\left[\left(\frac{\alpha_c}{1-\alpha_c-\alpha_n} \right)^{\frac{1}{\xi_c}} n^{\frac{1}{\xi_c}} + n \right]^{1-\xi_c}}$$

is larger for $n = 1$ than for $n = 2$. This provides an incentive to choose smaller number of children ($n = 1$) in (C-2). This incentive for reducing fertility is stronger for higher income I. Choosing high fertility ($n = 2$) may be optimal because of the second term in (C-4). Suppose that the parameters ε and ξ_n are large enough so that the two opposing effects described above are close to offsetting each other. Then, there would be a threshold income $\mathfrak{I}$ such that

$$N(I) = \begin{cases} 2 \text{ if } I < \mathfrak{I} \\ 1 \text{ if } I > \mathfrak{I} \end{cases} . \tag{C-5}$$

The fertility demand (C-5) implies the stylized negative relationship between income and fertility.

D Outline of the Numerical Solution Algorithm

1. Start with a guess for $\mathbf{P}_1^f(h_1^f, e^f, y)$, $\mathbf{P}_1^m(h_1^m)$, $\mathbf{P}_2^f(h_2^f, e^f)$, $\mathbf{P}_2^m(h_2^m)$, $\mathbf{P}_2(h_2^f, e^f, h_2^m, \gamma_{-1})$.

2. Calculate the value functions $\widetilde{V}_2^f(h_2^f, e^f, \widetilde{n}, s)$, $V_2^f(h_2^f, e^f)$, $V_2^m(h^m)$, $\widetilde{V}_2^{fm}(h_2^f, e^f, h_2^m, \gamma, \widetilde{n}, s)$, $\widetilde{V}_1^f(h_1^f, e^f, y, \widetilde{n}, s)$, $V_1^f(h_1^f, e^f, y)$, $V_1^m(h_1^m)$, $\widetilde{V}_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma, \widetilde{n}, s)$, $V_1^{fm}(h_1^f, e^f, y, h_1^m, \gamma)$ using the distributions and the normalized versions of the distributions for potential partners, that is $\widehat{\mathbf{P}}_1^f(h_1^f, e^f, y)$, $\widehat{\mathbf{P}}_1^m(h_1^m)$, $\widehat{\mathbf{P}}_2^f(h_2^f, e^f)$, and $\widehat{\mathbf{P}}_2^m(h_2^m)$.

3. Using the decision rules and distributions for singles and married, derive the distributions of teenagers and update the distributions for adults of the next generation.

4. Compare the resulting distributions $\widehat{\mathbf{P}}_1^f(h_1^f, e^f, y)$, $\widehat{\mathbf{P}}_1^m(h_1^m)$, $\widehat{\mathbf{P}}_2^f(h_2^f, e^f)$, and $\widehat{\mathbf{P}}_2^m(h_2^m)$ to the same distributions but derived from the initial guess (the ones used in Step 2.). If they are the same declare convergence, if not go to Step 1.