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Minority Schooling and Cultural Transmission

Chaim Fershtman[i], Sagit Bar-Gill[ii]

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[i] Chaim Fershtman - The Eitan Berglas School of Economics, Tel Aviv University. Email: fersht@post.tau.ac.il
[ii] Sagit Bar-Gill - The Eitan Berglas School of Economics, Tel Aviv University. Email: sagitb@post.tau.ac.il

Abstract

Modern-day societies are characterized by large degrees of heterogeneity, as cultural and ethnic minorities are intertwined with traditional majority groups in many countries. These cultural demographics are constantly changing, and receive growing attention from policy makers. The paper studies population dynamics, focusing on minorities' growth rate, in a model of cultural transmission with an endogenous choice of schooling. Two types of schooling are introduced into the cultural transmission framework – public schools and segregated schools. The latter contribute to minorities' socialization efforts, and the former provide stronger labor market skills but do not influence the cultural transmission process. Using this framework we show that assimilation policies aimed at equalizing the two school systems may have the opposite effect of increasing the minority's growth rate.

Minority Schooling and Cultural Transmission

1. Introduction

Modern-day western societies are characterized by a large degree of heterogeneity, as cultural and ethnic minorities often mix with the majority group. The fabric of these societies is ever changing, affected by immigration and assimilation processes, as well as by government-lead actions and regulations aimed at controlling cultural dynamics.

This work provides a framework for analyzing the effect of various assimilation policies on minorities' growth rate, in a model of cultural transmission with an endogenous choice of schooling. Cultural transmission refers to the intergenerational transmission of cultural traits, actively practiced by minority group members in order to preserve their identity.[2] The choice of schooling in our model will allow parents for another means to control their child's identity formation process.

We adopt the population dynamic framework presented by Bisin and Verdier (2001).[3] In this setting children's cultural identity is acquired by a socialization process that depends on parents' direct socialization effort. The level of direct socialization effort is a parental choice, which depends on the social environment as well as on the costs of such effort. This direct socialization is only the first step of the cultural transmission process. When direct socialization fails children are subject to "oblique socialization", whereby they adopt the cultural identity of a role model chosen at random from the entire population. This second phase socialization process affects the optimal parental socialization effort. For large majorities, it implies that parental incentives to engage in costly direct socialization are naturally low. For minority groups, on the other hand, these incentives are high as once parental socialization fails there is a high probability that the child will adopt the majority's cultural identity.

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[2] This behavior is a form of "paternalistic altruism", implying that parents perceive their children's best interests in light of their own preferences, and thus attempt to raise their children to have the same values and characteristics they possess.

[3] This population dynamics is based on earlier work in Anthropology by Cavalli-Sforza and Feldman (1973, 1981) and Boyd and Richardson (1985).

The focus of this paper is on the combination of cultural transmission and an endogenous choice of schooling and how together they affect the cultural mosaic. We thus introduce two types of schooling into the cultural transmission framework – public schools and segregated schools. Public schools in our model provide stronger labor market skills and do not influence the cultural transmission process, whereas segregated schools increase the probability of identity preservation by improved oblique socialization, resulting from the segregation that these types of schools provide.

Our model provides a framework for analyzing various types of governmental assimilation policies. These include: (1) Differential subsidies to minority and majority schools, represented in our model by cost differences between the public and segregated school systems; (2) Curriculum control in schools, represented by the difference in labor market skills acquired in public schools as compared to segregated schools; (3) General assimilation policies in the school system, as represented by the available level of segregation provided by the segregated schools in our model; and (4) Regional or country-wide assimilation policies, as represented by the changes in the cost of direct socialization in our model.

Our analysis shows that assimilation policies may "backfire". Specifically, decreasing the relative cost of public schools through differential subsidies may, in some cases result in a higher minority growth rate. Curriculum control which increases the difference in labor market skills taught in majority and minority schools may also accelerate minority growth, failing to produce a higher rate of assimilation. Furthermore, policies lowering the level of segregation available in minorities' schools and policies that raise the cost of direct parental socialization may also have the opposite effect of increasing the minority's growth rate.[4]

2. Cultural transmission – basic framework

Our model is based on the cultural transmission setting of Bisin and Verdier (2001) into which we will introduce an endogenous choice of schooling. In this

[4] Bar-Gill and Fershtman (2012) consider a cultural transmission setup with endogenous fertility and showed that integration policies may result if higher minority fertility and a larger minority size.

section we present the basic model without schooling, which will be added in Section 3.

Consider a society in which there are two cultural groups, the minority and the majority denoted by *r* and *m*. The fraction of the minority in the population is denoted by $q_r \in [0, 0.5)$ (and $q_m = 1 - q_r$). Consider an overlapping generations model in which each individual lives for two periods. In the first period, the childhood period, cultural identity is determined. In the second period, the adult period, individuals bear children and engage in socialization activities. Each adult chooses the level of socialization effort. We assume for simplicity that all individuals have one child.

We follow the literature and assume paternalistic altruistic preferences such that each individual would like his child to be of his own type. We let $V_i(j)$ be the utility of type *i* individual from having a child of type *j*. This utility includes all the costs of child bearing and rearing, as well as the enjoyment of having a child. We let $\beta = V_i(i) - V_i(j)$ where $\beta > 0$.

Cultural Transmission: There are two types of cultural transmission. The first is **direct socialization** which occurs inside the family, and is an outcome of parental effort. We denote the degree of direct socialization of members of group *i* (where $i \in \{m, r\}$) by τ_i, which represents the probability that a child of a type *i* parent becomes type *i* through the process of direct socialization. We assume that the cost of such direct socialization is $\alpha \tau_i^2 / 2$.

Children whose cultural type has not been determined via the direct socialization process are subject to **oblique socialization** - they randomly choose a role model from the entire population.

Letting $p_i(j)$ be the probability that a child of type *i* parent becomes type *j* individual (where $j \in \{m, r\}, i \neq j$) we have:

$$(1_a) \quad p_i(i) = \tau_i + (1 - \tau_i) q_i$$

$$(1_b) \quad p_i(j) = (1 - \tau_i)(1 - q_i)$$

2.1 The direct socialization effort choice

The utility from having one child and choosing the direct socialization level τ_i is given by:

$$(2) \quad u_i(\tau_i \mid q_i) \equiv [\tau_i + (1-\tau_i)q_i]V_i(i) + (1-\tau_i)(1-q_i)V_i(j) - \frac{1}{2}\alpha\tau_i^{\,2}$$

Maximizing (2) with respect to τ_i yields

$$(3) \quad \tau_i(q_i) = Min\left\{\frac{(1-q_i)(V_i(i)-V_i(j))}{\alpha};1\right\} = Min\left\{\frac{1-q_i}{\alpha}\beta;1\right\}$$

We assume that $\alpha > \beta$, which guarantees an interior solution for the direct socialization effort. As intuition suggests when q_i increases there are lower incentives to invest in direct socialization. This is the cultural substitution effect (see also Bisin and Verdier, 2001).

In the following section we introduce an optimal choice of schooling, which may be used to affect the socialization process.

3. Cultural transmission and the choice of schooling

We consider the choice of schooling by individuals who belong to the minority group. We assume that there are two types of schools; regular "public schools" and "segregated schools". The public schools provide labor market skills L while the segregated schools provide no such skills. This is clearly a simplified assumption as in most cases the segregated schools provide some labor market skills as well. We let φL denote the benefit from such skills, where φ, $\varphi \in [0,1]$, describes the individual skills which determine the benefits that she may gain from attending public schools. We assume that φ is uniformly distributed in the population.[5]

The cost of public schools and segregated schools may differ. There is no general rule that states which one is more expensive, as the cost difference depends on the level and type of school subsidies in each country. We denote by M the difference in cost between the public and segregated schools. This difference is

[5] Individuals in our model must select a schooling option, either public or segregated – a "no school" option does not exist. The option of no schooling or home schooling may be represented by segregated schools with maximal segregation.

positive if public schools are more expensive, but may also be negative when segregated schools are more expensive.

We allow for varying money/education tradeoffs depending on individuals' wealth. We assume that the relative cost of public schools for an individual is given by θM, where $\theta \in [0,1]$ represents the individual's money/education tradeoffs. A low θ represents a wealthier individual with a lower tradeoff between money and paternalistic preferences. We assume that θ is uniformly distributed in the population, and that φ and θ are not correlated.

3.1 Choice of schooling and the minority's size

Public schooling has no effect on cultural transmission. Children that attend such schools are subject to direct socialization or to the standard oblique socialization where they follow a role model at random from the entire population. The segregated schools, as their name suggests, provide an element of segregation. We capture this by assuming that a child that attends a segregated school has a lower probability of following a role model from the majority type. Specifically, we define a segregation parameter $\sigma \in [0,1]$, and assume that the probability of following a role model of the majority type is reduced to $(1-\sigma)q_j$. A segregated school that is characterized by $\sigma = 1$ represents a fully segregated school while for $\sigma = 0$ there is no effective segregation.

We assume that each individual has one child in his adult period, and focus on the individuals' schooling choice. We continue to assume paternalistic preferences; individuals prefer their child to be of their own type such that the utility from children is defined by $V_i(i)$ and $V_i(j)$ where $\beta \equiv V_i(i) - V_i(j)$.

Since individuals value labor market skills there is a tradeoff between the two schooling alternatives – the public schools provide valued labor market skills but offer no segregation, whereas the segregated schools increase the probability that the child maintains his parent's type, but do not teach labor market skills.

The transition probabilities between types depending on their schooling choice are provided by equation (4) where the superscript *P* denotes the public school and superscript *S* denotes the segregated school:

(4a) $p_i^P(i) = \tau_i + (1-\tau_i)q_i \;;\; p_i^P(j) = (1-\tau_i)q_j$

(4b) $p_i^S(i) = \tau_i + (1-\tau_i)\left(1-\left(1-\sigma\right)q_j\right) \;;\; p_i^S(j) = \left(1-\tau_i\right)\left(1-\sigma\right)q_j$

The utility from each schooling choice is thus given by:

(5a) $u_i^P(\tau_i \mid \varphi,\theta) = p_i^P(i)V_i(i) + p_i^P(j)V_i(j) + \varphi L - \theta M - \frac{\alpha}{2}\tau_i^2$

(5b) $u_i^S(\tau_i \mid \varphi,\theta) = p_i^S(i)V_i(i) + p_i^S(j)V_i(j) - \frac{\alpha}{2}\tau_i^2$

The optimal level of direct socialization for each choice of schooling is then derived:

(6a) $\tau^P(q_i) = \frac{\beta}{\alpha}(1-q_i)$

(6b) $\tau^S(q_i) = \frac{\beta}{\alpha}\left(1-\sigma\right)(1-q_i)$

The tradeoff between direct socialization effort and choosing segregated schooling is reflected in equation (6b). As the school segregation level, σ, increases, the optimal direct socialization level decreases reflecting the substitutability between school segregation and the direct socialization effort. When the school is segregated the benefit from direct socialization is clearly lower.

We now examine the effect of sending a child to a segregated school on $p_i^S(i)$, the probability that the minority identity is maintained. The direct effect of segregation is a higher probability of "successful" oblique socialization. However, choosing a segregated school implies a lower direct socialization effort $\tau^S(q_i)$. As a result of these conflicting effects choosing a segregated schooling may not necessarily yields a higher $p_i(i)$. The overall effect of schooling choice on the minority's growth rate is presented in Lemma 1.

Lemma 1 (schooling choice and the minority's growth rate):

For $\frac{\beta}{\alpha} \le \frac{1}{(2-\sigma)(1-q_i)}$ (condition C1) $p^S{}_i(i) \ge p^P{}_i(i)$, and thus choosing a segregated school implies a (weakly) higher $p_i(i)$, and a (weakly) higher growth rate for the minority whenever the fraction of minority members who choose segregated schools increases. When condition C1 does not hold, $p^S{}_i(i) < p^P{}_i(i)$, and the minority's growth rate decreases whenever there are more group members who choose segregated schools.

Proof: See the Appendix.

Lemma 1 implies that there is a critical segregation level $\sigma^*(\alpha, \beta, q_r)$ such that whenever $\sigma \ge \sigma^*(\alpha, \beta, q_r)$ condition C1 holds and sending children to segregated schools implies a higher $p_r(r)$.

Substituting the optimal direct socialization (6) into the utility (5), for each schooling alternative, we derive the utility from each schooling alternative as a function of the minority's fraction in the population:

$$\text{(7a)}\quad U^P(\varphi, \theta, q_r) = \frac{\beta^2}{2\alpha}(1-q_r)^2 + \beta q_r + V_r(m) + \varphi L - \theta M$$

$$\text{(7b)}\quad U^S(\varphi, \theta, \sigma, q_r) = \frac{\beta^2}{2\alpha}(1-\sigma)^2(1-q_r)^2 + \beta\big(1-(1-\sigma)(1-q_r)\big) + V_r(m)$$

The schooling choice is now made by comparing (7a) and (7b). The public school system will be chosen over the segregated one whenever $U^P(\varphi, \theta, q_r) > U^S(\varphi, \theta, \sigma, q_r)$. The partition of the (φ, θ) space according to schooling choice is defined by inequalities (8a,b), also depicted by the solid line in Figure 1. We will refer to this line as the "decision line":

$$\text{(8a)}\quad \theta < \frac{L}{M}\varphi - I, \qquad \text{for } M > 0$$

$$\text{(8b)}\quad \theta > -\frac{L}{|M|}\varphi + I, \qquad \text{for } M < 0$$

Where,

$$
(9)\ ^{6}I = \begin{cases} \dfrac{\beta}{M}\sigma\left(1-q_i\right)\left[1-\dfrac{\beta}{2\alpha}\left(2-\sigma\right)\left(1-q_i\right)\right] & \text{for } M>0 \\ \dfrac{\beta}{\left|M\right|}\sigma\left(1-q_i\right)\left[1-\dfrac{\beta}{2\alpha}\left(2-\sigma\right)\left(1-q_i\right)\right] & \text{for } M<0 \end{cases}
$$

When $M=0$ the schooling choice depends only on skills (Figure 1b). There is a threshold φ^* such that only children with high skills, $\varphi > \varphi^*$, are sent to public schools. When public schools are more expensive, $M>0$, the decision line is upward sloping (figure 1a), and the area below the line is the set of (φ, θ) values for which public schools are chosen. In this case the rich and the talented attend public schools, whereas relatively talented but children from poorer families are sent to the cheaper segregated school system.

When segregated schools are more expensive, $M<0$, the decision line is downward sloping (Figure 1c), and the area above the line is the set of (φ, θ) values for which public schools are chosen. In this case poor members of the minority attend the public school system, even when they are relatively untalented, and rich minority members send their children to public schools only when they are relatively talented.

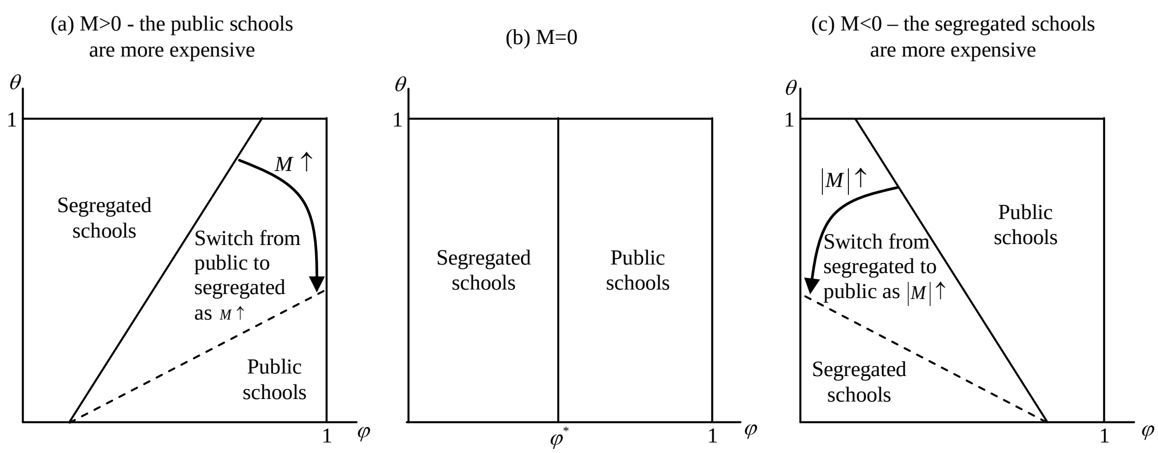

Figure 1 – Schooling choice and the effect of changes in *M*[7]

[6] The expression in square brackets is (weakly) positive, since $\frac{\beta}{2\alpha}\left(2-\sigma\right)\left(1-q_i\right) = 0.5\left(\tau^P + \tau^S\right) \le 1$.

[7] Note that if *I*<1 then (c) is misleading as the lines should be much lower – public schools are chosen by the most of the minority as they are both cheaper and provide labor market skills.

Note that individuals may choose segregated schools even when this lowers $p_i(i)$, i.e. when condition C1 does not hold. In these cases their choice could be motivated by either a low utility from labor market skills when their child is relatively untalented, a relatively high money/education tradeoffs, or by high costs of direct socialization. It is therefore possible that segregated schools are chosen by some minority members, and this decreases the overall growth rate of the minority.

3.2 The effect of changes in relative schooling costs and labor market skills taught on the schooling choice and minority growth rate

Possible policies that encourage cultural assimilation may include increasing the cost of segregated schools, lowering the cost of public schools, or increasing the gap in labor market skills taught in public schools as compared to segregated schools. These policies can be carried out in different ways; e.g., raising the cost of segregated schools may be achieved by limiting government subsidies only to public schools, or by adopting regulations that would make it relatively more costly to open segregated schools. In terms of our model such policies imply a lower $|M|$ or a higher *L*. We start by analyzing the effect of the relative costs of public and segregated schools, *M*, on individuals' choices and the resultant minority growth rate. The effects of changes in *M* are depicted in Figure 1.

(i) When $M > 0$, an increase in the cost of public schools will cause the relatively talented yet poor members of the minority group to switch to the segregated school system. This increases the minority's growth rate if C1 holds.

(ii) When $M < 0$, an increase in the cost of segregated schools will cause the relatively poor and less talented members of the minority to switch to the public school system. This lowers the minority's growth rate if C1 holds.

This is summarized in the following proposition.

Proposition 1 (the effect of decreasing the cost difference between public and segregated schools): An assimilation policy that lowers $|M|$ would result in a higher growth rate of the minority group whenever condition C1 does not hold, that is whenever $\frac{\beta}{\alpha} > \frac{1}{(2-\sigma)(1-q_i)}$.

Proof: see appendix.■

Lowering the cost of public schools, while inducing switching from segregated to public schools, may still increase the minority's growth rate. This is caused through the endogenously determined direct socialization effort, which the switching members choose to increase.

Changes in the magnitude of difference in labor market skills taught in the two schools (L) will also affect the individuals' decisions. This is depicted in Figure 4 and summarized in Proposition 2.

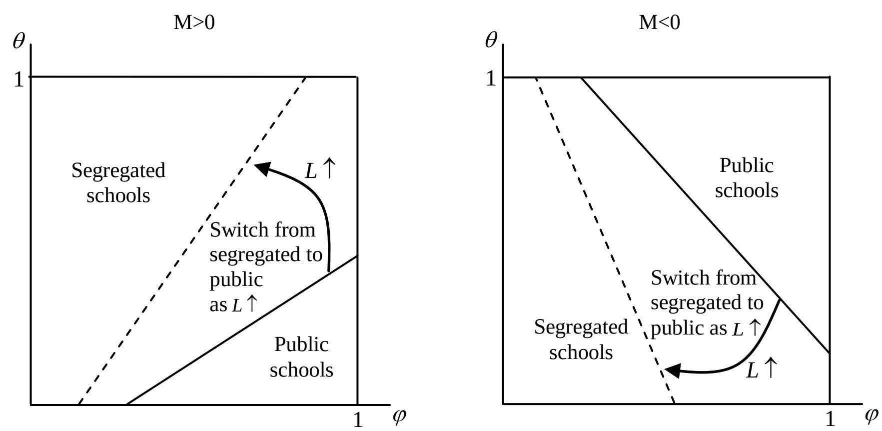

Figure 2 – Effect of changes in *L*

Proposition 2 (effect of an increase of in *L*): An increase (decrease) in L will cause a larger (smaller) fraction of relatively talented members of the minority group to choose the public school system; this will induce a decrease (increase) in the minority's growth rate if and only if C1 holds.

Proof: see appendix.■

As the labor market benefits from public schools increase, minority members become more inclined to choose these schools. The effect is proportional to individual talent, such that relatively talented individuals are more strongly affected. Policies that increase L, while resulting in minority group members' switching from segregated to public schools may still increase the minority's growth rate. This is because under certain conditions the switching group members will increase their direct socialization effort, to compensate for the forgone benefits of segregation.

3.3 Increasing the cost of direct socialization and lowering the available segregation levels

We proceed to examine the effect of changes in the relative size of the minority, the level of segregation in the segregated schools, and the cost of direct socialization (q_i, σ, α). The effect of these parameters is independent of the relative talent and money/education tradeoffs, and thus they have the same effect on minority members regardless of their φ, θ. Graphically, changes in these parameters translate to vertical shifts of the decision line. The effects of these changes are depicted in Figure 3 and summarized in the following proposition.

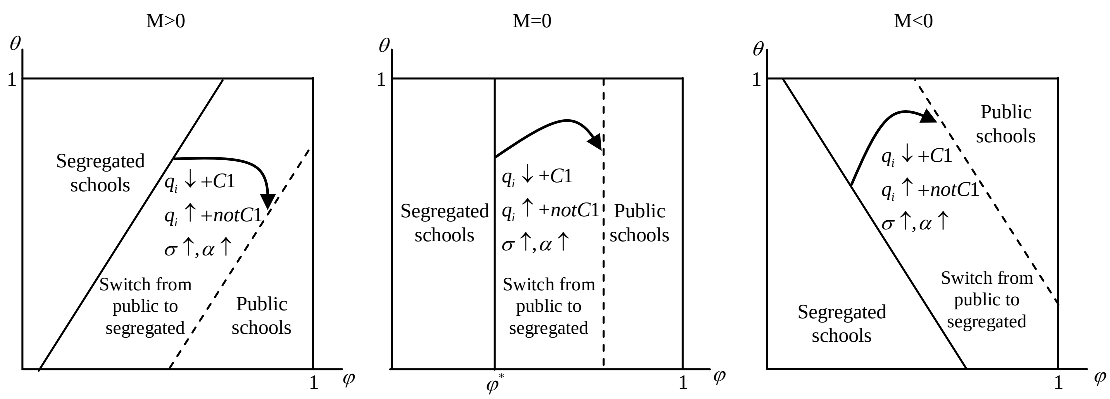

Figure 3 – Effect of changes in q_i, σ, α

Proposition 3 (effect of changes in q_i, σ, α):

(i) An increase (decrease) in q_i will cause a shift towards public (segregated) schools whenever C1 holds, and the opposite shift when C1 does not hold. When C1 holds, the increase (decrease) in q_i will result in a lower (higher) growth rate, and when C1 does not hold it will result in a higher (lower) growth rate for the minority.

(ii) As the level of segregation in the segregated schools increases (decreases), more (fewer) individuals will choose these schools, and the minority's growth rate will increase (decrease) if and only if C1 holds.

(iii) As the cost of direct socialization increases (decreases), more (fewer) individuals will choose the segregated schools, and the minority's growth rate will increase (decrease) if and only if C1 holds.

Proof: see appendix.■

The intuition for parts (ii) and (iii) of the proposition is straight forward. As segregated schools offer a higher level of segregation or as direct socialization becomes more costly, more individuals will choose the segregated school system, due to the improved chances of successful oblique socialization that they offer.

The intuition for part (i) is based on the different effects of increasing q_i. As the minority's relative size increases, the probability of successful oblique socialization increases while the direct socialization effort decreases for both schools. Whenever C1 holds, the socialization effort for both school choices is already small. Therefore further lowering the optimal effort as q_i increases has a smaller effect compared to the benefit of improved oblique socialization. This benefit is more pronounced when choosing public schools that offer no segregation. Thus when C1 holds the utility from public schools is more sensitive to changes in the minority's size, and increasing it will make the public schools more attractive, as the benefit of segregation becomes less needed. When C1 does not hold, the optimal effort is large enough, and the effect of decreasing the optimal effort and relying more on oblique socialization drives the minority towards segregated schools.

3.4 The effect of wealth inequality

We conclude by considering the effect of wealth inequality within the minority group. We model an increase in inequality as a mean preserving spread (MPS) of the money/education tradeoff parameter, θ. The effect of such an MPS is graphed in figure 4. The thick line in the figures is the decision line. This line does not change following the MPS. However, the relative location of individuals with respect to this "decision line" may change. As poor become poorer, and rich become richer, there will be individuals who change their schooling choice. The thin solid line in the figures represents the change for individuals who were previously indifferent between the two schooling choices. These individuals, previously located on the decision line, will now prefer the segregated school when they are relatively poor, and prefer the public school when they are relatively rich, as long as M is positive (and the opposite when M is negative). Results of an increase in wealth inequality are summarized in Proposition 4.

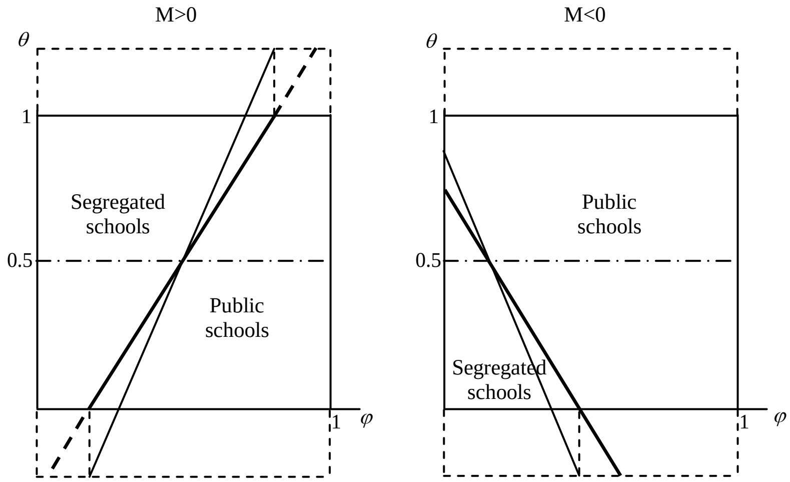

Figure 4 – Effect of an MPS of θ

Proposition 4: effect of an MPS of θ

(i) When $M > 0$: an MPS of θ causes relatively poor and talented individuals to switch from public to segregated schools, while relatively rich and less talented individuals switch from segregated to public schools.

(ii) When $M < 0$: an MPS of θ causes relatively poor and less talented individuals to switch from segregated to public schools, while relatively rich and talented individuals switch from public to segregated schools.

Proof: see appendix.■

4. Concluding Remark

We presented a model of cultural transmission with an endogenous choice of schooling, which offers individuals a means to affect their children's socialization process. We employ this framework to analyze assimilation policies implemented through the two available school systems. Our analysis calls for caution when using such policies, as these may lead to an increase in the minority growth rate.

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Appendix:

Proof of lemma 1: schooling choice and the minority's growth rate –
Denote by $s \in [0,1]$ the fraction of the minority that chooses segregated schools (when $s=0$ everyone chooses public schools and when $s=1$ everyone chooses the segregated school system). The minority's growth rate is given by:

$$q_{i,t+1} = \frac{sq_{i,t}\,p_i^S(i) + (1-s)\,q_{i,t}\,p_i^P(i) + q_{j,t}\,p_j(i)}{q_{i,t} + q_{j,t}}$$

$$\frac{q_{i,t+1}}{q_{i,t}} = sp_i^S(i) + (1-s)\,p_i^P(i) + \left(\frac{1}{q_{i,t}} - 1\right)p_j(i)$$

Therefore $p_i^S(i) > p_i^P(i)$ implies that $\frac{q_{i,t+1}}{q_{i,t}}$ is increasing in s.

We compare $p_i^S(i)$ and $p_i^P(i)$ to derive condition C1, using $\tau^S = \tau^P(1-\sigma)$ from equation (6a,b):

$$p_i^S(i) > p_i^P(i) \Leftrightarrow \tau^P(1-\sigma) + \left[1-\tau^P(1-\sigma)\right]\left[1-(1-\sigma)q_j\right] > \tau^P + \left(1-\tau^P\right)\left(1-q_j\right)$$

$$\Leftrightarrow \tau^P < \frac{1}{2-\sigma} \quad \Leftrightarrow \frac{\beta}{\alpha} < \frac{1}{(2-\sigma)(1-q_i)}$$

Thus, whenever condition C1 holds, an increase in the fraction of minority group members who choose segregated schools increases the minority's overall growth rate.■

Proof of proposition 1: effect of changes in *M* -

(i) For $M>0$: increasing M decreases the slope of the decision line $\frac{L}{M}$, and increases the intersect which is given by $-I$. Both decision lines, before and after the increase, cross the φ axis at the same point. To see this, denote by φ_0 the value that solves $\theta(\varphi_0)=0$, for the decision line before increasing *M*. This implies:

$$0 = \frac{L}{M}\varphi_0 - \frac{\beta}{M}\sigma(1-q)\left[1-\frac{\beta}{2\alpha}(2-\sigma)(1-q)\right]$$

Thus $0 = L\varphi_0 - \beta\sigma(1-q)\left[1-\frac{\beta}{2\alpha}(2-\sigma)(1-q)\right]$, which guarantees that

$$0 = \frac{L}{M+\Delta M}\varphi_0 - \frac{\beta}{M+\Delta M}\sigma(1-q)\left[1-\frac{\beta}{2\alpha}(2-\sigma)(1-q)\right]$$

and therefore $\theta(\varphi_0)=0$ holds after increasing *M* by ΔM.

The minority's growth rate will increase, whenever condition C1 is satisfied.

(ii) For $M < 0$: increasing M decreases the slope of the decision line $\frac{L}{|M|}$ (or increases $-\frac{L}{|M|}$), and decreases the intersect which is given by I. The remainder of the proof is analogous to (i). ■

Proof of proposition 2: effect of changes in *L*.

An increase in L increases the slope of the decision line, and does not change I. Thus more individuals will choose public schools. The remainder follows from lemma 1. ■

Proof of proposition 3: effects of changes in q_i, σ, α.

For $M > 0$, the intersect is given by $-I$. We focus on this case as $M < 0$ is analogous.

(i) Effect of q_i: $\frac{\partial(-I)}{\partial q_i} = -\frac{\beta}{M}\sigma\left[-1+\frac{\beta}{\alpha}(1-q_i)(2-\sigma)\right]$, therefore $\frac{\partial(-I)}{\partial q_i} > 0$ if and only if C1 holds. The remainder follows from lemma 1.

(ii) Effect of σ: $\frac{\partial(-I)}{\partial \sigma} = -\frac{\beta}{M}(1-q_i)\left[1-\frac{\beta}{\alpha}(1-\sigma)(1-q_i)\right] < 0$. The remainder follows from lemma 1.

(iii) Effect of α: $\frac{\partial(-I)}{\partial \alpha} = -\frac{\beta}{M}\sigma(1-q_i)\left[1+\frac{\beta}{2\alpha^2}(2-\sigma)(1-q_i)\right] < 0$. The remainder follows from lemma 1. ■

Proof of proposition 4: effects of an MPS of θ.

(i) For $M > 0$: Following an MPS of θ there exists a group of individuals with $\theta > 0.5$ (relatively poor) and high φ who switch from public to segregated schools. Similarly, there exists a group of individuals with $\theta < 0.5$ (relatively rich) and low φ who switch from segregated to public schools (see figure 4).

(ii) For $M < 0$: Following an MPS of θ there exists a group of individuals with $\theta > 0.5$ (relatively poor) and low φ who switch from segregated to public schools. Similarly, there exists a group of individuals with $\theta < 0.5$ (relatively rich) and high φ who switch from public to segregated schools (see figure 4). ■