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When Stolper-Samuelson Does Not Apply: International Trade and Female Labor#

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Abstract

Whenever a country specializes on industries that use female labor intensively, its female labor force participation should increase. This intuition, which bases on the Stolper-Samuleson Theorem, may fail in a three-factor, two-good model. We develop a model where capital, male and female work are distinct factors of production. We follow an established assumption and postulate that capital accumulation closes the gender wage gap. In this setup, the Stolper-Samuleson based intuition fails necessarily: the gender wage gap widens in countries that specialize on sectors intensive in female labor, and vice versa.

1 Introduction

International trade affects relative factor prices. This simple rule applies in particular to male and female wages. Economic intuition based on the well-known Stolper-Samuleson Theorem suggests, moreover, that the gender wage gap shrinks in countries, which specialize on sectors that are intensive in female labor.

We show that under standard assumptions this intuition fails to apply. To make our point, we develop a model where competitive firms convert capital, male labor and female labor into two tradable goods using generic constant returns to scale technologies. We impose one key assumption: a rise in the capital stock closes the gender wage gap, increasing the ratio of female over male wage. Under this assumption, a country's gender wage gap must increase, whenever the country specializes on sectors intensive in female labor, and *vice versa*.

The mechanism that drives this seemingly paradoxical result is best explained in two steps. First, accumulation of capital promotes production in the capital-intensive sector. Therefore, the factor (other than capital), which is used intensively in the capital-intensive sector, experiences an increase in marginal productivity and its factor reward rises. The latter factor must be female to comply with our key assumption about the effects of capital accumulation.

As a second step, we observe that our key assumption requires a relatively strong complementarity between capital and female labor. Now, an economy that specializes in the capital-intensive sector experiences a contraction of the sector intensive in male labor. Consequently, over-proportionally many male workers migrate to the expanding sector. This inflow of labor into the expanding sector depresses its capital-labor ratio and, given the relatively strong complementarity between capital and female labor, the returns to female labor decline and the gender wage gap widens. Finally, given that female labor supply is decreasing in the gender wage gap, female labor shares shrink.

It is worth stressing that we impose minimal restrictions on the setup of our analysis. Regarding production technologies, we merely impose constant returns to scales. In order to address the effects of international trade, our framework features two tradable goods. Capital, female labor and male labor are three distinct factors of production. We consequently deal with the — slightly unconventional — case of a two-good, three-factor model.

Concerning factor supply, we assume that female labor supply decreases in the gender wage gap. Supply of capital and male labor are both inelastic.[1] These assumptions do not only simplify the analysis, they also shut down income effects on female labor shares brought about by proportional changes in wages. Thus, movements in female labor shares can be genuinely attributed to changes in the gender wage gap.

[1]This setting comprises models in which household optimization induces women to split their time between non-market activities as child-rearing and formal employment on the labor market. See Galor and Weil (1996).

While our setup is quite general, the main assumptions of our model require a word of justification. First, we assume that female labor and male labor are imperfect substitutes, which makes them two distinct factors of production. Making this assumption, we refer to Acemoglu, Autor, and Lyle (2004), who utilize the large positive shock to demand for female labor induced by World War II to assess the substitutability between male and female labor. Their estimated elasticity of substitution ranges between 2.5 and 3.5.

Second, we assume strong complementarity between physical capital and female labor so that capital accumulation closes the gender wage gap. Doing so, we follow Goldin (1990), who argues that the rapid accumulation of physical capital was responsible for a dramatic increase in the relative wage of women. Indeed, Goldin writes:

> The labor market's rewards for strength, which made up a large fraction of earnings in the nineteenth century, ought to be minimized by the adoption of machinery, and its rewards for brain power ought to be increased (p. 59).

Galor and Weil (1996) build a theory of the demographic transition formalizing this mechanism. Their approach relies on the intrinsic difference of endowments of brains and brawn by male and female individuals and the relatively high complementarity between capital and mental labor.

Finally, we assume that female labor supply reacts positively to a decrease in the gender wage gap. This link between female labor supply and the gender wage gap is very well established. Blau and Kahn (2007) find that the increase in female labor supply during the period 1980–2000 is due to the decline in the gender wage gap. The authors write that "...married women's real wages increased in both the 1980s and 1990s, and these caused comparable increases in labor supply in each decade, given women's positively sloped labor supply schedules." Other empirical studies find that women's labor supply exhibits a positive elasticity regarding females' wages but a negative cross wage elasticity regarding males' wages (Goldin 1990, Killingsworth 1983, Juhn and Murphy 1997, Blundell and MaCurdy 1999, Devereux 2004). Thus, with the building blocks of our setup well secured, we can claim that our theory is based on accepted and 'standard' assumptions.

In the present study the general analytical framework is of the Heckscher-Ohlin type, as discussed in Helpman and Krugman (1985). Various studies have analyzed generalizations of the standard Heckscher-Ohlin framework. Thus, Chang (1979) considers the case of arbitrary numbers of goods and factors. Inoue (1981) analyzes the Stolper-Samuleson Theorem under variable returns to scale. We know from these studies that the Stolper-Samuelson Theorem does not necessarily generalize to such settings. Our own model is closest to the one in Jones and Easton (1983), who investigate effects of good price changes in a two-good, three-factor model. The authors show that in such a setting and under specific technical conditions, an expansion of a sector may actually imply a decrease in the price of its most intensively used factor. In particular, the sign of the movement of relative factor prices depends on factor intensities *as well as* on the elasticities of factor demand. We add to this literature by showing that under our key assumption the factor shares and demand elasticities automatically fulfill the conditions that imply the seemingly paradoxical result concerning factor (*i.e.*, female labor) shares. In this way, we substitute a set of technical assumptions in Jones and Easton (1983) that involve factor shares and elasticities with a simple, well-known and economically meaningful assumption.[2]

The rest of the paper is organized as follows. Section 2 formalizes our argument and section 3 presents some concluding remarks.

2 The Model

We aim to assess the effects of trade liberalization and international specialization. Since we are interested in domestic effects based on factor price changes that can ultimately be traced down to changes in good prices, it is sufficient to employ the framework of a small open economy.

2.1 The Setup

Regarding the framework of our model we try to be quite general. On the preference side we assume that female labor supply is a decreasing function of the gender wage gap, while supply of male labor is inelastic. Regarding production technologies, we merely assume constant returns to scales in two tradable sectors. Moreover, female labor, male labor and capital are distinct factors of production. We thus deal with the – slightly unconventional – case of a two-good, three-factor model.

2.1.1 Production

Firms transform three different factors K, F and M into two distinct consumption goods Q_1 and Q_2, using the technologies

$$Q_i = G^i(K, F, M) \qquad i = 1, 2. \tag{1}$$

The functions G^i exhibit constant returns to scale, *i.e.*, they are homogeneous of degree one. We assume that the functions G^i are twice continuously differentiable and satisfy

$$G^i_X > 0; \; G^i_{XY} \geq 0 \quad \text{for } X \neq Y; \qquad G^i_{XX} < 0 \tag{2}$$

where subscripts stand for partial derivatives and $X, Y \in \{K, F, M\}$. Finally, the usual Inada conditions are assumed to hold.

[2] Sauré and Zoabi (2011) provide evidence that increases in U.S.-Mexican trade volumes had a negative impact on both, female employment and female relative wage in the U.S.

Sectors differ in their demand for F-type labor relative to M-type labor. Without loss of generality the first sector is relatively more intensive in F, i.e.

$$F_1/\bar{F} > M_1/\bar{M} \tag{3}$$

holds under firm optimization, provided that $Q_1, Q_2 > 0$ is satisfied.

2.1.2 Factors

The variable K stands for physical capital and the variables F and M stand for female and male labor, respectively. In this case, under positive output in both sectors,

$$p_1 G_M^1 = p_2 G_M^2 \qquad \text{and} \qquad p_1 G_F^1 = p_2 G_F^2 \tag{4}$$

must hold, as the Inada conditions imply positive employment of all factors in all industries.

2.1.3 Preferences

Individuals consume the two goods Q_1 and Q_2. Concerning labor supply, we assume that (i) male labor is entirely inelastic and (ii) female labor supply depends only on the ratio of female to male wages, ω.[3] By the second assumption, we can write supply of female over male working hours as

$$R^s(\omega). \tag{5}$$

The superscript s indicates supply and ω stands for the ratio of F-factor price over M-factor price. The function R is assumed to be increasing in ω.

2.2 Inelastic Factor Supply

We begin our analysis by considering an economy with inelastic factor supply. Denoting the vector of factor endowments with $\bar{Z} = (\bar{K}, \bar{F}; \bar{M})^t$, we write $Z = (K_1, F_1, M_1)^t$ for the vector of factors employed in the Q_1-sector.

2.2.1 Factor Allocation

Competitive firms maximize their profits. In terms of factor allocation, such maximization is equivalent to the maximization of total revenues (see Mas-Colell, Whinston, and Green (1995)):

$$\max_Z \; p_1 G^1(Z) + p_2 G^2\left(\bar{Z} - Z\right) \tag{6}$$

[3]This feature may be the outcome of household optimization under home production of a third good. See Galor and Weil (1996) for a corresponding model.

We assume that the solution to (6) is unique and interior and we denoted it by $Z^*(\bar{Z})$.

Further, we introduce the notation w_X for the reward of factor X to formulate the following lemma.

Lemma 1 *Assume prices p_i are constant, then (2) implies*

$$\frac{d}{d\bar{X}} \ln\left(\frac{w_X}{w_Y}\right) < 0 \qquad X, Y = K, M, F \qquad Y \neq X \tag{7}$$

Proof. See Appendix. ■

The lemma states that an increase in aggregate supply of one factor decreases its price relative to the price of all other factors. Thus, the decreasing returns to each factor on the industry level translate, quite intuitively, to decreasing returns to the same factor on the aggregate, economy-wide level.

2.2.2 Effects of Capital Accumulation: the "Goldin-Condition"

Having derived some intuitive results in our setup of a small open economy, we now impose our key assumption on the modelling framework. Specifically, we assume that an increase in the capital stock raises the rewards of F more than that of M:

$$\frac{d}{d\bar{K}} \ln\left(\frac{w_F}{w_M}\right) > 0. \tag{8}$$

Following Goldin (1990), an important branch of the economics of demography have argued that the accelerating capital accumulation has helped to closed the gender wage gap. Referring to her seminal contribution, we will refer to this inequality as the "Goldin-Condition".[4]

It will prove useful to formulate the relations between equilibrium factor allocation and factor prices in terms of demand elasticities. Doing so, however, we need to account for the fact that under technologies with constant return to scale, the good- and factor-prices determine factor demand uniquely only up to a scaling factor. To regain unique factor demand, we thus consider relative factor demand relative to male labor: $k = K/M$ and $f = F/M$. The relation between factor allocation and factor prices is then

$$\begin{pmatrix} \Delta\hat{w}_K \\ \Delta\hat{w}_F \end{pmatrix} \equiv \begin{pmatrix} \hat{w}_K - \hat{w}_M \\ \hat{w}_F - \hat{w}_M \end{pmatrix} = \begin{pmatrix} \alpha_k^K & \alpha_f^K \\ \alpha_k^F & \alpha_f^F \end{pmatrix} \begin{pmatrix} \hat{k} \\ \hat{f} \end{pmatrix} \tag{9}$$

where we have set $\hat{X} = dX/X$ and $\alpha_y^X = \left[d\left(w_X/w_M\right)/dy\right] / \left[\left(w_X/w_M\right)/y\right]$.

[4]Below, we reformulate the "Goldin-Condition" in terms of terms of factor price elasticities.

In the terminology thus defined, the "Goldin-Condition" (8) becomes

$$\alpha_k^F > 0.$$

Moreover, setting $X = K, F$ and $Y = M$ in inequality (7) and using the system (9) translates into the following condition

$$\alpha_x^X < 0 \qquad \text{for} \quad X = K, F.$$

Finally, setting $X = M$ and $Y = K, F$ in inequality (7) leads to[5]

$$-\alpha_k^Y - \alpha_f^Y > 0 \qquad Y = K, F.$$

Together, these conditions imply that the determinant of the 2×2 matrix from (9) is positive[6]

$$D = \alpha_k^K \alpha_f^F - \alpha_f^K \alpha_k^F > 0.$$

We can thus invert the system (9), writing

$$\begin{pmatrix} \hat{k} \\ \hat{f} \end{pmatrix} = \begin{pmatrix} \sigma_K^k & \sigma_F^k \\ \sigma_K^f & \sigma_F^f \end{pmatrix} \begin{pmatrix} \Delta \hat{w}_K \\ \Delta \hat{w}_F \end{pmatrix} \tag{10}$$

According to Cramer's rule, $\sigma_K^k = \alpha_f^F / D$, $\sigma_F^f = \alpha_k^K / D$, $\sigma_F^k = -\alpha_f^K / D$ and $\sigma_K^f = -\alpha_k^F / D$ hold so that the above inequalities on the α_y^X are

$$\sigma_K^f < 0 \quad \text{and} \quad \sigma_Y^y < 0 \quad \text{and} \quad |\sigma_Y^y| > |\sigma_X^y| \quad (Y, X = K, F; \quad X \neq Y). \tag{11}$$

By definition, σ_X^y is the economy-wide elasticity of relative demand with respect to the relative factor price, i.e.

$$\sigma_X^y = \frac{(w_X / w_M)}{y} \frac{dy}{d\,(w_X / w_M)} \qquad X = K, F \quad y = k, f. \tag{12}$$

Hence the first of the inequalities in (11) constitutes the "Goldin-Condition" (8) expressed in terms of factor demand elasticities. The translation into factor price elasticities shows that the "Goldin-Condition" is equivalent to a relatively strong economy-wide complementarity between capital and female labor ($\sigma_K^f < 0$). For a better understanding of the equivalence between (8) and (11), observe that, as more capital K is added to the system, demand for female labor F must rise so as to increase its factor reward relative to M. This rise in demand for female labor F is achieved by a strong complementarity between F and K.

[5] Notice that, by definition of $k = K/M$ and $f = F/M$, a one percent increase in M is equivalent to a simultaneous one percent decrease in k and f.

[6] If $\alpha_f^K < 0$ this statement is true by the inequalities on the α_y^X above. If $\alpha_f^K > 0$, instead, use $-\alpha_k^X - \alpha_f^X > 0$ to verify $\alpha_k^K \alpha_f^F - \alpha_f^K \alpha_k^F > -\alpha_f^K \alpha_f^F - \alpha_f^K \alpha_k^F = \alpha_f^K(-\alpha_f^F + \alpha_k^F) > 0$.

2.2.3 Capital Intensity

Having stated our main assumption concerning wage-raising capital accumulation, we now turn to an important intermediate result, which concerns relative capital intensities of the two sectors.

Lemma 2 *If (2), (3) and (8) hold and $Z^*(\bar{Z})$, is interior, then*

$$K_1/\bar{K} > F_1/\bar{F} \tag{13}$$

Proof. As the solution to (6) is interior, we can write $w_X = G^1_X$ $(X = K, M, F)$. Observe that the uniqueness of the solution to (6), together with homogeneity of degree one of G^i, implies linear independence of Z^* and $\bar{Z} - Z^*$. Further, at constant p_i, an increase of the vector $\bar{Z}$ in the directions Z^* or $\bar{Z} - Z^*$ leaves factor prices unchanged. Thus, factor prices are constant under a marginal change of $\bar{Z}$ in the direction $\xi = Z^* - \gamma\left(\bar{Z} - Z^*\right)$ for all $\gamma \in \mathbb{R}$. The particular choice $\gamma = F_1/(\bar{F} - F_1)$ implies $\xi = (\xi_1, 0, \xi_3)$. Hence,

$$\left(\xi_1 \frac{d}{d\bar{K}} + \xi_3 \frac{d}{d\bar{M}}\right) \ln\left(\frac{G^1_F(z^*)}{G^1_M(z^*)}\right) = 0$$

holds. Therefore, by (7) with $X = M$ and $Y = F$ and (8), we infer that ξ_1 and ξ_3 have opposite sign. By (3) we have

$$\xi_3 = M_1 - (\bar{M} - M_1)F_1/(\bar{F} - F_1) < 0.$$

Therefore, $\xi_1 = K_1 - (\bar{K} - K_1)F_1/(\bar{F} - F_1) > 0$ holds, implying (13). ■

The Lemma shows that Q_1-production is relatively more K-intensive than F-intensive. Together with (3) we then have

$$\frac{K_1}{\bar{K} - K_1} > \frac{F_1}{\bar{F} - F_1} > \frac{M_1}{\bar{M} - M_1} \tag{14}$$

Interestingly, in a two-sector world Goldin's statement implies that the sector, which is intensive in female labor (relative to male labor), is necessarily even more intensive in capital. An intuition for this result obtains from the following considerations. Assume that X_2-production were K-intensive, violating (13), while (3) still implied that X_1-production is F-intensive. Under these assumptions, increases in the capital stock would spur production of the X_2-sector (presuming that an Rybczynski-like effect operates). In terms of factor prices, this advantage to the X_2-sector should benefit mainly the factor it uses most intensively – i.e., male labor. But this is ruled out by assumption (8). – It must be stressed that this explanation provides not more than an intuition. As shown further below, simple arguments relating factor intensities to movement of relative factor prices are not admissible. Instead, an important role is played by factor demand elasticities.

2.2.4 Price Changes

To analyze the effects of changes in goods prices, we adapt and extend the results from Jones and Easton (1983) to our current setting. For the time being, we keep the assumption that factors are inelastically supplied. We start by introducing the notation a_{Xj} for the (equilibrium) input requirement of factor $X = K, F, M$ to produce one unit of good $j = 1, 2$. With this notation, inequalities (14) become

$$\frac{a_{K1}}{a_{K2}} > \frac{a_{F1}}{a_{F2}} > \frac{a_{M1}}{a_{M2}}$$

Multiplying each a_{Xj} by the according factor price w_X and dividing by the respective good prices, p_j, leads to the expenditure share of factor X in sector j, which we denote by $\theta_{Xj} = w_X a_{Xj}/p_j$. Hence, the condition above is equivalent to

$$\frac{\theta_{K1}}{\theta_{K2}} > \frac{\theta_{F1}}{\theta_{F2}} > \frac{\theta_{M1}}{\theta_{M2}} \tag{15}$$

In a competitive economy with constant returns to scale

$$\sum\nolimits_X a_{Xj} w_X = p_j \qquad j = 1, 2 \tag{16}$$

is satisfied as long as both goods are produced in positive quantities.

Being interested in a change in relative price changes we next consider a marginal increase in p_j $(j = 1, 2)$. Differentiating expression on the left of (16) with respect to p_i, we apply the envelope theorem to cost minimization (taking partial derivatives of w_X only), which leads to

$$\sum\nolimits_X \theta_{Xj} \hat{w}_X = \delta_{ij} \qquad j = 1, 2 \tag{17}$$

where $\delta_{ii} = 1$, $\delta_{ij} = 0$ $(j \neq i)$ and $\hat{y} = (dy/dp_1)p_1/y$ as defined above.

Finally, the second line of the system (10) reads

$$\sigma_K^f \left(\hat{w}_K - \hat{w}_M\right) + \sigma_F^f \left(\hat{w}_F - \hat{w}_M\right) = \hat{f}. \tag{18}$$

Combining now (17) and (18) leads to

$$\begin{pmatrix} \theta_{K1} & \theta_{F1} & \theta_{M1} \\ \theta_{K2} & \theta_{F2} & \theta_{M2} \\ \sigma_K^f & \sigma_F^f & -\sigma_K^f - \sigma_F^f \end{pmatrix} \begin{pmatrix} \hat{w}_K \\ \hat{w}_F \\ \hat{w}_M \end{pmatrix} = \begin{pmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{f} \end{pmatrix} \tag{19}$$

We will now analyze a one percentage increase in p_1 at constant factor supply. To this aim, consider the exogenous change $(\hat{p}_1, \hat{p}_2, \hat{f})^t = (1, 0, 0)^t$ in (19). To solve this specific system, denote the determinant of the 3×3 matrix by Δ and use Cramer's Rule to compute (setting $\sigma_M^f \equiv -\sigma_K^f - \sigma_F^f$)

$$\begin{aligned}
\hat{w}_K &= \Delta^{-1} \det \begin{pmatrix} 1 & \theta_{F1} & \theta_{M1} \\ 0 & \theta_{F2} & \theta_{M2} \\ 0 & \sigma_F^f & \sigma_M^f \end{pmatrix} = \Delta^{-1} \left[\sigma_M^f \theta_{F2} - \sigma_F^f \theta_{M2} \right] \\
\hat{w}_F &= \Delta^{-1} \det \begin{pmatrix} \theta_{K1} & 1 & \theta_{M1} \\ \theta_{K2} & 0 & \theta_{M2} \\ \sigma_K^f & 0 & \sigma_M^f \end{pmatrix} = -\Delta^{-1} \left[\sigma_M^f \theta_{K2} - \sigma_K^f \theta_{M2} \right] \\
\hat{w}_M &= \Delta^{-1} \det \begin{pmatrix} \theta_{K1} & \theta_{F1} & 1 \\ \theta_{K2} & \theta_{F2} & 0 \\ \sigma_K^f & \sigma_F^f & 0 \end{pmatrix} = \Delta^{-1} \left[\sigma_F^f \theta_{K2} - \sigma_K^f \theta_{F2} \right]
\end{aligned}$$

Using $\sum_X \theta_{Xj} = 1$ and $\sum_X \sigma_X^f = 0$ (from $\sigma_M^f \equiv -\sigma_K^f - \sigma_F^f$) leads to

$$\begin{aligned}
\hat{w}_K &= -\Delta^{-1} \left[\sigma_K^f \theta_{F2} + \sigma_F^f (1 - \theta_{K2}) \right] \\
\hat{w}_F &= \Delta^{-1} \left[\sigma_F^f \theta_{K2} + \sigma_K^f (1 - \theta_{F2}) \right] \\
\hat{w}_M &= \Delta^{-1} \left[\sigma_F^f \theta_{K2} - \sigma_K^f \theta_{F2} \right]
\end{aligned} \tag{20}$$

Employ again $\sum_X \theta_{Xj} = 1$ and $\sum_X \sigma_X^f = 0$ to compute the determinant Δ:

$$\Delta = \det \begin{pmatrix} \theta_{K1} & 1 & \theta_{M1} \\ \theta_{K2} & 1 & \theta_{M2} \\ \sigma_K^f & 0 & -(\sigma_K^f + \sigma_F^f) \end{pmatrix} = (\theta_{M2} - \theta_{M1}) \sigma_K^K - (\theta_{K1} - \theta_{K2}) (\sigma_K^K + \sigma_F^K) \tag{21}$$

Combining (20) and (21) leads to

$$\frac{d}{dp_1} \ln \left(\frac{w_F}{w_M} \right) = \frac{\sigma_K^f}{(\theta_{F1} - \theta_{F2}) \sigma_K^f - (\theta_{K1} - \theta_{K2}) \sigma_F^f} \tag{22}$$

This identity implies that female relative wages w_F / w_M are decreasing in p_1 if and only if the expression on the right is negative. Now, using (15) together with $\sum_X \theta_{Xj} = 1$, implies $\theta_{K1} > \theta_{K2}$. Since further $\sigma_K^f < 0$ holds by (11), we can state that a necessary and sufficient condition for the expression above to be negative is

$$\frac{\theta_{F1} - \theta_{F2}}{\theta_{K1} - \theta_{K2}} \leq \frac{\sigma_F^f}{\sigma_K^f}$$

Finally, the condition formulated in (11) implies that the expression on the right exceeds one, while the expression on the left falls short of unity, by (15). This proves the following statement.

Proposition 1 *If (8) holds, then*

$$\frac{d}{dp_1} \ln \left(\frac{w_F}{w_M} \right) < 0$$

The proposition shows that, under the "Goldin-Condition" (8) the intuition based on the Stolper-Samuelson effect of a two-good two-factor economy *never* generalizes to F and M in the current setting. Any price increase of the good whose production uses F more intensively than M, decreases the reward for F relative to that of M.

The key condition, of course, is the "Goldin-Condition". In absence of it, the usual Stolper-Samuelson based intuition concerning the interplay of factor intensities, international specialization and relative factor prices may go through.

In a general setting, Jones and Easton (1983) show that in order for the Stolper-Samuelson intuition to fail, a combination of rather technical assumptions needs to be satisfied. With Proposition 1 we have refined the findings of this earlier work by formulating a simple and realistic condition with a clear economic meaning, under which the counter-intuitive effects operate.

2.3 Elastic F-Supply

It is now quick to translate these findings to a framework with elastic F-supply. The ratio of female wage over male wage is G_F^1/G_M^1. Therefore, the supply of female labor over male labor R^s from (5) is a function of relative factor prices $\omega = w_F/w_M = G_F^i/G_M^i$. As we have assumed above, the function $R^s(\omega)$ is increasing (see (5) in subsection 2.1.3).

Turning now to the demand for F, we maintain the assumption that the factors K and M are in inelastic supply. Thus, applying (7), we infer that an increase in $\bar{F}$ lowers the ratio of factor prices $\omega = w_F/w_M$. Inverting this relation implies that demand for $\bar{F}$, denoted by $R^d(\omega)$, is a decreasing function of ω.

The functions R^s and R^d are plotted in Figure 1 as solid lines – R^s as an increasing function and R^d as a decreasing function of ω. The figure also depicts the effects of an increase in p_1, which, by Proposition 1, decreases the ratio w_F/w_M for any given level of $\bar{F}$. This means that the increase of p_1 shifts the R^d-schedule to the left. Since the R^s-schedule is unaffected by the price change, the equilibrium employment of F drops from F^* to F^{**}.

Lemma 3 *If (8) holds, female labor shares drop whenever p_1/p_2 rises.*

The statement of lemma reformulates our main result from Proposition 1. To further translate it to the terminology of trade theory, we spell it out in terms international specialization.

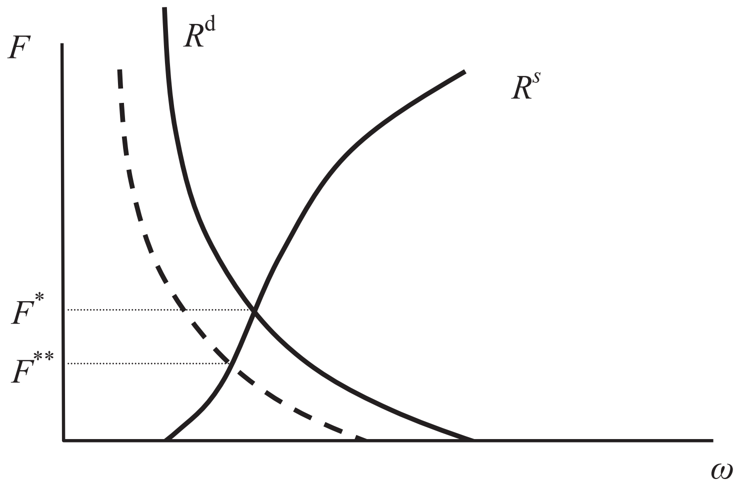

Figure 1: F-type labor - demand and supply.

2.4 International Specialization

Up to this stage, we have considered exogenous price changes and their consequence for a small open economy. In the following paragraphs, we will analyze the patterns of specialization that arise in equilibrium and their effect on female labor force participation. Nevertheless, we refrain from explicitly solving the general equilibrium of a world economy of many countries instead. Specifically, we assume that the world economy consists of a collection of countries of the type described above. We keep being general in terms of technologies and preferences over consumption goods, assuming that each country faces a set of production technologies (1) with which to produce the two consumption goods and individuals have preferences that give rise to F-supply (5). We do not require technologies or preferences to be identical across countries. This implies that international specialization may be driven by differences in technologies, in the per-household capital stocks, in demand for the consumption goods, or by a combination of all.

There are only two key assumptions we make. First, we assume that the "Goldin-Condition" (8) holds for each of the countries. Second, a drop in the relative price of a good is associated with a drop in this country's excess supply of the relevant good. Put differently, the Marshall-Lerner stability conditions are met by assumption.

Now, we say that a country *intensifies specialization in good* X_i if and only if its excess supply of X_i rises. With this terminology, the statement of Lemma 3 can be reformulated as follows: *given that the "Goldin-Condition" (8) holds, female labor shares drop in countries that intensify specialization on sectors intensive in female labor.*

Notice that this statement holds, whether the shift in excess supply and the associated price change originates from a removal of trade barriers, from demand shifts or from (foreign) technological change. Since all effects of trade ultimately operate through a shift in good prices, our result is independent of the actual source of the international pattern of specialization. In this sense, we claim that our finding, which runs counter to the well-established intuition derived from the Stolper-Samuelson Theorem, is very general.

3 Concluding Remarks

This paper has analyzes how expansions and contractions of sectors that use female labor intensively affect aggregate female labor force participation. Whenever a country specializes on industries that use female labor intensively, its female labor force participation should increase. This intuition, which bases on the Stolper-Samuelson Theorem, may fail in a three-factor, two-good model and previous studies have shown that under such settings the Stolper-Samuelson effect does not necessarily generalize. Specifically, Jones and Easton (1983) analyzed the effect of good price changes in a two-good, three-factor model and find that under specific technical conditions, an expansion of a sector may actually imply a decrease in the price of its most intensively used factor. We add to this literature by showing that under the "Goldin-Condition" the factor shares and demand elasticities automatically fulfill the conditions that imply the seemingly paradoxical result concerning factor (*i.e.*, female labor) shares. Thus we provide a realistic example, in which, the Stolper-Samuelson based intuition fails necessarily.

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Appendix

Proof of Lemma 1. (i) Show that Y_1/X_1 and Y_2/X_2 cannot simultaneously increase in $\bar{Z}$ for $Z \neq Y$. Assume that they do. In the first case, where

$$Y_1/X_1 > \left(\bar{Y} - Y_1\right) / \left(\bar{X} - X_1\right) \tag{23}$$

holds one has (dots indicate derivatives w.r.t. $\bar{Z}$)

$$\begin{array}{c} \dot{Y}_1/Y_1 > \dot{X}_1/X_1 \\ -\dot{Y}_1/(\bar{Y} - Y_1) > (\delta_{XZ} - \dot{X}_1)/(\bar{X} - X_1) \end{array} \tag{24}$$

($\delta_{XX} = 1$, $\delta_{XZ} = 0$ if $X \neq Z$). Together with (23) the second inequality in (24) implies

$$\dot{Y}_1/Y_1 < \left(-\delta_{XZ} + \dot{X}_1\right) / X_1$$

contradicting the first inequality in (24). In the second case, were (23) is violated, the above equation implies together with the second inequality of (24)

$$-\dot{Y}_1/Y_1 > (\delta_{XZ} - \dot{X}_1)/X_1$$

again contradicting the first inequality in (24).

(ii) Show that at most one of the four ratios K_i/M_i and F_i/M_i increases in $\bar{M}$ ($i = 1, 2$). By HD0 of ∇G^i the first order conditions to (6) can be written as

$$p_1 \nabla G^1 \begin{pmatrix} K_1/M_1 \\ 1 \\ F_1/M_1 \end{pmatrix} = p_2 \nabla G^2 \begin{pmatrix} K_2/M_2 \\ 1 \\ F_2/M_2 \end{pmatrix}$$

Assume that K_1/M_1 and F_1/M_1 increase in $\bar{M}$. By (i) this implies that K_2/M_2 and F_2/M_2 decrease in $\bar{M}$. Hence, by (2), $p_1 G^1_M$ increases and $p_2 G^2_M$ decreases. This contradicts the optimality condition $p_1 G^1_M = p_2 G^2_M$. Assume, instead, that K_1/M_1 and F_2/M_2 increase in $\bar{M}$, so that K_2/M_2 and F_1/M_1 decrease. Again by (2), $p_1 G^1_F$ increases and $p_2 G^2_F$ decreases, contradicting optimality. Switching indices covers the remaining cases.

(iii) Show $dG^i_M/d\bar{M} < 0$. By (ii), for each $i = 1, 2$, at least one of the ratios K_i/M_i and F_i/K_i decreases in $\bar{M}$. By (2) and

$$G^i_M \left((K_i, F_i, M_i)^t\right) = G^i_M \left((K_i/M_i, F_i/M_i, 1)^t\right)$$

this implies that G^i_M decreases in $\bar{M}$.

(iv) Show $dG^i_F/d\bar{M} > 0$. Applying (i) to K_i/F_i and F_i/K_i shows that the ratio K_i/F_i increases in $\bar{M}$ for exactly one i. Let wlog F_1/K_1 increase and F_2/M_2 decrease in $\bar{M}$. Now, write the first order conditions to (6) as

$$p_1 \nabla G^1 \begin{pmatrix} 1 \\ M_1/K_1 \\ F_1/K_1 \end{pmatrix} = p_2 \nabla G^2 \begin{pmatrix} 1 \\ M_2/K_2 \\ F_2/K_2 \end{pmatrix}$$

By (i), M_i/K_i increases for at least one i. In case that M_1/K_1 increases and M_2/K_2 decreases, (2) implies that G_K^1 increases while G_K^2 decreases, contradicting optimality. If M_1/K_1 decreases and M_2/K_2 increases, then G_F^1 decreases while G_K^2 increases contradicting optimality. Hence, M_i/K_i increase for $i = 1, 2$. Therefore, G_F^2 increases in $\bar{M}$.

(i) - (iv) prove (7) for $X = F$ and $Y = M$; the other cases follow by permutation of the factors. ■