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Seesaw and Disciplining Effects of Central Bank Reform On Redistribution and Labor Taxes in the Presence of Labor Unions

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ABSTRACT

This paper investigates the impact of central bank reform on redistribution, labor taxation and welfare in economies with a small number of wage setting labor unions and governments concerned with maximizing some combination of general welfare and redistribution. By raising central bank (CB) conservativeness such reforms directly reduce the premium of unions' wages over the competitive wage inducing reductions in both inflation and unemployment, and an increase in aggregate welfare.

But such reforms also induce government to adjust the tax rate on labor and redistribution. Depending on whether the tax rate goes down (a disciplining effect) or up (a seesaw effect) the direct beneficial effects of reform are either reinforced or moderated. On one hand, by raising the positive marginal impact of a labor tax on the wage premium, central bank reform deters government from raising the tax. On the other hand, by raising the tax base, such a reform encourages government to raise it. A disciplining effect arises when the reform raises the marginal impact of the tax on the wage premium by a lot and a seesaw effect arises when this increase is moderate. In the first case the beneficial effect of CB reform is amplified and in the second it is moderated opening the door for the possibility that flexible inflation targeting is optimal even in the absence of stabilization policy. Actual changes in the share of redistribution following CB reform are used to discriminate between those two cases.

Keywords and Phrases: Fiscal monetary policy interactions, seesaw and disciplining effects, labor unions, redistributive politics, central bank reform, social welfare

JEL Classification Codes: E5, H3, H5, H7, J5, E1

1 Introduction

This paper discusses the impact of central bank reform on government's tax policy and on social welfare in the presence of unionized labor markets. Most existing discussions of fiscal-monetary policy interactions posit competitive labor markets.[1] Frameworks with competitive labor markets may be reasonable for the US in which the fraction of the labor force covered by collective agreements is relatively small. But they clearly are counterfactual for European economies in most of which union membership is at least fifty percent.[2] The literature of the last ten to fifteen years has established that, contrary to competitive labor markets, in the presence of a small number of wage setters, the level of central bank conservativeness affects real variables along with inflation.[3] Consequently, the nature of interactions between fiscal and monetary policy in the presence of unionized labor differs substantially from their interaction under competitive labor markets.[4]

By delivering price stability more conservative central banks are making it easier for government to use fiscal policy for the pursuit of other objectives. With price stability assured by another institution government can use taxation more effectively to maximize social welfare but also to finance redistribution in favor of general and special interests. Acemoglu et al. (2008) argue that sensible reforms do not always generate their anticipated benefits because, in the presence of strong political constituencies, the new constraints imposed by reform in one area are often offset by more intensive use of distortionary instruments in other areas in

[1] See for example Dixit and Lambertini (2003). An exception is Cukierman and Dalmazzo (2006).

[2] Even in those countries in which membership is lower than fifty percent the fraction of the labor force actually **covered** by collective bargaining is substantially higher. This is achieved through various legal and other institutionalized extensions of union wages to segments of the labor force that are not members of unions. According to OECD (1997) coverage rates ranged from 68% in Spain to 92% in France at the beginning of the nineties.

[3] A non exhaustive list includes Skott (1977), Cukierman and Lippi (1999) and Lippi (2003).

[4] Ardagna (2007) considers the interactions between fiscal policy and wage-setting unions, abstracting from monetary policy.

order to satisfy the same politically powerful constituencies. They provide empirical support for this argument by showing that only a subset of the countries that upgraded the independence of their central bank achieved the anticipated benefits. Thus, the upgrading of central bank independence in Argentina and Columbia in 1991 was followed by a significant fall in inflation, as well as by increases in government expenditures as a share of GDP.

The literature on monetary policy in unionized economies has shown that, in the presence of a small number of wage setters, effective central bank conservativeness (CBC) or independence affects the equilibrium levels of employment and output. Since they have a stronger concern for price stability more conservative central banks react to union's wage increases with stronger contractions of monetary policy. This moderates union's real wage demands – lowering the premium of unions wage demands over the competitive real wage. And the lower wage premium induces higher levels of employment and economic activity (see Soskice and Iversen (2000)). Consequently, in unionized economies higher CBC is associated, in the long run, with both higher income and lower inflation implying that, in the absence of stabilization policy, strict inflation targeting is optimal (see Coricelli, Cukierman and Dalmazzo (2006)).

This paper revisits those results when tax rates chosen by partly redistribution minded governments are affected by the level of CBC. Such governments care about aggregate social welfare but also about the total volume of (general and targeted) redistribution.[5] Since the level of CBC affects total income central bank reform raises government's tax base, and with it the temptation to raise the tax and spend more on redistribution. On the other hand, since an increase in the tax rate reduces employment, income and social welfare, there is also a countervailing effect that induces government to reduce the tax rate in the aftermath of central bank reform. When the first effect dominates there is a **seesaw** reaction of fiscal policy as

[5] For simplicity the paper abstracts from the existence of public goods so that all tax receipts are used for either general or targeted redistribution.

argued, more broadly, by Acemoglu et. al. (2008).[6] When the second effect dominates the increase in CBC generates a **disciplining** effect on government's tax policy. In general both types of outcomes are possible. A main objective of the paper is to identify circumstances under which either one of those two outcomes arises. The paper also revisits the long run optimality of strict inflation targeting in unionized economies in the presence of endogenous fiscal policy.

The paper is organized as follows. Section 2 presents an overview of the model economy and the main results for the case of an exogenously given fiscal policy. In the interest of brevity all proofs underlying the results of this section are relegated to a separate Annex that is available upon request. Using the results in section 2 as a benchmark, section 3 endogenizes the fiscal policy of a government that partially caters to special interests of particular constituencies. Section 4 contains the main results of the paper. It identifies structural parameters leading to either seesaw or disciplining effects on government's tax rate and redistribution. It also examines the robustness of the long run social optimality of strict inflation targeting to the endogenization of fiscal policy. Most proofs of results in sections 3 and 4 appear in the appendix to the paper. This is followed by concluding remarks.

2 Overview of the model

The economy is composed of individuals, firms, a central bank and a fiscal authority (the government). There is a continuum of mass one of monopolistically competitive firms. Each firm is owned by an entrepreneur who earns profits. There are n labor unions that organize the entire labor force. Each union covers the labor force of a fraction $1/n$ of the firms. A quantity

[6] But the mechanisms leading to a seesaw effect in this paper and in Acemoglu et. al. (2008) are very different. In Acemoglu et. al. a seesaw effect arises (under implicitly competitive labor markets) due to a poor quality government's attempt to regain some of its lost seignorage revenues. When it arises in this paper a seesaw effect is due to the impact of central bank reform on the wage premium, employment and the tax base in a unionized economy, and through them on government's incentive to raise taxation and redistribution.

L_0 of workers is attached to each firm. Without loss of generality, all firms whose labor force is represented by union i are assigned to the contiguous subinterval $(\frac{i}{n}, \frac{i+1}{n})$ of the unit interval, where $i = 0, 1..., n-1$.

Individuals. Utility of an individual (be him a worker or an entrepreneur) in the economy is given by:[7]

$$U = \left(\frac{C}{\gamma}\right)^{\gamma}\left(\frac{M/P}{1-\gamma}\right)^{1-\gamma} + (1-\lambda)R, \;\; \gamma \in (0,1) \tag{1}$$

where C is the usual Dixit-Stiglitz (1977) consumption aggregator

$$C = \left(\int_0^1 C_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \tag{2}$$

of imperfectly substitutable consumption varieties, C_j, and θ is the elasticity of substitution between any pair of varieties. M denotes the nominal money stock held by the individual, and the price level, P, is given by:

$$P = \left(\int_0^1 P_j^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{3}$$

where P_j is the price of variety j. A worker can be either employed or unemployed. R denotes utility from leisure when unemployed, so that $\lambda = 0$ when the worker is unemployed and $\lambda = 1$ when he is employed. Since all entrepreneurs must forego leisure to manage their firms, we take $\lambda = 1$ for each entrepreneur. Each individual, whether worker or entrepreneur, possesses the same initial endowment of money, $\overline{M}$.

Let A_{cs} denote total nominal resources available to individual s in class c where $c = EW, UW, E$. Here EW, UW, E stand for "employed worker", "unemployed worker", and "entrepreneur" respectively. The budget constraint of individual s states that the nominal resources at his disposition are used to satisfy his consumption demands for the different varieties, C_{csj},

[7] The reader may recognize that the first part of this utility function is identical to the first part of equation (1) in chapter 8 of Blannchard and Fischer (1988). (See also Blanchard and Kiyotaki (1987)).

plus his demand for nominal money balances, M_{cs}:

$$A_{cs} = M_{cs} + \int_0^1 P_j C_{csj} dj, \quad c = EW, UW, E \tag{4}$$

where

$$A_{EWs} = W_{EWs} + \overline{M} + TR_W, \quad A_{UWs} = B + \overline{M} + TR_W, \quad A_{Es} = \Pi_s + \overline{M} + TR_E. \tag{5}$$

Here W_{EWs} is the net nominal wage earned by employed worker s, $B \geq 0$ is an unemployment benefit paid by government to each unemployed worker, Π_s is the profit received by firm owner s, and TR_W,and TR_E are governmental transfers to each worker (be him employed or unemployed) and employer respectively.[8] For simplicity we focus on the case of zero unemployment benefits ($B = 0$).[9] Each individual s in class c chooses the consumption C_{csj} of each variety j, with $j \in [0, 1]$, and nominal money balances M_{cs}, so as to maximize utility (1), subject to the budget constraint (4).

Government. Government raises taxes on labor and utilizes the proceeds to finance transfer payments. As in Alesina and Perotti (1997), there are two types of taxes. A social security tax paid by the employer (at rate σ), and an income tax (at rate υ). Denoting by W_g the gross wage paid to an employee, a firm bears a per-worker cost of labor equal to $(1 + \sigma)W_g$, while the worker receives a net wage equal to $W = (1 - \nu)W_g$. Thus, the ratio between the net wage and the cost of labor to the firm is given by $\frac{1-\nu}{1+\sigma} \equiv (1 - t)$, and the cost of labor to the firm can be written as $\frac{W}{(1-t)}$. Taking (natural) logarithms, the last equation can be reformulated as $\log W - \log(1 - t) \equiv w + \tau$, where $-\log(1 - t) \equiv \tau > 0$. Government tax revenues are used to finance transfer payments (other than unemployment benefits, since here $B = 0$), which

[8]Due to the homotheticity of preferences, transfers across individuals do not affect the size or composition of demand for goods at given prices (Alesina and Perotti (1997, p.924)).

[9]However all the proof in the separate Annex are developed for the more general case $B \geq 0$.

are TR_W per worker and TR_E per employer. Since the mass of employers is one, the total outlays for those transfers are $L_0 TR_W + TR_E$. Denoting by $\chi \geq 0$ the amount of such transfers as measured per-worker, the government pays out χL_0 where $\chi \equiv TR_W + \frac{1}{L_o} TR_E$. Total tax revenues are given by $\frac{tW}{1-t}(1-u)L_0$. We assume the budget is balanced implying that:

$$\frac{tW}{1-t}(1-u) = \chi \tag{6}$$

where u is the rate of unemployment.

Firms. Given taxation, the real profits of firm j, whose workforce belongs to union i are given by

$$\frac{\Pi_{ij}}{P} = \frac{P_j}{P} C_j^a - \frac{W_i}{P(1-t)} L_{ij}, \quad j \in [0,1] \tag{7}$$

where C_j^a denotes the aggregate demand for variety j. Taking the nominal wage W_i and the general price level P as given, each firm chooses the price, P_j, of the variety it sells so as to maximize profits subject to the production function

$$Y_j = L_{ij}^{\alpha}, \quad \alpha < 1. \tag{8}$$

Y_j is the amount of this variety that is produced and L_{ij} is the number of workers employed by firm j and covered by union i.

Central Bank. Monetary institutions are represented by a central bank (CB) that dislikes both inflation, π, and unemployment, u. As in Coricelli, Cukierman and Dalmazzo (2006), the CB chooses the money supply so to minimize the combined costs of inflation and of unemployment given by

$$\Gamma = u^2 + I \cdot \pi^2, \quad I \in [0, \infty). \tag{9}$$

As in Rogoff (1985), the parameter I measures the relative importance that the CB assigns to the objective of low inflation versus low unemployment. This parameter is also known as the degree of CB conservativeness.

Labor Unions. The probability that a member of union i will be unemployed is identical and independent across the union's members. Thus, the probability that any union member is unemployed is equal to the rate of unemployment among union members. Taking the nominal wages of other unions as given, each union, i, sets the nominal wage W_i for its members so as to maximize the expected utility of a representative member. This expected utility is given by

$$V_i = (1 - u_i) \cdot v_{EW} + u_i \cdot v_{UW} \tag{10}$$

where u_i is the unemployment rate among union i's members and, v_{EW} and v_{UW} are the individually optimal values of utility of employed and unemployed workers respectively. We postulate that the level of utility when employed is larger than utility when unemployed for all real wages higher than or equal to the competitive one. Thus

$$v_{EW} \geq v_{UW}. \tag{11}$$

This implies that all unemployment is involuntary. We refer to (11) as a "participation constraint".

2.1 Timing

We postulate the following sequence of events. In the first stage government sets fiscal policy parameters. Those parameters consist of labor taxes and of transfer payments that reflect government's politically motivated redistributive objectives. In the second stage each union chooses its nominal wage so as to maximize its objective function (10). When doing this, the union takes the nominal wages set by other unions as given, and anticipates the reactions of both the CB and the firms to its wage choice. In the third stage the CB chooses the nominal stock of money so as to minimize its loss function (9), taking as given the preset nominal wages and anticipating the reaction of firms to its choice. In the last stage, each firm takes the wage and the general price level as given and sets its own price so as to maximize real profits.

This timing sequence is meant to capture, within a static model, the fact that nominal wages are stickier than prices and that they normally are set for a period that is longer than the period for which monetary policy is set.[10] We view fiscal authorities as the first mover because tax rates and transfers are adjusted relatively infrequently.

General equilibrium is characterized by backward induction. We start by solving the firms' pricing problem, then the CB problem, and finally the unions' nominal wage decisions. First, we briefly characterize equilibrium in the last three stages for *given* values of transfer payments and taxes. Then, we discuss the choice of fiscal policy parameters by a politically motivated government in stage 1. In particular, we discuss the impact on macroeconomic equilibrium of redistributive policies in favor of the general public and/or "special interests".

2.2 Equilibrium with exogenous taxation

We take the actions of the government in stage 1 as exogenously given, and solve the model by backward induction starting with the typical firm's problem. Proofs of all results in the remainder of this section appear in the separate Annex.

Firm j's price-setting problem. When individuals maximize utility (1) with respect to each consumption variety, C_j, $j \in [0, 1]$ and money, M, subject to (4), the resulting aggregate

[10] Note that, since there are no shocks in the model the relative position of monetary policy and of price setting by firms within this timing sequence is immaterial for the nature of equilibrium. The reason is that, in the absence of shocks firms perfectly anticipate the subsequent choice of monetary policy by the CB. Hence they set the same prices as those they would have set when monetary policy precedes price setting - - leading to the same monetary policy and an identical equilibrium.

demand for variety j faced by producer j is equal to

$$C_j^a = \left(\frac{\gamma(1+L_0)}{1-\gamma}\right)\left(\frac{P_j}{P}\right)^{-\theta}\left(\frac{\overline{M}}{P}\right). \qquad (12)$$

Firm j maximizes profits (7), subject to technology (8) and demand (12). This yields the optimal pricing rule (in logs):

$$p_j - p = \psi + \frac{\alpha}{D}(w_i + \tau - p) + \frac{1-\alpha}{D}(\overline{m} - p) \qquad (13)$$

where $D \equiv \alpha + \theta(1-\alpha)$ and ψ are constants.

The Central Bank's money supply decision. The CB recognizes that (as shown in the Annex) both the inflation rate, $\pi \equiv p - p_{-1}$, and the unemployment rate, u, are : (i) increasing functions of the tax wedge τ, and (ii) increasing functions of nominal wages set in the economy. Moreover, inflation rises and unemployment falls when $\overline{m}$, the logarithm of money supply $\overline{M}$, goes up. The CB choose the money supply $\overline{m}$ so as to minimize the objective function (9), where $I \in [0, \infty)$. The solution to the central bank's problem yields the following reaction function:

$$\begin{aligned} \overline{m} &= \mu + \left[\frac{1-\alpha(1-\alpha)I}{K}\right]\cdot\tau + \\ &+ \left[\frac{(1-\theta)-D(1-\alpha)I}{(1-\theta)K}\right]\cdot\ln\left(\widehat{W}_1\right) - \left[\frac{1}{K}\right]\cdot\ln\left(\widehat{W}_2\right) \end{aligned} \qquad (14)$$

where $K \equiv 1 + (1-\alpha)^2 I > 0$, μ is a combination of exogenous parameter, $\widehat{W}_1 \equiv \int_0^1 W_j^{\frac{\alpha(1-\theta)}{D}} dj$ and $\widehat{W}_2 \equiv \int_0^1 W_j^{\frac{-\theta}{D}} dj$ are aggregations of individual wages.

Equation (14) implies that the sign of the response of the money supply to an increase in the tax wedge, τ, depends on the degree of Central Bank conservativeness (CBC), I. Depending on whether I is larger or smaller than $\frac{1}{\alpha(1-\alpha)}$, the CB reacts to an increase in the tax wedge by reducing or raising the money supply. The intuitive reason is that an increase in the tax wedge raises both inflation and unemployment. Although the CB dislikes those changes it cannot fully offset both since it has only one instrument. But, if the CB is sufficiently conservative, it will partially offsets the increase in inflation in spite of the fact that this aggravates unemployment.[11]

Union i's choice of nominal wage. Each monopolistic union i, $i \in \{1, 2, .., n\}$, sets the same nominal wage W_i for all its members so as to maximize a typical member's expected utility, (10). Thus, all firms whose workforce is controlled by union i pay the same nominal wage. In setting its wage, the union takes the demand for its workforce and the nominal wages set by other unions as given. The union also anticipates the impact of its action on subsequent monetary policy as summarized by the monetary policy reaction of the CB in equation (14). We define the wage premium ϕ as the logaritmic difference between the wage set by the union, W/P, and the competitive wage, $(W/P)_c$: thus, $\phi \equiv \log(W/P) - \log(W/P)_c$. In a symmetric equilibrium where all unions set the same wage ($W_i = W, \;\; i = 0, 1..., n$) a first-order Taylor approximation for the wage premium is given by

$$\phi \cong \frac{\left[\frac{Z_w}{Z_u} + \frac{R}{(1-t)W^g_{rc}} - 1\right] - \left(\frac{1-\gamma}{\gamma}\right)\frac{\alpha(1-\alpha)I}{nKZ_u}\left(\frac{J}{1-t}\right)}{\left[\left(\frac{1}{1-\alpha}\right)\frac{Z_w}{Z_u} + \frac{R}{(1-t)W^g_{rc}}\right] - \left(\frac{1-\gamma}{\gamma}\right)\frac{\alpha I}{nKZ_u}\left(\frac{J}{1-t}\right)}. \tag{15}$$

Here $W^g_{rc} \equiv \alpha L_0^{\alpha-1}\left(\frac{\theta-1}{\theta}\right)$ is the level of the gross competitive real wage, J is a constant, and Z_w and Z_u are given by:

$$Z_w \equiv 1 - \frac{1}{n\left[1 + (1-\alpha)^2 I\right]} > 0; \quad Z_u \equiv \frac{1}{n}\left[\frac{\theta(n-1)}{\alpha + \theta(1-\alpha)} + \frac{(1-\alpha)I}{1 + (1-\alpha)^2 I}\right] > 0. \tag{16}$$

Z_w measures the overall elasticity of the union's net real wage with respect to a change in its nominal wage, and Z_u measures the overall elasticity of the union's unemployment rate with

[11] It can also be shown that an increase in nominal wages will induce a contraction in the money supply when the CB is sufficiently conservative, that is, if and only if $I > \frac{1}{\alpha(1-\alpha)}$. A related analysis that abstracts from the existence of taxes appears after equation (16) in Coricelli, Cukierman and Dalmazzo (2006).

respect to a change in the union's nominal wage. These overall elasticities take into consideration the subsequent reactions of the money supply and of the price level to a change in the union's nominal wage policy. Since it is likely that the weight attached to utility from a unit of aggregate consumption is large in comparison to utility from a unit of real money balances, γ is likely to be close to one, implying that the ratio $\frac{1-\gamma}{\gamma}$ is relatively small.

Equilibrium Unemployment and Inflation rates. The equilibrium values of the economy-wide unemployment rate, u, and of the inflation rate, π, can be expressed as increasing linear functions of the wage-premium ϕ as follows:

$$u = \frac{\phi}{1-\alpha} \tag{17}$$

and

$$\pi = \frac{\phi}{(1-\alpha)^2 I}. \tag{18}$$

It can be shown that the wage premium, ϕ, is an increasing function of the tax wedge, t ($\frac{d\phi}{dt} > 0$) and a decreasing function of CBC, I ($\frac{d\phi}{dI} < 0$). Consequently both inflation and unemployment are increasing in t and decreasing in I.[12] Stated somewhat differently, an increase in the tax wedge reduces employment – a result supported by most empirical studies on unionized European economies like that of Nickell, Nunziata and Ochel (2005).[13]

The results described so far hold for any arbitrarily **given** level of the tax wedge t (or τ). The following subsection contains a preliminary analysis of some welfare implications for such an exogenously given fiscal stance. This analysis provides a partial benchmark for the main analysis in sections 3 and 4 in which the tax wedge is determined endogenously along with other

[12] Details and proofs appear in subsection 3.1 of the Annex. Those results provide robustness to similar results in Cukierman and Dalmazzo (2006).

[13] See also OECD (1997), Alesina and Perotti (1997), Daveri and Tabellini (2000) and Belot and van Ours (2001).

variables.

2.3 Social Welfare

This subsection presents the comparative statics effects of taxation and of Central Bank conservativeness (CBC) on welfare for a given fiscal policy stance. Average welfare per individual[14], denoted by $\widehat{v}$, can be expressed as the following function of the wage premium:

$$\widehat{v}(\phi) = \Psi \cdot [1-\phi]^{\frac{\alpha}{1-\alpha}} + \frac{L_0^{\alpha}}{1+L_0}\left[1 - \frac{\phi}{1-\alpha}\right]^{\alpha} + \frac{L_0}{1+L_0}\left(\frac{\phi}{1-\alpha}\right) \cdot R \tag{19}$$

where $\Psi > 0$ is a constant combination of parameters. An increase in the wage-premium, ϕ, affects $\widehat{v}(\phi)$ through three channels: (i) it reduces average utility by reducing income from production, (ii) it increases average utility from leisure by raising the number of unemployed, and (iii) it reduces average utility by reducing aggregate real money balances. We **assume that some positive employment is socially desirable** which implies that

$$\alpha\left[(1-u)L_0\right]^{\alpha-1} > R. \tag{20}$$

This condition states that the marginal contribution to output (and therefore to welfare from consumption) from an additional employee has to be greater than the value of leisure this worker foregoes when becoming employed. Under this condition an increase in the wage-premium, ϕ, unambiguously reduce welfare, i.e. $\frac{d\widehat{v}(\phi)}{d\phi} < 0$ (A proof appears in Appendix 6.1). Furthermore, given that some employment is socially desirable and taking the tax wedge, t, as exogenously given, it can be shown that:[15]

[14] Recall that the mass of individuals in the economy is given by $(1 + L_0)$ – that is; the mass of entrepreneurs (1) plus the mass of workers per firm (L_0)

[15] A proof appears in section 4 of the Annex.

Lemma: *Given the participation constraint, (11), and the social desirability of some positive employment, (20), and provided* $(1-\gamma)$ *is sufficiently small, then:*

(i) The higher the tax wedge, t*, the lower social welfare.*

(ii) The higher central bank conservativeness, I*, the higher social welfare.*

The Lemma states that higher taxation is detrimental to welfare.[16] **Given the tax wedge, t, it also implies that an ultra-conservative central banker (with $I \to +\infty$) is socially optimal.** This result which confirms, for unionized labor markets, Rogoff's (1985) conclusion in the absence of shocks, is common to several other papers that analyze the strategic interaction between the central bank and labor unions.[17] Roughly speaking it is a consequence of the fact that the more conservative is the central bank, the more it contracts the money supply in reaction to inflationary increases in nominal wages – leading to a more serious contraction in output and labor demand. Fearing the unemployment consequences of this stronger reaction unions tone down their real (and nominal) wage demands – which leads to an equilibrium with a lower wage premium and higher employment. This disciplining effect on union's wage demands is highest when the central bank is ultra-conservative. Essentially the deterring effect of central bank conservativeness on unions' wage demands is maximal in this case.[18]

Maintaining the conditions postulated in the Lemma the next section reconsiders, inter-alia, the issue of optimal central bank conservativeness when a government with some partisan leanings toward general or "special interests" redistribution sets the tax wedge, t, and, χ, endogenously.

[16] As shown in the Annex, the results in the Lemma also hold when there exists a strictly positive (but smaller than α) "replacement ratio".

[17] See, e.g., Soskice and Iversen (2000) and Coricelli et al. (2006) among others.

[18] During the seventies and the eighties the Bundesbank occasionally contracted the money supply in order to moderate unions' wage demands.

3 Fiscal policy and redistribution.

This section characterizes the first stage of the game in which a political authority picks fiscal instruments, anticipating the reactions of labor unions, the Central Bank and price setters in subsequent stages. Formally, the fiscal authority acts as a Stackelberg leader.

It is well known that the motives of political authorities and of social planners are not fully aligned. Although this does not necessarily mean that politicians do not care at all about social welfare, it usually implies that they *also* care about general redistribution, as well as about redistribution in favor of particular constituencies. We therefore endow fiscal authorities with an objective function that is a weighted average of social welfare and of total redistribution. In particular, government's objective function is given by:

$$\Upsilon = \delta \cdot \left[\frac{\chi}{P} L_0 \right] + (1 - \delta) \cdot \widehat{v} \tag{21}$$

The term $\frac{\chi}{P} L_0$ on the right-hand side of (21) represents the total amount of real transfer payments.[19] Thus, the parameter $\delta \in [0, 1]$ represents the weight assigned by government to total redistribution and $1-\ \delta$ represents the weight assigned to $\widehat{v}$, the indirect average utility of individuals in the economy in (19). The balanced-budget condition in (6) can be written as

$$\frac{\chi}{P} L_0 = \frac{W}{P} \frac{t(1-u)L_0}{1-t} \equiv T(t). \tag{22}$$

where the left hand side of (22) represents total real redistribution and the right hand side represents total real tax revenues.

The instrument of fiscal policy - the tax rate t - is chosen to maximize (21) subject to

[19] See also the discussion preceding equation (6). Since $\chi \equiv TR_W + \frac{1}{L_0} TR_E$ is the average transfer per worker in the economy, χL_0 represent total nominal transfers. Note that, although transfers are *measured* per worker, they can accomodate any pattern of transfers between workers and entrepeneurs.

(22). To develop some intuition about the mechanisms underlying government's choice, it is convenient to start with two extreme particular cases: (i) a government that has no interest in redistribution ($\delta = 0$) and, (ii) a government that only cares about redistribution ($\delta = 1$).

Case (i). A government that only cares about social welfare gives no weight to redistribution ($\delta = 0$) and sets t so as to maximize $\widehat{v}$. Part (i) of the Lemma implies that welfare is maximized when $t = 0$. Hence, like a Benthamite social planner, a government with no redistributional concerns will impose no taxes.[20]

Case (ii) A fully partisan government that only cares about the special interests of its favored constituency ($\delta = 1$) chooses the tax wedge, t, so as to maximize T, the amount of funds available for redistribution in (22). Formally such a government sets the tax wedge, t, so that the condition $\frac{dT(t)}{dt} = 0$ is satisfied.[21]

3.1 Characterization of t and of the size of government in the general case

To characterize the equilibrium values of t and of total redistribution in the intermediate case, in which $0 < \delta < 1$, we need to express all the components of T in (22) as a function of t. Since both the real wage and unemployment depend on t via the wage premium, we start by expressing T in terms of the wage premium $\phi(t)$, which itself is a function of the tax wedge. By exploiting the approximation $\frac{W}{P} \cong \frac{W^g_{rc}(1-t)}{(1-\phi(t))}$ in (22), expression (21) - the objective function of

[20] Obviously, this extreme conclusion is a consequence of the implicit assumption that utility from public goods is zero. In the presence of utility from public goods there will be, in this case, some taxation but only to finance the public good.

[21] Such a political equilibrium arises in Meltzer and Richard (1981) when the median voter in their model does not work.

the fiscal authority - can be expressed as a function of t:

$$\Upsilon(t) = \delta \cdot T + (1-\delta) \cdot \widehat{v} \;\cong\; \delta \;\; \frac{W_{rc}^{g} \cdot t \cdot L_0}{1-\phi(t)} \left[1 - \frac{\phi(t)}{1-\alpha}\right] + (1-\delta) \cdot \widehat{v}(t) \tag{23}$$

where the function $\widehat{v}(t)$ is given by (19). At an internal solution, the tax rate t^* that maximizes government's objective (23) satisfies the first-order condition

$$\frac{d\Upsilon(t^*)}{dt} = \delta \frac{dT(t^*)}{dt} + (1-\delta) \frac{d\widehat{v}(\phi(t^*))}{d\phi} \frac{d\phi}{dt} = 0 \tag{24}$$

By the Lemma $\frac{d\widehat{v}(\phi(t^*))}{d\phi} < 0$, and provided $1-\gamma$ is sufficiently small, $\frac{d\phi}{dt} > 0$. Hence the second term on the right hand side of (24) is negative. Consequently, $\frac{dT(t^*)}{dt} > 0$ for all $\delta < 1$ establishing that government operates on the efficient side of the Laffer curve. Thus, in equilibrium, the size of redistribution (and therefore the "size of government") is increasing in the tax wedge.

Application of the implicit function theorem to (24) yields

$$\frac{dt^*}{d\delta} = - \frac{\frac{dT(t^*)}{dt} - \frac{d\widehat{v}(\phi(t^*))}{d\phi} \frac{d\phi}{dt}}{SOC(t^*)}$$

where $SOC(t^*) < 0$ is the second order condition for government's decision problem. Condition (24) implies that $\frac{dT(t^*)}{dt} - \frac{d\widehat{v}(\phi(t^*))}{d\phi} \frac{d\phi}{dt} > 0$. This leads to the following proposition.

Proposition 1: *Governments with stronger redistributive concerns (higher δ) set higher tax wedges.*

Thus, governments with stronger redistributive motives set a higher tax wedge in order to raise general as well as "special interests" redistribution.

4 Seesaw and disciplining effects of central bank conservativeness on taxation, redistribution and social welfare

The impact of an increase in CBC on government's tax policy is generally ambiguous, since it triggers two opposing effects. On one hand, by raising the tax base for a given tax, an increase in I tends to increase the marginal impact of t on tax revenues. This effect encourages the government to *raise* the tax wedge due to its concern for redistribution. On the other hand, higher CBC, by magnifying the adverse effect of t on the wage premium and unemployment, encourages government to *reduce* the tax wedge. This effect operates via government's concern about general social welfare as well as through the reduction this causes in the tax base. Depending on which of those two effects dominates, central bank reform triggers either a seesaw or a disciplining effect on government's choice of tax policy. The following discussion identifies conditions for the existence of either effect.[22]

4.1 Seesaw and disciplining effects

Applying the implicit function theorem to (24)

$$\frac{dt^*}{dI} = \frac{\delta \frac{d^2T}{dtdI} + (1-\delta)\left[\frac{d^2\widehat{v}}{d\phi dI}\frac{d\phi}{dt} + \frac{d\widehat{v}}{d\phi}\frac{d^2\phi}{dtdI}\right]}{-\ SOC(t^*)}. \tag{25}$$

where $SOC(t^*) < 0$ is the second order condition for government's optimization. It is shown in Appendix 6.1 that $\frac{d\widehat{v}}{d\phi} < 0$ and in Appendix 6.2 that $\frac{d^2\phi}{dtdI} > 0$ and $\frac{d^2\widehat{v}}{d\phi dI} > 0$. Those inequalities imply that $\frac{d\widehat{v}}{d\phi}\frac{d^2\phi}{dtdI} < 0$ and, since $\frac{d\phi}{dt} > 0$, that $\frac{d^2\widehat{v}}{d\phi dI}\frac{d\phi}{dt} > 0$. Hence, the sign of the second term in the numerator on the right hand side (RHS) of (25) is generally ambiguous. Although the

[22] As in the earlier discussion this discussion relies on the assumption that utility from real money balances is small in comparison to utility from the total consumption basket (γ in equation (1) is close to one).

sign of $\frac{d^2T}{dtdI}$ is also ambiguous in general it can be shown that, when $\frac{d^2\phi}{dt \cdot dI}$ is not too large, $\frac{d^2T}{dtdI}$ is positive. Since in this case the second term in brackets in the numerator is also positive, $\frac{dt^*}{dI} > 0$. Those considerations underlie the following proposition.[23]

Proposition 2: *If the (non-negative) cross-derivative* $\frac{d^2\phi}{dt \cdot dI}$ *is not too large the tax wedge,* t^*, *set by fiscal authorities is* ***increasing*** *in the level of Central Bank conservativeness,* I.

Thus, under certain circumstances, higher CBC generates a "seesaw" effect on government's behavior, inducing it to raise the tax wedge. Acemoglu et al. (2008) argue that sensible reforms do not always generate the benefits that they promise because, in the presence of strong political demands, the new constraints imposed by reform in a particular area are often offset by more intensive use of other distortionary instruments to satisfy the same politically powerful constituencies. Taking the success of central bank independence in reducing inflation as an example they show empirically that this offsetting or seesaw effect is weaker in countries with intermediate quality levels of political institutions.

Proposition 2 provides a sufficient condition for the operation of a seesaw effect between central bank reform and fiscal policy in countries with highly unionized labor markets. Broadly speaking the political economy content of this condition is that such an effect is more likely to appear when the (non-negative) impact of an increase in CBC on $\frac{d\phi}{dt}$ is sufficiently small. This condition implies that the increase in CBC induces only a moderate increase in the adverse marginal impact of the tax rate on the wage premium. As a consequence, the increase in the adverse **direct** effect of t on marginal tax collections as well as on marginal social welfare are small in comparison to the increase in the **indirect positive** effect of t on marginal tax collections due to the increase in the tax base. As a result the second effect dominates, creating a seesaw effect.

[23] Further details appear in Appendix 6.2

The first part of the next proposition identifies conditions leading to a negative impact of central bank reform on government's choice of tax policy and the second identifies conditions under which this policy is independent of CBC.[24]

Proposition 3:

(i) *When the fiscal authority is concerned mainly with social welfare (*δ close to zero*), and* $\frac{d^2\phi}{dt \cdot dI}$ *is relatively large, the tax wedge* t^* *set by fiscal authorities is* ***decreasing*** *in the level of Central Bank conservativeness,* I*.*

(ii) For sufficiently high levels of I and for all $n > 3$ *the tax wedge,* t^*, *is* ***independent*** *of* I*.*

The first part of Proposition 3 suggests that, if $\frac{d^2\phi}{dt \cdot dI}$ is sufficiently large and government has only moderate redistributive objectives, an increase in CBC **disciplines** government by inducing it to **decrease** the tax wedge. Thus, in the presence of unionized labor markets, an increase in CBC generates a **seesaw** effect under some circumstances and a **disciplining** effect on government under other circumstances. The second part of Proposition 3 provides a sufficient condition for the borderline case in which both effects are absent.

We turn next to the impact of CBC on total government tax collection, $T(t)$, and redistribution. Holding the tax wedge, t, constant an increase in CBC raises tax collections by increasing the tax base. This effect operates through the moderating impact that higher CBC has on the wage premium. In the presence of a seesaw effect on t there is a further increase in tax collections and redistribution making the increase in redistribution even larger. But in the presence of a disciplining effect on t there is a moderating effect on tax collections and redistribution. As a consequence, in the presence of a disciplining effect, the impact of higher CBC on

[24] The proof appears in Appendix 6.3.

tax collections is generally ambiguous. This is summarized in the following proposition.[25]

Proposition 4:

(i) In the presence of a ***seesaw effect*** *of I on t^* the total impact of an increase in CBC on tax collections, $T(t)$, and redistribution is positive.*

(ii) *In the presence of a* ***disciplining effect*** *of I on t^* the total impact of an increase in CBC on tax collections and redistribution is ambiguous.*

The recent economic history of Argentina, in which union coverage is non negligible, is consistent with the first part of the proposition. In particular, the 1991 upgrading of central bank independence in this country was followed by both a fall in inflation and an increase in government expenditures as a percent of GDP. Note, however, that this fact alone does not discriminate between the seesaw and the disciplining effects of CBC. The reason is that, by the second part of the proposition, government expenditures can go up even in the presence of a disciplining effect provided this effect is sufficiently small in comparison to the direct positive effect of higher CBC on the tax base.

Nonetheless the change in redistribution in the presence of a seesaw effect is unambiguously larger than this change in the presence of a disciplining effect. Since, when a seesaw effect is present the impact on redistribution is always positive, this is obviously the case when redistribution goes down in the presence of a disciplining effect. When redistribution goes up it goes up by less than in the presence of a seesaw effect since the tax rate goes up in the presence of a seesaw effect and down in the presence of a disciplining effect. Since government operates in the efficient range of the Laffer curve total tax collections and redistribution are positively related implying that the increase in redistribution is larger in the presence of a seesaw than in

[25] The proof appears in Appendix 6.4.

the presence of a disciplining effect also when redistribution increases in the latter case.

These considerations can be used to provide an approximate criterion for determining whether, following upgrades in CBC seesaw or disciplining effects are more likely to be operating within a given country. When, following central bank reform, redistribution rises subsantially it is likely that a seesaw effect operates and when it goes down this is evidence in favor of a disciplining effect. In what follows this discriminatory criterion is briefly illustrated by applying it before and after the upgrading of central bank independence in unionized European economies. This is done by calculating the average GDP share of redistribution in some OECD countries before and after the upgrading of CBC. Since, the processes modeled here most likely operate in the long run, we compare those average shares in the period ending three years prior to central bank reform with the period starting three years after the reform.[26] We refer to those periods in the sequel as the periods "before" and "after" reform respectively.

Redistribution is measured by the item "Social contributions paid by government plus subsidies" from OECD data between 1990 and 2010. Two countries that stand out as strong candidates for the operation of a **seesaw** effect are Portugal and Greece. In Portugal (who joined EMU upon its creation on January 1 1999) the average percentage share of redistribution was 10.3 in the "before" 6 years ending at the beginning of 1996 and 15.2 in the "after" 9 years starting at the beginning of 2002. The corresponding numbers for Greece (who joined EMU on January 1 2001) are 13.4 in the "before" period and 18.1 in the "after" period. Some weaker evidence in favor of a **seesaw** effect appears in Italy for which the shares are 16.3 and 17.5 in the "before" and "after" periods respectively. At the other end of the spectrum we note the UK that granted independence to the Bank of England in May 1997. In this country the share of redistribution went down from 14.4 to 13.2 supporting the existence of a **disciplining** effect of

[26] An additional reason is to avoid contaminating our search for either seesaw or disciplining effects with the attempts made by potential entrants into EMU to live up to the Maastricht criteria during the final run-up years towards admittance into the monetary union.

central bank reform.

4.2 Central Bank conservativeness and social welfare

To this point we have been concerned with the impact of CBC on government's **tax policy**. A related important question is what is the impact of Central Bank reform on **social welfare** when fiscal policy is endogenous. The analysis in section 2 has shown that, given fiscal policy, such a reform – by raising CBC – always raises welfare. However, proposition 2 implies that, when fiscal instruments are allowed to react to the change in I, this may no longer be the case in line with the seesaw argument.

To explore this issue we focus on the following specific question: Under what conditions will a potential seesaw effect between CBC and the tax wedge reduce welfare to an extent that more than offsets the direct beneficial effect of an increase in CBC on welfare? To answer this question we first note that a change in I affects welfare only through the wage premium and that (by Appendix 6.1) welfare is a decreasing function of the wage premium. Hence in order to establish the overall impact of an increase in CBC on welfare in the presence of reactive fiscal policy it suffices to investigate the impact of I on the wage premium, ϕ. For a sufficiently small value of $(1-\gamma)$ in (15), the total derivative of ϕ with respect to I (which also accounts for the effect of I on the equilibrium tax wedge t^*) is:

$$\frac{d\phi(t^*)}{dI} = \frac{\widehat{K}}{1-\alpha}\left[\left[1-\alpha\frac{R}{(1-t)W_{rc}^g}\right]\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} + \frac{R}{(1-t)^2 W_{rc}^g}\left[(1-\alpha)+\alpha\frac{Z_w}{Z_u}\right]\frac{dt^*(I)}{dI}\right] \quad (26)$$

where $\widehat{K}$ is a positive constant, and $\left[1-\alpha\frac{R}{(1-t)W_{rc}^g}\right] > 0$ by the participation constraint. Using equation (16)

$$\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} = -\frac{(1-\alpha)(n-1)}{\left\{\theta(n-1)\frac{1+(1-\alpha)^2 I}{\alpha+\theta(1-\alpha)} + (1-\alpha)I\right\}^2 (\alpha+\theta(1-\alpha))}\left\{\alpha + (1-\alpha)^2 DI\right\} \tag{27}$$

which is negative for all $n > 1$. Together with (26) this confirms that (excluding the case $n = 1$), when the tax wedge does not change or goes down, welfare is monotonically increasing in CBC as was the case when fiscal policy was exogenous. But in the presence of a seesaw effect $\frac{dt^*(I)}{dI} > 0$ implying that the second term on the right hand side of (26) is positive. For a given value of the first term this raises the wage premium and reduces welfare. Thus, in the presence of a seesaw effect the overall impact of higher CBC on welfare depends on the relative magnitudes of the (negative) effect of the seesaw and of the direct (positive) effect of higher CBC on aggregate welfare.

5 Concluding remarks

Since the end of the eighties and mainly though the nineties there has been a worldwide trend of central bank reform. In practically all cases central banks were granted higher independence [27] A major feature of this process was the high priority assigned by law to the price stability objective in the central bank charter making central banks more effectively conservative. Acemoglu et. al. (2008) argue that, although the reforms reduced inflation, they triggered (mainly in countries with low government quality) offsetting adjustment in other areas of governmental activity. Thus, following the introduction of central bank independence in Columbia and Argentina in 1991, inflation went down and government expenditures went up. But the mechanisms inducing a seesaw effect in this paper and in Acemoglu et. al. (2008) are very different. In Acemoglu

[27] Cukierman (2008) provides a survey of this process.

et.al. the seesaw effect is due to government's desire to compensate for a fall in seignorage revenues in an economy with (implicitly) competitive labor markets. In this paper, a seesaw (or a disciplining) effect is due to the impact of central bank reform on the wage premium and employment in a unionized economy and through the latter on government's incentive to adjust the tax rate and redistribution.

This paper identifies political-economy channels through which an increase in central bank conservativeness (CBC) induces fiscal authorities to raise labor taxes in economies with unionized labor markets – creating a seesaw effect between central bank reform on one hand and tax cum redistribution policy on the other. A central message of the paper is that seesaw effects are neither the rule, nor the exception. Although such effects arise under some structural configurations, central bank reform triggers a decrease in labor taxes under other structural configurations – creating a disciplining effect of reform on taxation and redistribution. An important factor that determines whether, following reform, the tax rate will go up or down is the magnitude of the impact of higher CBC on the marginal effect of a higher tax rate on the wage premium.[28]

The paper shows that, invariably, an increase in conservativeness raises the impact of the tax rate on the wage premium. A seesaw or a disciplining effect arises depending on whether this increase is small or large. The intuition underlying this result follows from the observation that, by reducing income and the tax base, a higher marginal impact of labor taxes on the wage premium, has a stronger deterrent impact on government's tendency to raise the tax rate.[29] But at sufficiently high levels of CBC seesaw, as well as disciplining effects, become negligible. In these cases redistributions still goes up due to the direct upward impact of central bank reform on the tax base. As a matter of fact the size redistribution may go up even in the presence of

[28] In unionized economies the wage premium over the competitive wage is positive and is increasing in the labor tax rate.

[29] This deterrence effect operates through both government's concern about social welfare as well as about the volume of redistribution.

moderate disciplining effects. But, in the presence of seesaw effects redistribution always goes up following central bank reform.

Back of the envelope calculations presented in subsection 4.1 are consistent with the view that fiscal policies in Portugal and Greece were affected by seesaw effects after the entry of those countries into EMU. By contrast the behavior of redistribution in the UK before and after the granting of central bank independence in 1997 is consistent with the existence of a disciplining effect on fiscal policy.

Previous literature on the strategic interaction between monetary policymaking institution has shown that, in the absence of stabilization policy, strict inflation targeting is socially optimal (Soskice and Iversen (2000), Coricelli, Cukierman and Dalmazzo (2006).[30] But in those frameworks fiscal policy is given exogenously. When, government's reaction to central bank reform takes the form of a seesaw effect, the optimality of strict inflation targeting may no longer obtain. As with exogenous fiscal policy, central bank reform still **directly** increases welfare by lowering the wage premium and through it inflation and unemployment. However, the presence of a seesaw effect operates in the opposite direction opening the door for the possibility that flexible inflation targeting is optimal even in the absence of stabilization policy.

6 Appendix.

6.1 The impact of ϕ on $\widehat{v}(\phi)$

Differentiating $\widehat{v}(\phi)$ in (19) with respect to ϕ and rearranging

$$\frac{d\widehat{v}(\phi)}{d\phi} = -\Psi\left(\frac{\alpha}{1-\alpha}\right)[1-\phi]^{\frac{2\alpha-1}{1-\alpha}} - \frac{L_0}{(1+L_0)(1-\alpha)}\left[\alpha\left(L_0(1-u)\right)^{\alpha-1} - R\right] \tag{28}$$

[30] In the absence of stabilization policy this result already arises in Rogoff (1985) classic paper. But in Rogoff's paper the sole reason for this result is lower inflation whereas, in the presence of strategic interactions the optimality of strict inflation targeting arises because of both lower inflation aand higher employment.

Since $\Psi \equiv \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{L_0}{1+L_0}\right)\left[\frac{\theta\exp\{\alpha\}}{\alpha(\theta-1)}\right] > 0$ and γ is close to zero the first term vanishes. Condition (20) implies that the second term is negative implying that $\frac{d\widehat{v}(\phi)}{d\phi} < 0$.

6.2 Government's choice of the tax wedge and proof of Proposition 2

The proof starts with two intermediate claims.

Claim 1: $\frac{d^2\phi}{dt \cdot dI} > 0$.

Proof: For $(1-\gamma)$ sufficiently small,

$$\frac{d\phi}{dI} = \frac{1}{D_1^2(1-\alpha)}\left\{1-\alpha\left(\frac{R}{(1-t)W_{rc}^g}\right)\right\}\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} < 0$$

which, from (27), is negative.[31] Differentiating $\frac{d\phi}{dt}$ with respect to I and rearranging:

$$\frac{d^2\phi}{dIdt} = -Q_1\left\{\frac{\alpha}{1-\alpha}\frac{Z_w}{Z_u} + \left[4 - \frac{\alpha R}{(1-t)W_{rc}^g}\right]\right\}\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} \tag{29}$$

where Q_1 is a positive constant. The participation constraint (11) implies that $\frac{R}{(1-t)W_{rc}^g} \leq 1$ and, thus, that $\frac{\alpha R}{(1-t)W_{rc}^g} < 1$. The proof of Claim 1 is completed by noting that $\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} < 0$ implies $\frac{d^2\phi}{dIdt} > 0$, as claimed.

Claim 2: *If* $\frac{d^2\phi}{dt \cdot dI}$ *is not too large, then* $\frac{d^2T}{dt \cdot dI} > 0$.

Proof: Differentiating T in (22) with respect to t, one obtains that:

$$\frac{dT}{dt} = \frac{W_{rc}^g \cdot L_0}{1-\alpha}\left[\frac{1-\alpha-\phi}{(1-\phi)} - \frac{\alpha \cdot t}{(1-\phi)^2}\left(\frac{d\phi}{dt}\right)\right] \tag{30}$$

[31] D_1 is a positive combination of parameters whose explicit form is irrelevant for the argument.

where $1-\alpha-\phi>0$. Differentiating (30) with respect to I,

$$\frac{d^2T}{dt\cdot dI}\equiv H_1=\frac{\alpha\cdot W_{rc}^g\cdot L_0}{(1-\alpha)(1-\phi)^2}\left\{\left(-\frac{d\phi}{dI}\right)\left[1+\frac{2t}{(1-\phi)}\frac{d\phi}{dt}\right]-t\left(\frac{d^2\phi}{dt\cdot dI}\right)\right\}. \tag{31}$$

We saw (at the end of section 2.2) that $\frac{d\phi}{dt}>0$. Hence the first term in curly parentheses in (31) is positive. However, since $\frac{d^2\phi}{dt\cdot dI}>0$, the second term in curly parentheses is negative. It follows that the sign of (31) is positive whenever $\frac{d^2\phi}{dt\cdot dI}$ is not too large. This establishes Claim 2. Hence the first term in the numerator on the right hand side (RHS) of (25) is positive.

For $(1-\gamma)$ sufficiently small, and using equations (15) and (19),

$$\frac{d\widehat{v}}{dt}=\frac{d\widehat{v}}{d\phi}\times\frac{d\phi}{dt}=\frac{-L_0\left[\alpha\left[L_0(1-u)\right]^{\alpha-1}-R\right]}{(1+L_0)(1-\alpha)}\times\frac{d\phi}{dt}. \tag{32}$$

Differentiating the expression for $\frac{d\widehat{v}}{d\phi}$ in (32) with respect to I yields:

$$\frac{d^2\widehat{v}}{dId\phi}=\frac{d}{dI}\left[\frac{-L_0\left[\alpha\left[L_0(1-u)\right]^{\alpha-1}-R\right]}{(1+L_0)(1-\alpha)}\right]=\left[\frac{-\alpha L_0^{\alpha}(1-u)^{\alpha-2}}{(1+L_0)(1-\alpha)}\right]\times\frac{d\phi}{dI}>0. \tag{33}$$

Recall that $\frac{d\phi}{dt}>0$ and $\frac{d\phi}{dI}<0$ (see end of subsection 2.2). Equation (33) along with the second inequality implies $\frac{d^2\widehat{v}}{d\phi dI}>0$. Along with $\frac{d\phi}{dt}>0$ the last inequality implies that $\frac{d^2\widehat{v}}{d\phi dI}\frac{d\phi}{dt}>0$. Thus, the second term in the numerator on the right hand side (RHS) of (25) is also positive.

However, from Claim 1, $\frac{d^2\phi}{dtdI}>0$ and from Appendix 6.1, $\frac{d\widehat{v}}{d\phi}<0$. Hence $\frac{d\widehat{v}}{d\phi}\frac{d^2\phi}{dtdI}<0$ implying that the third term in the numerator on the right hand side (RHS) of (25) is negative.

Consequently, the sign of the last expression in brackets on the RHS of (25) is generally ambiguous. However, when $\frac{d^2\phi}{dt\cdot dI}$ is sufficiently small, the positive terms in the numerator dominate establishing that $\frac{dt^*}{dI}>0$.

6.3 Proof of proposition 3

(i) When the fiscal authority is concerned mainly with social welfare the first term in the numerator on the RHS of (25) is dominated by the second term. Since the cross derivative $\frac{d^2\phi}{dtdI}$ is relatively large this second term is dominated by the product $\frac{d\widehat{v}}{d\phi}\frac{d^2\phi}{dtdI}$ which is negative. Hence $\frac{dt^*}{dI} < 0$.

(ii) Following a substantial amount of messy algebra it can be shown that equation (25) may be rewritten

$$\begin{aligned} -SOC(t^*)\frac{dt^*}{dI} \quad &= \quad -\delta\frac{\alpha \cdot W_{rc}^g \cdot L_0}{(1-\alpha)(1-\phi)^2}\left[1+\frac{2t}{(1-\phi)}\frac{d\phi}{dt}\right]\frac{1}{D_1^2(1-\alpha)}\left\{1-\alpha\left(\frac{R}{(1-t)W_{rc}^g}\right)\right\}\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} \\ &\quad -(1-\delta)\frac{d\phi}{dt}\left[\frac{\alpha L_0^\alpha(1-u)^{\alpha-2}}{(1+L_0)(1-\alpha)}\right]\times\frac{1}{D_1^2(1-\alpha)}\left\{1-\alpha\left(\frac{R}{(1-t)W_{rc}^g}\right)\right\}\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} \\ &\quad +\{Q_2-Q_3\}\frac{d^2\phi}{dt\cdot dI} \end{aligned} \tag{34}$$

where

$$\left\{\begin{array}{l} Q_2 \equiv \delta\frac{d\phi}{dI}t\frac{\alpha\cdot W_{rc}^g\cdot L_0}{(1-\alpha)(1-\phi)^2} \\ Q_3 \equiv (1-\delta)\left\{\Psi\left(\frac{\alpha}{1-\alpha}\right)[1-\phi]^{\frac{2\alpha-1}{1-\alpha}}+\frac{L_0}{(1+L_0)(1-\alpha)}\left[\alpha\left(L_0(1-u)\right)^{\alpha-1}-R\right]\right\} \\ \frac{d^2\phi}{dIdt} = -Q_1\left\{\frac{\alpha}{1-\alpha}\frac{Z_w}{Z_u}+\left[4-\alpha\frac{R}{(1-t)W_{rc}^g}\right]\right\}\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} \end{array}\right\} \tag{35}$$

and where Q_1 and D_1 are bounded combinations of parameters that do not depend on $\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI}$. Equations (34) and (35) imply that $\frac{dt^*}{dI}$ may be rewritten as

$$\frac{dt^*}{dI} = \frac{CTI}{-SOC(t^*)}\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} \tag{36}$$

where CTI is a bounded combination of parameters that does not depend on $\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI}$. Differentiating (15) with respect to I

$$\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} = -\frac{(1-\alpha)(n-1)}{D\left\{\theta(n-1)\frac{\frac{1}{I}+(1-\alpha)^2}{D}+(1-\alpha)\right\}^2 I}\left\{\frac{1}{I}+(1-\alpha)^2 D\right\} < 0 \tag{37}$$

where $D \equiv \alpha + \theta(1-\alpha)$. It is easy to see from (37) that, when $I \to \infty$, $\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI}$ vanishes. Differentiating $\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI}$ again with respect to I and applying some further algebra it can be shown that for $n > 3$

$$\frac{d^2\left(\frac{Z_w}{Z_u}\right)}{dI^2} > 0$$

implying that the absolute value of $\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI}$ converges monotonically to zero as I increases. Hence for $n > 3$ and I sufficiently large $\frac{dt^*}{dI}$ in equation (36) becomes negligible implying that, at sufficiently large levels of CBC, seesaw and/or disciplining effects become negligible.

6.4 Proof of proposition 4

Let W_{rc}^g be the gross real competitive wage rate. The definition of the wage premium implies.

$$\frac{1}{1-t}\frac{W}{P} = \frac{W_{rc}^g}{(1-\phi)}.$$

Using this relation and (17) in (22) total tax revenues can be expressed as

$$T(t) = \frac{t \cdot W_{rc}^g \cdot L_0}{(1-\phi)}\left[1 - \frac{\phi\left(t(I), I\right)}{1-\alpha}\right] \tag{38}$$

where the arguments of the wage premium are written explicitly to highlight the fact that the premium depends on I directly trough the tax base as well as because CBC generally affects the choice of tax wedge. Differentiating totally with respect to I

$$\frac{dT}{dI} = W_{rc}^{g} \cdot L_0 \left\{ \begin{array}{c} \left[\frac{1}{1-\phi}\left(1-\frac{\phi}{1-\alpha}\right)\right]\frac{dt}{dI}+ \\ +\frac{t}{(1-\phi)^2}\left(1-\frac{\phi}{1-\alpha}\right)\left[\frac{\partial\phi}{\partial I}+\frac{\partial\phi}{\partial t}\cdot\frac{dt}{dI}\right]+ \\ -\frac{t}{(1-\phi)(1-\alpha)}\left[\frac{\partial\phi}{\partial I}+\frac{\partial\phi}{\partial t}\cdot\frac{dt}{dI}\right] \end{array} \right\}.$$

Rearranging:

$$\frac{dT}{dI} = \frac{W_{rc}^{g} \cdot L_0}{(1-\phi)(1-\alpha)}\left\{-\frac{\alpha t}{1-\phi}\left(\frac{\partial\phi}{\partial I}\right)+\left[(1-\alpha-\phi)-\frac{\alpha t}{1-\phi}\left(\frac{\partial\phi}{\partial t}\right)\right]\frac{dt}{dI}\right\} \tag{39}$$

The sign of (39) is determined by the sum of terms in curly brackets. The first-term in curly brackets is positive, since $\frac{\partial\phi}{\partial I} < 0$. Note that the second term in curly brackets, $\left[(1-\alpha-\phi)-\frac{\alpha t}{1-\phi}\left(\frac{\partial\phi}{\partial t}\right)\right]$, captures both the direct and the indirect effect (working through ϕ) of the tax-rate t on tax revenues, T. Since, from equation (22) government operates on the efficient side of the Laffer curve, this expression is positive. Hence, in the presence of a seesaw effect (i.e., $\frac{dt}{dI} > 0$) and $\frac{dT}{dI}$ must be positive. In the presence of a disciplining effect (i.e., $\frac{dt}{dI} < 0$) the second term in the curly brackets is negative implying that the sign of $\frac{dT}{dI}$ is generally ambiguous.

7 References

Acemoglu D., S. Johnson, P. Querubin and J. Robinson (2008), "When Does Policy Reform Work?: The Case of Central Bank Independence", **Brookings Papers on Economic Activity**, Spring, 351-418.

Alesina A. and R. Perotti (1997), ”The Welfare State and Competitiveness”, **American Economic Review**, 87, 921-39.

Ardagna S. (2007), "Fiscal Policy in Unionized Labor Markets", **Journal of Economic Dynamics and Control**, 31, 1498-1534.

Belot M. and J. van Ours (2001), ”Unemployment and Labor Market Institutions. An Empirical Analysis”, **Journal of the Japanese and International Economies**, 15, 403-18.

Blanchard O. and S. Fischer (1988), **Lectures on Macroeconomics**, The MIT Press, Cambridge, MA.

Blanchard O. and N. Kiyotaki (1987), "Monopolistic Competition and the Effects of Aggregate Demand", **American Economic Review**, 77, 647-676, September.

Coricelli F., A. Cukierman and A. Dalmazzo, (2006) "Monetary Institutions, Monopolistic Competition, Unionized Labor Markets and Economic Performance", **Scandinavian Journal of Economics**, 108, 39-63, March.

Cukierman A. (2008), "Central Bank Independence and Monetary Policymaking Institutions - Past, Present and Future" , **European Journal of Political Economy**, 24, 722-736.

Cukierman A. and A. Dalmazzo (2006), "Fiscal-Monetary Policy Interactions in the Presence of Unionized Labor Markets", **International Tax and Public Finance**, 13, 411-435.

Cukierman A. and A. Dalmazzo (2007), "Fiscal Policy, Labor Unions and Monetary Institutions: Their Long Run Impact on Unemployment, Inflation and Welfare", CEPR Discussion Paper, No. 6429, August.

Cukierman A. and F. Lippi (1999), "Central Bank Independence, Centralization of Wage Bargaining, Inflation and Unemployment - Theory and Some Evidence", **European Economic Review**, 43, 1395-1434, May.

Daveri F. and G. Tabellini (2000), ”Unemployment, Growth and Taxation in Industrial Countries”, **Economic Policy**, 15, 48-104.

Dixit A. and J. Stiglitz (1977), "Monopolistic Competition and Optimal Product Diversity", **American Economic Review**, 67, June. 297-308.

Dixit A. and L. Lambertini (2003), Interactions of Commitment and Discretion in Monetary and Fiscal Policies”, **American Economic Review**, 93, 1522-1542, December.

Lippi, F. (2003), "Strategic Monetary Policy with Non-Atomistic Wage Setters", **Review of Economic Studies**, *70*, 909-919.

Meltzer A. and S. Richard (1981). "A Rational Theory of the Size of Government", **Journal of Political Economy**, 89, 914-927, October.

Nickell S., L. Nunziata and W. Ochel (2005), "Unemployment in the OECD Countries since the 1960s; What do we Know?", **The Economic Journal**, 115, 1-27, January.

OECD (1997), "Economic Performance and the Structure of Collective Bargaining", **Employment Outlook**, 65-91.

Rogoff K. (1985), "The Optimal Degree of Commitment to a Monetary Target", **Quarterly Journal of Economics**, 100, 1169-1190.

Skott, P. (1997), "Stagflationary Consequences of Prudent Monetary Policy in a Unionized Economy", **Oxford Economic Papers**, 49, 609-622.

Soskice D. and T. Iversen (2000), "The Non Neutrality of Monetary Policy with Large Price or Wage Setters", **Quarterly Journal of Economics,** 115, 265-284.

ANNEX

Technical Appendix - Seesaw and Disciplining Effects of Central Bank Reform on Labor Taxes and Redistribution in the Presence of Labor Unions

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Abstract

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1 The structure of the model

Section 2 of the paper presents the equations that summarize the structure of the model. We report these expressions for convenience.[1] Moreover, for the sake of generality, we allow for the possibility (ruled out in the paper) that unemployment benefits are greater than zero ($B \geq 0$).

Utility of an individual in the economy is given by:

$$U = \left(\frac{C}{\gamma}\right)^{\gamma}\left(\frac{M/P}{1-\gamma}\right)^{1-\gamma} + (1-\lambda)R, \quad \gamma \in (0,1) \tag{1}$$

where

$$C = \left(\int_0^1 C_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \tag{2}$$

and the price level P **is given by**:

$$P = \left(\int_0^1 P_j^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{3}$$

The budget constraint of individual s **is given by**

$$A_{cs} = M_{cs} + \int_0^1 P_j C_{csj} dj, \quad c = EW, UW, E \tag{4}$$

where

$$A_{EWs} = W_{EWs} + \overline{M} + TR_W, \quad A_{UWs} = B + \overline{M} + TR_W, \quad A_{Es} = \Pi_s + \overline{M} + TR_E. \tag{5}$$

Government's revenues come from labor taxes: Total tax revenues are given by $tW_g(1-$

[1] An index of definitions of various constants used throughout this Annex appears at the end of the Annex.

$u)L_0 = \frac{tW}{1-t}(1-u)L_0$. We assume the budget is balanced implying that:

$$\frac{tW}{1-t}(1-u) = Bu + \chi. \tag{6}$$

Real profits of firm j are given by

$$\frac{\Pi_{ij}}{P} = \frac{P_j}{P}C_j^a - \frac{W_i}{P(1-t)}L_{ij}, \quad j \in [0,1] \tag{7}$$

where C_j^a denotes the aggregate demand for variety j. The **production function** is given by:

$$Y_j = L_{ij}^{\alpha}, \quad \alpha < 1. \tag{8}$$

The **central bank chooses the money supply so to minimize the loss-function**:

$$\Gamma = u^2 + I \cdot \pi^2, \;\; I \in [0, \infty). \tag{9}$$

The parameter I is also known as the degree of central bank conservativeness (CBC).

Each union, i, **sets the nominal wage,** W_i **to maximize the expected utility of a representative member:**

$$V_i = (1-u_i) \cdot v_{EW} + u_i \cdot v_{UW} \tag{10}$$

where u_i is the unemployment rate among union i's members and, v_{EW} and v_{UW} are the maximum values of utility of employed and unemployed workers. We assume that the "replacement ratio" $\frac{B}{W_i}$ is constant and equal to $\overline{\beta} < 1$ so that $\frac{B}{P} = \overline{\beta}\frac{W_i}{P}$. We also postulate that

$$v_{EW} \geq v_{UW}. \tag{11}$$

The constraint in (11) is referred to as a "participation constraint".

2 General equilibrium

General equilibrium is characterized by backward induction. First, the price choice of each firm, given nominal wages and the money supply, is derived. Second the choice of the money supply by the Central Bank, given nominal wages, is characterized. Finally, the choice of nominal wage by each union is calculated. Characterization of price setting decisions by each firm requires knowledge of the demand facing each firm. Since this demand depends on the behavior of consumers, we start with the maximization problem of a typical consumer.

2.1 The individual consumer.

Each individual s in class $c = EW, UW, E$ maximizes utility (1) with respect to each consumption variety, C_j, $j \in [0,1]$ and money, M, subject to (4). Omitting without any risk of ambiguity both the individual index s and the class index c, and using the first-order conditions with respect to varieties j and z, we obtain

$$\frac{C_j}{C_z} = \left(\frac{P_j}{P_z}\right)^{-\theta}, \quad \text{for any } (j, z). \tag{12}$$

Solving for C_j from (12), substituting it into (4) and using (3), we obtain

$$C_j = \left(\frac{P_j}{P}\right)^{-\theta} \left(\frac{A - M}{P}\right). \tag{13}$$

Raising both sides of (13) to power $\left(\frac{\theta-1}{\theta}\right)$, integrating over j and raising the result to power $\left(\frac{\theta}{\theta-1}\right)$, one also obtains that $C = \frac{A-M}{P}$. Hence

$$C_j = \left(\frac{P_j}{P}\right)^{-\theta} C. \tag{14}$$

Combining the first-order conditions with respect to variety j and money, M, we obtain $\frac{\gamma C_j^{-\frac{1}{\theta}}}{C^{\frac{\theta-1}{\theta}}} = \frac{(1-\gamma)P_j}{M}$. Using (14) to substitute C_j out from this expression and rearranging

$$C = \frac{\gamma}{1-\gamma}\left(\frac{M}{P}\right). \tag{15}$$

Each individual's demand for variety j, given by (12), can therefore be rewritten as

$$C_j = \left(\frac{P_j}{P}\right)^{-\theta}\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{M}{P}\right). \tag{16}$$

The demand for nominal money is obtained by substituting this expression for C_j in the individual's budget constraint (4) and by rearranging:

$$M = (1-\gamma)A \tag{17}$$

Substituting (17) into (15), the consumption aggregator, C, can be expressed as

$$C = \gamma\frac{A}{P}. \tag{18}$$

Similarly, using (17), we can rewrite (16) as

$$C_j = \gamma\left(\frac{P_j}{P}\right)^{-\theta}\left(\frac{A}{P}\right). \tag{19}$$

Summarizing, the demand of an individual for variety j is given by

$$C_j = \gamma\left(\frac{P_j}{P}\right)^{-\theta}\left(\frac{A_c}{P}\right), \quad j \in [0,1] \tag{20}$$

Aggregating (19) over the mass of individuals in the economy and using equations (4) and (5), total demand for variety j is given by

$$C_j^a = \gamma \left(\frac{P_j}{P}\right)^{-\theta} \frac{A_a}{P} = \gamma \left(\frac{P_j}{P}\right)^{-\theta} \left[(1+L_0)\frac{\overline{M}}{P} + Y\right]. \tag{21}$$

Finally, by substituting (17) and (18) into the utility function (1), one obtains the indirect utility function of the representative individual within each class:

$$v_c = \frac{A_c}{P} + (1-\lambda)R, \quad c = EW, UW, E \tag{22}$$

where $\lambda = 0$ for $c = UW$ and $\lambda = 1$ otherwise. Equation (22) in conjunction with equation (5) implies that the indirect utility functions of each of the three types of individuals (employed, unemployed and capitalists) are given respectively by

$$\begin{aligned} v_{EW} &= \frac{W + \overline{M} + TR_W}{P} \equiv A_{EW}, \\ v_{UW} &= \frac{B + \overline{M} + TR_W}{P} + R \equiv A_{UW} + R, \\ v_E &= \frac{\Pi + \overline{M} + TR_E}{P} \equiv A_E. \end{aligned} \tag{23}$$

2.2 Aggregate equilibrium conditions and the demand facing an individual firm

From (17) aggregate demand for money is equal to:

$$M_a^d = (1-\gamma)\left[A_{EW} + A_{UW} + A_E\right] \equiv (1-\gamma)A_a. \tag{24}$$

Using the definition $\chi \equiv TR_W + \frac{1}{L_o} TR_E$ and government's budget constraint in (6), total demand for money may be rewritten as

$$M_a^d = (1-\gamma)\left[(1+L_0)\overline{M} + (1-u)L_0 W\left(1+\frac{t}{1-t}\right) + \Pi\right] = (1-\gamma)\left[(1+L_0)\overline{M} + PY\right] \quad (25)$$

where the second equality follows from the fact that the last two terms inside the brackets of the middle expression are equal to total nominal income, PY. Money market equilibrium requires that total money supply, given by $M_a^s = (1+L_0)\overline{M}$, is equal to total money demand or

$$(1+L_0)\overline{M} = (1-\gamma)\left[(1+L_0)\overline{M} + PY\right] \quad (26)$$

Rearranging, total output, Y, can be expressed as a function of total real money balances.

$$Y = \frac{\gamma}{1-\gamma}(1+L_0)\frac{\overline{M}}{P}. \quad (27)$$

Equation (27) implies that total demand for variety j can be expressed as a function of real money balances. Inserting (27) into (21) and rearranging, one obtains this demand as a function of real money balances:

$$C_j^a = \left(\frac{\gamma(1+L_0)}{1-\gamma}\right)\left(\frac{P_j}{P}\right)^{-\theta}\left(\frac{\overline{M}}{P}\right). \quad (28)$$

which is **Equation (12)** in the text.

2.3 Price setting by firm j and its derived demand for labor

The real profits of firm j are given by equation (7). Using the production function (8) to obtain the derived demand for labor, L_{ij}^d, the profit function may be expressed as

$$\frac{\Pi_{ij}}{P} = \frac{P_j}{P}C_j^a - \frac{W_i}{P(1-t)}(C_j^a)^{\frac{1}{\alpha}}, \quad j \in [0,1] \quad (29)$$

where demand for variety j (which is, C_j^a) is given by (28). Maximization of this expression subject to (28) and a given value of P, yields the following equilibrium relative price for the good of firm j

$$\frac{P_j}{P} = \Psi_1' \left(\frac{\theta}{\theta - 1}\right)^{\frac{\alpha}{D}} \left(\frac{W_i}{P(1-t)}\right)^{\frac{\alpha}{D}} \left(\frac{\overline{M}}{P}\right)^{\frac{1-\alpha}{D}} \tag{30}$$

where $D \equiv \alpha + \theta(1-\alpha) > 0$ and $\Psi_1' \equiv \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{D}} \left(\frac{\gamma(1+L_0)}{1-\gamma}\right)^{\frac{1-\alpha}{D}} > 0$. Inspection reveals that the equilibrium relative price is an increasing function of the gross real wage and of the level of real money balances. Taking logs of (30) one obtain:

$$p_j - p = \psi + \frac{\alpha}{D}(w_i + \tau - p) + \frac{1-\alpha}{D}(\overline{m} - p) \tag{31}$$

which corresponds to **Equation (13)** in the text, where $\psi \equiv \log\left[\Psi_1' \left(\frac{\theta}{\theta-1}\right)^{\frac{\alpha}{D}}\right]$.

The demand for labor by firm j can be obtained by using the production function in (8) to express the demand for labor in terms of C_j^a and by substituting out $\frac{P_j}{P}$ from the expression for C_j^a by means of the profit maximizing price in (30). This yields

$$L_{ij}^d = \Psi_2 \left(\frac{W_i}{P(1-t)}\right)^{-\frac{\theta}{D}} \left(\frac{\overline{M}}{P}\right)^{\frac{1}{D}} \tag{32}$$

where $\Psi_2 \equiv \left(\frac{\theta}{(\theta-1)\alpha}\right)^{-\frac{\theta}{D}} \left(\frac{\gamma(1+L_0)}{1-\gamma}\right)^{\frac{1}{D}}$. Thus, labor demanded by the firm is a decreasing function of the gross real wage and an increasing function of real money balances.

2.4 Choice of money supply by the central bank

To derive the reaction function of the central bank we have to express inflation and unemployment in terms of the money supply.

To obtain an expression for inflation we raise the equilibrium relative price of each firm in (30) to the power of $(1-\theta)$ and integrate over firms. Using (3) and taking logs of the resulting expression, we obtain:

$$0 = \log \Psi_1 + \frac{1-\alpha}{D}(\overline{m} - p) + \frac{\alpha}{D}(\tau - p) + \frac{1}{1-\theta} \log \left(\int_0^1 W_j^{\frac{\alpha(1-\theta)}{D}} dj \right) \tag{33}$$

where $\tau \equiv -\log(1-t)$, $\overline{m}$ and p denote the natural logarithms of $\overline{M}$ and P respectively, and $\Psi_1 \equiv \left[\frac{\theta}{(\theta-1)\alpha} \left(\frac{\gamma(1+L_0)}{1-\gamma} \right)^{\frac{1-\alpha}{\alpha}} \right]^{\frac{\alpha}{D}}$. The price-level, p, is obtained by rearranging (33). Substituting the resulting expression for p it into the definition of inflation, $\pi \equiv p - p_{-1}$, one obtains:

$$\pi = [D \log \Psi_1] + (1-\alpha)\overline{m} + \alpha\tau + \frac{D}{1-\theta} \log \left(\int_0^1 W_j^{\frac{\alpha(1-\theta)}{D}} dj \right) - p_{-1} \tag{34}$$

To obtain the rate of unemployment, u, we aggregate (32) over firms and take logs. Defining $\log(L_0) \equiv l_0$, we obtain:

$$u \equiv l_0 - l^d = [l_0 - \log \Psi_2] - \frac{1}{D}(\overline{m} - p) - \frac{\theta}{D}(p - \tau) - \log \left(\int_0^1 W_j^{\frac{-\theta}{D}} dj \right) \tag{35}$$

Equation (36) below is obtained by using (34) to substitute out $p = \pi + p_{-1}$ in (35):

$$u = [l_0 - \log \Psi_2 + (1-\theta) \log \Psi_1] - \overline{m} + \tau + \log \left(\widehat{W}_1 \right) - \log \left(\widehat{W}_2 \right) \tag{36}$$

where $\overline{m}$ is the log of $\overline{M}$, W_j is the nominal wage paid by firm j and $\widehat{W}_1 \equiv \int_0^1 W_j^{\frac{\alpha(1-\theta)}{D}} dj$, $\widehat{W}_2 \equiv \int_0^1 W_j^{\frac{-\theta}{D}} dj$. Note that an increase in the tax wedge, τ, raises both inflation and unemployment.

Substituting (34) and (36) into (9), the Central Bank's objective function can be rewritten as:

$$\Gamma = \left\{[l_0 - \log \Psi_2 + (1-\theta)\log \Psi_1] - \overline{m} + \tau + \log\left(\widehat{W}_1\right) - \log\left(\widehat{W}_2\right)\right\}^2 + \\ +I \cdot \left\{[D \log \Psi_1] + (1-\alpha)\overline{m} + \alpha\tau + \frac{D}{1-\theta}\log\left(\widehat{W}_1\right) - p_{-1}\right\}^2 \tag{37}$$

The reaction function of the central bank is obtained by minimizing (37) with respect to $\overline{m}$. The first-order condition for the central bank's problem, $\frac{\partial \Gamma}{\partial \overline{m}} = 0$, yields the following optimal reaction function:

$$\overline{m} = \mu + \left[\frac{1-\alpha(1-\alpha)I}{K}\right] \cdot \tau + \\ + \left[\frac{(1-\theta) - D(1-\alpha)I}{(1-\theta)K}\right] \cdot \ln\left(\widehat{W}_1\right) - \left[\frac{1}{K}\right] \cdot \ln\left(\widehat{W}_2\right) \tag{38}$$

where $K \equiv 1 + (1-\alpha)^2 I > 0$, and the constant μ is given by

$$\mu \equiv \frac{l_0 - \log \Psi_2 + [(1-\theta) - (1-\alpha)ID]\log \Psi_2 + (1-\alpha)Ip_{-1}}{1+(1-\alpha)^2 I}$$

Equation (38) is the central bank's reaction function reported in the text as **Equation (14)**. To illustrate the reaction of the money supply rule in (38) to nominal wages consider the case in which all nominal wages in the economy increase by the factor $\xi > 1$. Staightforward calculations then show that the elasticity of the money supply with respect to ξ (defined as $\frac{d\overline{m}}{d\xi/\xi}$) is equal to $\frac{1-\alpha(1-\alpha)I}{1+(1-\alpha)I^2}$. Thus, an increase in nominal wages induces a contraction in the money supply if and only if the CB is sufficiently conservative or, more formally, if and only if $I > \frac{1}{\alpha(1-\alpha)}$.

2.5 Union i's choice of nominal wage and symmetric equilibrium.

Each monopolistic union i, $i \in \{1, 2, .., n\}$, sets the same nominal wage W_i for all its members so as to maximize the typical member's expected utility, (10). All firms whose workforce is controlled by union i pay the same nominal wage. Each union takes the nominal wages of other unions as given and calculates the impact of its choice on the real wage and the unemployment rate among its members. When doing that the union acts as a Stackelberg leader in the sense that it takes into consideration the implications of its nominal wage choice for the subsequent choice of money supply by the central bank through equation (38) as well as the price setting behavior of firms – which jointly determine the rate of inflation in (34).

The solution for the choice of nominal wage by each union can be decomposed into three steps. First, we let union i maximize (10) with respect to W_i, taking as *given* the wages set by other unions. Second, since all unions are identical, we focus on a symmetric solution in which $W_i = W$, $i = 0, 1..., n$. Third, by using the approximation $\log(1 - x) \cong -x$, we express the symmetric solution in terms of a wage-premium, ϕ. This premium is defined as the percentage (positive) deviation of the equilibrium real wage induced by unions' actions from the competitive wage level. Formally

$$\phi \equiv (w + \tau - p) - (w_c + \tau - p) \equiv w_r^g - w_{rc}^g \tag{39}$$

where w_r^g and w_{rc}^g denote respectively the (logarithms of the) gross equilibrium real wage rate in the presence of unions and its counterpart under a competitive labor market. The competitive gross real wage is defined as the real wage at which the labor market clears so that the rate of unemployment is zero.[2]

(i) Union i chooses W_i so as to maximize a member's expected utility (10). The unem-

[2] An explicit expression is derived in equation (50) below.

ployment rate among members, u_i, is equal to

$$u_i = \log \frac{L_0}{n} - \log L_i^d = l_0 + \log \frac{1}{n} - l_i^d = \tag{40}$$
$$= [l_0 - \log \Psi_2] + \frac{\theta}{D}[w_i - p + \tau] - \frac{1}{D}(\overline{m} - p)$$

where the second equality follows from (32) and the fact that union i controls $\int_{\frac{i-1}{n}}^{\frac{i}{n}} L_0 dj = \frac{L_0}{n}$ workers, and the number of union i members demanded by firms is $L_i^d = \int_{\frac{i-1}{n}}^{\frac{i}{n}} L_{ij}^d dj = \frac{1}{n}\Psi_2 \left(\frac{W_i}{P(1-t)}\right)^{-\frac{\theta}{D}} \left(\frac{\overline{M}}{P}\right)^{\frac{1}{D}}$. Differentiating (10) with respect to W_i, we obtain the union's first-order condition:

$$\left[\frac{1-(1-\overline{\beta})u_i}{P}\right]\left[1 - \frac{W_i}{P}\frac{dP}{dW_i}\right] + \left[R - (1-\overline{\beta})\frac{W_i}{P}\right]\frac{du_i}{dW_i} + \frac{1}{P}\frac{d\overline{M}}{dW_i} - \frac{\overline{M}}{P^2}\frac{dP}{dW_i} = 0 \tag{41}$$

where

$$\frac{du_i}{dW_i} = \frac{1}{D \cdot W_i}\left\{\theta\left[1 - \frac{W_i}{P}\frac{dP}{dW_i}\right] - \left[\frac{W_i}{\overline{M}}\frac{d\overline{M}}{dW_i} - \frac{W_i}{P}\frac{dP}{dW_i}\right]\right\} \tag{42}$$

(ii) Symmetric equilibrium in wages. Symmetry in wages (i.e., $W_i = W$ for all $i's$) implies symmetry in prices (i.e., $P_j = P$), which implies, by (30), that:

$$\frac{\overline{M}}{P} = (\Psi_1)^{\frac{-D}{1-\alpha}}\left(\frac{W}{P(1-t)}\right)^{\frac{-\alpha}{1-\alpha}}. \tag{43}$$

Imposing symmetry in wages also on equation (40) and using (43), it follows that:

$$u_i = u = \Psi_3 + \frac{w + \tau - p}{1-\alpha}, \quad i = 1, ...n \tag{44}$$

where $\Psi_3 \equiv l_0 - \log \Psi_2 + \frac{\log \Psi_1}{1-\alpha}$, and w and p are the logarithms of W and P, respectively. Moreover, by exploiting equation (34) and equation (42) under symmetry one obtains, respectively, that:

$$1-\frac{W_i}{P}\frac{dP}{dW_i}=1-\frac{1}{n\left[1+(1-\alpha)^2 I\right]}\equiv 1-\frac{1}{nK}\equiv Z_w \tag{45}$$

$$\frac{du_i}{dW_i}=\frac{1}{W_i}\frac{\theta(n-1)+(1-\alpha)\left[n(1-\alpha)\theta+\alpha\right]I}{nKD}\equiv\frac{1}{W_i}Z_u. \tag{46}$$

Finally, from the CB reaction function in (38)

$$\frac{d\overline{M}}{dW_i}\frac{W_i}{\overline{M}}=\frac{1-\alpha(1-\alpha)I}{nK}. \tag{47}$$

Using (45), (46), and (47), the first-order condition (41) evaluated in a symmetric equilibrium of the nominal wage setting game can be rewritten as:

$$\left[1-\left(1-\overline{\beta}\right)\left(\Psi_3+\frac{w+\tau-p}{1-\alpha}\right)\right]\cdot Z_w+\left[\frac{P}{W}R-\left(1-\overline{\beta}\right)\right]\cdot Z_u-\frac{\overline{M}}{W}\left(\frac{\alpha(1-\alpha)I}{nK}\right)=0 \tag{48}$$

(iii) Equation (48) implicitly determines the real wage. But since it involves both the level and the logarithm of the real wage it cannot be solved explicitly. For tractability reasons it is convenient to reformulate (48) in terms of the wage-premium, ϕ, defined by (39). This procedure requires three additional sub-steps. First, we express $\frac{\overline{M}}{W}$ in terms of the real wage by rewriting (43) as:

$$\frac{\overline{M}/P}{W/P}=\frac{(\Psi_1)^{\frac{-D}{1-\alpha}}}{(1-t)}\cdot\left(\frac{W}{P(1-t)}\right)^{\frac{-1}{1-\alpha}}. \tag{49}$$

Second, we characterize the level of the *competitive* real wage in terms of the model's parameters. Noting that, under symmetry, the rate of unemployment among union i's members (40) is equal to the economy-wide unemployment rate u, we can set $u=0$ in (44) to determine the logarithm of the gross competitive real wage, $\log\left(W_{rc}^g\right)\equiv w_{rc}^g$. After some algebra, this yields

$$w_{rc}^g\equiv(w_c+\tau-p_c)=-(1-\alpha)\Psi_3=\log\left[\frac{\theta-1}{\theta}\frac{\alpha}{(L_0)^{1-\alpha}}\right]. \tag{50}$$

Third, we consider the identity

$$\frac{P}{W} = \frac{1}{(W/P)_c}\left[\frac{P}{W}\left(\frac{W}{P}\right)_c\right] \tag{51}$$

where $(W/P)_c$ is the net real competitive wage. Letting $x = \frac{P}{W}\left(\frac{W}{P}\right)_c$, using the approximation $x \cong 1 + (\log x)$ and the definition of the wage premium in (39) one gets

$$\frac{P}{W}\left(\frac{W}{P}\right)_c \cong 1 + (w_c - p_c) - (w - p) \equiv 1 - \phi. \tag{52}$$

Substituting (52) into (51) yields

$$\frac{P}{W} \cong \frac{1}{(W/P)_c}\left[1 + (w_c - p_c) - (w - p)\right] \equiv \frac{1}{(W/P)_c}\left[1 - \phi\right]. \tag{53}$$

Substituting (49), (50) and (53) into (48), we obtain that the wage premium, ϕ, is determined implicitly by the following equation:

$$f(\phi) = a\phi + b\left(1 - \phi\right)^{\frac{1}{1-\alpha}} + c = 0 \tag{54}$$

where

$$\begin{aligned}
a &\equiv -\left[Z_w\left(\frac{1-\overline{\beta}}{1-\alpha}\right) + \frac{R \cdot Z_u}{(1-t)W_{rc}^g}\right] \\
b &\equiv -\left(\frac{1-\gamma}{\gamma}\right)\frac{\alpha\left(1-\alpha\right)I}{nK}\frac{J}{(1-t)} \\
c &\equiv Z_w + \frac{R \cdot Z_u}{(1-t)W_{rc}^g} - (1-\overline{\beta})Z_u.
\end{aligned}$$

where W_{rc}^g is the level of the gross real competitive wage, $J \equiv \frac{1}{1+L_0}\left[\frac{(\theta-1)\alpha}{\theta}\right]^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{W_{rc}^g}\right)^{\frac{1}{1-\alpha}} > 0,$

Z_w and Z_u are given by:

$$Z_w \equiv 1 - \frac{1}{n\left[1+(1-\alpha)^2 I\right]} > 0; \quad Z_u \equiv \frac{1}{n}\left[\frac{\theta(n-1)}{\alpha+\theta(1-\alpha)} + \frac{(1-\alpha)I}{1+(1-\alpha)^2 I}\right] > 0. \tag{55}$$

Z_w measures the overall elasticity of the union's net real wage with respect to a change in its nominal wage, and Z_u measures the overall elasticity of the union's unemployment rate with respect to a change in the union's nominal wage.

Taking a first-order Taylor approximation of the function $f(\phi)$ in (54) around $\phi = 0$, setting the approximated value of $f(\phi)$ equal to zero and rearranging, we obtain the following explicit approximation for ϕ:

$$\phi \cong \frac{\left[\frac{Z_w}{Z_u} + \frac{R}{(1-t)W_{rc}^g} - (1-\overline{\beta})\right] - \left(\frac{1-\gamma}{\gamma}\right)\frac{\alpha(1-\alpha)I}{nKZ_u}\left(\frac{J}{1-t}\right)}{\left[\left(\frac{1-\overline{\beta}}{1-\alpha}\right)\frac{Z_w}{Z_u} + \frac{R}{(1-t)W_{rc}^g}\right] - \left(\frac{1-\gamma}{\gamma}\right)\frac{\alpha I}{nKZ_u}\left(\frac{J}{1-t}\right)}. \tag{56}$$

This expression corresponds to **Equation (15)** in the text. The parameter γ is likely to be close to one, implying that the ratio $\frac{1-\gamma}{\gamma}$ is likely to be relatively small.

Finally, the equilibrium values of the economy-wide unemployment rate, u, and of the inflation rate, π, can be expressed as linear functions of the wage-premium ϕ:

$$u = \frac{\phi}{1-\alpha} \tag{57}$$

and

$$\pi = \frac{\phi}{(1-\alpha)^2 I}. \tag{58}$$

These expressions correspond, respectively, to **Equation (17)** and **Equation (18)** in the text. Equation (57) is obtained by noting that, in a symmetric equilibrium, the unemployment rate among union i's members, u_i, is equal to u, and by using (44) and (50) in (40). To obtain (58) note that the first-order condition of the Central Bank problem (see equation (9)) implies that $\frac{\partial u}{\partial m}u + I\frac{\partial \pi}{\partial m}\pi = 0$, where $\frac{\partial u}{\partial m} = -1$ and $\frac{\partial \pi}{\partial m} = (1-\alpha)$ (this can be seen from equations (36) and (34)). Substituting those terms into $\frac{\partial u}{\partial m}u + I\frac{\partial \pi}{\partial m}\pi = 0$ and rearranging one obtains (58).

3 General equilibrium comparative statics with exogenous taxation.

This section summarizes some main comparative static results concerning the impact of fiscal parameters and CBC on the wage premium. Then, one can use equations (57) and (58) to assess the impact on unemployment and inflation. The following results are intermediate steps towards the claim made in the **Lemma** in the text.

3.1 The impact of fiscal parameters and central bank conservativeness

The propositions in this subsection focus on the impact of fiscal policy instruments on the wage premium, employment and inflation.

Result 1: *Provided that $1-\gamma$ is sufficiently small and $\alpha > \overline{\beta} \geq 0$,*[3] *a higher tax wedge, t, is associated with a higher wage premium, lower employment and higher inflation.*

Result 2: *Provided that $1-\gamma$ is sufficiently small, higher unemployment benefits, as represented by a higher replacement ratio, $\overline{\beta}$, are associated with a higher wage premium, lower employment and higher inflation.*

Result 3: *Provided that $1-\gamma$ is sufficiently small and the participation constraint in (11) is satisfied, a more conservative Central Bank (a higher I) is associated with a lower wage*

[3] Table 1 in Ardagna (2007) suggests that the replacement ratio in European countries is about 0.25 and that it varies between a minimum of 0.17 and a maximum of 0.32.

premium, higher employment and lower inflation.

Proofs. The derivations of the impacts on the wage premium are obtained by differentiating the expression for the wage premium in equation (56). The impacts on unemployment and inflation are inferred from equations (57) and (58). In establishing the impact of a given variable s, $s = t, \overline{\beta}, I$, some similar expressions appear. Those expressions are derived once in the early stages of the proof and used subsequently, as appropriate, to complete the proof of each separate proposition. It is convenient to rewrite (56) as

$$\phi \cong \frac{N_1 - \left(\frac{1-\gamma}{\gamma}\right) N_2}{D_1 - \left(\frac{1-\gamma}{\gamma}\right) D_2} \tag{59}$$

where

$$\begin{aligned} N_1 &\equiv \frac{Z_w}{Z_u} + \frac{R}{(1-t)W_{rc}^g} - (1-\overline{\beta}), \quad N_2 \equiv \frac{\alpha\,(1-\alpha)\,I}{nKZ_u}\left(\frac{J}{1-t}\right) \\ D_1 &\equiv \left(\frac{1-\overline{\beta}}{1-\alpha}\right)\frac{Z_w}{Z_u} + \frac{R}{(1-t)W_{rc}^g}, \quad D_2 \equiv \frac{\alpha I}{nKZ_u}\left(\frac{J}{1-t}\right). \end{aligned} \tag{60}$$

Differentiating ϕ with respect to the dummy index, s, and rearranging

$$\frac{d\phi}{ds} \cong \frac{D_1\frac{dN_1}{ds} - N_1\frac{dD_1}{ds} + \frac{1-\gamma}{\gamma}Q}{\left(D_1 - \left(\frac{1-\gamma}{\gamma}\right)D_2\right)^2} \tag{61}$$

where

$$Q \equiv \left(\frac{1-\gamma}{\gamma}D_2 - D_1\right)\frac{dN_2}{ds} - \left(\frac{1-\gamma}{\gamma}N_2 - N_1\right)\frac{dD_2}{ds} + N_2\frac{dD_1}{ds} - D_2\frac{dN_1}{ds} \tag{62}$$

Under the assumption that $\frac{1-\gamma}{\gamma}$ is relatively small an approximate expression for the impact of s on the wage premium is

$$\frac{d\phi}{ds} \cong \frac{D_1\frac{dN_1}{ds} - N_1\frac{dD_1}{ds}}{D_1^2} \tag{63}$$

Proof of Result 1: Differentiating N_1 and D_1 with respect to t

$$\frac{dN_1}{dt} = \frac{dD_1}{dt} = \frac{R}{(1-t)^2 W_{rc}^g}. \tag{64}$$

Substituting (64) into (63) and rearranging

$$\frac{d\phi}{dt} = \frac{R}{D_1^2\,(1-t)^2 W_{rc}^g}\left[\left(\frac{\alpha-\overline{\beta}}{1-\alpha}\right)\frac{Z_w}{Z_u} + 1 - \overline{\beta}\right] \tag{65}$$

Since $\alpha \geq \overline{\beta}$ and $(\alpha, \overline{\beta}) < 1$ this expression is positive. Hence, an increase in the tax wedge, t, raises the wage premium, reduces employment (see (57)) and raises inflation (see (58)). QED

Proof of Result 2: Immediate from inspection of (59) and (60) and by using (57) and (58). QED

Proof of Result 3: Letting $s = I$ in (63) and using the definitions in (60) to evaluate the resulting expression we obtain after some rearrangement

$$\frac{d\phi}{dI} = \frac{1-\overline{\beta}}{D_1^2(1-\alpha)}\left\{(1-\overline{\beta}) - \frac{\alpha-\overline{\beta}}{1-\overline{\beta}}\left(\frac{R}{(1-t)W_{rc}^g}\right)\right\}\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI}. \tag{66}$$

The expression in large curly parenthesis in (66) is positive since the participation constraint implies that $(1-\overline{\beta})(1-t)W_{rc}^g - \frac{\alpha-\overline{\beta}}{1-\overline{\beta}}R > 0$, as the following argument shows. By using equation (23) in (11), one can rewrite

$$(1-t)W_{rc}^g > \overline{\beta}\cdot(1-t)W_{rc}^g + R \tag{67}$$

or

$$\overline{\beta} + \frac{R}{(1-t)W_{rc}^g} < 1. \tag{68}$$

Simple algebra then leads to the following inequality:

$$\overline{\beta}+\frac{\alpha-\overline{\beta}}{1-\overline{\beta}}\left(\frac{R}{(1-t)W_{rc}^{g}}\right)<1. \tag{69}$$

as claimed above. Thus, the sign of $\frac{d\phi}{dI}$ is identical to the sign of $\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI}$, where

$$\frac{Z_w}{Z_u}=\frac{1-\frac{1}{nK}}{\frac{\theta}{D}(1-\frac{1}{n})+\frac{(1-\alpha)I}{K}\frac{1}{n}}. \tag{70}$$

Differentiating (70) with respect to I and rearranging, one obtains

$$\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI}=-\frac{(1-\alpha)(n-1)}{\left\{\theta(n-1)\frac{K}{D}+(1-\alpha)I\right\}^2 D}\left\{\alpha+(1-\alpha)^2 DI\right\} \tag{71}$$

which is unambiguously negative, establishing that $\frac{d\phi}{dI}<0$.

The negative association between u and π on one hand, and I on the other, follows from (57) and 58). QED

3.2 Cross effects between Central Bank conservativeness and the tax wedge

Result 3 above shows that higher Central Bank conservativeness exerts a moderating effect on the wage premium. The following proposition shows that, in the presence of a higher tax wedge t, this moderating effect is weaker.

Result 4: *Provided that $1-\gamma$ is sufficiently small, $\alpha>\overline{\beta}\geq 0$ and the participation constraint in (11) is satisfied, $\frac{d\phi}{dI}$ is smaller in absolute value when the tax wedge, t, is higher.*

Proof: Since, by Result 3 $\frac{d\phi}{dI}$ is negative, the statement can be established by showing that

$$\frac{d}{dt}\left(\frac{d\phi}{dI}\right) = \frac{d^2\phi}{dIdt} > 0 \tag{72}$$

Differentiating (65) with respect to I and rearranging

$$\frac{d^2\phi}{dIdt} = -Q_1\left\{(\alpha-\overline{\beta})\left(\frac{1-\overline{\beta}}{1-\alpha}\right)\frac{Z_w}{Z_u} + \left[(2-\overline{\beta})^2 - (\alpha-\overline{\beta})\frac{R}{(1-t)W_{rc}^g}\right]\right\}\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} \tag{73}$$

where $Q_1 \equiv \frac{R}{D_1^3(1-\alpha)(1-t)^2W_{rc}^g} > 0$. Since $1-\overline{\beta} \leq 1$, condition (69) also implies that $(2-\overline{\beta})^2 - (\alpha-\overline{\beta})\frac{R}{(1-t)W_{rc}^g} > 0$. Since all the remaining terms in curly parenthesis are also positive, the sign of $\frac{d^2\phi}{dIdt}$ is opposite to the sign of $\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI}$. But equation (71) implies that $\frac{d\left(\frac{Z_w}{Z_u}\right)}{dI} < 0$, establishing (72). QED

4 Welfare analysis.

Social welfare is characterized by **average welfare per individual**. To obtain this measure of social welfare we start by evaluating the sum, denoted by $\widehat{v}^a$, of the indirect utility functions of all individuals in the general equilibrium. Aggregate welfare equals the welfare of employed workers plus the welfare of unemployed workers, each weigthed by its appropriate proportion in the labor force, plus the welfare of employers:

$$\widehat{v}^a = ((1-u)\cdot v_{EW} + u\cdot v_{UW})\cdot L_0 + v_E. \tag{74}$$

Substituting (23) into (74) and rearranging

$$\begin{aligned}\widehat{v}^a &= (1+L_0)\frac{\overline{M}}{P} + \frac{Bu+\chi}{P}L_0 + (1-u)\frac{W}{P}L_0 + RuL_0 + \frac{\Pi}{P} \\ &= (1+L_0)\frac{\overline{M}}{P} + \left\{(1-u)\frac{tW}{(1-t)P} + (1-u)\frac{W}{P}\right\}L_0 + \frac{\Pi}{P} + uRL_0 \end{aligned} \tag{75}$$

where the last equality follows from the government budget constraint in (6) and the reader is reminded that $\chi \equiv TR_W + \frac{1}{L_o} TR_E$. Gross real income is equal to the sum of (real) taxes, net wages and profits. Hence

$$Y = \left\{ (1-u) \frac{tW}{(1-t)P} + (1-u) \frac{W}{P} \right\} L_0 + \frac{\Pi}{P}. \tag{76}$$

Using (76) in (75), **average welfare per individual** can be expressed as

$$\widehat{v} \equiv \frac{\widehat{v}^a}{1+L_0} = \frac{\overline{M}}{P} + \frac{Y + RuL_0}{1+L_0}. \tag{77}$$

Equation (77) can be rearranged as follows. First, by substituting (53) into (43) and rearranging, one obtains

$$\frac{\overline{M}}{P} = \Psi \cdot \{1-\phi\}^{\frac{\alpha}{1-\alpha}} \tag{78}$$

where $\Psi \equiv \left(\frac{1-\gamma}{\gamma}\right) \left(\frac{L_0}{1+L_0}\right) \left[\frac{\theta \exp\{\alpha\}}{\alpha(\theta-1)}\right]$. Notice that total output Y is given by

$$Y = \int_0^1 C_j^\alpha dj = \int_0^1 (L_j)^\alpha dj = \int_0^1 (L_0(1-u))^\alpha dj = (L_0)^\alpha \left(1 - \frac{\phi}{1-\alpha}\right)^\alpha \tag{79}$$

The first equality follows from the continuum of differentiated goods on the zero-one interval, the second is obtained by using the production function, the third by specializing to a symmetric equilibrium and the last, by using equation (57). Substituting equations (57), (78) and (79) into (77), **average welfare per individual**, $\widehat{v}$, can be expressed as the following function of the wage premium:

$$\widehat{v}(\phi) = \Psi \cdot [1-\phi]^{\frac{\alpha}{1-\alpha}} + \frac{L_0^\alpha}{1+L_0} \left[1 - \frac{\phi}{1-\alpha}\right]^\alpha + \frac{L_0}{1+L_0} \left(\frac{\phi}{1-\alpha}\right) \cdot R \tag{80}$$

Expression (80) corresponds to **Equation (19)** in the text. In what follows we report all the steps leading to the **Lemma** in the text.

Impact of ϕ on $\widehat{v}(\phi)$. Differentiating $\widehat{v}(\phi)$ in (80) with respect to ϕ and rearranging

$$\frac{d\widehat{v}(\phi)}{d\phi} = -\Psi\frac{\alpha}{1-\alpha}\left[1-\phi\right]^{\frac{2\alpha-1}{1-\alpha}} - \frac{L_0}{(1+L_0)(1-\alpha)}\left[\alpha\left(L_0(1-u)\right)^{\alpha-1} - R\right] \tag{81}$$

From equation (57), it follows that $\phi = (1-\alpha)u$. This implies that $1-\phi > 0$. Thus, since $\Psi > 0$, the first term on the right hand side of equation (81) is negative. Condition (20) in the text, which requires

$$\alpha\left[(1-u)L_0\right]^{\alpha-1} > R, \tag{82}$$

implies that the second term (81) is also negative.[4] This leads to the conclusion that $\frac{d\widehat{v}(\phi)}{d\phi} < 0$

Based on (80) we can find how the tax wedge, the replacement ratio, and CBC affect welfare by totally differentiating $\widehat{v}(\phi)$ with respect to $\left(t, \overline{\beta}, I\right)$. The results are summarized in the following **Lemma,** which appears in the text (Since we assume, in the text, that $\overline{\beta} = 0$ the lemma as formulated there abstracts from the role of the replacement ratio):

Lemma: *For $(1-\gamma)$ sufficiently small, the higher the replacement ratio, $\overline{\beta}$, the lower social welfare.*

If, in addition, the participation constraint and both the conditions $\alpha > \overline{\beta} \geq 0$ and (82) are satisfied, then:

(i) The higher the tax wedge, t, the lower social welfare.

(ii) The higher CBC, I, the higher social welfare.

Proof. Inspection of equation (80) suggests that each of the parameters $t, \overline{\beta}, I$ affects

[4] Condition (82) postulates that some positive employment is socially desirable. It requires that the marginal contribution to output (and therefore consumption) from an additional employed individual has to be greater than the value of leisure foregone by the individual.

welfare **only** through its effect on the wage premium, ϕ. Hence

$$\frac{d\widehat{v}(\phi)}{ds} = \frac{d\widehat{v}(\phi)}{d\phi}\frac{d\phi}{ds}, \quad s = t, \overline{\beta}, I. \tag{83}$$

Since $\frac{d\widehat{v}(\phi)}{d\phi} < 0$, the signs of $\frac{d\widehat{v}(\phi)}{ds}$, $s = t, \overline{\beta}, I$ are the opposite of the signs of $\frac{d\phi}{ds}$. The proof of the proposition follows by noting that the signs of $\frac{d\widehat{v}(\phi)}{ds}$, $s = t, \overline{\beta}, I$ are given, under the appropriate restrictions (carried over to this proposition) in Results 1-3. QED

All the discussion in sections 3 and 4 of the text assumes that the conditions required in the Lemma above are satisfied. In addition the Lemma in the text is specialized to the case $\overline{\beta} = 0$.

5 Definitions of constants

1. θ - elasticity of substitution between any two consumption varieties
2. γ - exponent of consumption aggregator in utiliy
3. $1 - \gamma$ - exponent of real money balances in utiliy
4. α - exponent of labor in production function
5. L_0 - number of workers per firm
6. R - value of leisure when unemployed
7. TR_c, $c = EW, UW, E$ - transfer to individual of class c.
8. EW - index for employed worker
9. UW - index for unemployed worker
10. E - index for employer
11. I - Central Bank conservativeness or independence
12. $\overline{\beta}$ - replacement ratio
13. t - tax wedge

14. $D \equiv \alpha + \theta(1-\alpha)$

15. $K \equiv 1 + (1-\alpha)^2 I$

16. $Z_w \equiv 1 - \frac{1}{n[1+(1-\alpha)^2 I]} > 0;$

17. $Z_u \equiv \frac{1}{n}\left[\frac{\theta(n-1)}{\alpha+\theta(1-\alpha)} + \frac{(1-\alpha)I}{1+(1-\alpha)^2 I}\right]$

18. $\Psi_1' \equiv \left[\frac{1}{\alpha}\left(\frac{\gamma(1+L_0)}{1-\gamma}\right)^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{D}} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{D}}\left(\frac{\gamma(1+L_0)}{1-\gamma}\right)^{\frac{1-\alpha}{D}} > 0$

19. $\Psi_1 \equiv \left[\frac{\theta}{(\theta-1)\alpha}\left(\frac{\gamma(1+L_0)}{1-\gamma}\right)^{\frac{1-\alpha}{\alpha}}\right]^{\frac{\alpha}{D}} = \left(\frac{\theta}{\theta-1}\right)^{\frac{\alpha}{D}}\Psi_1' > 0$

20. $\Psi_2 \equiv \left(\frac{\theta}{(\theta-1)\alpha}\right)^{-\frac{\theta}{D}}\left(\frac{\gamma(1+L_0)}{1-\gamma}\right)^{\frac{1}{D}} > 0.$

21. $\mu \equiv \frac{l_0 - \log\Psi_2 + [(1-\theta)-(1-\alpha)ID]\log\Psi_2 + (1-\alpha)Ip_{-1}}{1+(1-\alpha)^2 I}$

22. $\Psi_3 \equiv l_0 - \log\Psi_2 + \frac{\log\Psi_1}{1-\alpha}$

23. $\Psi \equiv \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{L_0}{1+L_0}\right)\left[\frac{\theta\exp\{\alpha\}}{\alpha(\theta-1)}\right]$

24. $J \equiv \frac{1}{1+L_0}\left[\frac{(\theta-1)\alpha}{\theta}\right]^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{W_{rc}^g}\right)^{\frac{1}{1-\alpha}}$