Expectations and Fluctuations: The Role of Monetary Policy *

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Abstract

How does the economy respond to shocks to expectations? This paper addresses this question within a cashless, monetary economy. A competitive economy features producers and consumers with *asymmetric* information. Only consumers observe current productivity and hence they perfectly anticipate prices, whereas all agents observe a noisy signal about long-run productivity. Information asymmetries imply that monetary policy and consumers' expectations have real effects. Non-fundamental, purely expectational shocks are conventionally thought of as demand shocks. While this remains a possibility, expectational shocks can also have the characteristics of supply shocks: if positive, they increase output and employment, and lower inflation. Whether expectational shocks manifest themselves as demand or supply shocks depends on the monetary policy pursued. Forward-looking policies generate multiple equilibria in which the role of consumers' expectations is arbitrary. Optimal policies restore the complete information equilibrium. They do so by manipulating prices so that producers' revenue becomes independent of productivity. I design targets for forward-looking interest-rate rules which restore the complete information equilibrium for any policy parameters. Inflation stabilization per se is typically sub-optimal as it can at best eliminate uncertainty arising through prices. This offers a motivation for the Dual Mandate of central banks.

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1 Introduction

Recent empirical work suggests that shocks to expectations contribute significantly to economic fluctuations.[1] But how so? This is a recurrent question for academics, practitioners, and op-ed columnists. There is a growing consensus that if, for instance, consumers overstate the economy's fundamentals, the economy will boom at the cost of inflation. A recent literature has formalized this idea:[2] non-fundamental, purely expectational shocks behave like demand shocks. When positive, they increase output and employment, and are inflationary. Stabilizing inflation emerges then as a natural policy recommendation.[3]

Nevertheless, Figures 1-4 show that the US economy was characterized by high cyclical employment and relatively low inflation in the mid-80s and the second half of the 90s, which are recalled as periods of exuberant optimism. Notably, Figures 3 and 4 reveal that consumer sentiment and inflation are negatively correlated.[4] An interpretation of expectational shocks as demands shocks does not seem to fit.

This paper reconsiders the nature of expectational shocks in a monetary, cashless economy where producers and consumers have asymmetric information about fundamentals and prices (inflation). I show that expectational shocks can have implications for the business cycle associated with supply shocks:[5] when positive, they increase output and employment, and they lower inflation, which is incompatible with the Phillips curve.[6] Nonetheless, the possibility that expectational shocks manifest themselves as demand shocks remains. I propose that the characteristics of expectational shocks (demand or supply) indeed reflect the monetary policy pursued.

A natural question that emerges concerns the role of monetary authority and its optimal response to shocks. With flexible prices, producers' incomplete information is the unique source of inefficiency. Asymmetric, as opposed to incomplete but symmetric, information about prices implies that monetary policy has real effects. Optimal policies restore the complete information equilibrium by rendering producers' revenue constant across states. I design targets for forward-looking policies

[1]Empirical studies on the contribution of changes in expectations to business cycle fluctuations include Beaudry and Portier (2006), Schmitt-Grohe and Uribe (2008), Blanchard et al. (2009)), Beaudry and Lucke (2010) and Barsky and Sims (2011a,b).

[2]See for example Blanchard (2009), Angeletos and La'O (2009), and especially Lorenzoni (2009, 2011).

[3]See Lorenzoni (2009).

[4]At a quarterly basis (Figure 3), the correlation of consumer sentiment and inflation is -0.53. Data are described in Appendix B.

[5]Gali (1992) explores the effects of demand and supply shocks on the US business cycle.

[6]A discussion of the Phillips curve can be found in Mankiw (2001).

which restore the complete information equilibrium for any chosen policy parameters. Further, I argue that inflation stabilization per se is typically suboptimal as it at best eliminates uncertainty arising through prices without removing producers' incomplete information.

The competitive (neoclassical) economy features two agents, a consumer/worker and a producer, and a monetary authority. Agents are price-takers. Productivity consists of a permanent and a temporary component. It is specific and known to the worker, while the producer faces uncertainty about it. Hence, there is asymmetric information. The monetary authority sets the riskless short-term nominal interest rate. I consider two interest-rate rules: a forward-looking one and a "contemporaneous" one. When discussing the business cycle, I restrict attention to "active" policies, which involve interest-rate rules with a weight on inflation greater than one.

Each period is split into two stages: In the first stage, the worker realizes his current productivity -not its individual components-, both agents observe a noisy public signal about the permanent (long-run) productivity component, and the labor market opens. In the second stage, the commodity and the nominal bond markets open, and all payments materialize.

The nominal wage, announced in stage 1, reflects the producer's expectations about productivity as well as stage 2 prices (or inflation). With constant returns to scale, the scale of production is pinned down by labor supply. The worker has complete information, so his labor decision and, consequently, production, depend on the wage and the prices he knows will prevail in stage 2.

Prices, in turn, depend on productivity, on the producer's expectations about it, and the consumer's expectations about permanent (equivalently, long-run) productivity in a way decided by monetary policy. Asymmetric information leads agents to form heterogeneous expectations about the prices to prevail; this opens the door to monetary policy. Further, to the extent that prices depend on the consumer's expectations about long-run productivity, the producer needs to second-guess the consumer. Then, in a way decided by monetary policy, the consumer's expectations will also have real effects, indirectly through prices. Therefore, that prices are announced after the labor market has cleared not only prevents productivity from being revealed, but, in combination with asymmetric information, it implies monetary policy and consumer's expectations have real effects.

Taking these into account, the producer's expectations, directly and indirectly through prices, along with productivity are the driving forces on the real side of the economy; the nominal side is affected by both agents' expectations and productivity. Monetary policy connects the two sides.

Prices perfectly reveal aggregate productivity. Agents update their beliefs about permanent productivity over time in a Bayesian way. The distinction between permanent and temporary shocks implies that their effects persist, as in Woodford (2001) and Lorenzoni (2009).

The *first* set of results is on the real side of the economy.[7] Positive, non-fundamental, shocks to the producer's expectations (henceforth, *expectational shocks*) increase employment and output (positive co-movement) temporarily. Positive permanent productivity shocks cause output to gradually increase towards its higher steady-state level, whereas employment falls temporarily.

In the former case, the producer overstates the economy's fundamentals and a higher than the marginal product of labor real wage prevails. This induces the worker to increase his labor supply. In the latter case, the producer's expectations underreact, a lower real wage prevails, and labor supply falls.

It should not come perhaps as a surprise that incomplete information manifests itself as a distortion in the *labor wedge* originating from the labor demand side. The labor wedge is defined as the ratio of the marginal product of labor to the marginal rate of substitution of leisure for consumption.[8] Chari et al. (2007) find that it is countercyclical and accounts for more than half of the US output variance. When the real wage exceeds the marginal product of labor, the labor wedge falls. Positive expectational shocks, then, induce a countercyclical labor wedge.[9]

The *second* set of results is on the nominal side of the economy. When the monetary authority targets inflation in the following period, prices depend positively on output. Combining this with the results above, it follows that a positive expectational shock increases prices temporarily, whereas a positive permanent productivity shock causes a gradual increase of prices towards their higher long-run level. The implications for inflation are attributed to the Bayesian evolution of expectations; as time evolves, the producer's expectations become more aligned with the underlying state. Following a positive expectational shock, price levels exhibit a non-monotonic pattern:[10] they jump on impact and gradually return to their long run level afterwards. Thus, positive expectational shocks cause an inflationary pressure on impact and a deflationary pressure from the following period onwards. By the same logic, permanent productivity shocks are inflationary, until they reach their higher steady-state level.

When the monetary authority targets current variables, the nominal results are more sensitive

[7] Results depend on the monetary policy parameters. An active monetary policy is sufficient for the results here.

[8] See for example Hall (1997), Chari et al. (2007) and Shimer (2009).

[9] Related papers generating a countercyclical labor wedge in response to expectational shocks include Angeletos and La'O (2009), La'O (2010) and Venkateswaran (2011). Unlike these papers, the present paper emphasizes the connection of monetary policy with the labor wedge.

[10] A similar result is obtained in Lorenzoni (2005), though not associated with disinflation.

to the policy parameters. A key difference is that a "contemporaneous" interest-rate rule specifies inflation rather than price levels. For the standard "Taylor rule," permanent productivity shocks increase inflation temporarily. However, positive expectational shocks lower inflation, just like textbook supply shocks do. Put differently, expectational shocks can push in a direction other than the one dictated by the Phillips curve. Nevertheless, if a sufficiently low weight is placed on the output gap, the nominal implications are overturned: positive expectational shocks, just like demand shocks, are inflationary, whereas positive permanent productivity shocks are disinflationary, as in Lorenzoni (2009). This leads to the central thesis of the paper: the characteristics of expectational shocks reflect the monetary policy pursued.

There are two channels for expectational shocks: the producer's expectations about current productivity and the consumer's expectations about long-run productivity. Positive expectational shocks via the consumer's expectations are inflationary. The rationale is a permanent income hypothesis one: a consumer overstating the long-run prospects of the economy raises his current demand which generates an inflationary pressure. If the producer had complete information, the consumer's expectations would have no real effects as prices would adjust. Hence it is the producer's incomplete information that brings the consumer's expectations into play. When maximizing profits, the producer forms expectations about inflation, which, as I argued, is affected by the consumer's expectations. Hence, the producer needs to second-guess the consumer which implies that the consumer's expectations have real effects indirectly through the inflation channel. Positive expectational shocks via the producer's expectations are typically disinflationary. Whether expectational shocks are inflationary (consumer effect dominates) or disinflationary (producer effect dominates) depends on the monetary policy parametrization. In particular, the greater the policy response to the output gap, the less inflationary they will be.

The *third* set of results concerns monetary policy. It was implicit earlier that, when monetary policy is forward-looking, the consumer's expectations play no, nominal or real, role. However, this is only one possibility. I show that forward-looking policies generate a continuum of equilibria across which the consumer's expectations have an arbitrary role. Importantly, this happens independently of the monetary policy parameters. In fact, I do not discuss determinacy in the sense, for example, of Clarida et al. (2000) or Bullard and Mitra (2002).[11] Furthermore, the short-run volatility of expectational shocks under forward-looking policies is considerably higher than under "contemporaneous" ones. These results can contribute to the discussion about the desirability of

[11] This discussion has recently been revived with Cochrane (2011).

forward-looking policies.[12]

The producer's incomplete information is the unique source of inefficiency, hence optimal monetary policies restore the complete information equilibrium. This can be achieved by rendering stage-2 revenue independent of productivity, a consequence of prices inversely related to productivity. In other words, optimal policies restore the complete information equilibrium not by eliminating producer's uncertainty about productivity, but by rendering it irrelevant. Inflation stabilization per se typically fails to do so as it at best eliminates the indirect (price/inflation) channel of expectations. I subsequently propose forward-looking interest-rate rules which restore the complete information equilibrium. The rules "punish" deviations of expected inflation and expected growth from targets designed with reference to their complete-information levels. These policies are different from the one proposed in Weiss (1980). In Weiss (1980), prices perfectly reveal the unknown fundamentals. Here prices are observed with a delay, so this possibility is non-existent.

This paper shares with Weiss (1980), King (1982) and Lorenzoni (2010) the idea that monetary policy is non-neutral when there is asymmetric information about variables the monetary authority will respond to.[13] Crucially, it is asymmetric, rather than incomplete but symmetric, information that breaks the policy irrelevance, proposed in Sargent and Wallace (1975, 1976).

In an extension to the main framework, I endow a forward-looking monetary authority with superior information about the following period's state. To prevent the nominal interest rate from being fully revealing, I assume that the monetary authority communicates its information with noise, which could be either a measurement error or a "surprise" monetary policy shock. The nominal interest rate serves then as an endogenous public signal. To the extent that prices depend positively on productivity, I show that a positive measurement error/monetary policy shock raises the producer's expectations about the following period's productivity, which implies that output and prices increase. However, these effects are one-period lived as the producer observes productivity at the end of each period.

[12] Clarida et al. (1999, 2000) and Giannoni and Woodford (2003) are papers on forward-looking monetary policies, however in different settings. The Bank of England is suggested to follow a forward-looking policy (see for example Nelson (2000) for an account of the period $1992-1997$).

[13] Recent papers studying monetary policy in environments with informational frictions include Adam (2007), Paciello and Wiederholt (2011) and Angeletos and La'O (2011b).

1.1 Further notes on the literature

The idea that changes in expectations affect the business cycle has its origins at least in Pigou (1926) and has recently been revived by Beaudry and Portier (2004).[14] Expectational shocks are shown to be disinflationary also in Christiano et al. (2010)[15] in a different (New-Keynesian) framework. However, this strand of literature distinguishes between shocks to current and future productivity, whereas I emphasize the distinction between fundamental and non-fundamental shocks.

This paper lies in the literature following Phelps (1970) and Lucas (1972) which has formalized the idea that incomplete information can open the door to non-neutralities of non-fundamental factors.[16] The closest paper, as I have already noted, is Lorenzoni (2009). The key difference is that Lorenzoni (2009) restricts attention to the consumer side within a New-Keynesian framework, whereas I additionally consider the producer side.[17] To the best of my knowledge, the present paper is the first to emphasize the diverse business cycle patterns caused by expectational shocks and their connection with monetary policy.

The structure of the paper is as follows. Section 2 presents the model. Equilibrium results are collected in Section 3. In an extension to the main framework, Section 4 endows the monetary authority with superior information. Section 5 discusses the role of monetary policy and proposes optimal policies. Section 6 concludes.

2 Environment

The competitive economy features two agents: a representative consumer/worker supplying labor to a representative firm he owns and a producer managing the firm. The firm produces a non-storable commodity. Agents are price-takers in both the labor and the commodity market. The economy is cashless and the only relevant financial market is a nominal bond market; a monetary authority

[14] See for example Beaudry and Portier (2006, 2007) and Jaimovich and Rebelo (2009).

[15] It has also been suggested in the empirical work of Barsky and Sims (2011b).

[16] Polemarchakis and Weiss (1977), Weiss (1980), King (1982), Bulow and Polemarchakis (1983) and, especially, Grossman and Weiss (1982) are related papers of the early literature. The literature has been revived with Woodford (2001), Morris and Shin (2002), Mankiw and Reis (2002) and Sims (2003). Hellwig (2008), Mankiw and Reis (2010), Lorenzoni (2011) and Chapter 9 in Veldkamp (2011) offer excellent overviews of the literature.

[17] A strand of the related literature, which includes for instance Angeletos and La'O (2009), Angeletos and La'O (2011a) and La'O (2010), also considers both sides however within non-monetary "Lucas-islands" frameworks featuring Dixit-Stiglitz monopolistic competition. It emphasizes the link between dispersed information and strategic complementarities across islands, which I abstract from.

sets the price of a riskless short-term nominal bond according to a "Taylor-type" rule.[18] Time is discrete and infinite commencing in period 0. Each period comprises two stages: in stage 1 only the labor market opens, whereas in stage 2 the commodity and the nominal bond markets open.

Consumer's preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t) \,,$$

with period-t utility

$$U(C_t, N_t) \;=\; \log C_t \;-\; \frac{1}{1+\zeta} \, N_t^{1+\zeta} \,.$$

C_t and N_t denote consumption and employment in period t, respectively, and $\zeta > 0$ denotes the inverse of the constant marginal utility of wealth ("Frisch") elasticity of labor supply. The consumer's time-preference is parametrized by $\beta \in (0, 1)$.

The consumer faces a sequence of budget constraints given by

$$P_t \, C_t \;+\; Q_t \, B_{t+1} \;=\; B_t \;+\; W_t \, N_t \;+\; \Pi_t \,,$$

where Q_t and B_{t+1} denote the price and holdings of nominal bonds maturing in $t+1$, respectively, P_t and W_t the commodity price and the nominal wage in t, respectively, and Π_t the firm's profits that accrue to the consumer.

The firm's technology is

$$Y_t \;=\; A_t \, N_t \,,$$

where A_t denotes the worker's productivity.

Productivity is specific and known to the worker, whereas the producer faces uncertainty about it.[19] It consists of a permanent and a temporary component (henceforth lowercase letters will denote natural logarithms),

$$a_t \;=\; x_t \;+\; u_t \,, \tag{1}$$

where x and u denote the permanent and temporary components, respectively.

The permanent component x_t follows a random walk stochastic process

$$x_t \;=\; x_{t-1} \;+\; \epsilon_t \,, \tag{2}$$

[18] Chapter in Woodford (2003) provides a treatment of cashless monetary economies.

[19] It may be argued that it is in the worker's best interest to reveal his type as he is the firm's owner. This is only a simplifying assumption. An economy with many islands and complete financial markets which preserves the asymmetry of information within an island generates similar implications.

where ϵ_t is an i.i.d shock and $\epsilon \sim N(0, \sigma_\epsilon^2)$. The temporary component u_t is i.i.d. and $u \sim N(0, \sigma_u^2)$.

All agents have costless access to a public signal about the permanent productivity component

$$s_t = x_t + e_t , \tag{3}$$

where e_t is i.i.d. and $e \sim N(0, \sigma_e^2)$. Shocks u_t , ϵ_t , and e_t are mutually independent.

The distinction between permanent and temporary productivity introduces persistence in the shock effects.

2.1 Timeline

As outlined above, each period is divided into two stages. In stage 1, the consumer realizes his temporal productivity a_t, agents observe the public signal s_t about the permanent productivity component, and the labor market opens. In stage 2, the consumption-good and the nominal-bond markets open. All payments materialize in stage 2 and are perfectly enforceable.

I specify the role of the monetary authority in the next section.

3 Equilibria

The producer's labor demand in stage 1 maximizes the firm's expected profits, $E_t^p[\lambda_t \Pi_t \mid I_{t,1}^p]$, conditional on the producer's information set in stage 1 $I_{t,1}^p$ and subject to the firm's technology. Profits are evaluated according to the consumer/owner's period-t Lagrange multiplier, λ_t . Henceforth, the producer's expectations will always refer to his expectations as of stage 1. Constant returns to scale imply the producer accommodates any labor supply at the following wage:[20].

$$W_t = \frac{E_t^p[\lambda_t P_t A_t]}{E_t^p[\lambda_t]} . \tag{4}$$

The consumer has complete information about the state of the economy and, as a result, makes all decisions in stage 1. He chooses consumption, labor supply, and bond holdings to maximize his expected utility subject to his sequence of budget constraints and a usual no-Ponzi-scheme constraint. Nominal bonds are in zero net supply, hence market clearing in the nominal bond market requires $B_{t+1} = 0$ for all t. As such, I suppress bond holdings from the state of the economy.

[20]It is central in the paper that the nominal wage be such that the producer's *expected* evaluated profits are zero. However, once the state of the economy is realized, the real wage will generally be higher or lower than productivity, yielding losses or profits, respectively

The consumer's optimality conditions are[21]

$$N_t^{\zeta} = \frac{W_t}{P_t C_t} \tag{5}$$

$$C_t = \frac{Q_t}{\beta P_t} E_t^c \left[P_{t+1} C_{t+1}\right] , \tag{6}$$

where $E_t^c[\cdot]$ refers to the consumer's expectations conditional on his information set I_t^c.

3.1 Linear equilibria

I focus on linear equilibria.[22] All equilibria are Rational Expectations equilibria. In log-linear form the optimality equations are[23]

$$w_t = E_t^p[a_t] + E_t^p[p_t] \tag{7}$$

$$\zeta n_t = w_t - p_t - c_t \tag{8}$$

$$c_t = -\log\beta + \log Q_t + E_t^c[c_{t+1} + \pi_{t+1}] . \tag{9}$$

Combining (7) and (8) results in

$$\zeta n_t = E_t^p[a_t] + E_t^p[p_t] - p_t - c_t . \tag{10}$$

I use the optimality conditions (9) and (10) in the rest of the analysis.

The existence of a monetary policy rule gets round the equilibrium indeteterminacy, nominal or real depending on whether agents have complete information or not, that would have prevailed in its absence.

Monetary authority. The monetary authority sets the gross nominal interest rate (equivalently, the inverse of the logarithm of the nominal bond price), $i_t = -\log Q_t$, according to an interest-rate rule. Two commonly used rules will be considered in sequence, a forward-looking one (henceforth,

[21] Appendix A.1 offers an analytical demonstration of the agents' problems.

[22] I ignore whether non-linear equilibria exist.

[23] Where applicable, approximations are first-order around the stochastic steady-state to be characterized below.

rule 1) and a contemporaneously-looking one (henceforth, rule 2):[24]

$$i_t \,=\, -\log\beta \,+\, \phi_\pi\, E_t^m[\pi_{t+1}] \tag{Rule 1}$$

$$i_t \,=\, -\log\beta \,+\, \phi_\pi\,\pi_t \,+\, \phi_y\,(y_t - a_t)\,. \tag{Rule 2}$$

where i_t denotes the nominal interest rate and π_t denotes inflation in period t, defined as $\pi_t := p_t - p_{t-1}$. In the case of rule 2, the monetary authority targets the output gap which is defined as the deviation of output from its complete information counterpart.

The monetary authority's information is solely based on the sequence of public signals as well as information extraction from wages and prices. In Section 4, I let it be endowed with superior information when it follows rule 1 and subsequently study the information extraction problem of the agents. I consider more rules in Section 5, in which I explicitly study the optimal monetary policies in the current framework.

State. The state of the economy as of period t coincides with the the entire history $\Psi_t = \{(a_\tau)_{\tau=0}^t, (s_\tau)_{\tau=0}^t\}$. Past shocks are part of the current state due to the agents' formation of expectations, which I analyze in Section 3.2.4. I show below that the producer's information set in stage 1 is $I_{t,1}^p \,=\, \{(a_\tau)_{\tau=0}^{t-1}, (s_\tau)_{\tau=0}^t\}$ as prices or inflation perfectly reveal productivity at the end of each period. For the same reason the monetary authority's information set when it steps in, I_t^m, is the same as the consumer's I_t^c, which coincides with the state.

3.2 Equilibrium under rule 1

3.2.1 Complete information benchmark

Consider the case where the state of the economy is publicly known. Then, the real side of the economy is determined irrespectively of the public signal and the pursued monetary policy; it can be confirmed that $n_t^* \,=\, 0$ and $y_t^* \,=\, a_t$.

The Euler equation (9) becomes

$$E_t^c[a_{t+1}] \,-\, a_t \,=\, (\phi_\pi - 1)(E_t^c[p_{t+1}] \,-\, p_t)\,,$$

[24]Rule 1 has been suggested in Clarida et al. (1999, 2000). Nelson (2000) proposes that a forward-looking rule fits well the Bank of England's policy in 1992-1997. Rule 2 has been suggested by Taylor (1993, 1999) to fit Fed's policy in the period 1987-1992.

a solution to which is $p_t^* = \frac{1}{\phi_\pi - 1} a_t$.[25]

3.2.2 Incomplete information

Conjecture that

$$c_t = \xi_1 E_t^p[a_t] + \xi_2 a_t \tag{C1}$$

$$p_t = \kappa_1 E_t^p[a_t] + \kappa_2 a_t \,. \tag{C2}$$

Conjectures (C1) and (C2) imply that the state of the economy is viewed as $\Psi_t = \{E_t^p[a_t],\, a_t\}$.[26] The information set of the producer at the beginning of stage 1 can be restated as $I_t^p = \{E_t^p[a_t]\}$.[27] The monetary authority can fully extract the current state by observing the public signal in stage 1 and the commodity price (alternatively, quantity or employment) in stage 2, which by conjecture (C2) (respectively, (C1)), perfectly reveals productivity a_t. In other words, when the monetary authority steps in at the beginning of stage 2, it has the same information set as the consumer; this applies to the producer's information set in stage 2 as well, i.e. $I_t^m = I_{t,2}^p = I_t^c = \Psi_t$.

As I show in Appendix A.2, combining conjectures (C1) and (C2) with (10) and the market-clearing condition, $y_t = c_t$, yields

$$y_t = \frac{1}{\phi_\pi + \zeta(\phi_\pi - 1)} \left(\phi_\pi E_t^p[a_t] + \zeta(\phi_\pi - 1)\, a_t\right) \tag{11}$$

$$n_t = \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} \left(E_t^p[a_t] - a_t\right) \tag{12}$$

$$p_t = \frac{1}{\phi_\pi - 1}\, y_t \,. \tag{13}$$

[25]Since $E_t^c[a_{t+1}] = E_t^c[x_t] \neq a_t$ (see also Section 3.2.4), the possibility of prices being fixed in equilibrium appears only as a limit case for $\phi_\pi \to \infty$. It is also a possibility in the special case where $\sigma_e^2 = \sigma_u^2 = 0$. Constant prices could also have prevailed (as a unique non-explosive path) if either productivity a_t evolved as a random walk, or if the economy were static.

[26]In Section 3.2.5 I consider an enlarged state and show the existence of other linear equilibria.

[27]Combining conjectures (C1) and (C2) with (7) implies $w_t = (1 + \kappa_1 + \kappa_2)\, E_t^p[a_t]$; the nominal wage perfectly reveals $E_t^p[a_t]$ to the consumer and the monetary authority in stage 1. Hence, if the signal were instead privately observed by the producer, the nominal wage would generally perfectly communicate it. Exception would be the case where $\kappa_1 + \kappa_2 = 1$ which replicates the complete information equilibrium. Section 5 explores this case.

It follows from (11) and (13) that

$$\pi_t = \frac{1}{(\phi_\pi - 1)\left[\phi_\pi + \zeta(\phi_\pi - 1)\right]} \left(\phi_\pi \left(E_t^p[a_t] - E_{t-1}^p\left[a_{t-1}\right]\right) + \zeta(\phi_\pi - 1)\left(a_t - a_{t-1}\right)\right). \qquad (14)$$

Equation (11) shows that output is a weighted average[28] of productivity and the producer's expectations about it. The respective weights depend on the Frisch elasticity of labor supply, parametrized by ζ, and the monetary policy parameter ϕ_π. Equation (12) shows that employment depends proportionally on the wedge between the producer's expectations about productivity and productivity itself. By (13), prices are a monotone transformation of output.[29] For a monetary policy response to inflation greater than one-to-one ($\phi_\pi > 1$), output and prices depend positively on the producer's expectations about productivity and productivity itself, whereas employment depends positively on their distance.[30] Inflation, by equation (14), is a weighted average of the change in producer's expectations and the change in productivity in the last two periods.

Let me conclude this section with some remarks. First, each value of ϕ_π is associated with a unique equilibrium; the equilibrium with constant prices is obtained as a limit case for $\phi_\pi \to \infty$. Second, observe that the optimal monetary policy given that rule 1 is followed, on which I elaborate in Section 5, is a zero-response to expected inflation policy, $\phi_\pi = 0$. In this case all variables are at their complete information (efficient) levels. In Section 5 I further consider an enriched version of policy rule 1 in which the authority places weight on the deviations of expected inflation and expected growth rate from targets related to their complete information counterparts. Last, note that for $E_t^p[a_t] = a_t$ the complete information benchmark equilibrium prevails.

3.2.3 Labor wedge

Following Chari et al. (2007) and Shimer (2009), the labor wedge is defined as the ratio of the marginal product of labor to the marginal rate of substitution of leisure for consumption, by con-

[28]This is a direct consequence of logarithmic preferences in consumption.

[29]Output, employment, and prices are non-stationary. Stationarity can be restored by normalizing all variables with the permanent productivity component. For instance, in the case of output we could instead use $Y_t^s = \frac{Y_t}{e^{x_t}}$ ($y_t^s = y_t - x_t$ in logs). However, throughout the paper I use the non-normalized variables.

[30]For $\frac{1}{1+\zeta} < \phi < 1$ output depends positively on the producer's expectations about productivity and negatively on productivity, whereas for $0 < \phi < \frac{1}{1+\zeta}$ it depends negatively on the producer's expectations and positively on productivity. The opposite relations are true for price levels. Employment has the same sign as the weight of expectations in output.

struction equal to $\frac{1}{1-\tau_{n,t}}$ in the expression below:

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\tau_{n,t})\, MP_{n,t}$$

where $U_{n,t}$ and $U_{c,t}$ denote the marginal disutility of labor and marginal utility of consumption, respectively, and $MP_{n,t}$ the marginal product of labor in period t. The above expression becomes here

$$N_t^{-(1+\zeta)} = \frac{1}{1-\tau_{n,t}}\,.$$

Under complete information, $N_t^* = 1$ and the labor wedge is equal to 1.

Under incomplete information this will generally not be the case; taking logs and using (12) implies

$$-\log(1-\tau_{n,t}) = -\frac{\phi_\pi(1+\zeta)}{\phi_\pi + \zeta(\phi-1)}\,(E_t^p[a_t] - a_t)\,. \tag{15}$$

For $\phi_\pi > \frac{1}{1+\zeta}$, in case $E_t^p[a_t] > a_t$, the log-labor wedge is negative, and positive, otherwise. In addition, it is decreasing in the monetary policy parameter, ϕ_π [31] and becomes zero for $\phi_\pi = 0$. One can observe that purely expectational shocks induce a countercyclical labor wedge. This is in line with the observed countercyclicality of the labor wedge documented in Chari et al. (2007) and Shimer (2009).

3.2.4 Equilibrium dynamics

The producer's and the consumer's expectations about productivity evolve, respectively, as

$$E_t^p[a_t] = E_{t,1}^p[x_t] = (1-\mu)\,E_{t-1,2}^p[x_{t-1}] + \mu\, s_t \tag{16}$$

$$E_{t,2}^p[x_t] = E_t^c[x_t] = (1-k)\,E_{t-1}^c[x_{t-1}] + k\,[\theta\, s_t + (1-\theta)\,a_t]\,, \tag{17}$$

where μ, k, θ depend on the variances $\sigma_\epsilon^2, \sigma_e^2, \sigma_u^2$ and are in $(0,1)$. The first equality in (17) is due to agents having the same information set in stage 2. Appendix A.3 offers an explicit treatment of the formation of expectations.

Before turning to the impulse response functions, let me make three remarks. First, the *stochastic* steady-state is pinned down by the permanent productivity component x_t, which by (2) evolves as a random walk (see also fn. 29). The steady-state is typically different from the efficient, complete information level of the economy which is pinned down by aggregate productivity a_t. In the analysis

[31] Note that there is a discontinuity for $\phi_\pi = \frac{1}{1+\zeta}$.

of the impulse response functions below, the economy has already reached its steady-state which is assumed to coincide with its complete information counterpart before any shocks hit. As such, the two will remain coincidental after a permanent productivity or an expectational shock and they will differ only on impact following a temporary productivity shock. Second, the signs I report in the equations refer to $\phi_\pi > 1$; that is the monetary authority follows an "active" policy, along the lines of Taylor (1999).[32] Third, Figures 5-10 show the impulse responses. I follow the parametrization in Lorenzoni (2009) (one may check the references therein). The calibrated parameters are collected in Table 1. The parametrization implies the Kalman gain terms, μ and k, are 0.22 and 0.23, respectively, whereas the relative weight the consumer places on the public signal is 0.96. In all figures, impulse responses are for one standard deviation shocks. Periods, appearing on the horizontal axis, are interpreted as quarters.

If a shock to the permanent productivity component $\epsilon_t = 1$ arises, the consumer's expectations about productivity in period $t+s$ increase by $1 - (1-k)^{s+1}$ as implied by (17), whereas the producer's expectations increase by $1 - (1-\mu)[1-(1-k)^s]$ as implied by (16). The impulse response functions are

$$\frac{dy_{t+s}}{d\epsilon_t} = 1 - (1-k)^s \frac{\phi_\pi (1-\mu)}{\phi_\pi + \zeta(\phi_\pi - 1)} \in (0,1) \tag{18}$$

$$\frac{dn_{t+s}}{d\epsilon_t} = -(1-k)^s \frac{\phi_\pi (1-\mu)}{\phi_\pi + \zeta(\phi_\pi - 1)} < 0 \tag{19}$$

$$\frac{d\pi_t}{d\epsilon_t} = \frac{1}{\phi_\pi - 1}\left(1 - \frac{\phi_\pi (1-\mu)}{\phi_\pi + \zeta(\phi_\pi - 1)}\right) > 0 \tag{20}$$

$$\frac{d\pi_{t+s}}{d\epsilon_t} = (1-k)^{s-1} \frac{\phi_\pi (1-\mu)\, k}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} > 0 \quad \text{for } s \geq 1\,. \tag{21}$$

A unit increase in the permanent productivity shock causes an equivalent change in steady-state output and no change in steady-state employment. However, we can see from (18) and (19) (see also Figure 5) that a positive permanent productivity shock causes output to increase by less

[32] For $\frac{\zeta}{1+\zeta} < \phi_\pi < 1$, positive shocks to expectations generate positive co-movement, are disinflationary on impact, and inflationary afterwards, while permanent productivity shocks cause negative co-movement, are inflationary on impact, and disinflationary afterwards. For $0 < \phi_\pi < \frac{\zeta}{1+\zeta}$, positive shocks to expectations generate negative co-movement, are inflationary on impact, and disinflationary afterwards, while permanent productivity shocks generate positive co-movement, are disinflationary on impact, and inflationary afterwards.

Table 1: Calibrated parameters

Inverse of Frisch elasticity of labor supply	ζ	0.5
Monetary policy weight on inflation	ϕ_π	1.5
Standard deviation of permanent productivity shock	σ_ϵ	0.0077
Standard deviation of temporary productivity shock	σ_u	0.15
Standard deviation of expectational shock	σ_e	0.03

than one and employment to fall temporarily. By (15), the labor wedge increases temporarily. This happens because expectations underreact after a positive permanent productivity shock. As a result, labor demand shifts inwards and the real wage falls relative to its efficient level.[33] Equation (21) suggests productivity shocks are inflationary (see also Figure 6). The positive dependence of prices on expectations for $\phi_\pi > 1$, as (11) and (13) imply, underlies this result. Hence, as expectations converge to the new permanent productivity level, prices get closer to their steady-state level, implying inflation along the way. This last result is in sharp contrast with Lorenzoni (2009), in which permanent productivity shocks are disinflationary.

To pin down the impulse responses of the nominal and the real interest rate (Figure 6) I need to specify the impulse response of expected inflation:

$$\frac{dE^c_{t+s}[\pi_{t+s+1}]}{d\epsilon_t} = (1-k)^s \, \frac{\phi_\pi \,(k-\mu) - \zeta(\phi_\pi - 1)(1-k)}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} \,. \tag{22}$$

The nominal rate is $i_{t+s} = \phi_\pi \, E^c_{t+s}[\pi_{t+s+1}]$ and the real rate $r_{t+s} = (\phi_\pi - 1)\, E^c_{t+s}[\pi_{t+s+1}]$.[34]

Figure 6 shows that following a permanent productivity shock inflation expectations fall and so are the nominal and the real interest rate. In the limit as $s \to \infty$, expectations converge to the new productivity level, output converges to its new steady-state, whereas the remaining variables return to their pre-shock levels.

If a unit shock to the noise component of the public signal, e_t, arises, the consumer's expectations in period $t+s$ increase by $(1-k)^s \, k\,\theta$. The producer's expectations increase on impact by μ and

[33] Gali (1999) and Basu et al. (2006) also argue that positive technology shocks are contractionary.

[34] In addition, notice that $E^c_t[y_{t+s}] = E^c_t[x_t]$ and $E^c_t[\pi_{t+s}] = 0$ for $s \geq 1$. These results follow from (11), (14), and (17) together.

in period $t+s$ for $s \geq 1$ by $(1-k)^{s-1}(1-\mu)\,k\,\theta$. The impulse response functions are

$$\frac{dy_t}{de_t} = \frac{dn_t}{de_t} = \frac{\phi_\pi\,\mu}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0, \tag{23}$$

$$\frac{dy_{t+s}}{de_t} = \frac{dn_{t+s}}{de_t} = (1-k)^{s-1}\frac{\phi_\pi\,(1-\mu)\,k\,\theta}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0, \quad \text{for } s \geq 1 \tag{24}$$

$$\frac{d\pi_t}{de_t} = \frac{\phi_\pi\,\mu}{(\phi_\pi - 1)\,[\phi_\pi + \zeta(\phi_\pi - 1)]} > 0 \tag{25}$$

$$\frac{d\pi_{t+1}}{de_t} = -\,\frac{\phi_\pi\,[\mu - (1-\mu)\,k\,\theta]}{(\phi_\pi - 1)\,[\phi + \zeta(\phi_\pi - 1)]} \tag{26}$$

$$\frac{d\pi_{t+s}}{de_t} = -\,(1-k)^{s-2}\frac{\phi_\pi\,(1-\mu)\,k^2\,\theta}{(\phi_\pi - 1)[\phi + \zeta(\phi_\pi - 1)]} < 0 \quad \text{for } s \geq 2\,. \tag{27}$$

Equations (23) and (24) demonstrate the positive co-movement result (also Figure 7): output and employment increase in response to a positive expectational shock. The result is due to the producer overstating the worker's productivity. This results in an outward shift in labor demand and a higher, relative to the efficient level, real wage.[35] In the limit as $s \to \infty$, expectations converge to the true level of productivity implying both output and employment return to their steady-state levels.

As I argued previously, for $\phi_\pi > 1$, the prices response to shocks is related positively to the producer's expectations. Hence, a positive expectational shock causes an increase in the price levels (Figure 8). Naturally, inflation is caused on impact; however, as agents update their beliefs over time, their expectations become more aligned with fundamentals and, hence prices return monotonically to their steady-state value, generating thereby disinflation as reflected in (27).[36] The previous remarks combined suggest a non-monotonic response of price levels to a positive expectational shock.[37] All effects vanish as $s \to \infty$.

That positive expectational shocks are disinflationary is a central result in the paper. It supports

[35] Even though on expectation profits are zero, firms make negative profits if producers overstate the state of the economy. Negative profits are subtracted in a lump-sum fashion from the consumer/owner's budget constraint.

[36] Whether there is inflation or deflation in period $t+1$ depends on the variances of the shocks. The parametrization here implies the latter.

[37] A similar result in a different, however, framework is obtained in Lorenzoni (2005).

my argument that expectational shocks can cause effects often associated with supply shocks[38] and comes in sharp contrast with Lorenzoni (2009), in which expectational shocks cause effects that pertain to demand shocks. One might claim that since I focus on the producer's side such a result could have been expected. This is incorrect. Price levels increase relative to their efficient levels in response to positive expectational shocks. However they do so at a decreasing rate over time and as, by definition, inflation is a dynamic variable, disinflation is caused from the following period onwards. In Section A.4 I present an example in which price levels fall in response to positive expectational shocks.

The impulse response function of the consumer's inflation expectations (Figure 8) is

$$\frac{dE_t^c[\pi_{t+1}]}{de_t} = \frac{\zeta(\phi_\pi - 1)\, k\, \theta \;-\; \phi_\pi\, (\mu - k\, \theta)}{(\phi_\pi - 1)[\phi_\pi \,+\, \zeta(\phi_\pi - 1)]} \tag{28}$$

$$\frac{dE_{t+s}^c[\pi_{t+s+1}]}{de_t} = (1-k)^{s-1} \frac{\phi_\pi\, (\mu - k) \,+\, [\zeta(\phi_\pi - 1)\,(1-k)]\, k\, \theta}{(\phi_\pi - 1)[\phi_\pi \,+\, \zeta(\phi_\pi - 1)]} \quad \text{for } s \geq 1\,. \tag{29}$$

Inflation expectations increase, given the parametrization, resulting in higher nominal and real interest rates. The former is, perhaps, associated with demand rather than supply shocks, however the increase in the real rate is a typical effect of a supply shock.

A temporary productivity shock causes responses similar on impact to those in the permanent productivity shock case; from the following period onwards the shock only affects the agents' expectations, hence the responses resemble the ones in the expectational shock case. In particular, the consumer's expectations in period $t+s$ increase by $(1-k)^s\, k\,(1-\theta)$, whereas the producer's expectations are unchanged on impact, as changes in the temporary productivity component affect their expectations with one period lag, and increase by $(1-k)^{s-1}\,(1-\mu)\, k\,(1-\theta)$ in period $t+s$ for $s \geq 1$. In particular, in period t

$$\frac{dy_t}{du_t} = \frac{\zeta(\phi_\pi - 1)}{\phi_\pi \,+\, \zeta(\phi_\pi - 1)} \;\in (0,1) \tag{30}$$

$$\frac{dn_t}{du_t} = -\frac{\phi_\pi}{\phi_\pi \,+\, \zeta(\phi_\pi - 1)} \;<\; 0 \tag{31}$$

$$\frac{d\pi_t}{du_t} = \frac{\zeta}{\phi_\pi \,+\, \zeta(\phi_\pi - 1)} \;>\; 0\,. \tag{32}$$

[38]Gali (1992) considers the textbook IS-LM model coupled with a Phillips curve and studies the business cycle effects of demand and supply shocks.

In the subsequent periods the responses are

$$\frac{dy_{t+s}}{du_t} = \frac{dn_{t+s}}{du+_t} = (1-k)^{s-1} \frac{\phi_\pi (1-\mu) k (1-\theta)}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0, \quad \text{for } s \geq 1 \tag{33}$$

$$\frac{d\pi_{t+1}}{du_t} = - \frac{\zeta(\phi_\pi - 1) - \phi_\pi (1-\mu) k(1-\theta)}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} \tag{34}$$

$$\frac{d\pi_{t+s}}{du_t} = -(1-k)^{s-2} \frac{\phi_\pi (1-\mu) k^2 (1-\theta)}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} < 0 \quad \text{for } s \geq 2\,. \tag{35}$$

The response of inflation expectations is given by

$$\frac{dE_t^c[\pi_{t+1}]}{du_t} = \frac{\phi_\pi k (1-\theta) - \zeta(\phi_\pi - 1)[1 - k(1-\theta)]}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} \tag{36}$$

$$\frac{dE_{t+s}^c[\pi_{t+s+1}]}{du_t} = (1-k)^{s-1} \frac{[\phi_\pi (\mu - k) + \zeta(\phi - 1)(1-k)] k (1-\theta)}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} \quad \text{for } s \geq 1\,. \tag{37}$$

Figures 9 and 10 display the impulse response functions.

3.2.5 Multiple equilibria

Consider the more general conjectures:[39]

$$c_t = \hat{\xi}_1 E_t^p[a_t] + \hat{\xi}_2 a_t \tag{C1'}$$

$$p_t = \hat{\kappa}_1 E_t^p[a_t] + \hat{\kappa}_2 E_t^c[x_t] + \hat{\kappa}_3 a_t\,. \tag{C2'}$$

Conjectures (C1') and (C2') imply the state can sufficiently be described by $\Psi_t = \{E_t^p[a_t], E_t^c[x_t], a_t\}$.

[39] See the analysis in the case of rule 2 on why I do not include $E_t^c[x_t]$ in (C1').

Taking the same steps as before (see Appendix A.2.1) yields

$$\hat{\xi}_1 \;=\; \frac{1 \,+\, \hat{\kappa}_3 \,+\, k\,(1-\theta)\,\hat{\kappa}_2}{1+\zeta} \tag{38}$$

$$\hat{\xi}_2 \;=\; 1 \,-\, \hat{\xi}_1 \tag{39}$$

$$\hat{\xi}_1 \;=\; (\phi_\pi - 1)\,\hat{\kappa}_1 \tag{40}$$

$$\hat{\kappa}_1 \,+\, \hat{\kappa}_3 \;=\; \frac{1}{\phi_\pi - 1}\;. \tag{41}$$

There are 4 equations and 5 unknowns.

Combining (38)-(41) yields

$$\hat{\xi}_1 \;=\; \frac{\phi_\pi}{\phi_\pi \,+\, \zeta(\phi_\pi - 1)} \,+\, \frac{(\phi_\pi - 1)\,k\,(1-\theta)}{\phi_\pi \,+\, \zeta(\phi_\pi - 1)}\,\hat{\kappa}_2\,. \tag{42}$$

Prices can be expressed

$$p_t \;=\; \frac{1}{\phi_\pi - 1}\,y_t \,+\, \hat{\kappa}_2\,E^c_t[x_t]\,. \tag{43}$$

A key difference with before is that, despite prices being flexible, the consumer's expectations have real effects as the presence of $\hat{\kappa}_2$ in (42) attests (for $\hat{\kappa}_2 \neq 0$). This is because of asymmetric information which implies that agents form heterogeneous expectations about prices. I elaborate on this in the next part.

Each value of $\hat{\kappa}_2$ corresponds to a different equilibrium. An immediate implication for monetary policy is that targeting expected inflation invites multiple (linear) equilibria in which the role of the consumer's expectations is arbitrarily specified.[40] Notably, this result holds for any value ϕ_π in the interest-rate rule. Setting $\hat{\kappa}_2 \,=\, 0$ pins down the equilibrium given by equations (11)-(14).

As expected, depending on $\hat{\kappa}_2$, expectational shocks can cause positive or negative co-movement and raise or lower price levels, resulting in lower or higher inflation, respectively. Last, observe that the volatility due to shocks to expectations increases in the absolute value of $\hat{\kappa}_2$, as (42) suggests.

[40]Under complete information, like before, $y^*_t \,=\, a_t$, however prices depend arbitrarily on the consumer's expectations; one solution is $p^*_t \,=\, \frac{1}{\phi_\pi - 1}\,a_t \,+\, \hat{\kappa}_2\,E^c_t[x_t]\,.$

3.3 Equilibrium under rule 2

3.3.1 Complete information benchmark

Like in the previous case, on the real side $y_t^* \,=\, a_t$ and $n_t^* \,=\, 0$. However, the nominal side generally differs. Conjecture that $\pi_t \,=\, \vartheta_1\, E_t^c[x_t] \,+\, \vartheta_2\, a_t$ and confirm that $\pi_t^* \,=\, \frac{1}{\phi_\pi}\,(E_t^c[x_t] - a_t)$.

3.3.2 Incomplete information

Conjecture that

$$c_t \,=\, \xi_3\, E_t^p[a_t] \,+\, \xi_4\, a_t \tag{C3}$$

$$\pi_t \,=\, \kappa_3\, E_t^p[a_t] \,+\, \kappa_4\, E_t^c[x_t] \,+\, \kappa_5\, a_t\,. \tag{C4}$$

The state of the economy can be expressed as $\Psi_t \,=\, \{E_t^p[a_t],\, E_t^c[x_t],\, a_t\}$ and the information sets of the agents and the monetary authority remain like in the previous case.

The labor market optimality condition (10) can be written

$$\zeta n_t \,=\, E_t^p[a_t] \,+\, E_t^p[\pi_t] \,-\, \pi_t \,-\, c_t\,. \tag{44}$$

Combining the above conjectures with the optimality conditions, (9) and (44), and market-clearing (Appendix A.2 collects the derivations) yields

$$y_t \,=\, \frac{1}{\phi_\pi\,(1+\zeta) - (1+\phi_y)}\,\left([\phi_\pi - 1 + k(1-\theta)]\, E_t^p[a_t] \,+\, [\phi_\pi\,\zeta - \phi_y - k\,(1-\theta)]\, a_t\,\right) \tag{45}$$

$$\pi_t \,=\, \frac{1}{\phi_\pi}\left(-\,\frac{(1+\phi_y)\,[\phi_\pi - 1 + k\,(1-\theta)]}{\phi_\pi\,(1+\zeta) - (1+\phi_y)}\, E_t^p[a_t] \,+\, E_t^c[x_t] \,+\, \frac{\phi_\pi\,(\phi_y - \zeta) + (1+\phi_y)\,k(1-\theta))}{\phi_\pi\,(1+\zeta) - (1+\phi_y)}\, a_t\right), \tag{46}$$

where k, θ are parameters associated with the learning problem of the consumer introduced previously and derived in Appendix A.3.

A key remark is that, unlike in the baseline case of rule 1 and like in equilibria characterized by (38)-(41), the consumer's expectations have an explicit and well-defined role. One could conjecture that consumption in (C3) also depends on the consumer's expectations and, subsequently, verify that indeed consumer's expectations do not enter equilibrium consumption directly. This happens because what matters for the labor decision in stage 1, and hence the real side of the economy, is productivity and the producer's -not the consumer's- expectations about it as well as about inflation, as (44) attests.

Nevertheless, the consumer's expectations have real effects indirectly through inflation, a direct implication of asymmetric information. To the extent that inflation depends on the consumer's expectations, the producer needs to second-guess the consumer when forming expectations about inflation. In particular, as (90) in Appendix A.4 shows,

$$E_t^p[E_t^c[x_t]] \,=\, E_t^c[x_t] \,+\, k\,(1-\theta)\,(E_t^p[a_t] \,-\, a_t)\,.$$

However, what matters for the labor decision is the *wedge* between the producer's and the consumer's expectations about inflation. Given conjecture (C4) and the fact that $E_t^c[p_t] \,=\, p_t$, it follows that $E_t^p[p_t] \,-\, p_t \,=\, [\kappa_4\, k(1-\theta) \,+\, \kappa_5]\,(E_t^p[a_t] \,-\, a_t)\,.$ Hence, we can confirm that the consumer's expectations do not affect the labor decision directly, but they do so indirectly, as the presence of the parameter κ_4 attests.

Crucially, what lies in the common information of the agents (for example, the producer's expectations) and what lies outside both of the agents' information sets (possibly, non-fundamental shocks) has *no* real effect through the inflation channel.

Before continuing, let me point out that

$$\kappa_3 \,+\, \kappa_4 \,+\, \kappa_5 \,=\, 0 \tag{47}$$

$$\kappa_3 \,+\, \kappa_5 \,=\, -\frac{1}{\phi_\pi}\,. \tag{48}$$

Combining (47) and (48) implies $\kappa_4 \,=\, \frac{1}{\phi_\pi}$, which we can see in (46); the consumer's expectations are positively related to inflation, and, consequently, indirectly through inflation positively related to output. The logic underlying this result is a permanent hypothesis one: if the consumer overstates the long-run prospects of the economy, consumption smoothing implies higher demand in the current period, which in turn causes an inflationary pressure.

Turning to the producer, we can see from (45) and (46) that $\phi_\pi \,>\, \max\{\frac{1+\phi_y}{1+\zeta}\,,\, 1\}$ is a sufficient condition for the producer's expectations to be positively related to output and negatively related to inflation. However, this does not necessarily imply that a positive expectational shock lowers inflation, as an expectational shock also affects the consumer's expectations which point to the opposite direction, as I argued above. Supposing that the expectational shock affects the agents' expectations in the same way,[41] it follows that a positive expectational shock lowers inflation as long as $\kappa_3 + \kappa_4 < 0$. By (47) and (48), this is equivalent to requiring $\kappa_5 > 0$. Inspecting (46), we can see that the coefficient κ_5 increases in the monetary policy weight on the output gap. Its sign depends on how the policy response to the output gap relates to the Frisch elasticity of labor supply. In particular, a value of ϕ_y equal to the inverse Frisch elasticity of labor supply ζ is a sufficient condition for κ_5 to be positive and, consequently, expectational shocks to be negatively related to inflation. The picture that emerges then is that expectational shocks can manifest themselves as supply or demand shocks depending on the pursued monetary policy.

Turning to productivity, maintaining the assumption that expectational shocks affect the agents' expectations in the same way, a direct implication of (47) and (48) is that productivity and expectational shocks cannot both increase or lower inflation. In addition, $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}, \frac{(1+\phi_y)\,k\,(1-\theta)}{\zeta-\phi_y}\}$ is a sufficient condition for productivity to be positively related to output.

Last, when $E_t^p[a_t] = a_t$, the complete information equilibrium is pinned down.

3.3.3 Labor wedge

It follows from (45) that

$$n_t = \frac{\phi_\pi - 1 + k(1-\theta)}{\phi_\pi\,(1+\zeta) - (1+\phi_y)}\ \left(E_t^p[a_t] - a_t\right). \tag{49}$$

Taking the same steps as in the case of rule 1, the labor wedge in logs is given by

$$-\log\left(1-\tau_{n,t}\right) = -\frac{[\phi_\pi - 1 + k(1-\theta)]\,(1+\zeta)}{\phi_\pi\,(1+\zeta) - (1+\phi_y)}\ \left(E_t^p[a_t] - a_t\right). \tag{50}$$

Maintaining that $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}, 1\}$, any shock that implies $E_t^p[a_t] > a_t$ induces a countercyclical labor wedge.

3.3.4 Equilibrium dynamics

I deal with this case numerically, even though a closed-form representation of the dynamics can be obtained along the lines of Section 3.2.4. The baseline parametrization remains as in Table 1. In addition, I initially set the response to the output gap $\phi_y = 0.5$.[42]

[41] That is I assume $k\,\theta = \mu$ in the learning problems of the agents. This is a good approximation if the temporary productivity shock has a high variance relative to the expectational shock, which is consistent with the parametrization in Table 1.

[42] The monetary policy parameters are based on Taylor (1993).

Like in the case of rule 1, positive permanent productivity shocks (Figure 11) cause an increase in output, a fall in employment, and raise inflation. The labor wedge is procyclical, while the nominal and the real interest rates fall.

The real implications of positive expectational shocks (Figure 12) are also unchanged: output and employment increase, the labor wedge falls, and the real interest rate increases. However, the implications for the nominal side are different. This could have been expected as (46) pins down inflation, whereas (13) pins down price levels. Hence, price levels cannot exhibit a non-monotonic response, as they do under rule 1. In particular, for the considered parametrization, price levels fall after a positive expectational shock. They do so at a decreasing rate over time until they die out in the long-run.[43] Crucially, what is common under both interest-rate rule considerations is that positive expectational shocks lower inflation (from the following period onwards under rule 1). In addition, they both increase the nominal interest rate.

The impulse responses to a temporary productivity shock (Figure 13) are initially similar to the ones of a permanent productivity shock and, subsequently, to the ones of an expectational shock. As argued above, this is because they affect productivity only on impact, whereas from the following period onwards they only affect expectations.

As I pointed out above, the impulse responses when rule 2 is followed are generally sensitive to the specification of the monetary policy rule. For instance, consider the case in which the authority does not respond to the output gap, that is $\phi_y = 0$, with all other parameters left unchanged. While everything else remains unchanged, the implications for inflation are reversed. In particular, positive permanent productivity shocks lower inflation whereas positive expectational shocks increase inflation. Figures 14-16 show the impulse responses to (one standard-deviation) positive permanent productivity, expectational, and temporary productivity shocks, respectively. These results are perfectly in line with Lorenzoni (2009), in which the monetary authority also targets only current inflation. The central message that emerges then is that how shocks affect the economy depends on the type of interest-rate rule followed as well as its parametrization. More precisely, expectational shocks can well manifest themselves as supply or demand shocks depending on how the monetary authority acts. Notably, unlike in Lorenzoni (2009), this happens in a perfectly competitive environment where agents are price-takers and the real interest rate can freely adjust.

A natural question is why. The reason is that I assign an explicit role to the producer's ex-

[43] An additional difference between the equilibria under rules 1 and 2 is that in the latter case expected inflation is always zero as can be seen in Figures 7 and 8.

pectations. In particular, I replace the nominal rigidities in Lorenzoni (2009) with the producer's uncertainty about productivity. Shocks then affect the expectations of both the consumer and the producer. Like in Lucas (1972), producers' uncertainty (under rational expectations) opens the door to monetary policy, whose specification, as I claimed, is key to how shocks demonstrate themselves. However, unlike Lucas (1972),[44] Lorenzoni (2009), Angeletos and La'O (2009) and the related literature in which information is common on an island and differs across islands, here agents have asymmetric information within the same island (local economy). In fact, this is what pushes monetary policy and consumer's expectations through the door: given that prices (inflation) depend on monetary policy, heterogeneity of expectations about the prices to prevail in stage 2, implies monetary policy and consumer's expectations have real effects.

Turning back to expectational shocks, on the real side, positive expectational shocks induce positive co-movement for reasonable monetary policy parametrizations. As I have argued the effects work through the labor demand channel. It is key that the consumer has full information as in way wealth effects are muted.[45] On the nominal side (given the real-side results), the consumer's expectations, like in Lorenzoni (2009), point to a demand-shock interpretation, whereas the producer's expectations point to a supply-shock interpretation. The monetary authority decides which one dominates.

3.4 Short-run volatility

I compare the short-run (one-period) volatility attributed to expectational shocks among equilibria.[46] The parametrization is the one in Table 1. I normalize the short-run volatility generated by rule 2 for $\phi_y = 0$ to one to make comparisons easier. Table 2 reports the results.
We can see that the baseline case of rule 1 generates considerably higher short-run volatility than the considered cases of rule 2. Assigning a role to the consumer's expectations increases volatility further (see also (42)). Within the considered equilibria for rule 2, supply shocks ($\phi_y = 0.5$)

[44] In Lucas (1972) "old" know but they have a passive role.

[45] This is the subject of Beaudry and Portier (2004) and Jaimovich and Rebelo (2009) among other papers.

[46] Short-run volatility is

$$\frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)}\,\mu\,\sigma_e^2$$

$$\frac{\phi_\pi - 1 + k(1-\theta)}{\phi_\pi\,(1+\zeta) - (1+\phi_y)}\,\mu\,\sigma_e^2\,,$$

in the cases of rule 1 and 2, respectively.

Table 2: Short-run volatility

Rule 1 (baseline)	4.43
Rule 2 ($\phi_y = 0.5$)	2.78
Rule 2 ($\phi_y = 0$)	1

generate considerably higher volatility than demand shocks ($\phi_y = 0$).

4 Monetary Authority with Superior Information

In this section I lift the assumption that the monetary authority has no superior information compared to the agents. Instead, I assume that the monetary authority possesses information about the following period's state. To prevent the forward-looking[47] nominal interest rate from being fully revealing about the following period' state, I require that the monetary authority either reports the following period's price with a measurement error or transmits "surprise" monetary policy shocks. In both cases the nominal interest rate serves as a public signal about the following period's productivity. However, in the former case the monetary authority misreports the following period's prices unintentionally, as opposed to intentionally in the latter. The aim of this section is twofold: first, to analyze the informational implications *per se* when the monetary authority communicates its superior information with noise; second, to equip the monetary authority with an additional monetary policy tool and pin down its equilibrium effects. I further explore monetary policy shocks in Section 5.

When the monetary authority reports the following period's price with a measurement error, the prevailing nominal interest rate in $t-1$ is

$$i_{t-1} = \phi_\pi \tilde{\pi}_t \,, \tag{51}$$

where $\tilde{\pi}_t \equiv \tilde{p}_t - p_{t-1}$, with

$$\tilde{p}_t = p_t + w_t \,. \tag{52}$$

The error term is i.i.d with $w_t \sim N(0, \sigma_w^2)$ and is independent of the shocks ϵ_t, e_t, and u_t.

[47] Since agents have complete information about the current state when the monetary authority steps in, there can only be information extraction if the monetary authority is forward-looking. Therefore, I restrict attention only to the case in which rule 1 is followed.

In terms of observables as of stage 2 in period $t-1$, this can be expressed as

$$\tilde{p}_t \,=\, \frac{1}{\phi_\pi}\big(i_{t-1} \,+\, \phi_\pi p_{t-1}\big)\,.$$

In the case of "surprise" monetary policy shocks the nominal interest rate is

$$i_{t-1} \,=\, \phi_\pi \pi_t \,+\, \omega_t\,, \tag{53}$$

where ω is i.i.d. with $\omega_t \sim N(0, \sigma_\omega^2)$ and is independent of the shocks ϵ_t, e_t, u_t, and w_t.

Agents now extract

$$\hat{p}_t \,=\, \phi_\pi p_t + \omega_t\,, \tag{54}$$

which in terms of observables in stage 2 of period $t-1$ can be expressed as

$$\hat{p}_t \,=\, i_{t-1} \,+\, \phi_\pi p_{t-1}\,.$$

4.1 Linear equilibria

Like in Section 3.1, equilibrium is given by equations (11)-(14). The state of the economy is now augmented by the public signal about period t's productivity which the monetary authority transmits. I denote this by z_t in the first case and $\hat{z}_t$ in the second one. The state can sufficiently be described then by $\Omega_t \,=\, \big(\{a_\tau\}_{\tau=0}^{t}\,,\, \{s_\tau\}_{\tau=0}^{t}\,,\, z_t\big)$, replacing z_t with $\hat{z}_t$ in the monetary policy shock case. What distinguishes the two cases is the information set of the monetary authority; in the case of measurement errors it is $I_t^m \,=\, \Omega_t \setminus \{z_t\}$, whereas in the case of monetary policy shocks it is $I_t^m \,=\, \Omega_t$. That is, in the latter case, the monetary authority takes into account the effects of the signal it transmits. I assume it is common knowledge what the case is each time a shock hits. As I show in Appendix A.5, the endogenous public signals associated with each case, respectively, are

$$z_t \,=\, a_t \,+\, \frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\zeta}\, w_t \tag{55}$$

$$\hat{z}_t \,=\, a_t \,+\, \frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\phi_\pi\, \zeta}\, \omega_t\,. \tag{56}$$

Agents disentangle the endogenous public signals upon the realization of the public signal s_t in stage 1 of period t.[48] The producer's information set then becomes $I_{t,1}^p \,=\, \Omega_t \setminus \{a_t\}$, whereas the

[48]This happens because they know the stochastic process of prices, given by (13).

consumer's $I_t^c = \Omega_t$. As I show in Appendix A.5, the producer's expectation of productivity is

$$E^p[a_t \,|\, I_{t,1}^p] = \delta E_t^p \left[x_t \mid I_{t,1}^p \setminus \{z_t\}\right] + (1-\delta) z_t \,, \tag{57}$$

where δ is a coefficient in $(0,1)$ (respectively $\hat{\delta}$ for $\hat{z}_t$). Importantly, δ depends on the monetary policy parameter ϕ_π.[49]

It is apparent from (55) and (56) that the economy's response to measurement errors and "surprise" shocks is very similar. In particular, for $\phi_\pi > 1$ positive interest rate shocks raise the producer's expectations about productivity in the following period. This happens because for $\phi_\pi > 1$ prices are positively related to productivity. Therefore, a higher nominal interest rate overstates the following period's price and leads the producer to partially attribute it to an increase in productivity.[50]

4.2 Equilibrium dynamics

The dynamics when shocks ϵ_t, e_t, and u_t are realized are very similar to the ones in Section 3.2.4.

Unlike in these cases, the effects of a measurement error or a monetary policy shock last only one period. This is because it generates a signal about a_t, which consumers learn and producers realize once the labor decision is made. If a shock $w_t = 1$ arises, the impact responses are

$$\frac{dy_t}{dw_t} = \frac{dn_t}{dw_t} = \frac{\phi_\pi(1-\delta)}{\zeta} > 0 \tag{58}$$

$$\frac{dp_t}{dw_t} = \frac{d\pi_t}{dw_t} = -\frac{d\pi_{t+1}}{dw_t} = \frac{\phi_\pi(1-\delta)}{\zeta(\phi_\pi - 1)} > 0\,. \tag{59}$$

It can be seen from (58) and (59) that interest rate shocks boost output and prices, causing thereby an inflationary pressure on impact and a disinflationary one in the following period. These

[49] The case analyzed in Section 3 corresponds to $\delta = 1$, which would prevail if the conditional variance of the endogenous signals was infinite.

[50] In case the monetary authority has no superior information and this is common knowledge, like in Section 3, monetary policy shocks have no real effects because they are unanticipated by both agents, hence they have no effect on the labor decision in stage 1. They can immediately be extracted by the agents which implies they have no effect on the consumer's inflation and output expectations for the following period. As a result, they only affect the current price in a co-monotone way for $\phi_\pi > 1$. On the contrary, in the superior information case agents extract monetary policy shocks with one period lag, hence their nominal effects are realized in the following period. In addition, monetary policy shocks have real effects since they are not simultaneously fully extracted by both agents. Section 5 elaborates on the heterogeneity of expectations and its connection with monetary policy.

responses are in the same direction as the ones after a shock to the public signal s_t. This is because in both cases the producer's expectations about productivity increase.

The impact responses to a policy shock $\omega_t = 1$ are scaled by ϕ_π as (56) suggests:

$$\frac{dy_t}{d\omega_t} = \frac{dn_t}{d\omega_t} = \frac{(1-\delta)}{\zeta} > 0 \tag{60}$$

$$\frac{dp_t}{d\omega_t} = \frac{d\pi_t}{d\omega_t} = -\frac{d\pi_{t+1}}{d\omega_t} = \frac{(1-\delta)}{\zeta(\phi_\pi - 1)} > 0\,. \tag{61}$$

The previous comments apply. However, in the next section I show that partly different monetary policy implications apply to the two cases.

5 Monetary Policy

The equilibrium nominal wage in stage 1 is given by

$$w_t = E_t^p[a_t] + E_t^p[p_t]\,.$$

Consequently, through the nominal wage, the real side of the economy reflects the producer's expectations about productivity. The producer's expectations enter the nominal wage both directly and indirectly through prices (in the case of rule 1) or inflation (in the case of rule 2). Monetary policy has real effects through the indirect channel. To see this, observe that the equilibrium labor condition (10) can more generally be written

$$\zeta n_t = E_t^p[a_t] + E_t^p[p_t] - E_t^c[p_t] - E_t^c[c_t]\,.$$

As long as agents have *heterogeneous* expectations about the prices to prevail in stage 2, that is $E_t^p[p_t] \neq E_t^c[p_t]$, monetary policy will have real effects. By construction, this is the case here. Crucially, incomplete but symmetric information would imply a neutral monetary policy.

A policy implying $E_t^p[p_t] = E_t^c[p_t] = \bar{p}$, like an infinitely aggressive policy on inflation does here, only removes the indirect, price (inflation) channel. As a result, it will typically be suboptimal as the producer's expectations still have real effects through the direct channel.

Optimal monetary policy restores the complete information equilibrium. This happens if and only if

$$E_t^p[a_t] + E_t^p[p_t] = 0 \tag{62}$$

$$E_t^p[a_t] + E_t^p[\pi_t] = 0 , \tag{63}$$

for rules 1 and 2, respectively.

That is monetary policy succeeds, not by removing the producer's uncertainty, but rather by making it irrelevant. To see this, note that prices depend on productivity and agents' expectations in a way decided by monetary policy. Optimal monetary policy manipulates prices in such a way that the price channel of expectations precisely offsets the direct one. Effectively, optimal policy implies that prices and productivity are inversely related to each other. In other words, it implies the producer's stage-2 revenue is *independent* of the worker's productivity. As a result, producer's uncertainty about productivity becomes irrelevant.

One would argue that the inefficiency here arises exactly because of agents's asymmetric information; if agents had incomplete but symmetric information, then the complete information equilibrium would prevail. However, this is true only because of logarithmic preferences in consumption; in more general environments, incomplete information would suffice. Nevertheless, it is asymmetric, rather than incomplete but symmetric, information in combination with the existence of a nominal bond market that enable the monetary authority to drive the economy closer to the complete information equilibrium. If a real bond market was in the place of the nominal bond market, then the price (inflation) channel would be absent, and there would be no way to drive the economy to the first-best.

Optimal policy has different implications from the ones in Weiss (1980) which implies that prices perfectly communicate fundamentals. By construction, this is a nonexistent possibility here. However, this paper shares with Weiss (1980), King (1982) and Lorenzoni (2010) the feature that it is asymmetric, rather than incomplete but symmetric, information about variables the monetary policy is based on at the time labor decisions are made that breaks the policy irrelevance proposed in Sargent and Wallace (1975, 1976). It is implicit in this that the monetary authority perfectly observes the variables in question when it steps in.

Below I consider both interest-rate rules and explore how the monetary authority can mitigate the incomplete information effect and drive the economy closer to its complete information counterpart in each case. In the context of rule 1, I design a more general policy rule which restores the complete information equilibrium for any policy parameters.

5.1 Rule 1

A first policy implication generated by the equilibrium analysis (see Section 3.2.5) is that a forward-looking rule, like rule 1, invites multiple equilibria in which the consumer's expectations is arbitrarily specified, which is not the case when a contemporaneously-looking rule is followed.[51] Second, as I showed in Section 3.4, the short-run volatility of expectational shocks is substantially higher for forward-looking rules than for contemporaneously-looking ones, for the parametrization in Table 1.

I initially consider the baseline equilibrium in which the consumer's expectations have no role.[52] This corresponds to setting $\hat{\kappa}_2 = 0$ in (42). Observe in (11) that the weight of output placed on producer's expectations is $\kappa_1 = \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)}$, whereas the weight placed on productivity is $1 - \kappa_1$. The former decreases in ϕ_π (see also fn. 31). Intuitively, the greater ϕ_π, the weaker the indirect (price) channel of expectations will be. In the limit as $\phi_\pi \to \infty$, $p_t \to 0$; prices are constant and the indirect channel of prices is muted. However, even in this limit case, the producer's expectations continue to matter via the direct channel. Hence, stabilizing inflation can at best eliminate the uncertainty arising through the price channel.

The focus so far has been on active policies, which correspond to the monetary authority setting $\phi_\pi > 1$. However, setting $\phi_\pi = 0$ in (11) and (13) returns $y_t = a_t$ and $p_t = -a_t$; a Friedman-rule policy completely eliminates the role of expectations and maintains economy at its complete-information level. For $\phi_\pi < \frac{1}{1+\zeta}$, the price effect is negative, which implies the indirect channel effect mitigates the direct one. For $\phi_\pi = 0$, the two effects precisely offset each other, rendering, therefore, incomplete information irrelevant in equilibrium. Consequently, a Friedman-rule emerges as an optimal policy. However, if an active policy is to be pursued, then it should be as aggressive on inflation as possible.

These results are different from the ones in Lorenzoni (2009) in which the limit $\phi_\pi \to \infty$ restores the efficient equilibrium. In Lorenzoni (2009), producers know current fundamentals, however nominal rigidities allow consumers' expectations to have real effects. In that environment, fixed prices allow the nominal interest rate to behave like the real rate in the flexible price equilibrium. As demonstrated, this is not the case here, as the producer has incomplete information about fundamentals too; eliminating the price channel does not remove his uncertainty.

[51] At least, I have failed to find other linear equilibria in this case.

[52] The logic for the other equilibria is the same and is exposed in detail in fn. 54.

Next, I analyze monetary policy when the monetary authority has superior information. As we saw earlier, the monetary authority can either -unintentionally- report prices with a measurement error or, intentionally, fuel the economy with "surprise" monetary policy shocks. A straightforward option for a "benevolent" monetary authority in the latter case is to use monetary policy shocks to "kill" the producer's expectational errors. However, I focus on the monetary policy parameters that can insulate the economy against measurement errors and can serve as a commitment device against monetary policy shocks.

One can see from (58) and (60) and Appendix A.5 that the monetary policy parameter ϕ_π affects the equilibrium not only directly, but also indirectly by affecting the precision of the public signal, z_t or $\hat{z}_t$, it generates. The precision of the public signal is inversely related to δ ($\hat{\delta}$ for the monetary policy shock).

Considering the case where the authority reports prices with a measurement error, in the limit as $\phi_\pi \to \infty$, the precision of the public signal becomes zero and $\delta \to 1$; hence, the public signal is ignored and has no real effects. Alternatively, a Friedman-rule policy ensures immunity to measurement errors as well, for the reasons outlined above. Hence, both extreme policies imply measurement errors have no real effects.

In case of "surprise" monetary policy shocks, ϕ_π matters only through the parameter δ as can be seen from (60). Appendix A.5 shows that the variance of the signal z_t tends to infinity only when a Friedman-rule policy is pursued, which is the unique optimal policy in this case allowing the monetary authority to commit against "surprise" shocks. Even though, for $\phi_\pi > 1$ (a sufficient condition), the variance of the signal increases in ϕ_π, in the limit $\phi_\pi \to \infty$ the public signal's variance is still finite, hence $\hat{\delta} \neq 1$. This implies that a policy infinitely aggressive on inflation cannot serve as a commitment device against "surprise" shocks.

5.1.1 Optimal monetary policies

In this section I design forward-looking interest-rate rules which restore the complete-information equilibrium for all chosen policy parameters (ϕ_π , ϕ_y). I start with the baseline case of rule 1 and subsequently deal with the general form that equilibria can have when a forward-looking policy is followed, given by (38)-(41).

Baseline equilibrium. I will follow a reverse engineering process. The optimal policy suggested above requires setting $\phi_\pi = 0$. It is straightforward to check that this implies $y_t = a_t$ and

$p_t = -a_t$ (see also fn. 54).

Consider the rule

$$i_t = -\log\beta + \phi_\pi E_t^m[\pi_{t+1} - \hat{\pi}_{t+1}] + \phi_y E_t^m \Delta[y_{t+1} - \hat{y}_{t+1}], \tag{64}$$

where the target levels of prices and output are set equal to their above identified levels: $\hat{\pi}_{t+1} = -E_t^m[\Delta a_{t+1}]$ and $\hat{y}_t = a_t$. This rule involves the monetary authority "punishing" deviations from the efficient inflation and growth rates.[53]

Taking the same steps as in the derivations of (11)-(13) shows that *any* chosen coefficients (ϕ_π, ϕ_y) can drive the economy to its efficient level. The Friedman rule is a special case obtained by setting $\phi_\pi = \phi_y = 0$.

Multiple equilibria. Consider the interest-rate rule given by (64). Make the following modification:

$$\hat{\pi}_{t+1} = \hat{\kappa}_4 E_t^m[\Delta a_{t+1}] \tag{65}$$

$$\hat{\kappa}_4 = -1 + \frac{\phi_\pi - 1}{\phi_\pi} k(1-\theta)\hat{\kappa}_2. \tag{66}$$

This rule drives the economy to its complete information counterpart.[54] As (66) shows the rule adjusts for each value of ϕ_π chosen,[55] whereas it is invariant to changes in ϕ_y. Appendix A.6

[53] As already emphasized, the authority has complete information when it sets the nominal interest rate. Orphanides (2003) discusses the advantages of targeting output growth rather than the output gap.

[54] To get an intuition for this, recall that the efficient equilibrium requires $\hat{\xi}_1 = 0$; this implies $\hat{\kappa}_1 = 0$ by (40) and $\hat{\kappa_3} = \frac{1}{\phi_\pi - 1}$ by (41). Given these, we can see in (42) that $\hat{\xi}_1 = 0$ prevails for $\hat{\kappa_2} = -\frac{1}{k(1-\theta)} \frac{\phi_\pi}{\phi_\pi - 1}$. Then, the equilibrium is

$$y_t = a_t \tag{67}$$

$$p_t = \frac{1}{\phi_\pi - 1} a_t - \frac{1}{k(1-\theta)} \frac{\phi_\pi}{\phi_\pi - 1} E_t^c[x_t]. \tag{68}$$

The rule given by (64)-(66) and $\hat{y}_t = a_t$ yields

$$y_t = a_t \tag{69}$$

$$p_t = -[1 + k(1-\theta)\hat{\kappa}_2] a_t + \hat{\kappa}_2 E_t^c[x_t]. \tag{70}$$

Setting $\hat{\kappa_2} = -\frac{1}{k(1-\theta)} \frac{\phi_\pi}{\phi_\pi - 1}$ returns (67)-(68).

[55] The rule will not adjust to changes in ϕ_π for $\hat{\kappa}_2 = 0$, as already shown. Further, the rule cannot be specified for $\phi_\pi = 0$ which leads to infinite inflation (see also (46)).

collects the derivations.

The monetary authority can extract the role of the consumer's expectations, parametrized by $\hat{\kappa}_2$, and productivity a_t by observing output and prices (see also (43)) when it steps in; subsequently, it can invoke the rule given by (64)-(66) and $\hat{y}_t = a_t$ and restore the complete information equilibrium for any choice of policy parameters (ϕ_π, ϕ_y).

Observe that setting $\hat{\kappa}_2 = 0$ returns $\hat{\kappa}_3 = -1$ and $\hat{\kappa}_4 = -1$, which corresponds to the baseline rule (64).

5.2 Rule 2

If monetary policy rule 2 is followed, setting $\phi_\pi = 1 - k(1-\theta)$ is optimal; however, this policy is unappealing as it requires the monetary authority to be fully aware of the agents' learning problems which is hardly realistic. Like before, a policy infinitely aggressive on inflation is suboptimal because it only mutes the inflation channel,[56] through which the consumer's expectations also operate.

Perhaps not surprisingly, in the limit $\phi_y \to \infty$, the economy is at its complete information counterpart.

6 Conclusion

This paper has reconsidered the nature of purely expectational shocks within a competitive, cashless, monetary economy. Asymmetric information is the driving force in the model. Informational asymmetries lead agents to form heterogeneous expectations about prices (inflation); this implies monetary policy and consumers' expectations have real effects through prices. Traditionally, expectational shocks are viewed as Keynesian demand shocks: when positive, they increase output, employment and inflation. I have shown that this interpretation remains a possibility but is not the only one; for commonly considered interest-rate rules, expectational shocks can cause business cycle patterns associated with supply shocks: when positive, they increase output and employment and they lower inflation. Such an interpretation is in line with the low inflation and high employment in the mid-80s and the second half of the 90s, which are recalled as periods of exuberant optimism. Whether expectational shocks manifest themselves as demand or supply shocks reflects

[56] As ϕ_π increases, the sign of the change in the weight of the producer's expectations is given by $-(1+\phi_y) + [1 - k(1-\theta)](1+\zeta)$ (see also (45)). It will be negative for a high enough ϕ_y relative to the inverse Frisch elasticity ζ, while there is a discontinuity at $\frac{1+\phi_y}{1+\zeta}$.

the monetary policy pursued.

I have considered different interest-rate rules and shown that forward-looking rules generate multiple equilibria in which consumers' expectations have an arbitrary role. Further to this, forward-looking interest-rate rules generate substantially higher volatility than "contemporaneous" ones. Inflation stabilization per se is typically suboptimal, as it can at best eliminate uncertainty arising through prices. Optimal monetary policies manipulate prices so that producers' revenue becomes constant across states. In this way, producer's incomplete information becomes irrelevant. I have designed targets for forward-looking interest-rate rules which restore the complete information equilibrium for any chosen policy parameters.

Recovering purely expectational shocks from the data will shed light on their seemingly shifting nature. Of course, the literature on the identification of expectational shocks remains far from settled (for example, Beaudry and Portier (2006), Blanchard et al. (2009), and Barsky and Sims (2011a,b)). On the policy front, introducing capital, investment and credit constraints is a rather natural extension with potentially promising monetary policy implications.

A Omitted derivations

A.1 Agents' problems

Producer's problem. Stage 2 profits of the consumer-owned firm are given by $\Pi_t = (P_t A_t - W_t) N_t$, where $Y_t = A_t N_t$.

In stage 1, firm chooses $N_t \geq 0$ to maximize the firm's expected profits:

$$E_t^p [\lambda_t \Pi_t] .$$

Expectations are with respect to the information set of the producer, specified in the main text. The maximization problem does not yield a solution if $W_t < \frac{E_t^p [\lambda_t P_t A_t]}{E_t^p [\lambda_t]}$, whereas any labor supply is accommodated if

$$W_t = \frac{E_t^p [\lambda_t P_t A_t]}{E_t^p [\lambda_t]} . \tag{71}$$

The case where the LHS of (71) is greater than the RHS implies $N_t = 0$ which in turn implies $C_t = 0$. This is a well-known issue in production economies with variable labor supply. As such, it is dismissed here. Hence, $C_t , N_t > 0$.

Consumer's Problem. Consumer solves the following problem:

$$\max_{\{C_t\,,N_t\,,B_{t+1}\}_{t=0}^{\infty}} E_0^c \sum_{t=0}^{\infty} \left(\log C_t \,-\, \frac{N_t^{1+\zeta}}{1+\zeta} \right)$$

subject to the sequence of budget constraints

$$P_t\, C_t \,+\, Q_t\, B_{t+1} \,=\, B_t \,+\, W_t\, N_t \,+\, \Pi_t\,,$$

and a no-Ponzi-scheme constraint. The FOCs with respect to C_t, N_t, and B_{t+1} respectively are:

$$\frac{1}{C_t} \,=\, \lambda_t\, P_t \tag{72}$$

$$N_t^{\zeta} \,=\, \lambda_t\, W_t \tag{73}$$

$$Q_t \,=\, \beta\, \frac{E_t^c[\lambda_{t+1}]}{\lambda_t}\,, \tag{74}$$

where λ_t is the current-value Lagrange multiplier associated with the period-t budget constraint. Expectations are with respect to the information set of the consumer, specified in the main text. No distinction has been made between stages 1 and 2 as the information structure implies the consumer has the same information set in both stages. Combining (72) with (73) and (72) with (74) yields, respectively,

$$N_t^{\zeta} \,=\, \frac{W_t}{P_t\, C_t} \tag{75}$$

$$C_t \,=\, \frac{Q_t}{\beta\, P_t} E_t^c[C_{t+1}\, P_{t+1}]\,. \tag{76}$$

In addition, the no-Ponzi-scheme condition and the fact that nominal bonds are in zero net-supply imply $B_{t+1} \,=\, 0$ in equilibrium. Note that it has implicitly been assumed so far that $B_0 \,=\, 0$.

A.2 Equilibrium under rule 1

Combining (10) with conjectures (C1) and (C2) implies equilibrium labor supply is

$$n_t \,=\, \frac{1}{\zeta}(1 \,+\, \kappa_2 \,-\, \xi_1)\, E_t^p[a_t] \,-\, \frac{1}{\zeta}\,(\kappa_2 \,+\, \xi_2)\, a_t\,.$$

Combined with firm's technology, $y_t = a_t + n_t$, the above implies

$$y_t = \frac{1}{\zeta}(1 + \kappa_2 - \xi_1) E_t^p[a_t] + \left(1 - \frac{1}{\zeta}(\kappa_2 + \xi_2) a_t\right). \tag{77}$$

Given the market-clearing condition $y_t = c_t$, matching coefficients in (C1) and (77) implies

$$\xi_1 = \frac{1 + \kappa_2}{1+\zeta} \tag{78}$$

$$\xi_2 = \frac{\zeta - \kappa_2}{1 + \zeta}. \tag{79}$$

The Euler equation (76) combined with market-clearing and the fact that $I_t^c = I_t^m$ can be written as

$$E_t^c[y_{t+1}] - y_t = (\phi_\pi - 1)(E_t^c[p_{t+1}] - p_t),$$

which implies (13). It follows then that

$$\xi_1 = (\phi_\pi - 1)\kappa_1 \tag{80}$$

$$\xi_2 = (\phi_\pi - 1)\kappa_2. \tag{81}$$

Solving the system (78)-(81) yields the coefficients in (11) and (13):

$$\xi_1 = \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} \tag{82}$$

$$\xi_2 = \frac{\zeta(\phi_\pi - 1)}{\phi_\pi + \zeta(\phi_\pi - 1)} \tag{83}$$

$$\kappa_1 = \frac{1}{\phi_\pi - 1}\xi_1 \tag{84}$$

$$\kappa_2 = \frac{1}{\phi_\pi - 1}\xi_2. \tag{85}$$

Equation (12) follows from $n_t = y_t - a_t$.

A.2.1 Multiple equilibria

Equations (38)-(39) can be obtained by combining the equilibrium labor decision (10) with the firm's technology and market-clearing. I derive these explicitly in Appendix A.4 below as they coincide with (91)-(93).

Turning to the Euler equation, conjectures (C1')-(C2') imply $E^c_t[c_{t+1}] = (\hat{\xi}_1 + \hat{\xi}_2)\, E^c_t[x_t]$ and $E^c_t[p_{t+1}] = (\hat{\kappa}_1 + \hat{\kappa}_2 + \hat{\kappa}_3)\, E^c_t[x_t]$. Then the LHS and the RHS of the Euler equation (9), respectively, become

$$E^c_t[c_{t+1}] - c_t = (\hat{\xi}_1 + \hat{\xi}_2)\, E^c_t[x_t] - \hat{\xi}_1\, E^p_t[a_t] - \hat{\xi}_2\, a_t \tag{86}$$

$$i_t - E^c_t[\pi_{t+1}] = (\phi_\pi - 1)\, E^c_t[\pi_{t+1}] = (\phi_\pi - 1)\, [(\hat{\kappa}_1 + \hat{\kappa}_3)\, E^c_t[x_t] - \hat{\kappa}_1\, E^p_t[a_t] - \hat{\kappa}_3\, a_t]\,. \tag{87}$$

Matching coefficients in (86)-(87) and using (39) yields (40)-(41).

A.3 Kalman filter

Let me start with the consumer's case which is easier to handle. The consumer's information set is $I^c_t = I^c_{t-1} \cup \{s_t, a_t\}$. Suppose the consumer's prior in period t is $x_t \,|\, I^c_{t-1} \sim N(0\,,\, \widetilde{\sigma}^2_x)$, where $\widetilde{\sigma}^2_x \equiv Var^c_{t-1}[x_t]$. Before applying Bayes' Law, recall that all shocks are serially uncorrelated, mutually independent, and normally distributed. The consumer's posterior is

$$x_t \,|\, I^c_{t-1} \sim N\left(0\,,\, \left(\frac{1}{\widetilde{\sigma}^2_x} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_u}\right)^{-1}\right)\,.$$

This implies the following period's prior is

$$x_{t+1} \,|\, I^c_t \sim N\left(0\,,\, \left(\frac{1}{\widetilde{\sigma}^2_x} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_u}\right)^{-1} + \sigma^2_\epsilon\right)\,.$$

Using $\hat{\sigma}^2_x$ to denote $Var^c_t[x_{t+1}]$, it follows that

$$\hat{\sigma}^2_x = \left(\frac{1}{\widetilde{\sigma}^2_x} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_u}\right)^{-1} + \sigma^2_\epsilon\,. \tag{88}$$

I will assume the consumer's prior in period 0 is $x_{0\,|\,-1} \sim N(0\,,\, \sigma^2_x)$, where σ^2_x is the fixed point in the Riccati equation (88). This implies the learning problem of the agents is at its steady-state when time commences. Turning to the coefficients in (17), let

$$k \equiv \frac{\frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_u}}{\frac{1}{\sigma^2_x} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_u}}\,,$$

denote the Kalman gain term, that is the precision of new information relative to the prior's precision. This is time-invariant due to the consumer's learning problem being at its steady-state. In addition, let $\theta \equiv \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_u^2}}$, denote the relative precision of the signal s_t within the new information $\{s_t, a_t\}$.

Turning to the producer's learning problem, recall from the analysis in the main text that agents have the same information set at the end of each period, that is $I^p_{t-1,2} = I^c_{t-1}$. As as result, they have the same prior in the following period. However, their information sets differ in stage 1. In particular, the producer's information set is $I^p_{t,1} = I^p_{t-1,2} \cup \{s_t\}$. Letting $\mu \equiv \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_x^2}+\frac{1}{\sigma_e^2}}$ yields the coefficient in (16).

A thorough demonstration of the Kalman filter can be found in Anderson and Moore (1979), Harvey (1989), and Technical Appendix B in Ljungqvist and Sargent (2004).

A.4 Equilibrium under rule 2

Let me elaborate first on the filtering problems of the agents.

The producer's and the consumer's expectations, respectively, are (see also (16) and (17)):

$$E^p_t[a_t] = E^p_{t,1}[x_t] = (1-\mu)\, E^p_{t-1,2}[x_{t-1}] + \mu\, s_t$$

$$E^p_{t,2}[x_t] = E^c_t[x_t] = (1-k)\, E^c_{t-1}[x_{t-1}] + k\,[\theta\, s_t + (1-\theta)\, a_t]\,.$$

Then, the consumer's expectations in period t of the producer's expectations in $t+1$ are given by

$$E^c_t\,[\,E^p_{t+1}[a_{t+1}]\,] = E^c_t[x_t]\,, \tag{89}$$

and the producer's expectations in period t of the consumer's expectations in t are given by

$$E^p_t\,[\,E^c_t[x_t]\,] = (1-k)E^c_{t-1}[x_{t-1}] + k\,[\theta\, s_t + (1-\theta)\, E^p_t[a_t]] = E^c_t[x_t] + k\,(1-\theta)(E^p_t[a_t] - a_t)\,. \tag{90}$$

Substituting conjectures (C3),(C4), and (90) in (44) implies

$$\zeta n_t = (1 + \kappa_5 - \xi_3)\, E^p_t[a_t] + \kappa_4\, k\,(1-\theta)\,(E^p_t[a_t] - a_t) - (\kappa_5 + \xi_4)\, a_t\,.$$

Substituting the firm's technology, using market-clearing, and, subsequently, matching coefficients with conjecture (C3) yields

$$\xi_3 \;=\; \frac{1 \,+\, \kappa_5 \,+\, \kappa_4\, k\,(1-\theta)}{1+\zeta} \tag{91}$$

$$\xi_4 \;=\; \frac{\zeta \,-\, \kappa_5 \,-\, \kappa_4\, k\,(1-\theta)}{1+\zeta}\,. \tag{92}$$

Observe that

$$\xi_3 \,+\, \xi_4 \;=\; 1\,, \tag{93}$$

a direct consequence of preferences logarithmic in consumption.

Turning to the Euler equation (76), conjectures (C3) and (C4) combined with (89) imply

$$E_t^c[c_{t+1}] \,-\, c_t \;=\; -\,\xi_3\, E_t^p[a_t] \,+\, (\xi_3 \,+\, \xi_4)\, E_t^c[x_t] \,-\, \xi_4\, a_t\,,$$

and

$$i_t \,-\, E_t^c[\pi_{t+1}] \;=\; (\phi_y\, \xi_3 + \phi_\pi\, \kappa_3)\, E_t^p[a_t] + [-\kappa_3 + (\phi_\pi - 1)\kappa_4 \,-\, \kappa_5]\, E_t^c[x_t] + [\phi_y\, \xi_4 + \phi_\pi\, \kappa_5 \,-\, \phi_y]\, a_t\,.$$

Matching coefficients yields

$$-\,\xi_3 \;= \phi_y\, \xi_3 \,+\, \phi_\pi\, \kappa_3 \tag{94}$$

$$\xi_3 \,+\, \xi_4 \;=\; -\,\kappa_3 \,+\, (\phi_\pi - 1)\kappa_4 \,-\, \kappa_5 \tag{95}$$

$$-\,\xi_4 \;= \phi_y\, \xi_4 \,+\, \phi_\pi\, \kappa_5 \,-\, \phi_y\,. \tag{96}$$

Summing (94)-(96) across sides and using (93) yields

$$\kappa_3 \,+\, \kappa_4 \,+\, \kappa_5 \;=\; 0\,, \tag{97}$$

whereas summing across (94) and (96) and again using (93) yields

$$\kappa_3 \,+\, \kappa_5 \;=\; -\frac{1}{\phi_\pi}\,, \tag{98}$$

which are equations (47) and (48), respectively.

Solving (92), (93) and (94)-(96) for ξ_3, ξ_4, κ_3, κ_4, κ_5 yields the coefficients in (45) and (46).

A.5 Omitted derivations in Section 4

First, I deal with the case in which the monetary authority reports the following period's prices with a measurement error. Next, I follow the same process in the case of a monetary policy shock. Recall that what differs among the two cases is the information set of the monetary authority.

Measurement error. Suppose at the end of period $t-1$ the nominal interest rate serves as a noisy signal about the price in t, as in (51). Agents extract

$$\tilde{p}_t \,=\, p_t \,+\, w_t \tag{99}$$

where $\tilde{p}_t \,\equiv\, \frac{i_{t-1} \,+\, \phi_\pi p_{t-1}}{\phi_\pi}$. The monetary authority's information set is $I^m_{t-1} \,=\, \Omega_t \setminus \{z_t\}$, where z_t is the public signal about period-t productivity which I derive below and Ω_t denotes the state of the economy in t. The latter is $\Omega_t \,=\, \big(\{a_\tau\}^t_{\tau=0}\,,\,\{s_\tau\}^t_{\tau=0}\,,\,z_t\big)$. Using (13), (99) becomes

$$\frac{\tilde{p}_t \,-\, \kappa_1\, E^p_t[a_t \,|\, I^p_{t,1} \setminus \{z_t\}]}{\kappa_2} \,=\, a_t \,+\, \frac{1}{\kappa_2}\, w_t\,, \tag{100}$$

where $\kappa_1,\ \kappa_2$ are coefficients given by (82)-(85). The producer's information set in stage 1 is $I^p_{t,1} \,=\, \Omega_t \setminus \{a_t\}$. The LHS in (100) is the endogenous public signal in stage 1 of t which I have denoted by z_t. It follows then that

$$z_t \,\equiv\, \frac{[\phi_\pi \,+\, \zeta(\phi_\pi - 1)]\tilde{p}_t \,-\, \frac{\phi_\pi}{\phi_\pi - 1}\, E^p_t[a_t \,|\, I^p_{t,1} \setminus \{z_t\}]}{\zeta} \,=\, a_t \,+\, \frac{\phi_\pi \,+\, \zeta(\phi_\pi - 1)}{\zeta}\, w_t\,. \tag{101}$$

The conditional variance of productivity is then $\sigma^2_z \,\equiv\, Var[a_t \,|\, z_t] \,=\, \Big(\frac{\phi_\pi \,+\, \zeta(\phi_\pi - 1)}{\zeta}\Big)^2\, \sigma^2_w\,.$

Turning back to the producer, suppose for a moment that z_t is not part of his information set. Then, the producer's posterior distribution of a_t is

$$a_t \,|\, I^p_t \setminus \{z_t\} \,\sim\, N\left(E^p_t\left[x_t \,|\, I^p_{t,1} \setminus \{z_t\}\right]\,,\, \sigma^2_x \,+\, \sigma^2_u\right)\,,$$

where $E^p_t\left[x_t \,|\, I^p_{t,1} \setminus \{z_t\}\right]$ is given by (16) and σ^2_x is the fixed point in (88). Taking z_t into account, the producer's posterior becomes

$$a_t \,|\, I^p_{t,1} \sim\, N\left(\delta\, E^p_t\left[x_t \,|\, I^p_{t,1} \setminus \{z_t\}\right] \,+\, (1-\delta)\, z_t\ ,\ \sigma^2_a\right)\,, \tag{102}$$

where $\delta \,=\, \Big(\frac{1}{\sigma^2_x \,+\, \sigma^2_u}\Big) \,/\, \Big(\frac{1}{\sigma^2_x \,+\, \sigma^2_u} \,+\, \frac{1}{\sigma^2_z}\Big)$ and $\sigma^2_a \,=\, \Big(\frac{1}{\sigma^2_x \,+\, \sigma^2_u} \,+\, \frac{1}{\sigma^2_z}\Big)^{-1}$.

Monetary policy shock. In case the monetary authority transmits monetary policy shocks, its information set additionally includes z_t, that is $I_t^m = \Omega_t$. Taking the same steps as before, agents observe $\hat{p}_t = \phi_\pi p_t + \omega_t$, where $\hat{p}_t \equiv i_{t-1} + \phi_\pi p_{t-1}$. The monetary authority transmits the public signal

$$\hat{z}_t = a_t + \frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\phi_\pi \zeta} \omega_t ,$$

where

$$\hat{z}_t \equiv \frac{[\phi_\pi + \zeta(\phi_\pi - 1)]\, \hat{p}_t - \frac{\phi_\pi^2}{\phi_\pi - 1} E_t^p[a_t \,|\, I_{t,1}^p]}{\phi_\pi \zeta} . \tag{103}$$

The conditional variance of productivity is $\sigma_{\hat{z}}^2 \equiv Var[a_t \,|\, \hat{z}_t] = \left(\frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\phi_\pi \zeta}\right)^2 \sigma_\omega^2$. The producer's posterior is

$$a_t \,|\, I_{t,1}^p \sim N\left(\hat{\delta}\, E_t^p\left[x_t \,|\, I_{t,1}^p \setminus \{\hat{z}_t\}\right] + (1 - \hat{\delta})\hat{z}_t \;,\; \sigma_{\hat{a}}^2\right) , \tag{104}$$

where $\hat{\delta} = \left(\frac{1}{\sigma_x^2 + \sigma_u^2}\right) / \left(\frac{1}{\sigma_x^2 + \sigma_u^2} + \frac{1}{\sigma_{\hat{z}}^2}\right)$ and $\sigma_{\hat{a}}^2 = \left(\frac{1}{\sigma_x^2 + \sigma_u^2} + \frac{1}{\sigma_{\hat{z}}^2}\right)^{-1}$.

Observe that unlike in (101), the producer's expectations in (103) are conditional on the entire information set of the producer. To fully extract z_t use (104) to get

$$\hat{z}_t \equiv \frac{\phi_\pi - 1}{\phi_\pi} \hat{p}_t - \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} \hat{\delta}\, E_t^p \left[x_t \,|\, I_{t,1}^p \setminus \{\hat{z}_t\}\right] .$$

A.6 Derivations in Section 5.1

Consider the interest-rate rule

$$i_t = -\log \beta + \phi_\pi E_t^m[\pi_{t+1} - \hat{\pi}_{t+1}] + \phi_y E_t^m \Delta[y_{t+1} - \hat{y}_{t+1}] ,$$

where $\hat{\pi}_{t+1} = \kappa_4 (a_{t+1} - a_t)$ and $\hat{y}_t = a_t$. A reverse engineering process will pin down κ_4.

The labor market optimality condition implies

$$\hat{\xi}_1 = \frac{1 + \hat{\kappa}_3 + k\,(1 - \theta)\,\hat{\kappa}_2}{1 + \zeta}$$

$$\hat{\xi}_2 = 1 - \hat{\xi}_1 ,$$

which correspond to equations (38)-(39) in the main text. Taking familiar steps, the Euler equation implies

$$1 - \phi_y \;=\; (\phi_\pi - 1)\,(\hat{\kappa}_1 \,+\, \hat{\kappa}_3) \;-\; \phi_\pi\,\hat{\kappa}_4 \;-\; \phi_y \tag{105}$$

$$(1 - \phi_y)\,\hat{\xi}_1 \;=\; (\phi_\pi - 1)\,\hat{\kappa}_1 \tag{106}$$

$$(1 - \phi_y)\,\hat{\xi}_2 \;=\; (\phi_\pi - 1)\,\hat{\kappa}_3 \;-\; \phi_\pi\,\hat{\kappa}_4 \;-\; \phi_y\,. \tag{107}$$

Setting

$$\hat{\kappa}_4 \;=\; \frac{(\phi_\pi - 1)\,\hat{\kappa}_3 - 1}{\phi_\pi} \tag{108}$$

implies $\hat{\xi}_1 = 0$ and $\hat{\xi}_2 = 1$ as required, $\hat{\kappa}_1 = 0$ and $\hat{\kappa}_3 = -\left[1 + k\,(1-\theta)\,\hat{\kappa}_2\right]$. Combining the latter with (108) yields (66) in text.

B Data

Data in Figures 1-4 are collected from the St. Louis Fed and refer to the US economy for the period 1965 : 1 – 2010 : 1. Data in Figures 1 and 3 are quarterly, whereas in Figures 2 and 4 they are annual. Employment refers to "All Employees: Total Nonfarm Employees (Thousands of Persons)" (series PAYEMS) and is seasonally adjusted. It is logged and HP-filtered with penalty 1600 for quarterly and 100 for annual data, respectively. Figures 1-4 show its cyclical component (scaled up by 50). Inflation in Figures 1 and 3 refers to percent changes in the "Gross Domestic Product: Implicit Price Deflator" (series GDPDEF) and is seasonally adjusted. Inflation in Figures 2 and 4 refers to percent changes in the "Consumer Price Index for All Urban Consumers: All Items" (series CPIAUCSL). Consumer Sentiment refers to "University of Michigan: Consumer Sentiment" (series UMCSENT1, UMCSENT) and is not seasonally adjusted. It is scaled down by 25 in Figure 3 and by 10 in Figure 4.

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Figure 1: Changes in GDP Deflator and Cyclical Employment: 1965-2010 (quarterly)

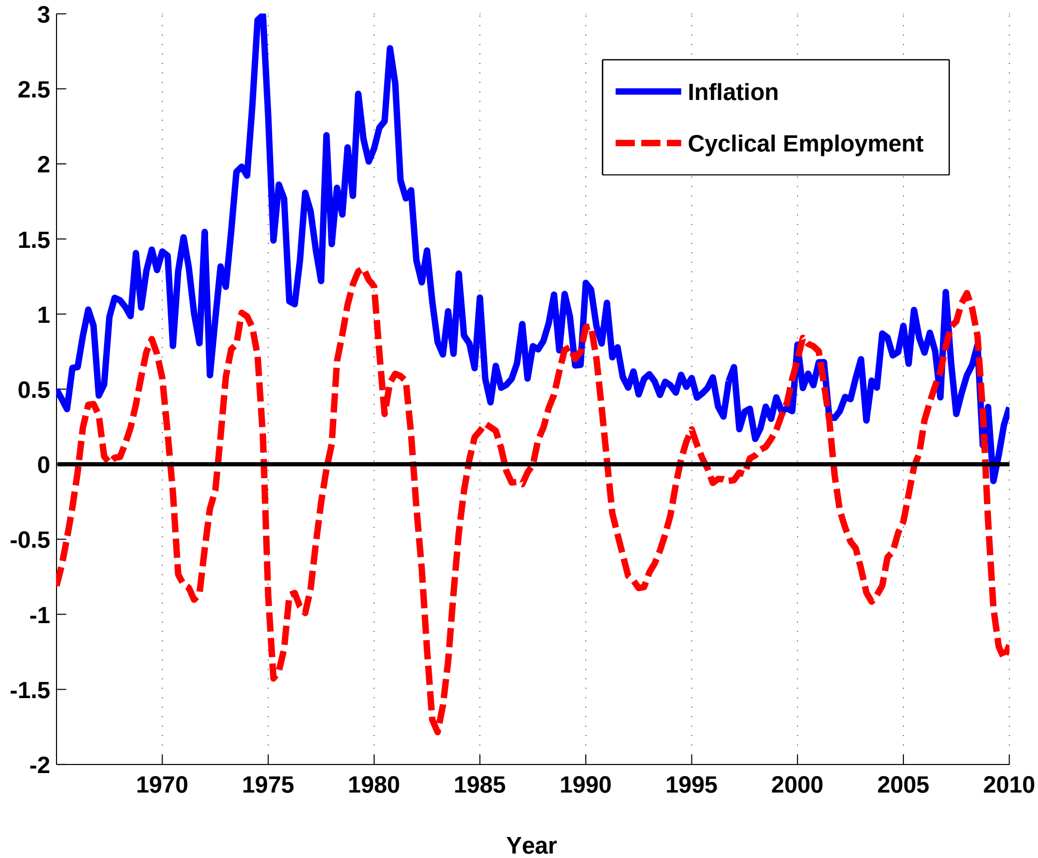

Figure 2: CPI Inflation and Cyclical Employment: 1965-2010 (annual)

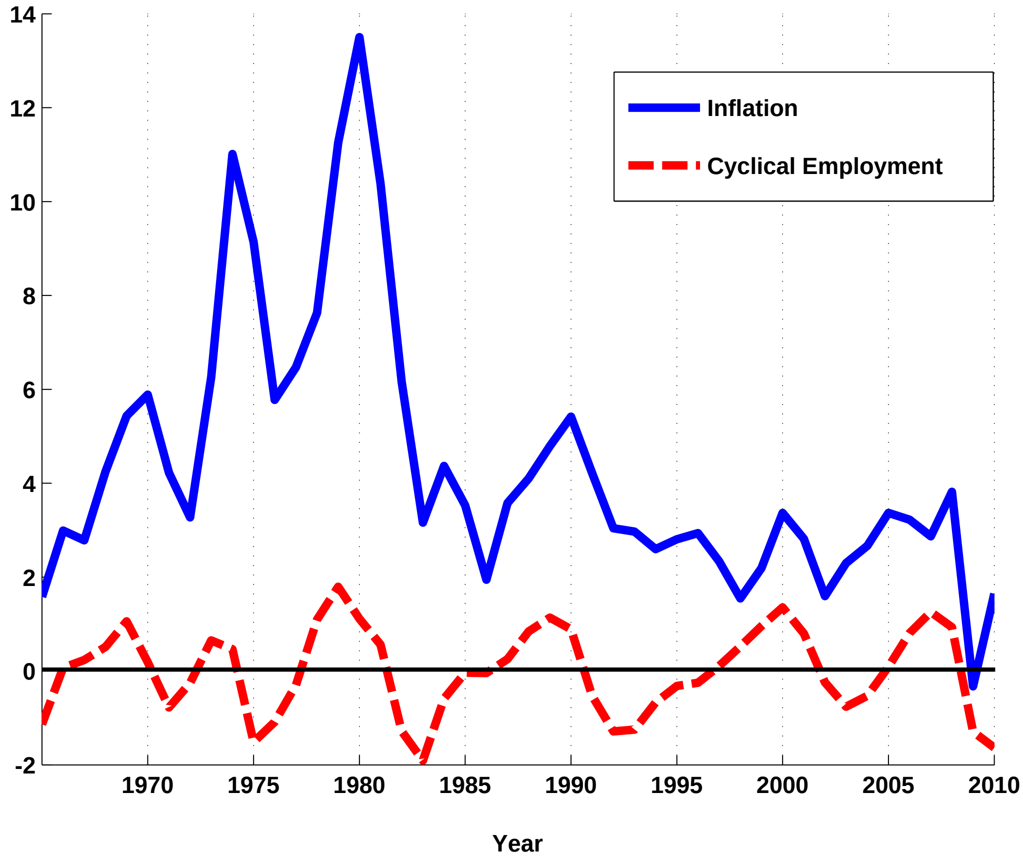

Figure 3: Changes in GDP Deflator, Cyclical Employment and Consumer Sentiment: 1965-2010 (quarterly)

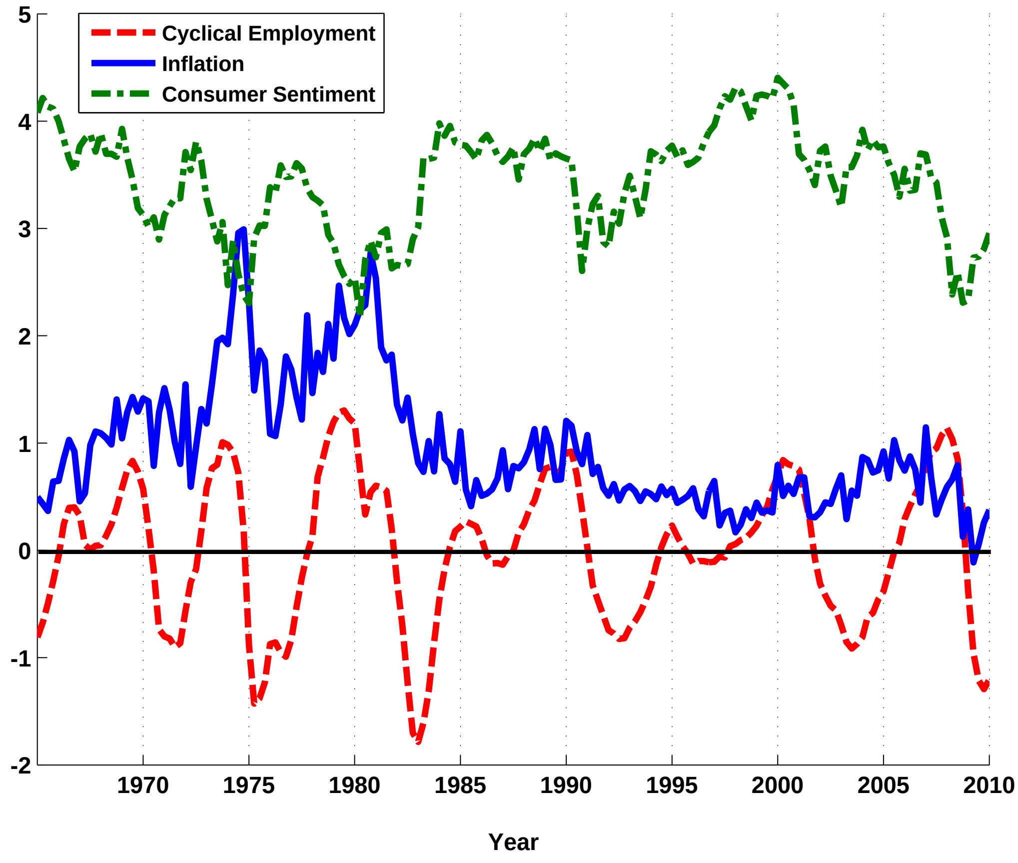

Figure 4: CPI Inflation, Cyclical Employment and Consumer Sentiment: 1965-2010 (annual)

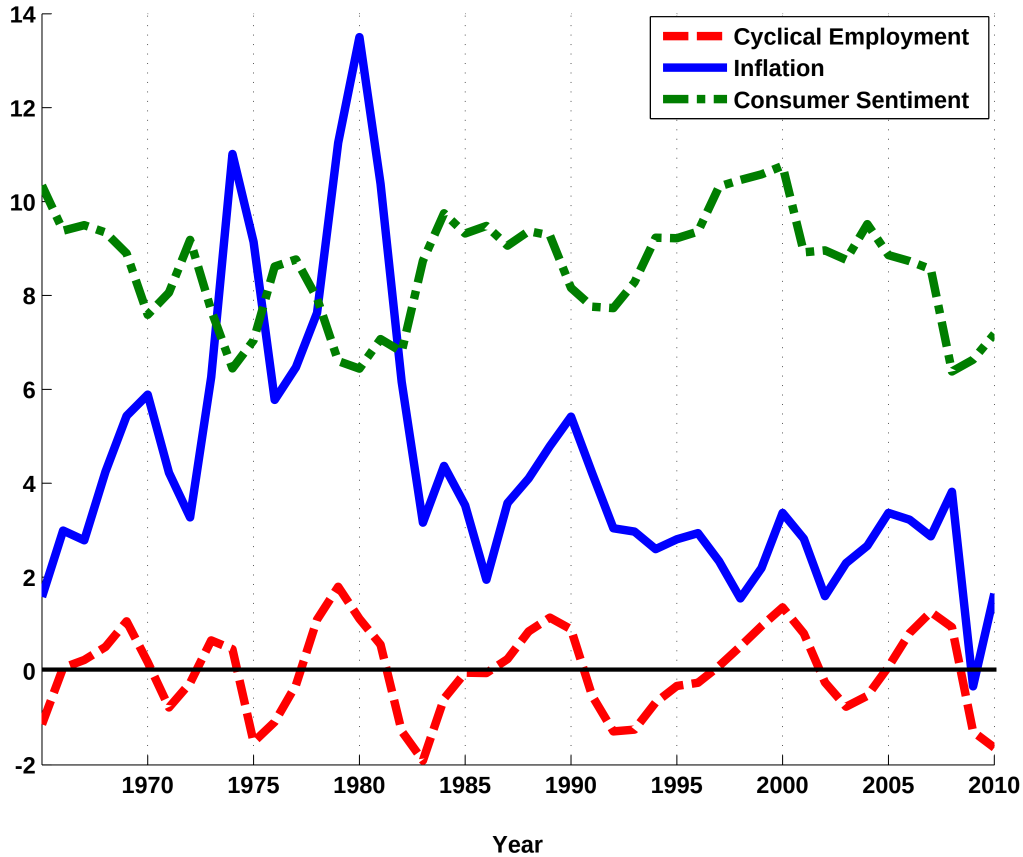

Figure 5: Impulse responses to a positive permanent productivity shock when rule 1 is followed (1)

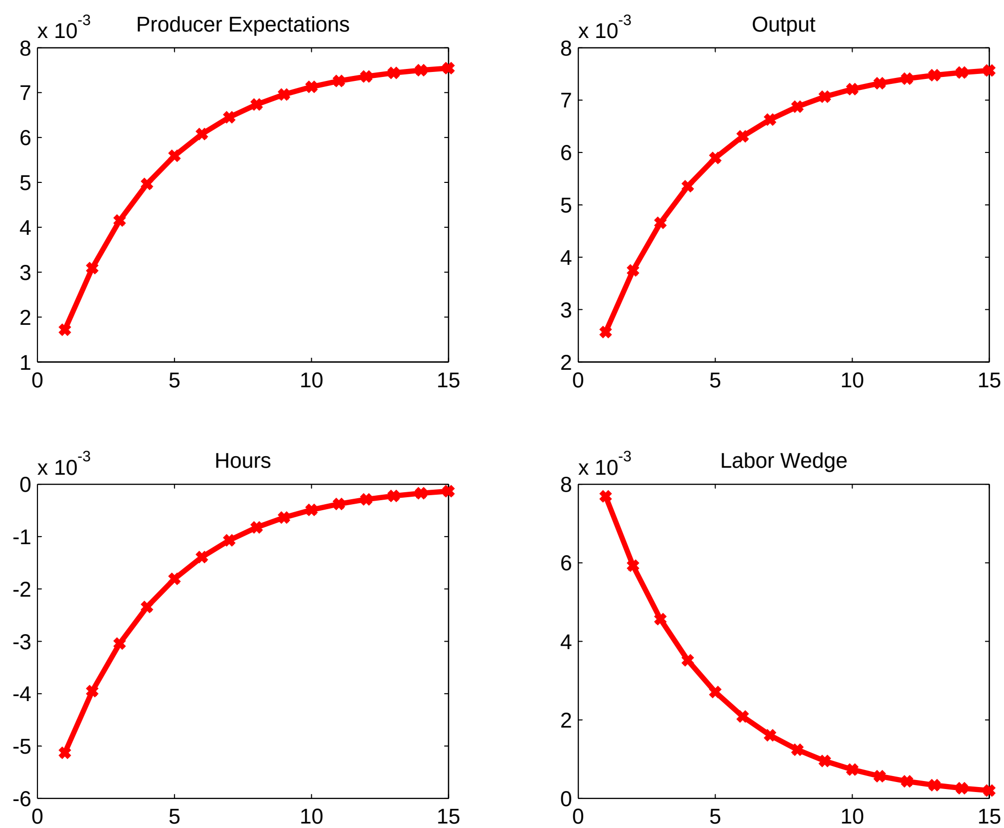

Figure 6: Impulse responses to a positive permanent productivity shock when rule 1 is followed (2)

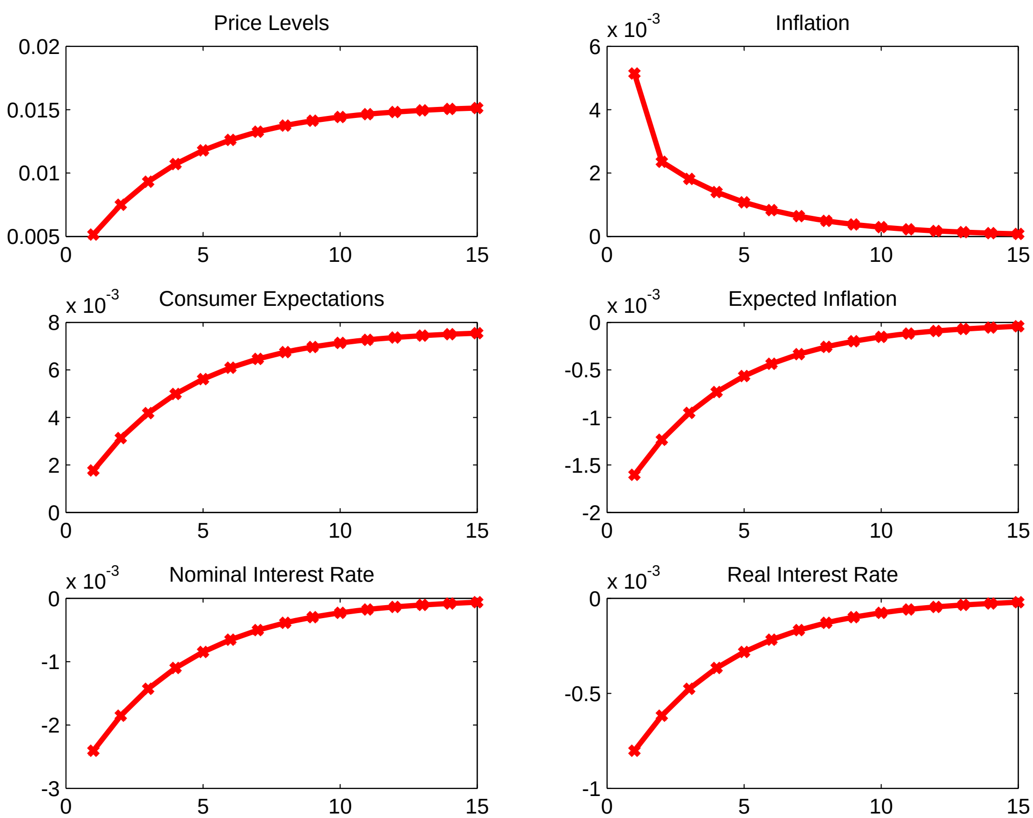

Figure 7: Impulse responses to a positive expectational shock when rule 1 is followed (1)

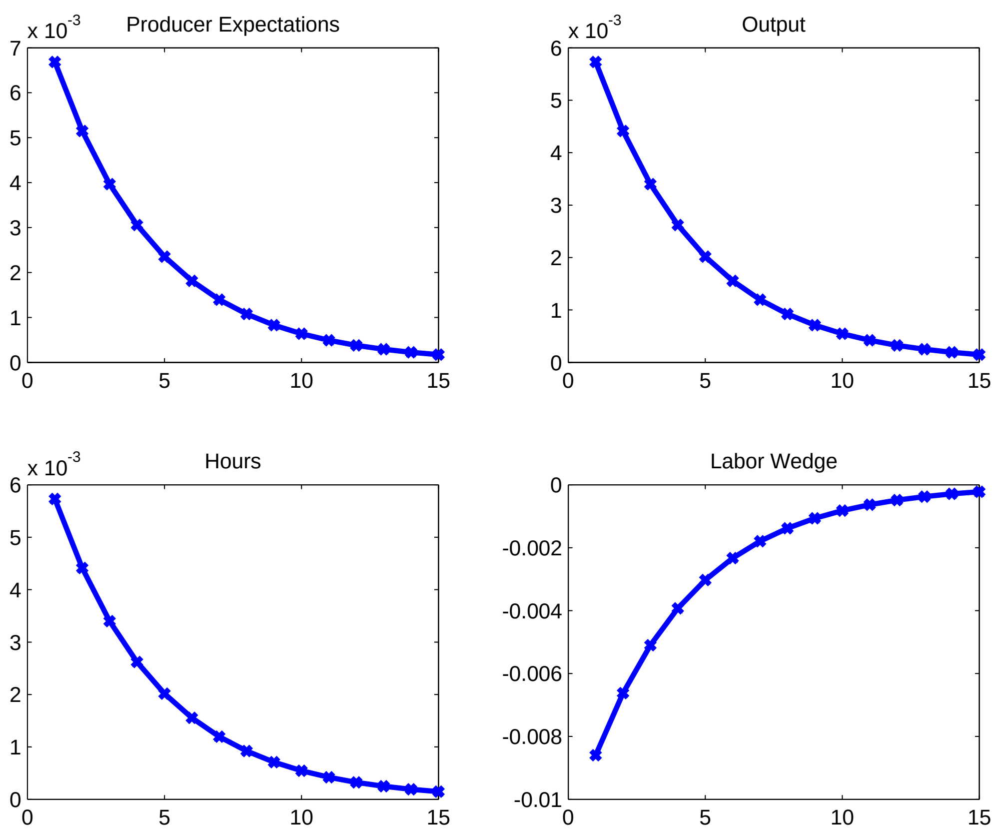

Figure 8: Impulse responses to a positive expectational shock when rule 1 is followed (2)

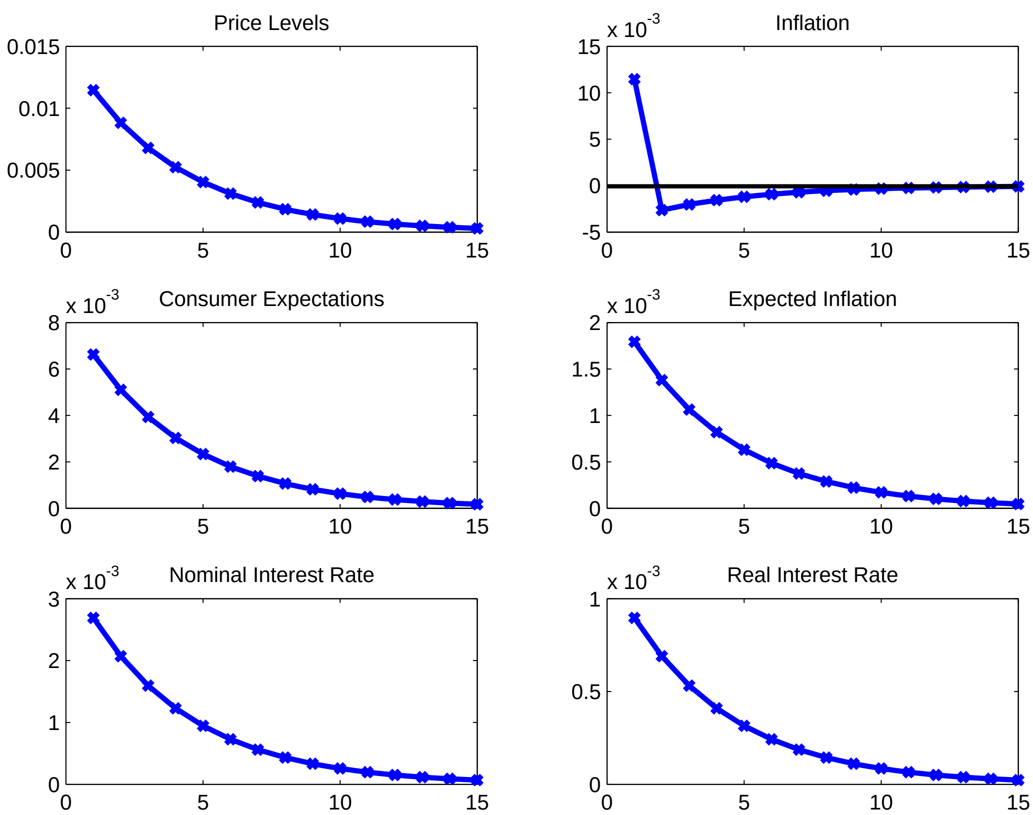

Figure 9: Impulse responses to a positive temporary productivity shock when rule 1 is followed (1)

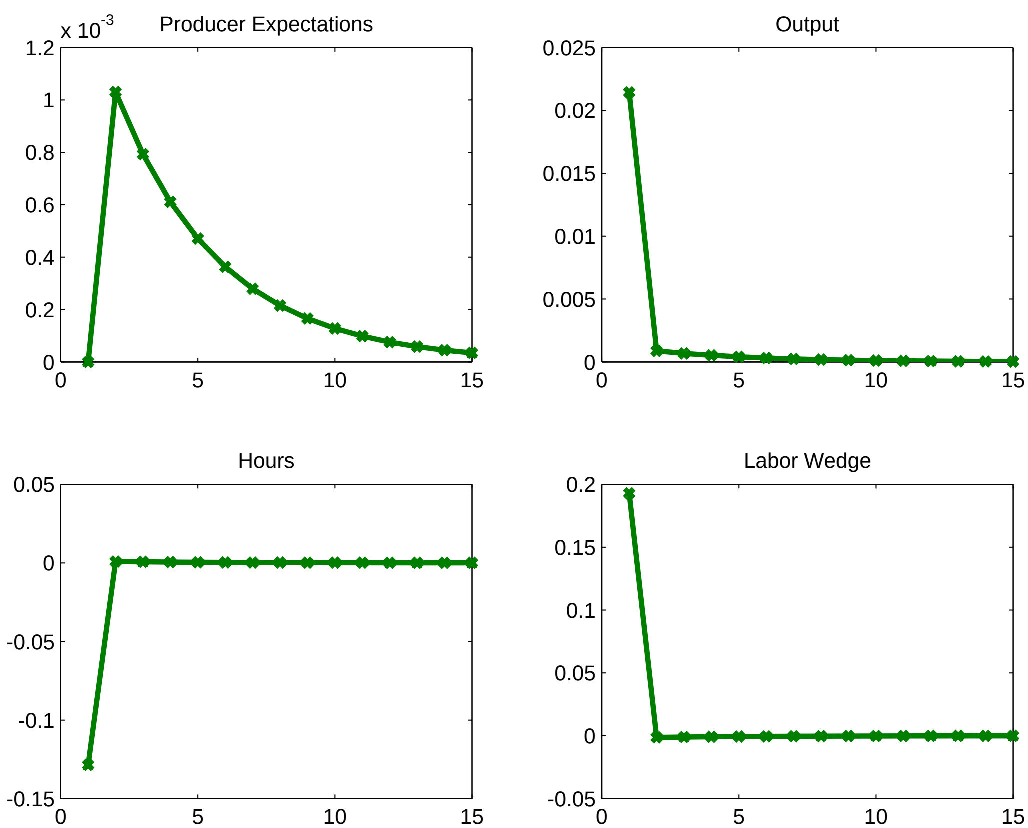

Figure 10: Impulse responses to a positive temporary productivity shock when rule 1 is followed (2)

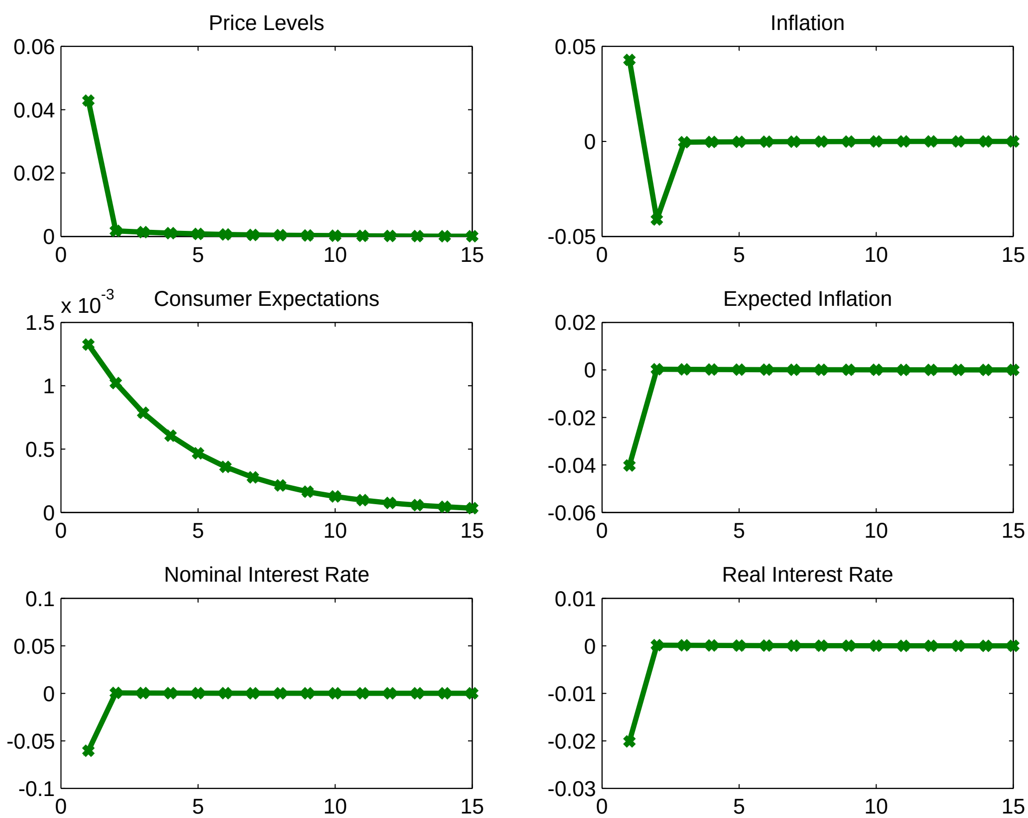

Figure 11: Impulse responses to a positive permanent productivity shock when rule 2 is followed with $\phi_y = 0.5$

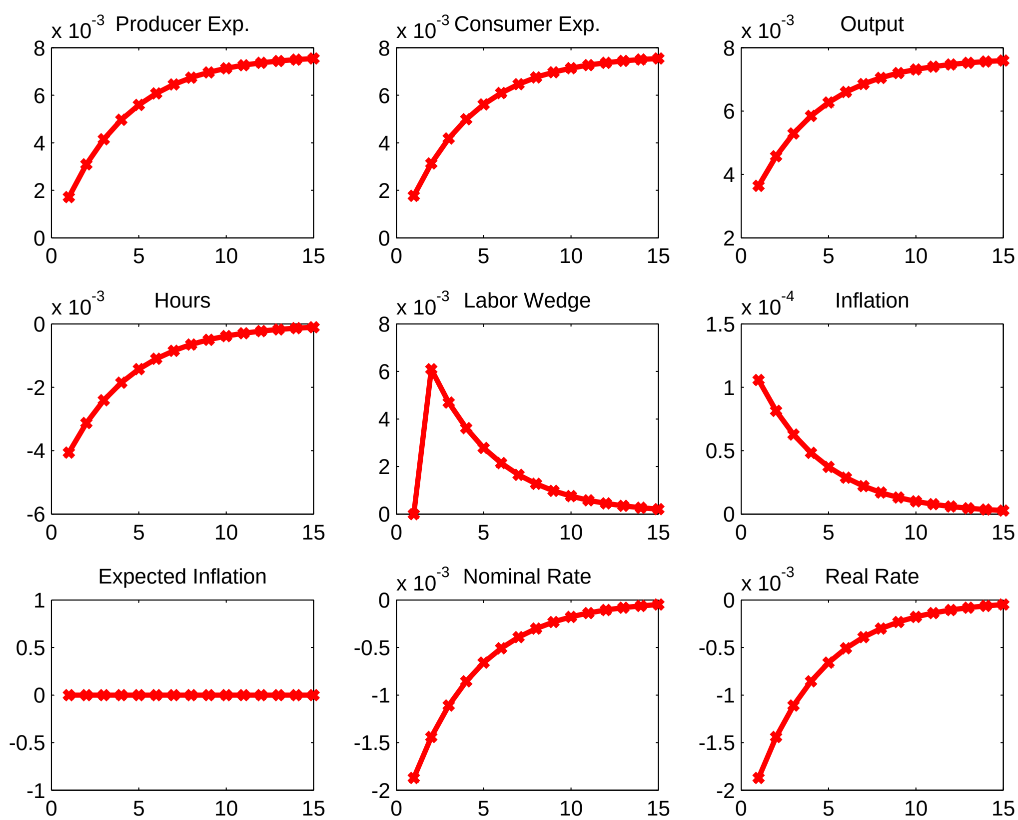

Figure 12: Impulse responses to a positive expectational shock when rule 2 is followed with $\phi_y = 0.5$

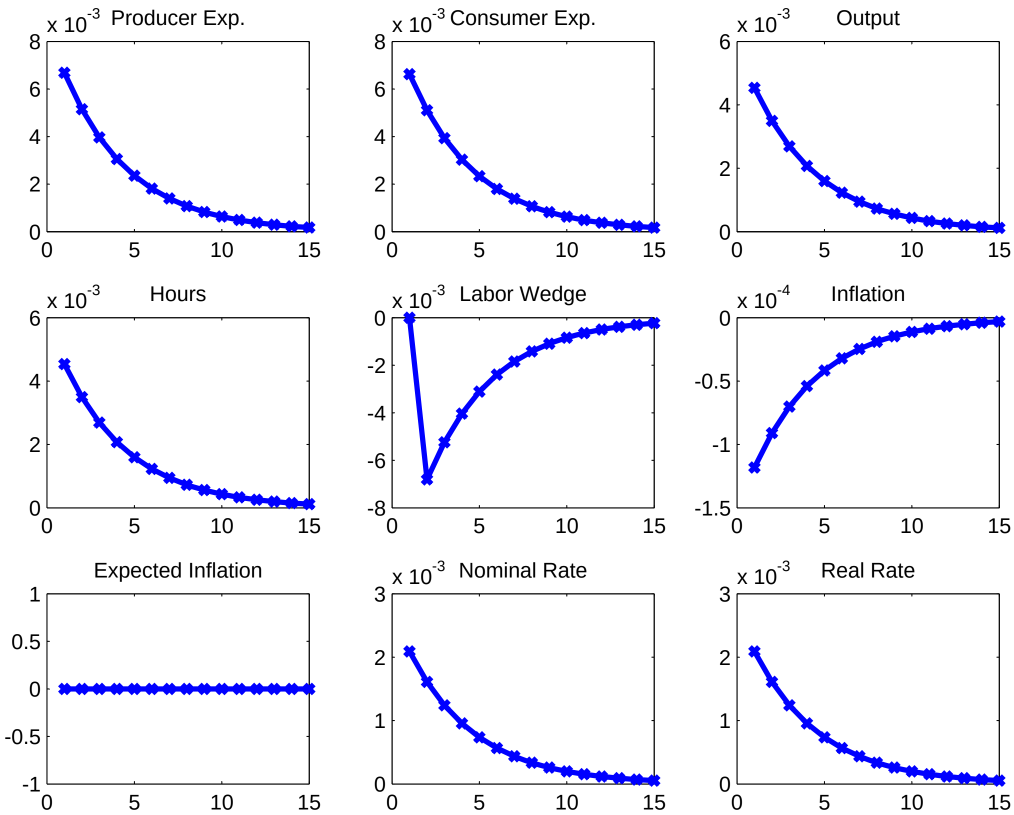

Figure 13: Impulse responses to a positive temporary productivity shock when rule 2 is followed with $\phi_y = 0.5$

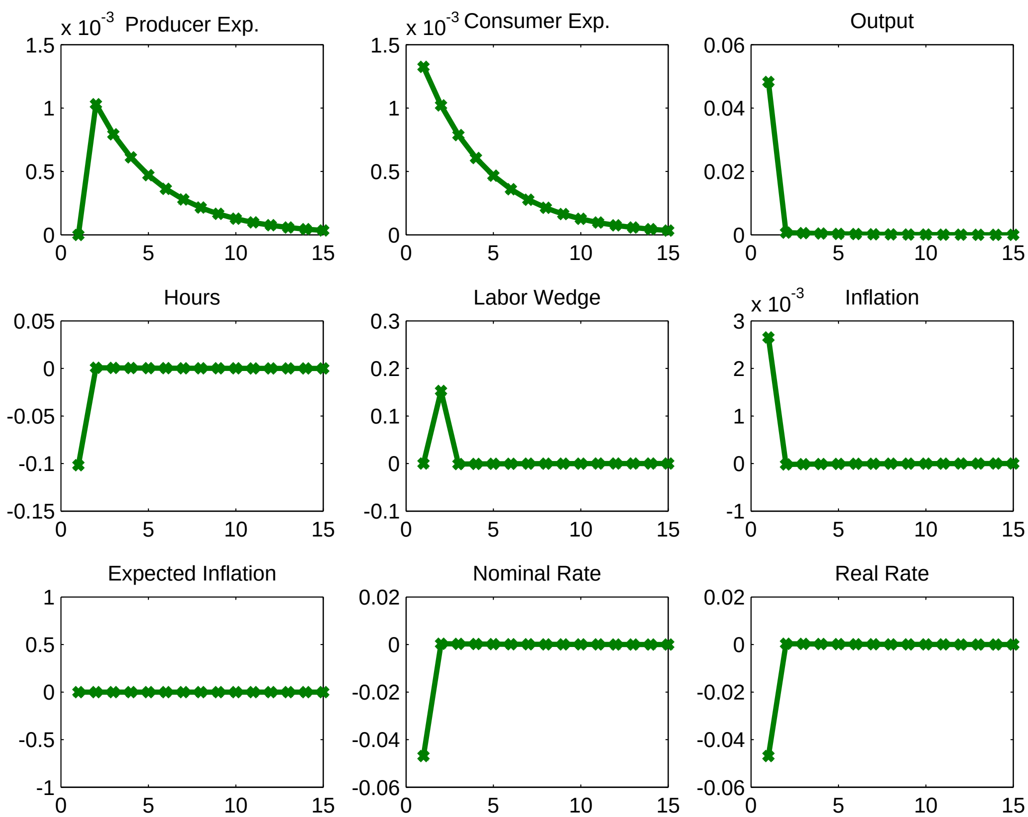

Figure 14: Impulse responses to a positive permanent productivity shock when rule 2 is followed with $\phi_y = 0$

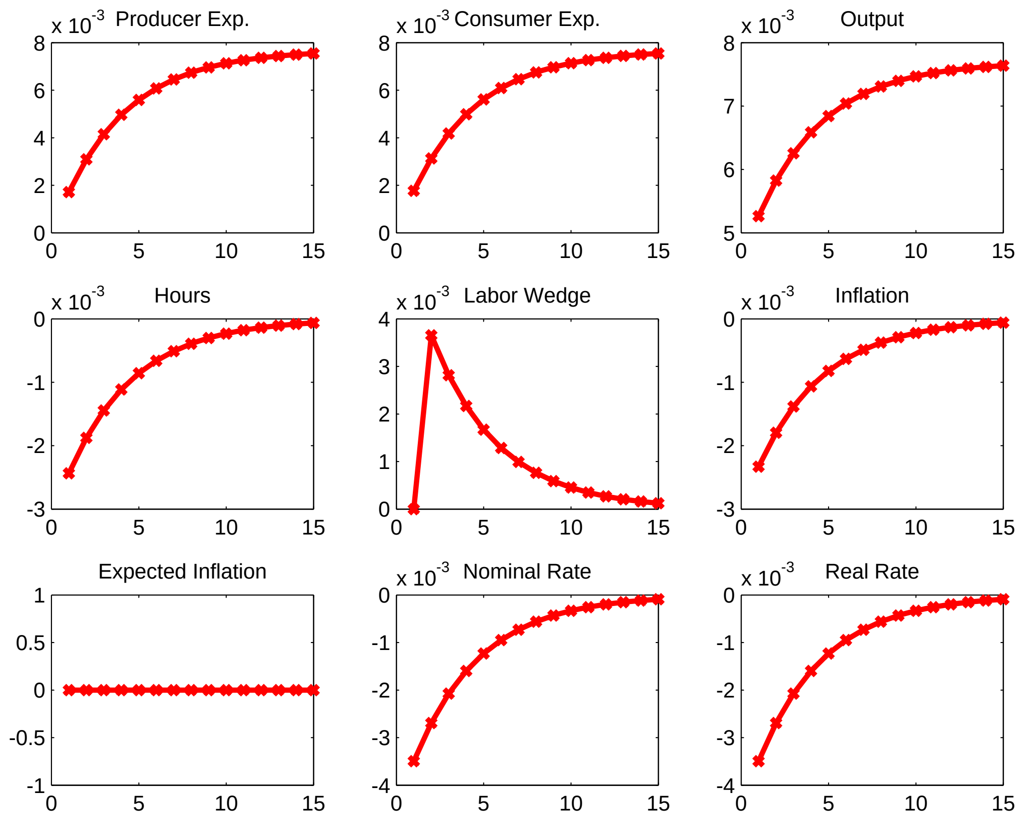

Figure 15: Impulse responses to a positive expectational shock when rule 2 is followed with $\phi_y = 0$

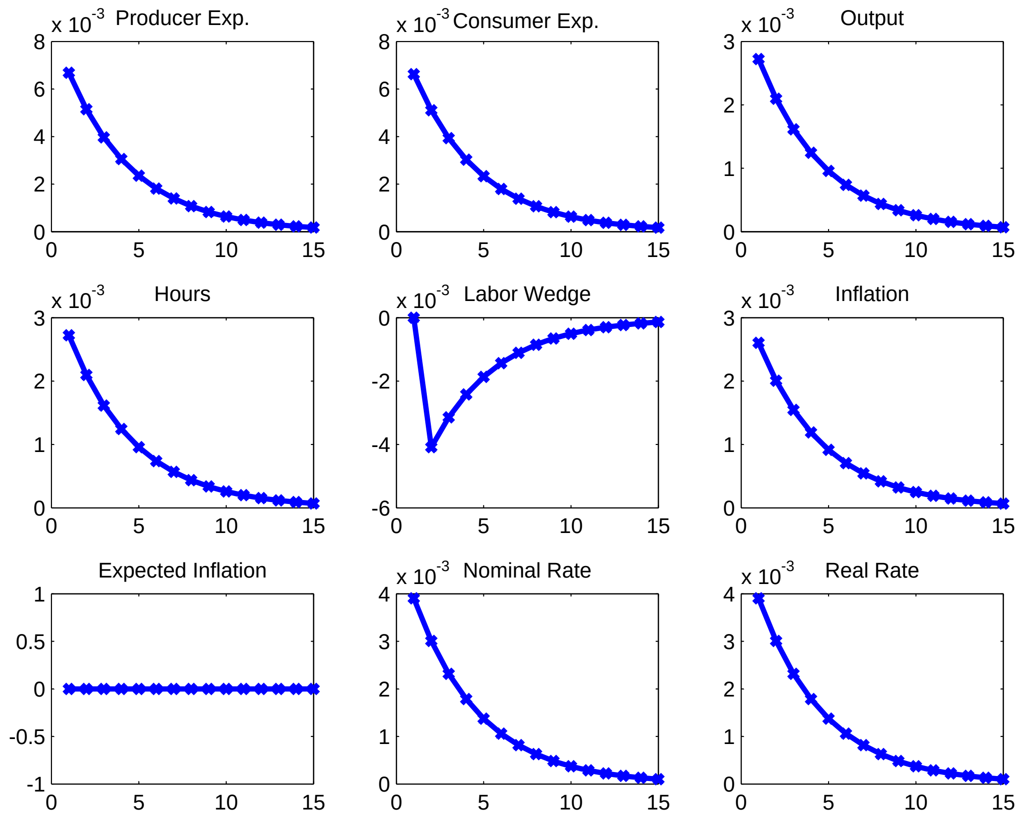

Figure 16: Impulse responses to a positive temporary productivity shock when rule 2 is followed with $\phi_y = 0$