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Investment in Schooling and the Marriage Market

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ABSTRACT

We present a model with pre-marital schooling investment, endogenous marital matching and spousal specialization in homework and market production. Investment in schooling raises wages and generates two kinds of returns in our framework: a labor-market return and a marriage-market return because education can affect the intra-marital share of the surplus one can extract from marriage. When the returns to education and household roles are gender neutral, men and women educate in equal proportions and there is pure positive assortative matching in the marriage market. But if men and women have different market returns or household roles, then there may be mixing in equilibrium where some educated individuals marry uneducated spouses and those who educate less extract a relatively larger share of the marital surplus. The existence of large and frictionless marriage markets creates competition among potential spouses, precludes bargaining and generates premarital investments that are efficient. Given that the gender wage gap narrows with the level of education, women's labor-market return from schooling is higher than that of men. Moreover, women's household time obligations have declined over time, raising their marriage-market return from schooling. Combining these two effects, we explain why women now attain higher schooling levels than men

1 Introduction

One of the salient trends in recent decades is the increased investment in education by women and the closing of the gap in schooling between men and women. In several developed countries, women now have more schooling than men. Goldin et al. (2006) discuss the rise in investments in schooling of men and women and show that starting with the 1970 birth cohort; women have attained *higher* college graduation rates than men in the United States. Of the 17 OECD countries with sufficient data, they document that tertiary school enrollment rates of women were below those of men in 13 countries in the mid-1980s, but by 2002, women's college enrollment rates exceeded those of men in 15 countries.

It is well documented that the market return to schooling has risen, especially in the second half of the 20th century. Thus, it is not surprising that women's demand for education has risen. What is puzzling, however, is the differential response of men and women to the changes in the returns to schooling. Women still receive lower wages in the labor market and spend more time at home than men, although these gaps have narrowed over time. Hence, one could think that women should invest in schooling less than men, because education appears to be less useful for women both at home and in the market. In fact, while women considerably increased their investment in education in the last four decades, men hardly responded to the higher returns to schooling since the 1970s, eventually enabling women to overtake them in educational attainment.[1] To help solve this puzzle, we introduce marriage market considerations as an additional motivation for investment in schooling.

Couples sort according to education and, therefore, changes in the aggregate supply of educated individuals have affected who marries whom.[2] Although the proportion of couples among whom the spouses have the same education level remained constant at about 50 percent, there was a reversal of the gaps in education between the spouses. While in the early cohorts, the husband was more educated than his wife for about 30 percent of the couples, in the later cohort of 2005, the wife is more

[1] Since the late-1970s, the returns to schooling have risen steadily for men too. Still, men's college graduation rates have peaked for the cohort born in the mid-1940s (i.e., around the mid-1960s). And, after falling for the cohorts that followed, men's college graduation rates have plateaued for the most recent cohorts at levels slightly below their peak. See Goldin (1997) and Goldin et al. (2006).

[2] This process is driven not only by the mutual gain from marriage and the availability of partners with different levels of schooling in the population, but also by the likelihood of meeting potential mates in school or the workplace (see Lewis and Oppenheimer, 2000). The "meeting technology," which is an essential part of search models of the marriage market, is not considered in the frictionless model that we discuss here.

educated than her husband for about 30 percent of the couples. Among younger couples, more women marry down and fewer women marry up now than they did in the 1970s. In contrast, young men with college degrees are now more likely to marry up (matching with women with advanced degrees) and less likely to marry down (pairing up with women with some college or high school degrees).[3]

Changes in the aggregate supply of educated individuals also affect the division of the gains from marriage. Although they are not directly observable, these changes affect marriage and investment patterns. Presumably, agents who invest in schooling form expectations of the potential returns for schooling within marriage and, together with the expected returns in the labor market, take them into account in making their education decisions. The gains from schooling within marriage strongly depend on the decisions of others to acquire schooling. However, since much of schooling happens before marriage, partners cannot coordinate their investments. Rather, men and women make their choices separately, based on the anticipation of marrying a "suitable" educated spouse with whom schooling investments are expected to generate higher returns.

The purpose of this paper is to provide a simple general equilibrium framework for the *joint* determination of pre-marital schooling and marriage patterns of men and women. Unlike other attributes such as race and ethnic background, schooling is an *acquired* trait and within some limits subject to choice. In our model, the returns to pre-marital investments can be decomposed into two parts: First, higher education raises ones wage rate and increases the payoff from time on the job (the *labor-market* return). Second, it can improve the intra-marital share of the surplus one can extract from marriage (the *marriage-market* return). Educational attainment influences intra-marital spousal allocations directly (due to the fact that education raises household income) and indirectly (by raising the prospects of marriage with an educated spouse and also changing the spousal roles within marriage).

The basic ingredients of our model are as follows. We consider a frictionless marriage market in which, conditional on the predetermined spousal schooling levels, the assignments are stable. That is, there are no men or women (married or single)

[3] Goldin (1997) compares female college graduates of several birth cohorts. Women who graduated from college during the early part of the twentieth century (1900 -1920) had sacrificed family to pursue a career; 50 percent of them had no children by age 35-44 and 30 percent of them never married. Women of later cohorts were better able to mix family life with a career, but they altered the timing of their career and family-life choices. Those who graduated from college prior 1965 gradually raised their marriage and fertility rates and they typically had children before entering the labor market. In contrast, women who graduated from college after 1965 had lower marriage and fertility rates and they tended to start to work before having children.

who wish to form a new union and there are no men or women who are married but wish to be single. We then assume transferable utility between the spouses to characterize the stable assignment. We further assume that men and women can be divided into schooling classes (high and low) and the interactions between married spouses depend only on their education classes. In particular, although men and women have idiosyncratic preferences for marriage and investment in schooling, they all have the same ranking over spouses of the opposite sex which depend only on their schooling. Thus, every educated man (woman) and every uneducated man (woman) has a perfect substitute. The absence of rents allows us to pin down the shares of the marital surplus of men and women in each schooling class based on competition alone, without resorting to bargaining. These shares, together with the known returns as singles, are sufficient to determine the investments in schooling of men and women.

When the market return to education and household roles are gender neutral, men and women acquire education in equal proportions and, under the assumption that the schooling of the two spouses complement each other, a strictly positive assortative matching arises in the marriage markets. That is, educated men marry only educated women and uneducated men marry only uneducated women. Regardless of whether they are educated or not, married couples in such equilibrium share their marital surplus equally. But if the market returns or household roles are not gender neutral, then the shares within marriage adjust and, for a sufficiently large gender gap, a mixed equilibrium arises where some educated individuals of the gender that has a higher overall return to schooling marry "down" with uneducated spouses. In such an asymmetric equilibrium, the gender-education class that is in short supply obtains the upper bound on the return from schooling in marriage, which is the marginal contribution of an educated man (woman) to an educated spouse.

We use this simple model to explain why women may *overtake* men in schooling despite their lower market wage rate and higher amount of housework compared with men. Our explanation relies on two phenomena. First, we hypothesize that the increase in the levels of schooling investment by women to and above the levels of men is a consequence of the higher return that women receive for schooling, reflecting lower labor market "discrimination" at higher levels of schooling.[4] The essence of the argument we make is that education can serve as a means to escape discrimination.[5]

[4] Mincer and Polachek (1974) and Weiss and Gronau (1981) provided explanations for the main patterns of the gender wage gap even in the absence of any discrimination based on lower investments on the job resulting from expected interruptions in participation. At that time, women also acquired less schooling. The current reversal in the schooling gender gap poses a challenge to this approach.

[5] Discrimination here simply means that conditioned on their level of schooling, women expect lower wages than men during their work careers. This outcome can result from a variety of causes

Therefore, although women today still receive lower wages and spend more time in the household than men, women may acquire *more* schooling than men because of their higher returns to schooling in the labor market. In the past, the higher market return was washed out by the lower returns for schooling that women received within marriage.

The second factor that contributes to women's education overtaking men is the weak response by men to increased labor-market returns to schooling. In terms of our model, a possible reason for this phenomenon is that the returns to education within marriage have risen for women substantially more than they did for men. In particular, our model suggests that, if women become more educated than men, some of them have to marry down to match with uneducated men. Due to spousal competition in the marriage markets, this can raise the marital surplus of uneducated men in all marriages. Consequently, uneducated women who can only marry uneducated men may suffer a reduction in their marital surplus. Thus, men's returns from schooling within marriage declines (or does not rise much) while women's returns rises. In this manner, marriage market considerations can explain the divergent patterns of educational attainments of both women and men.

2 Background

We begin with a brief description of the main facts that we wish to address. Figure 1 describes the time trends in levels of school completion for men and women, aged 30 to 40, in the United States. As seen, the proportions of women with some college education, college completion and advanced degrees (M.A., Ph.D.) have increased much faster than the corresponding proportions for men. By 2003, women had overtaken men in all of these three categories. Goldin et al. (2006) present trends for college graduation by gender and show that, starting with the 1970 birth cohort, women have attained higher college graduation rates than men.

As seen in Figure 2, couples sort positively according to schooling and for about 50 percent of the married couples, the husband and wife have the same level of schooling (when broadly classified into 5 groups). However, the changes in the aggregate number of educated men and women had a marked influence on who marries whom; 30 percent of the couples in the earlier cohorts had husbands who were more educated,

including self-selection of women into part-time jobs with lower wages and weaker incentives for women to acquire, or for employers to provide, on the job training.

whereas 30 percent of the couples in recent cohorts had wives with higher levels of educational attainment.

Figure 3.a shows the distribution of spousal education levels by husbands and wives with different level of schooling for young couples aged 30 to 40 in the period $1970-1979$. Figure 3.b displays the same distribution for the period $1996-2005$. At low schooling levels, each gender mainly marries with individuals of the opposite sex with similar education levels during both time periods. However, at higher levels of schooling, the two time periods display very different marriage patterns. For instance, among the most educated segments, we see that a woman with an advanced degree had a 64 percent chance of marrying a man with an advanced degree in the 1970s, while this likelihood had declined to 46 percent in the late 1990s and early 2000s. That is, because of the increase in the proportion of highly-educated women, some had to marry down and match with less-educated men, mainly college graduates. On the other hand, highly-educated men benefited from the increase in the number of educated women: the probability that a man with an advanced degree married a woman with an advanced degree rose from 19 percent in the 1970s to 37 percent between $1996-2005$, although it was still more likely for a highly-educated man to marry down than for a highly-educated woman to do so.

For intermediate education categories, we see a large increase in the proportion of marriages in which both the husband and wife have some college education, reflecting the sharp increase in the number of women with some college training. In addition, husbands with some college education have replaced wives with high school degrees with wives who have college degrees, while wives with some college education replaced men who have college or higher degrees with men with high school degrees. Specifically, the proportion of men with some college education who were married to women with high school degrees has declined from 53 percent in the period $1970-79$ to 25 percent in $1996-2005$, while the proportion of men with some college education married to women with college degrees has risen from 10 percent to 20 percent over the same time interval. In contrast, the proportion of women with some college education married to men with high school degrees has risen from 23 percent in $1970-79$ to 29 percent in $1996-2005$, while the proportion of women married to men with college degrees has declined from 26 percent to 18 percent.

Among the possible reasons for the changes in investment patterns of men and women are the changes in their market return from schooling and the household work that they perform. Figure 4 presents the time trends in the hourly wage differentials by schooling for men and women in the United States adjusted for potential work

experience and with Heckman correction for participation.[6] As seen, women receive a higher increase in wages than men when they acquire college or advanced degrees. The gender differences in the return for advanced degrees narrows over time. However, when we aggregate college and advanced degrees, the female advantage of remains stable over time, because the proportion of college graduates that acquire advanced degrees rises faster for women.[7] If we restrict the sample to "full time, full year" workers (who reported at least 35 hours a week and 51 weeks of work last year), the female advantage in the returns to schooling in the CPS is reduced and becomes insignificant at the most recent years, indicating that additional schooling favors women more than men mainly in terms of access (or commitment) to full-time jobs.[8]

Table 1 brings evidence on time allocation of married men and women (aged 20-59) in the US who have young children for the years 1975 and 2003. It is seen that women spend a substantially larger amount of time than men in non market work.[9] However, over time, the gap declined as women have increased their market work and reduced their non-market work, while men have reduced their market work and increased their non-market work. In 1975, married women with children 1-5 years old spent about 80 percent of their total working hours on non-market work, while this percentage for men was only 20 percent. By 2003, married women with young children had reduced their share of non-market work to 68 percent while married men had increased it to 32 percent. In 2003, the total amount of work performed by married men and women with children is quite similar, 9.35 and 8.72 hours per day, respectively.[10]

[6] These figures are for salaried whites workers aged 25-54. Hourly wage observations of less than 2 dollars or more than 200 dollars are considered as missing. The reported coefficients are for the school level dummies in a Mincerian wage equation which is quadratic in (potential) years of experience and includes region dummies. Additional variables that explain selection into work (i.e. non missing wage) are children and marital status.

[7] The gender differences for advanced degree or college and advanced degrees combined are statistically significant in most years, with few exceptions at the end of the sample period.

[8] A familiar conceptual issue is which variables should be held fixed when one considers the impact of schooling. It seems that for the analysis of schooling investment, variables such as hours of work and job characteristics, and perhaps even experience, should be allowed to vary. Mulligan and Rubinstein (2006) show that, in the CPS, the gender wage gap declines with schooling if one compares men and women who work full time without controlling for experience. Dougherty (2005) and O'Neill and O'Neill (2006) show, using NLSY data, that the gender differences in the impact of schooling are eliminated when detailed employment and occupational characteristics are added. Gronau (1998) shows, using PSID data, that education strongly affects access to on-the-job-training opportunities, but the difference between men and women in this regard is not significant.

[9] Similar findings for the US are reported by Aguiar and Hurst (2006). Browning et al. (2006, chapter 1) and Borda et al. (2006) present similar results for several countries.

[10] Borda et al. (2006) show that the similarity of total work performed by married men and women is a common phenomenon that holds in several countries and over different time periods.

3 The Basic Model

We begin with a benchmark model in which men and women are completely symmetric in their preferences and opportunities. However, by investing in schooling, agents can influence their marriage prospects and labor market opportunities. Competition over mates determines the assignment (i.e., who marries whom) and the shares in the marital surplus of men and women with different levels of schooling, depending on the aggregate number of women and men that acquire schooling. In turn, these shares together with the known market wages guide the individual decisions to invest in schooling and to marry. We investigate the rational-expectations equilibrium that arises under such circumstances.

3.1 Definitions

When man i and woman j form a union, they generate some aggregate *material output* ζ_{ij} that they can divide between them and the utility of each partner is linear in the share he\she receives (transferable utility). Man i alone can produce ζ_{i0} and woman j alone can produce ζ_{0j}. The *material surplus* of the marriage is defined as

$$z_{ij} = \zeta_{ij} - \zeta_{i0} - \zeta_{0j}. \tag{1}$$

In addition, there are emotional gains from marriage and the total *marital surplus* generated by a marriage of man i and woman j is

$$s_{ij} = z_{ij} + \theta_i + \theta_j, \tag{2}$$

where θ_i and θ_j represent the non-economic gains of man i and woman j from their marriage.

3.2 Assumptions

There are two equally large populations of men and women to be matched.[11] Individuals live for two periods. Each person can choose whether to acquire schooling or not and whether and whom to marry. Investment takes place in the first period of life and marriage in the second period. Investment in schooling is lumpy and takes one period so that a person who invests in schooling works only in the second period, while a person who does not invest works in both periods. To simplify, we assume

[11] We address the impact of the sex ratio in a separate section below.

no credit markets.[12] All individuals with the same schooling and of the same gender earn the same wage rate, but wages may differ by gender. We denote the wage of educated men by w_2^m and the wage of uneducated men by w_1^m, where $w_2^m > w_1^m$. The wage of educated women is denoted by w_2^w and that of uneducated women by w_1^w, where $w_2^w > w_1^w$. Market wages are taken as exogenous and we do not attempt to analyze here the feedbacks from the marriage market and investments in schooling to the labor market. We shall discuss, however, different wage structures.

We denote a particular man by i and a particular woman by j. We represent the schooling level (class) of man i by $I(i)$ where $I(i) = 1$ if i is uneducated and $I(i) = 2$ if he is educated. Similarly, we denote the class of woman j by $J(j)$ where $J(j) = 1$ if j is uneducated and $J(j) = 2$ if she is educated. An important simplifying assumption is that the material surplus generated by a marriage of man i and woman j depends only on the class to which they belong. That is,

$$s_{ij} = z_{I(i)J(j)} + \theta_i + \theta_j. \tag{3}$$

We assume that the schooling levels of married partners complement each other so that

$$z_{11} + z_{22} > z_{12} + z_{21}. \tag{4}$$

Except for special cases associated with the presence of children, we assume that the surplus rises with the schooling of both partners. When men and women are viewed symmetrically, we also have $z_{12} = z_{21}$.

The per-period material utilities of man i and woman j as singles also depend on their class, that is $\zeta_{i0} = \zeta_{I(i)0}$ and $\zeta_{0j} = \zeta_{0J(j)}$ and are assumed to increase in $I(i)$ and $J(j)$. Thus, a more educated person has a higher utility as a single. Men and women who acquire no schooling and never marry have life time utilities of $2\zeta_{10}$ and $2\zeta_{01}$, respectively. A person that invests in schooling must give up the first period utility and, if he\she remains single, the life time utilities are ζ_{20} for men and ζ_{02} for women. Thus, the (absolute) return from schooling for never married men and women are $R^m = \zeta_{20} - 2\zeta_{10}$ and $R^w = \zeta_{02} - 2\zeta_{01}$, respectively.[13] The return to schooling of never married individuals depends only on their own market wages and we shall refer to it as the *labor-market* return. However, investment in schooling raises the probability of marriage and those who marry have an additional return from schooling investment in the form of increased share in the material surplus,

[12] Allowing borrowing and lending raises issues such as whether or not one can borrow based on the income of the future spouse and enter marriage in debt (see Browning et al., in progress, ch. 7).

[13] Because we assume away the credit market, the rate of return from schooling investment depends on consumption decisions and is in utility terms.

which we shall refer to as the *marriage-market return* to schooling. In addition to the returns in the labor market or marriage, investment in schooling is associated with idiosyncratic costs (benefits) denoted by μ_i for men and μ_j for women.

The idiosyncratic preference parameters are assumed to be independent of each other and across individuals. We denote the distributions of θ and μ by $F(\theta)$ and $G(\mu)$ and assume that these distributions are symmetric around their zero means. This specification is rather restrictive because one might expect some correlations between the taste parameters and the observable attributes. For instance, individuals that have a low cost of schooling may also have a high earning capacity and individuals may derive different benefits from marriage depending on the observed quality of their spouses. One may also expect a correlation between the emotional valuations of the marriage by the two spouses. Thus, the model is very basic and intended mainly as an illustration of the possible feedbacks between the marriage market and investment in schooling.

3.3 The Marriage Market

Any *stable* assignment of men to women must maximize the *aggregate surplus* over all possible assignments (Shapley and Shubik, 1972).[14] The dual of this linear programming problem posits the existence of non-negative shadow prices associated with the constraints of the primal that each person can be either single or married to one spouse. We denote the shadow price of woman j by u_j and the shadow price of man i by v_i. The complementarity slackness conditions require that

$$z_{I(i)J(j)} + \theta_i + \theta_j \leq v_i + u_j, \tag{5}$$

with equality if i and j are married and inequality otherwise.

The complementarity slackness conditions are equivalent to

$$\begin{aligned} v_i &= Max\{\underset{j}{Max}[z_{I(i)J(j)} + \theta_i + \theta_j - u_j], 0\} \\ u_j &= Max\{\underset{i}{Max}[z_{I(i)J(j)} + \theta_i + \theta_j - v_i], 0\}, \end{aligned} \tag{6}$$

which means that the assignment problem can be *decentralized*. That is, given the shadow prices u_j and v_i, each agent marries a spouse that yields him\her the highest

[14] Note that the maximization of the aggregate *surplus* is equivalent to the maximization of aggregate *output* because the utilities as singles are independent of the assignment.

share in the marital surplus. We can then define $\bar{u}_j = u_j + \zeta_{0j}$ and $\bar{v}_i = v_i + \zeta_{i0}$ as the *reservation utility* levels that woman j and man i require to participate in *any marriage*. In equilibrium, a stable assignment is attained and each married person receives his\her reservation utility, while each single man receives ζ_{i0} and each single woman receives ζ_{0j}.

Our specification imposes a restrictive but convenient structure in which the interactions between agents depend on their group affiliation only, i.e., their levels of schooling. Assuming that, in equilibrium, at least one person in each class marries, the endogenously-determined shadow prices of man i in $I(i)$ and married j in $J(j)$ can be written in the form,

$$v_i = Max(V_{I(i)} + \theta_i, 0) \qquad \text{and} \qquad u_j = Max(U_{J(j)} + \theta_j, 0) \tag{7}$$

where

$$V_I = \underset{J}{Max}[z_{IJ} - U_J] \qquad \text{and} \qquad U_J = \underset{I}{Max}[z_{IJ} - V_I] \tag{8}$$

are the *shares* that the partners receive from the *material surplus* of the marriage (not accounting for the idiosyncratic effects θ_i and θ_j). All agents of a given type receive the *same* share of the material surplus z_{IJ} no matter whom they marry, because all the agents on the other side rank them in the same manner. Any man (woman) of a given type who asks for a higher share than the "going rate" cannot obtain it because he (she) can be replaced by an equivalent alternative.

Although we assume equal numbers of men and women in total, it is possible that the equilibrium numbers of educated men and women will differ. We shall assume throughout that there are some uneducated men who marry uneducated women and some educated men who marry educated women. This means that the equilibrium shares must satisfy

$$U_2 + V_2 = z_{22}, \tag{9}$$

$$U_1 + V_1 = z_{11}. \tag{10}$$

We can then classify the possible matching patterns as follows: Under *strict* positive assortative mating, educated men marry only educated women and uneducated men marry only uneducated women. Then,

$$U_1 + V_2 \geq z_{21}, \tag{11}$$

$$U_2 + V_1 \geq z_{12}. \tag{12}$$

If there are more educated men than women among the married, some educated men will marry uneducated women and condition (11) also will hold as equality. If

there are more educated women than men among the married, equation (12) will hold as equality. It is impossible that all four conditions will hold as equalities because this would imply

$$z_{22} + z_{11} = z_{12} + z_{21}, \tag{13}$$

which violates assumption (4) that the education levels of the spouses are complements. Thus, either educated men marry uneducated women or educated women marry uneducated men but not both.

When types mix and there are more educated men than educated women among the married, conditions (9) through (11) imply

$$\begin{aligned} U_2 - U_1 &= z_{22} - z_{21}, \\ V_2 - V_1 &= z_{21} - z_{11}. \end{aligned} \tag{14}$$

If there are more educated women than men among the married, then conditions (9), (10) and (12) imply

$$\begin{aligned} V_2 - V_1 &= z_{22} - z_{12}, \\ U_2 - U_1 &= z_{12} - z_{11}. \end{aligned} \tag{15}$$

One may interpret the differences $U_2 - U_1$ and $V_2 - V_1$ as the (additional) *return to schooling in marriage* for women and men, respectively.[15] The quantity $z_{22} - z_{21}$, which reflects the contribution of an educated woman to the material surplus of a marriage with an educated man, provides an *upper bound* on the return that a woman can obtain through marriage, while her contribution to a marriage with an uneducated man, $z_{12} - z_{11}$, provides a *lower bound*. When there are more educated women than men, analogous bounds apply to men. When types mix in the marriage market equilibrium, we see that the side that is in short supply receives the marginal contribution to a marriage with an educated spouse, while the side in excess supply receives the marginal contribution to a marriage with an uneducated spouse.

One issue of interest is whether the material surplus shares are non-negative. In principle, it is possible that the non-monetary gains from marriage are sufficiently high so that *all* men or women of a certain class who marry in equilibrium are willing

[15] The total return for schooling in terms of output that men receive is R^m if they remain single and $R^m + V_2 - V_1$ if they marry. Similarly, the total return for schooling in terms of output that women receive is R^w if they remain single and $R^w + U_2 - U_1$ if they marry.

to give up in marriage some of the material output that they have as singles. In the appendix, we provide sufficient conditions which ensure that, in equilibrium, V_I and U_J are all strictly positive and, in the analysis hereafter, we shall provide examples in which the material shares are strictly positive in equilibrium.[16]

3.4 Investment Decisions

We assume rational expectations so that, in equilibrium, individuals know V_I and U_J, which are sufficient statistics for investment decisions. Given these shares and knowledge of their own idiosyncratic preferences for marriage, θ, and costs of schooling, μ, agents know for sure whether or not they will marry in the second period, conditional on their choice of schooling in the first period.

Man i chooses to invest in schooling if

$$\zeta_{20} - \mu_i + Max(V_2 + \theta_i,\ 0) > 2\zeta_{10} + Max(V_1 + \theta_i,\ 0). \tag{16}$$

Similarly, woman j chooses to invest in schooling if

$$\xi_{02} - \mu_j + Max(U_2 + \theta_j,\ 0) > 2\zeta_{01} + Max(U_1 + \theta_j,\ 0). \tag{17}$$

Figure 5 describes the choices made by different men. Men for whom $\theta < -V_2$ do not marry and invest in schooling if and only if $\mu < R^m \equiv \zeta_{20} - 2\zeta_{10}$. Men for whom $\theta > -V_1$ always marry and they invest in schooling if and only if $\mu < R^m + V_2 - V_1$. Finally, men for whom $-V_2 < \theta < -V_1$ marry if they acquire education and do not marry if they do not invest in schooling. These individuals will acquire education if $\mu < R^m + V_2 + \theta$. In this range, there are two motives for schooling: to raise future earning capacity and to enhance marriage. We shall assume that the variability in θ and μ is large enough to ensure that *all* these regions are non-empty in an equilibrium with positive V_I and U_J. In particular, we assume that, irrespective of marital status, there are some men and women who prefer not to invest in schooling and some men and women who prefer to invest in schooling. That is, $\mu_{\max} > \max[R^m + z_{22} - z_{12},\ R^w + z_{22} - z_{21}]$ and $\mu_{\min} < \min[R^m,\ R^w]$. We shall also assume that $\theta_{\min} < -z_{22}$ so that, irrespective of the education decision, there are some individuals who wish not to marry. Note, finally, that because the support of $F(.)$ extends into the positive range, there are always some educated men and women who marry and some uneducated men and women who marry.

[16] Another issue is whether the material *output* shares are non-negative. If the only means to transfer utility between spouses is via the transfer of private consumption goods then the non-negativity constraints on consumption may bind, and utility is no longer transferable. Because positive surplus shares imply positive output shares, the sufficient conditions in the appendix eliminate this problem.

The proportion of men who invest in schooling is

$$G(R^m)F(-V_2) + [1 - F(-V_1)]G(R^m + V_2 - V_1) + \int_{-V_2}^{-V_1} G(R^m + V_2 + \theta)f(\theta)d\theta, \quad (18)$$

the proportion of men who marry is

$$[1 - F(-V_1)] + \int_{-V_2}^{-V_1} G(R^m + V_2 + \theta)f(\theta)d\theta, \quad (19)$$

and the proportion of men who invest and marry is

$$[1 - F(-V_1)]G(R^m + V_2 - V_1) + \int_{-V_2}^{-V_1} G(R^m + V_2 + \theta)f(\theta)d\theta. \quad (20)$$

The higher are the returns from schooling in the labor market, R^m, and in marriage, $V_2 - V_1$, the higher is proportion of men who acquire schooling. A common increase in the levels V_2 and V_1 also raises investment because it makes marriage more attractive and schooling obtains an *extra* return within marriage. For the same reason, an increase in the market return R^m raises the proportion of men that marry. Analogous expressions hold for women.

3.5 Equilibrium

In the marriage market equilibrium, the numbers of men and women who marry must be the same. Using equation (19) and applying symmetry, we can write this condition as

$$F(V_1) + \int_{V_1}^{V_2} G(R^m + V_2 - \theta)f(\theta)d\theta = F(U_1) + \int_{U_1}^{U_2} G(R^w + U_2 - \theta)f(\theta)d\theta. \quad (21)$$

Under strictly positive assortative mating, the numbers of men and women in each education group are equal. Given that we impose condition (21), it is necessary and sufficient to require that the numbers of men and women who marry but do *not* invest in schooling are the same. Using condition (20) and symmetry, we can derive this condition as

$$F(V_1)G(-R^m + V_1 - V_2) = F(U_1)G(-R^w + U_1 - U_2). \tag{22}$$

Together with conditions (9) and (10), conditions (21) and (22) yield a system of four equations in four unknowns that are, in principle, solvable.

If there is some mixing of types, equation (22) is replaced by an inequality and the shares are determined by the boundary conditions on the returns to schooling within marriage for either men or women, whichever is applicable. If there are more educated men than women among the married,

$$F(V_1)G(-R^m + V_1 - V_2) < F(U_1)G(-R^w + U_1 - U_2) \tag{22a}$$

and educated women receive their maximal return from marriage while men receive their minimal return so that condition (14) holds. Conversely, if there are more educated women than men among the married we have

$$F(V_1)G(-R^m + V_1 - V_2) > F(U_1)G(-R^w + U_1 - U_2) \tag{22b}$$

and educated men receive their maximal return from marriage while educated women receive their minimal return so that condition (15) holds. Together with conditions (9) and (10), we have four equations in four unknowns that are again, in principle, solvable.[17]

The two types of solutions are described in Figures 6 and 7, where we depict the equilibrium conditions in terms of V_1 and V_2 after we eliminate U_1 and U_2 using (9) and (10). The two positively-sloped and parallel lines in these figures describe the boundaries on the returns to schooling of men within marriage. The negatively-sloped red line describes the combinations of V_1 and V_2 that maintain equality in the numbers of men and women who wish to marry. The positively-sloped blue line describes the combinations of V_1 and V_2 that maintain equality in the numbers of men and women that acquire no schooling and marry. The slopes of these lines are determined by the following considerations: An increase in V_1 (and a reduction in U_1), keeping V_2 and U_2 constant, induces more men and fewer women to prefer marriage. An increase in V_2 holding V_1 has a similar effect. Thus, V_1 and V_2 are substitutes in terms of their impact on the incentives of men to marry and U_1 and U_2 are substitutes in terms of their impact on the incentives of women to marry. Therefore, equality in the number of men and women who wish to marry can be maintained only if V_2

[17] Note the system of equations consisting of (9), (10) and (14) and the system consisting of (9), (10) and (15) impose only *three* independent requirements.

declines when V_1 rises.[18] At the same time, an increase in V_1 (and a reduction in U_1), keeping V_2 and U_2 constant, increases the number of men that would not invest and marry and reduces the number of women who wish to acquire no schooling and marry. Therefore, equality in the numbers of uneducated men and women who wish to marry can be maintained only if V_2 rises when V_1 rises so that the rates of return to education within marriage are restored.[19]

As long as the model is completely symmetric, that is $R^m = R^w$ and $z_{12} = z_{21}$, the equilibrium is characterized by equal sharing: $V_2 = U_2 = z_{22}/2$ and $U_1 = V_1 = z_{11}/2$. With these shares, men and women have identical investment incentives. Hence, the number of educated (uneducated) men equals the number of educated (uneducated) women, both among the singles and the married. Such a solution is described by point e in Figure 6, where the lines satisfying conditions (21) and (22) intersect. There is a unique symmetric equilibrium. However, with asymmetry, when either $R^m \neq R^w$ or $z_{12} \neq z_{21}$, there may be a mixed equilibrium where the line representing condition (21) intersects either the lower or upper bound on $V_2 - V_1$ so that condition (22) holds as an inequality. Such a case is illustrated by the point $e\prime$ in Figure 7. In this equilibrium, educated men obtain the lower bound on their return to education within marriage, $z_{21} - z_{11}$. The equilibrium point $e\prime$ is on the lower bound and above the blue line satisfying condition (22), indicating excess supply of educated men.

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[18] Differentiating (21),

$$\begin{aligned} 0 &= \{f(V_1)[1 - G(R^m + V_2 - V_1)] + f(z_{11} - V_1)[1 - G(R^w + z_{22} - z_{11} - (V_2 - V_1)]\}dV_1 \\ &\quad +\{G(R^m)f(V_2) + G(R^w)f(z_{22} - V_2)] \\ &\quad +[\int_{V_1}^{V_2} g(R^m + V_2 - \theta)f(\theta)d\theta + \int_{U_1}^{U_2} g(R^w + U_2 - \theta)f(\theta)d\theta]\}dV_2 \end{aligned}$$

implying that

$$\frac{dV_2}{dV_1} < 0.$$

[19] The slope line satisfying condition (22) must exceed 1 because

$$\begin{aligned} & f(V_1)G(R^m - (V_1 - V_2)) + f(z_{11} - V_1)G(R^m - (z_{22} - z_{11}) + (V_1 - V_2)]dV_1 \\ = \; & F(V_1)g(R^m - (V_1 - V_2) + F(z_{11} - V_1)g(R^w - (z_{22} - z_{11}) + (V_1 - V_2)]d(V_2 - V_1) \end{aligned}$$

and therefore

$$\frac{d(V_2 - V_1)}{dV_1} = \frac{dV_2}{dV_1} - 1 > 0.$$

3.5.1 The Impact of the Sex Ratio

Although we assume in this paper that there are equal numbers of men and women in the population, one can extend the analysis to examine the impact of an uneven sex ratio on the marriage market equilibrium. Let $r \gtreqless 1$ represent the ratio of men to women in the population. Then we modify equations (21) and (22) as follows, respectively:

$$rF(V_1) + r\int_{V_1}^{V_2} G(R^m + V_2 - \theta)f(\theta)d\theta = F(U_1) + \int_{U_1}^{U_2} G(R^w + U_2 - \theta)f(\theta)d\theta. \quad (21c)$$

$$rF(V_1)G(-R^m + V_1 - V_2) = F(U_1)G(-R^w + U_1 - U_2). \quad (22c)$$

Note that, even if $R^m = R^w$ and $z_{12} = z_{21}$, the equilibrium with an uneven sex ratio will not be characterized by equal sharing. For example, if $r > 1$ and there are more men than women in the population, then (21c) implies that V_2 and U_1 will need to decline and V_1 and U_2 will need to rise to ensure that there are equal numbers of men and women who want to marry. As a result, the marriage-market return for the sex in excess supply (men) will fall and that of the sex in short supply (women) will rise, regardless of whether the marriage market equilibrium is strict or mixed. For r closer to unity, equation (22c) may still hold, implying a strict sorting equilibrium with equal numbers of educated men and educated women among the married. However, with more uneven sex ratios, equation (22c) may not hold even if $R^m = R^w$ and $z_{12} = z_{21}$. Then, when $r > 1$ ($r < 1$) there will be a mixed equilibrium where the line representing condition (21c) intersects the lower (upper) bound on $V_2 - V_1$. In such cases, condition (22c) will no longer hold as equality.

3.5.2 Efficiency

An important issue is whether premarital investments in education are efficient. The concern arises when ex-post bargaining within marriage determines the division of the gains between the two partners. Because each person bears the full cost of his\her investment prior to marriage and receives only part of the gains, there is a potential for under investment. This is known as the "hold-up problem." In contrast, models that allow endogenous assignments or intra-marital time allocation can generate over-investment in schooling, if the intra-marital allocation depends on the outside options of the spouses (which are in turn influenced by their educational attainment). Nonetheless, due to our assumptions that marriage markets are large and operate

without frictions, we can demonstrate that individuals' pre-marital investments are *efficient*.

Consider, first, a mixed equilibrium in which some married men are more educated than their wives and consider a particular couple (i, j) such that the husband is educated and the wife is not. The question is whether by *coordination* this couple could have gained, i,e., by changing investments and allowing redistribution between them.

If woman j had gotten educated, the partners together would have gained $\zeta_{22} - \zeta_{21}$ in terms of marital output but the cost of schooling for woman j would have been her forgone earnings in the first period ζ_{01} plus her idiosyncratic non-monetary cost, μ_j. The couple would gain from such a shift only if $\mu_j + \zeta_{01} < \zeta_{22} - \zeta_{21}$ or, equivalently,

$$\mu_j < z_{22} - z_{21} + R^w. \tag{23}$$

But, in the assumed marriage market configuration, $z_{22} - z_{21} = U_2 - U_1$ and, by assumption, woman j chose not to invest and marry. Therefore, by (17),

$$\mu_j > Max(U_2 + \theta_j, 0) - U_1 - \theta_j + R^w \geq U_2 - U_1 + R^w = z_{22} - z_{21} + R^w. \tag{24}$$

We thus reach a contradiction, implying that there is no *joint* net gain from such a rearrangement of investment choices. Nor is it profitable from the point of view of the couple that the husband would have refrained from schooling. The couple could gain from such a rearrangement only if the reduction in the costs of the husband's schooling exceeds the lost marital output, $\mu_i + \zeta_{10} > \zeta_{21} - \zeta_{11}$, or equivalently,

$$\mu_i > z_{21} - z_{11} + R^m. \tag{25}$$

But, in the assumed marriage market configuration, $z_{21} - z_{11} = V_2 - V_1$ and, by assumption, man i chose to invest and marry. Therefore, by (17)

$$\mu_i < R^m + V_2 + \theta_i - Max(V_1 + \theta_i, 0) \leq V_2 - V_1 + R^m = z_{21} - z_{11} + R^m. \tag{26}$$

So, again, we have a contradiction, implying that there is no *joint* net gain from such a rearrangement of investment choices. Similar arguments hold if we consider a mixed equilibrium in which some educated women marry uneducated men.

Next, consider a strictly assortative equilibrium and a married couple (i, j) such that neither spouse is educated. Could this couple have been better off had the partners coordinated their educational investments so that they both had acquired education? This would be profitable if the joint gain $\zeta_{22} - \zeta_{11}$ in terms of marital output exceeds the total costs of the two partners $\zeta_{01} + \zeta_{10} + \mu_j + \mu_i$. That is, if

$$\mu_j + \mu_i < z_{22} - z_{11} + R^m + R^w. \tag{27}$$

But, by assumption, man i and woman j married and did not invest, implying that

$$\begin{aligned} \mu_j &> U_2 - U_1 + R^w, \\ \mu_i &> V_2 - V_1 + R^m. \end{aligned} \tag{28}$$

By adding up these two inequalities, and using the equilibrium conditions $z_{22} = U_2 + V_2$ and $z_{11} = U_1 + V_1$, we see that it is impossible to satisfy (27). Hence, there is no joint gain from such a rearrangement of investments. By similar arguments, there is no joint gain for a couple in which both partners are educated from a coordinated reduction in their investments.

We conclude that the equilibrium shares that individuals expect to receive within marriage induce them to fully internalize the *social* gains from their premarital investments. An important piece of this argument is that the marriage market is large in the sense that individual perturbations in investment do not affect the equilibrium shares. In particular, a single agent cannot tip the market from excess supply to excess demand of educated men or women. This efficiency property of large and frictionless marriage markets has been noted by Cole et al. (2001), Felli and Roberts (2002) Peters and Siow (2002) and Iyigun and Walsh (forthcoming). In contrast, markets with frictions or small number of traders are usually characterized by inefficient premarital investments (Lommerud and Vagstad, 2000, Baker and Jacobsen, 2005).[20]

4 Gender Differences in the Incentive to Invest

In this section, we discuss differences between women and men that can cause them to invest at different levels. We discuss two possible sources of asymmetry:

- In the labor market, women may receive lower wages than men; this would lower the schooling return for working women.
- In marriage, women may be required to take care of the children; this would lower the schooling return for married women.

[20] Peters (2005) formulates premarital investments as a Nash game in which agents take as given the actions of others rather than the expected shares (as in a market game). In this case, inefficiency can persist even as the number of agents approaches infinity. The reason is that agents play mixed strategies that impose on other agents the risk of being matched with an uneducated spouse, leading to under-investment in schooling.

Either of the above causes can induce women to invest less in schooling. Therefore, the lower incentives of women to invest can create equilibria with mixing, where educated men are in excess supply and some of them marry less-educated women.

To illustrate these effects we shall perform several comparative statics exercises, starting from a benchmark equilibrium with strictly positive assortative matching, resulting from a complete equality between the sexes in wages and household roles such that $w_1^m = w_1^w = w_1$, $w_2^m = w_2^w = w_2$ and $\tau = 0$.

4.1 The Household

We use a rudimentary structural model to trace the impact of different wages and household roles of men and women on the marital output and surplus. We assume that, irrespective of the differences in wages or household roles, men and women have the same preferences given by

$$u = cq + \theta, \tag{29}$$

where c is a private good, q is a public good that can be shared if two people marry but is private if they remain single, and θ is the emotional gain from being married (relative to remaining single). The household public good is produced according to a household production function

$$q = e + \gamma t, \tag{30}$$

where e denotes purchased market goods, t is time spent working at home and γ is an efficiency parameter that is assumed to be independent of schooling.[21]

This specification implies transferable utility between spouses and allows us to trace the impact of different market wages or household roles on the decisions to invest and marry. Time worked at home is particularly important for parents with children. To simplify, we assume that all married couples have one child and that rearing it requires a specified amount of time $t = \tau$, where τ is a constant such that $0 \leq \tau < 1$. Initially, we shall assume that, due to social norms, all the time provided at home is supplied by the mother. Also, individuals who never marry have no children and for them we set $\tau = 0$.[22]

[21] A plausible generalization is to allow the mother's schooling level to affect positively child quality. This would be consistent with the findings of Behrman (1997) and Glewwe (1999), for example. However, the qualitative results will be unaffected as long as schooling has a larger effect on market wages than on productivity at home. The fact that educated women participate more in the labor market than uneducated women supports such an assumption.

[22] We make no distinction here between cohabitation and marriage. So either no one cohabitates, or, if two individuals cohabitate, they behave as a married couple.

If man i of class I with wage $w^m_{I(i)}$ marries woman j of class J with wage $w^w_{J(j)}$, their joint income is $w^m_{I(i)}+(1-\tau)w^w_{J(j)}$. Any efficient allocation of the family resources maximizes the partners' sum of utilities given by $[w^m_{I(i)}+(1-\tau)w^w_{J(j)}-e](e+\tau\gamma)+\theta_i + \theta_j$. In an interior solution with a positive money expenditure on the public good, the maximized material output is

$$\zeta_{ij} = \frac{[w^m_{I(i)} + \tau\gamma + (1-\tau)w^w_{J(j)}]^2}{4}. \tag{31}$$

Note that the wages of the husband and wife complement each other in generating marital output, which is a consequence of sharing the public good.[23]

An unmarried man i solves

$$\underset{e_i, c_i}{Max} \;\; c_i e_i \tag{32}$$

subject to

$$c_i + e_i = w^m_{I(i)}, \tag{33}$$

and his optimal behavior generates a utility level of $\zeta_{i0} = (w^m_{I(i)}/2)^2$. A single woman j solves an analogous problem and obtains $\zeta_{0j} = (w^w_{J(j)}/2)^2$. Therefore, the total *marital surplus* generated by the marriage in the second period is

$$s_{ij} \;\; = \;\; \frac{[w^m_{I(i)} + \tau\gamma + (1-\tau)w^w_{J(j)}]^2 - (w^m_{I(i)})^2 - (w^w_{J(j)})^2}{4} + \theta_i + \theta_j \;\; \equiv z_{I(i)J(j)} + \theta_i + \theta_j \;. \tag{34}$$

The surplus of a married couple arises from the fact that married partners jointly consume the public good. If the partners have no children and $\tau = 0$, the gains arise solely from the pecuniary expenditures on the public good. In this case, the surplus function is *symmetric* in the wages of the two spouses. If the couple has a child, however, and the mother takes care of it, then the mother's contribution to the household is a weighted average of her market wage and productivity at home. We assume that $w^w_2 > \gamma > w^w_1$ so that having children is costly for educated women but

[23] The first-order condition for e is

$$[w^m_{I(i)} + (1-\tau)w^w_{J(j)} - e] - (e + \tau\gamma) \leq 0.$$

Hence, $e = [w^m_{I(i)}+(1-\tau)w^w_{J(j)}-\tau\gamma]$ / 2 in an interior solution. The maximized material output in this case is $[w^m_{I(i)}+\tau\gamma+(1-\tau)w^w_{J(j)}]^2$ / 4. If $e = 0$, the maximal material output is $[w^m_{I(i)}+(1-\tau)w^w_{J(j)}]\tau\gamma$, which would imply an additive surplus function, contradicting our assumption of complementarity. A sufficient condition for a positive e is $w^m_1 + (1-\tau)w^w_1 > \tau\gamma$ if the wife works at home and $w^w_1 + (1-\tau)w^m_1 > \tau\gamma$ if the husband works at home. We assume hereafter that these conditions hold.

not for uneducated women. The surplus function in (34) maintains complementarity between the wages of the husband and wife, which is a consequence of sharing the public good. However, the assumed asymmetry in household roles between men and women implies that a higher husband's wage always raises the surplus but a higher mother's wage can reduce the surplus. In other words, it may be costly for a high-wage woman to marry and have a child because she must spend time on child care, while if the mother does not marry, her utility as a single remains $w_{J(j)}^2/4$. In addition, it is no longer true that $z_{21} = z_{12}$.[24]

Since we have assumed here that, due to social norms, all the time provided at home is supplied by the mother, all the gains from marriage arise from sharing a public good and the wages of the partners complement each other so that $z_{11} + z_{22} > z_{12} + z_{21}$. In later sections, we discuss endogenous specialization whereby couples act efficiently and the partner with lower wage works at home. For sufficiently low time requirements, i.e., τ close to 0, complementarity continues to hold. However, for τ close to 1, the wages of the two partners become substitutes, that is, $z_{11} + z_{22} < z_{12} + z_{21}$, because wage differentials between spouses increase the gain from specialization (see Becker, 1991, ch. 2). Thus, whether couples act efficiently or according to norms influences the equilibrium patterns of assortative mating.[25]

4.2 The Impact of the Wage Gap

We are now ready to examine the implications of gender wage differences. The gender difference in wages can be an outcome of discrimination associated, for instance, with

[24] For instance, when the wages of men and women are equal but $\tau > 0$, we have

$$z_{21} - z_{12} = \frac{\tau(w_2 - w_1)}{2}[(1-\tau)\frac{w_2 + w_1}{2} + \tau\gamma] > 0.$$

[25] For fixed household roles, the second cross derivative of the surplus function with respect to wages are positive, implying complementarity. But with endogenous household roles, the relevant measure of complementarity is embedded in the *maximized* marital gains that can change discontinuously as household roles change. Suppose that $w_2^m > w_2^w > w_1^m$. Let

$$\begin{aligned} f(\tau) &\equiv 4(z_{11} + z_{22} - z_{12} - z_{21}) \\ &= [w_1^m + \tau\gamma + (1-\tau)w_1^w]^2 + [w_2^m + \tau\gamma + (1-\tau)w_2^w]^2 \\ &\quad -[w_2^w + \tau\gamma + (1-\tau)w_1^m)]^2 - [w_2^m + \tau\gamma + (1-\tau)w_1^w]^2. \end{aligned}$$

Then, $f(\tau) > 0$ if $\tau = 0$ and $f(\tau) < 0$ if $\tau = 1$, where $\forall\ \tau \in [0, 1]$, $f'(\tau) < 0$.

fewer opportunities for investment on the job. Such discrimination can reduce or increase the incentives of women to invest, depending on whether discrimination is stronger at the low or high levels of schooling.

Define the (relative) wage gap among educated individuals as $d_2 = w_2^w \ / \ w_2^m$ and let the gender wage gap between uneducated individuals be $d_1 = w_1^w \ / \ w_1^m$. Starting from the benchmark equilibrium with strictly positive assortative mating and equal shares (point e in Figure 8), we examine the impact of a difference in the market returns from schooling of women and men. Specifically, we consider an increase in the wage of educated men, w_2^m, combined with a reduction in the wage of educated women, w_2^w, holding the wage of uneducated men at the benchmark value, w_1. To isolate the role of market returns, we assume that the increase in the wage of educated men exactly compensates the reduction in the wage of educated women so that marital output is unaffected and symmetry is maintained.[26] In other words, the change in wages affect directly only the returns as singles, R^m and R^w. For now, we assume that discrimination is uniform across schooling levels so that $d_1 = d_2 \equiv d < 1$ and women have a lower market return from schooling investment than men.[27] Later, we shall discuss a case in which discrimination against educated women is weaker so that $d_1 < d_2 < 1$.

With uniform discrimination, the returns to investment in schooling for never married men and women, respectively, are

$$R^m = z_{20}^m - 2z_{10}^m = (\frac{w_2^m}{2})^2 - 2(\frac{w_1^m}{2})^2, \tag{35}$$

and

$$R^w = z_{02}^w - 2z_{01}^w = (\frac{w_2^w}{2})^2 - 2(\frac{w_1^w}{2})^2 = d^2 R^m < R^m. \tag{36}$$

The higher market return from schooling of men encourages their investment in schooling and also strengthens their incentives to marry, because schooling obtains an additional return within marriage. In contrast, the lower return to schooling for women reduces their incentives to invest and marry. These changes create excess

[26] When wages change $z_{I(i)J(j)}$ usually changes. Also, when wages differ by gender, we generally do *not* maintain symmetry in the contribution of men and women to marriage so that $z_{12} \neq z_{21}$. It is only in the special case in which the product $w_{I(i)}^m w_{J(j)}^w$ remains invariant under discrimination that the marital surplus generated by all marriages is intact. The qualitative results for shares are not affected by this simplification.

[27] In standard human capital models where the only cost of investment is forgone earnings and the only return is higher future earnings, uniform discrimination has *no* impact on investment. In this model, however, the absolute market returns are added to the returns within marriage, which together determine investment decisions (see equations (16) and (17)). Therefore, the absolute market returns to schooling matter in our model.

supply of men who wish to invest and marry. Consequently, to restore equilibrium, the rates of returns that men receive within marriage must decline implying that, for any V_1, the value of V_2 that satisfies conditions (21) and (22) must decline. These shifts in the equilibrium lines are represented by the broken blue and red lines in Figure 8.

For moderate changes in wages, strictly positive assortative mating continues to hold. However, the equilibrium value of V_2 declines and educated men receive a *lower* share of the surplus than they do with equal wages in any marriage. That is, as market returns of men rise and more men wish to acquire education, the marriage market response is to reduce the share of educated men in *all* marriages. When the gap between R^m and R^w becomes large, the equilibrium shifts to a mixed equilibrium, where some educated men marry uneducated women. That is, because of their higher tendency to invest, some educated men must "marry down." This equilibrium is represented by the point $e\prime$ in Figure 8, where the broken red line representing equality in the numbers of men and women that wish to marry (condition (21)) intersects the green line representing the lower bound on the share that educated men obtain in the marital surplus, $z_{21} - z_{11}$. As seen, *both* V_1 and V_2 are lower in the new equilibrium so that *all* men (women), educated and uneducated, receive lower (higher) shares of the material surplus when men have stronger market incentives to invest in schooling than women.

These results regarding the shares of married men and women in the material surplus must be distinguished from the impact of the shares in the material output. If men get a higher return from schooling as singles (due to the fact that their labor-market return from schooling is higher than that of women), then their share of the material output can be higher even though they receive a lower share of the surplus. The same remark applies to our subsequent analysis as well; one can obtain sharper comparative static results on shares of the material surplus than those on shares of the material output.

4.3 The Impact of Household Roles

Recall that we assume that the wife alone spends time on child care. To investigate the impact of this constraint, we start again at the benchmark equilibrium and examine the impact of an increase in τ, holding the wages of men and women at their benchmark values, that is $w_1^m = w_1^w = w_1$ and $w_2^m = w_2^w = w_2$. Such an increase reduces the contribution that educated women make to marital output and raises the contribution of uneducated women. That is, z_{11} and z_{21} rise because uneducated

women are more productive at home, $\gamma > w_1$, while z_{12} and z_{22} decline because educated women are less productive at home, $\gamma < w_2$. Consequently, both equilibrium lines corresponding to conditions (21) and (22) shift down so that V_2 is lower for any V_1. At the same time, the boundaries on the rate of returns from schooling that men can obtain within marriage shift as $z_{21} - z_{11}$ rises and $z_{22} - z_{12}$ declines. These changes are depicted in Figure 9.

For moderate changes in τ, strictly positive assortative mating with equal sharing continues to hold. As long as a symmetric equilibrium is maintained, the returns to schooling that men and women receive within marriage, $V_2 - V_1$ and $U_2 - U_1$, are equal. Hence, men and women have the same incentives to invest. But because the material surplus (and consequently utilities within marriage) of educated men and women, $z_{22}/2$, declines with τ, while the material surplus of uneducated men and women, $z_{11}/2$, rises, both men and women will reduce their investments in schooling by the same degree.

As τ rises further, the difference in the contributions of men and women to marriage can rise to the extent that an educated man contributes to a marriage with uneducated woman more than an educated woman contributes to a marriage with an educated man.[28] That is,

$$z_{21} - z_{11} > z_{22} - z_{21}. \tag{37}$$

Condition (37) implies that the *lower* bound on the return to schooling that men receive within marriage exceeds the *upper* bound on the return to schooling that woman receive within marriage. In this event, the symmetric equilibrium in Figure 9 is eliminated and instead there is a mixed equilibrium with some educated men marrying uneducated women (point $e\prime$ in Figure 9). This outcome reflects the lower incentive of educated women to enter marriage and the stronger incentive of men to

[28] Consider the expression

$$\begin{aligned} h(w_1, w_2, \tau) &\equiv 2z_{21} - z_{11} - z_{22} = 2[w_2 + \tau\gamma + (1-\tau)w_1]^2 \\ &-[w_1 + \tau\gamma + (1-\tau)w_1]^2 - [w_2 + \tau\gamma + (1-\tau)w_2]^2 \end{aligned}$$

as a function of w_1 and w_2 and τ. For $w_1 = w_2 = \gamma$, $h(\gamma, \gamma, \tau) = 0$ and

$$\begin{aligned} h_1(\gamma, \gamma, \tau) &= -4\gamma\tau, \\ h_2(\gamma, \gamma, \tau) &= 4\gamma\tau. \end{aligned}$$

Therefore, for a positive τ, w_1 slightly below γ and w_2 slightly above γ, $h(w_1, w_2, \tau) > 0$. Also

$$h_3(w_1, w_2, \tau) = (w_2 - w_1)[w_2(4 - 2\tau) + 2\tau(2\gamma - w_1)] > 0$$

and for all $w_2 > \gamma > w_1$, $h(w_1, w_2, 0) < 0$ and $h(w_1, w_2, 1) > 0$. Hence, the larger is τ the broader will be the range in which $h(w_1, w_2, 0) > 0$.

invest because their return from schooling within marriage, $V_2 - V_1 = z_{21} - z_{11}$, exceeds the return to schooling that women can obtain within marriage. Consequently, some educated men must "marry down" and match with uneducated women.

4.4 Division of Labor and Career Choice

We can further refine the family decision problem by letting the partners decide who shall take care of the children. Reinterpreting τ as a temporal choice, imagine that one of the partners must first spend τ units of time during marriage on the child and later enter the labor market and work for the remainder of the period (length $1 - \tau$).

An important idea of Becker (1991, ch. 2) is that wage differences among identical spouses can be created endogenously and *voluntarily* because of learning by doing and increasing returns. Thus, it may be *optimal* for the household for *one* of the spouses to take care of the child and for the other to enter the labor market immediately, thereby generating a higher wage in the remainder of the period. Thus, by choosing schooling ahead of marriage one can influence his\her household role within marriage.

Because we assume transferable utility between spouses, household roles will be determined efficiently by each married couple, as long as there is ability to commit to a transfer scheme, whereby the party that sacrifices outside options when he\she acts in a manner that raises the total surplus is compensated for his/her action. In particular, the partners will assign the spouse with the lower wage to take care of the child. In the previous analysis, there was no need for such a commitment because the division of the surplus was fully determined by attributes that were determined prior to marriage via competition over mates who could freely replace partners. However, if time spent on child care affects one's labor market wages subsequently, the cost of providing childcare can differ between the two spouses. Thus, implementing the efficient outcome might require some form of commitment even if (re)matching is frictionless. A simple, enforceable, prenuptial contract is one in which both partners agree to pay the equilibrium shares V_I to the husband and U_J to the wife in case of divorce. By making those shares the relevant threat points of each spouse, this contract sustains the equilibrium values V_I and U_J in marriage, which is sufficient to attain the efficient household division of labor.

If there is discrimination against women and they receive lower market wages than men, then the wife will be typically assigned to stay at home, which will erode her future market wage and reinforce the unequal division of labor. Similarly, if there are predetermined household roles such that women must take care of their child, then women will end up with lower market wages. Thus, inequality at home and the

market are interrelated.[29] Models of statistical discrimination tie household roles and market wages through employers' beliefs about female participation. Typically, such models generate multiple equilibria and inefficiency (Hadfield, 1999, Lommerud and Vagstad, 2002). Here, we do not require employers' beliefs to be correct. Instead, we think of household roles and discrimination as processes that evolve slowly and can be taken as exogenous in the medium run.

4.5 Why Women May Acquire More Schooling than Men

We have examined two possible reasons why women may invest less than men in schooling. The first is that women may receive lower return from schooling investment in the market because of discrimination. The second reason is that women may receive a lower return to schooling in marriage because of the need to take care of children (due to social and cultural norms or the biological time requirements of child care).

Over time, fertility has declined and women's wages have risen in industrialized countries, a pattern being replicated in many developing countries too. This is consistent with increased investment in education by women. The fact that women are now slightly more educated than men, on average, appears surprising given the fact that women still earn substantially less than men. However, in dealing with investments in education, the crucial issue is whether the gender wage gap rises or declines with schooling, or equivalently, whether women obtain a higher rate of return from schooling. There is some evidence that this is indeed the case and that the gender wage gap declines with schooling (Dougherty, 2005).

Now consider a comparison of the following two situations. An "old" regime in which married women must spend a relatively large fraction of their time at home and a "new" regime in which, because of reductions in fertility and improved technology in home production, married women spend less time at home and work more in the market (Greenwood, Seshadri and Yorukoglu, 2005). Assume further that women suffer from statistical discrimination because employers still expect them to invest less on the job. However, this discrimination is weaker against educated women because they are expected to stay longer in the labor market than uneducated women. Finally, assume that in the old regime norms were relevant but in the new regime the roles are determined efficiently. It is then possible that in the new regime women will invest in schooling *more* than men. The presence of discrimination *raises* the return

[29] For related papers that emphasize the same dual-feedback mechanism between the intensity of home work and labor market wages we discuss here, see Albanesi and Olivetti (2005, 2006) and Chichilnisky (2005).

of women relative to men because schooling serves as an instrument for women to escape discrimination. The fact that women are still tied up in home work lowers their return from schooling relative to men because women obtain lower returns from schooling within marriage. However, as women raise their labor force participation, due to technological changes or break of norms, this second effect weakens and the impact of discrimination can dominate.

In Figure 10, we display the transition between the two regimes. We assume that $d_2 > d_1$ so that discrimination against women is lower at the higher level of schooling. This feature generates stronger incentives for women than men to invest in schooling. However, the fact that women must spend time working at home has the opposite effect. We then reduce the amount of time that the mother has to spend at home, τ, and raise the wage that educated women receive (so that d_2 rises), which strengthens the incentives of women to invest in schooling and to marry. Therefore, holding the marriage surplus z_{IJ} constant, an increase in V_2 relative to V_1 is required to maintain equality between the number of men who wish to invest and marry and the number of women who wish to invest and marry. This effect is represented by the upwards shifts in the broken red and blue lines in Figure 10.[30] The impact is assumed to be large enough to generate an equilibrium in which the two equilibrium requirements–equality of the numbers of men and women who acquire no schooling and marry (the broken blue line) and equality of the total numbers of men and women who wish to marry (the broken red line)–yield an intersection above the upper bound on the returns from schooling that men can receive within marriage. Therefore, strictly positive assortative mating cannot be sustained as an equilibrium and the outcome is a mixed equilibrium in which there are more educated women than men among the married and some educated women marry uneducated men. This new mixed equilibrium is indicated by the point e'' in Figure 10.

4.5.1 A Numerical Example

Suppose that μ and θ are uniformly and independently distributed on the interval $[-4, 4]$ and that the parameter of productivity at home is $\gamma = 2$. These parameters do not vary across regimes. Although wages vary across the two regimes, we assume that in both regimes, educated women are more productive in the market and uneducated

[30] Because the marital surplus matrix, z_{IJ}, also changes, it is not always the case that the equilibrium curves shift up. In fact, for the parameters of Figure 10, there is a range over which the equilibrium line representing market-clearing in the marriage market shifts down. This, however, has no bearing on the equilibrium outcome.

women are more productive at home. We further assume that in both regimes, men earn more than women with the same schooling level but educated women earn more than uneducated men. Finally, in both regimes, women have a higher market return from schooling. The transition from the old regime to the new regime is characterized by three features: (i) women are required to work less at home; (ii) men and women obtain higher market returns from schooling; and (iii) couples move from a traditional mode to an efficient one in which the high wage spouse works in the market. The parameters displayed in Table 2 reflect these assumptions.

Table 2: Parameters in the old and the new regimes

Parameter	Old Regime	New Regime
Wage of uneducated men	$w_1^m = 2$	$w_1^m = 2.1$
Wage of uneducated women	$w_1^w = 1.2$	$w_1^w = 1.26$
Wage of educated men	$w_2^m = 3$	$w_2^m = 3.2$
Wage of educated women	$w_2^w = 2.4$	$w_2^w = 2.56$
Wage difference among the educated	$d_1 = .6$	$d_1 = .6$
Wage difference among the uneducated	$d_2 = .8$	$d_2 = .8$
Market return to schooling, men	$R^m = .25$	$R^m = .36$
Market return to schooling, women	$R^w = .72$	$R^w = .84$
Work requirements	$\tau = .8$	$\tau = .3$
Norms	Wife at home	Efficient

The marriage market implications of these changes are summarized in Tables 3-5 below.

Table 3: Impact of parameter changes on marital surplus

Old regime

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 1.72$
Educ. husband	$z_{21} = 3.25$	$z_{22} = 2.76$

New Regime

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 1.71$	$z_{12} = 2.62$
Educ. husband	$z_{21} = 2.52$	$z_{22} = 3.62$

A decrease in the amount of time worked at home, raises the contribution of an educated woman to the material surplus and lowers the contribution of an uneducated woman. In this example, we take an extreme case such that in the old regime with $\tau = .8$, the material surplus *declines* with the education of the wife, while in the new regime with $\tau = .3$, it rises. This happens because educated women are more productive in the market than uneducated women but, by assumption, equally productive at home. In the old regime, if an educated wife would marry an uneducated man (which does not happen in equilibrium) she would be assigned to household work even though she has a higher wage than her husband. In the new regime, couples act efficiently, household roles are reversed and educated women *do* marry uneducated men.

Table 4: Impact of parameter changes on the equilibrium shares

Old regime

	Uneducated	Educated
Men	$V_1 = .76$	$V_2 = 1.68$
Women	$U_1 = 1.57$	$U_2 = 1.09$

New Regime

	Uneducated	Educated
Men	$V_1 = .85$	$V_2 = 1.85$
Women	$U_1 = .86$	$U_2 = 1.77$

Compared with the old regime, educated women receive a higher share of the marital surplus in the new regime, while uneducated women receive a lower share. These changes reflect the higher (lower) contributions to marriage of educated (uneducated) women.

The implied returns from schooling within marriage in the old regime are

$$U_2 - U_1 = 1.09 - 1.57 = z_{22} - z_{21} = 2.76 - 3.25 = -.49 \ ,$$

$$V_2 - V_1 = 1.68 - .76 = z_{21} - z_{11} = 3.25 - 2.33 = .92 \ .$$

That is, men receive the *lower* bound on their return from schooling within marriage while women receive the *upper* bound on their return from schooling. This pattern

is reversed in the new regime:

$$U_2 - U_1 = 1.77 - .86 = z_{12} - z_{11} = 2.61 - 1.71 = .90$$

$$V_2 - V_1 = 1.85 - .85 = z_{22} - z_{12} = 3.62 - 2.62 = 1.00 \ ,$$

where women receive their lower bound and men receive their upper bound. Both men and women receive a higher return from schooling within marriage in the new regime, reflecting the increased efficiency although the rise for women is much sharper.

Table 5: Impact of parameter changes on the investment and marriage rates*

Old Regime

	Married	Unmarried	All
Educ.	.452, .335	.154, .215	.606, .550
Uned.	.211, .327	.183, .123	.394, .450
All	.663, .663	.337, .337	1

New Regime

	Married	Unmarried	All
Educ.	.482, .512	.146, .169	.628, .681
Uned.	.200, .170	.172, .149	.372, .319
All	.682, .682	.318, .318	1

$*$ First and second entries in each cell refer to men and women resp.

In the old regime, more men invest in schooling than women and some educated men marry down to match with uneducated women. This pattern is reversed in the new regime and women invest in schooling more than men and some educated women marry down to join uneducated men. That is, women increase their investment in schooling more than men. Although market returns have risen for both men and women, the returns for schooling within marriage have risen substantially more for women. The basic reason for that is the release of married women from the obligation to spend most of their time at home, due to the reduction in the time requirement of childcare and the change in norms that allow educated women who are married

to uneducated men to enter the labor market. Uneducated men gain a higher share in the surplus in all marriages because of their new opportunity to marry educated women, while uneducated women lose part of their share in the marital surplus in all marriages because they no longer marry educated men. Notice that in the old regime, a fraction $.215/.550 = .39$ of the educated women remain single, while in the new regime only a fraction of $.169/.681 = .25$ of the educated women remain single. The higher propensity of educated women to marry raises the overall marriage rate only slightly from 66 percent to 68 percent.

We can use these examples to discuss the impact of norms. To begin with, suppose that in the old regime couples acted efficiently and if the wife was more educated than her husband, she went to work full time and the husband engaged in childcare. Comparing Tables 3 and 6, we see that the impact of such a change on the surplus matrix is only through the rise in z_{12}. Because women receive lower wages than men at all levels of schooling, the household division of labor is not affected by the norms for couples with identically educated spouses; for all such couples, the husband works in the market and the wife takes care of the child. However, the norm does affect the division of labor for couples among whom the wife has a higher education level than her husband. This is due to our assumptions that educated women have a higher wage than uneducated men in the labor market and their market wage exceeds their productivity at home. In contrast to the case in which the mother always works at home, we see in Table 6 that the education levels now become substitutes, namely $z_{11} + z_{22} < z_{12} + z_{21}$, implying that we can no longer assume that there will be some educated men married to educated women and some uneducated men married to uneducated women. More specifically, an educated woman contributes more to an uneducated man than she does to an educated man (i.e. $z_{12} - z_{11} > z_{22} - z_{21}$) so that uneducated men can bid away the educated women from educated men. Thus changes in norms can influence the patterns of assortative mating.

Table 6: Impact of norms on material surplus

Old regime, efficient

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 2.40$
Educ. husband	$z_{21} = 3.25$	$z_{22} = 2.76$

New Regime with norms

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 1.71$	$z_{12} = 2.30$
Educ. husband	$z_{21} = 2.52$	$z_{22} = 3.62$

Consider, next, the possibility that the norms persist also in the new regime and the mother must work at home even if she is more educated than her husband. Again, the norm bites only in those marriages in which the wife is more educated than the husband. In the new regime, positive assortative mating persists independently of the norms. However, the mixing equilibrium in which some educated women marry uneducated men is replaced by strict assortative mating in which educated men marry only educated women and uneducated men marry only uneducated women. Thus, again, norms can have a qualitative impact on the type of equilibrium that emerges.

The new marriage and investment patterns are presented in the lower panel of Table 7. Women invest less in education and educated women are less likely to marry when the norm calls for women to work at home. Men, however, increase their investment in schooling and uneducated men are less inclined to marry, due to the loss of efficiency in mixed marriages in which the wife is more educated than her husband.

Table 7: Impact of norms on investment and marriage rates (new regime)*

	Married	Unmarried	All
Educ.	.482, .512	.146, .169	.628, .681
Uned.	.200, .170	.172, .149	.372, .319
All	.682, .682	.318, .318	1

Wife always works at home

	Married	Unmarried	All
Educ.	.497, .497	.143, .172	.640, .669
Uned.	.185, .185	.175, .145	.360, .331
All	.682, .682	.318, .318	1

$*$ The first and second entry in each cell refer to men and women resp.

Consider, finally, the impact on the shares in the material surplus when norms are replaced by an efficient allocation in the new regime (see Table 8). The removal of social norms does not benefit all women (or harm all men). While educated women and uneducated men gain, uneducated women and educated men lose. This example illustrates the differences between the predictions of general equilibrium models with frictionless matching, like the one we present here, and partial equilibrium models that rely on bargaining. The latter would predict that no woman would lose from the removal of norms that force women in general to stay at home and take care of the child, but as this example demonstrates, market competition could benefit some women and hurt others depending on their level of education.

Table 8: Impact of norms on the equilibrium shares in the new regime

Efficient allocation

	Uneducated	Educated
Men	$V_1 = .85$	$V_2 = 1.85$
Women	$U_1 = .86$	$U_2 = 1.77$

Wife always works at home

	Uneducated	Educated
Men	$V_1 = .75$	$V_2 = 1.90$
Women	$U_1 = .96$	$U_2 = 1.72$

5 Conclusions

In standard models of human capital, individuals invest in schooling with the anticipation of being employed at a higher future wage that would compensate them for the current foregone earnings. In this paper, we add another consideration: the anticipation of being married to a spouse with whom one can share consumption and coordinate work activities. Schooling has an added value in this context because of complementarity between agents, whereby the contribution of the agents' schooling to marital output rises with the schooling of his\her spouse. In the frictionless marriage market considered here, the matching pattern is fully predictable and supported by a unique distribution of marital gains between partners. Distribution is governed by competition, because for each agent there exists a perfect substitute that can replace him\her in marriage. There is thus no scope for bargaining and, therefore, premarital investments are efficient. This simple framework allows us to jointly determine investment and marriage patterns as well as the welfare of men and women under a variety of circumstances.

From the perspective of family economics, gender differences in investment in schooling are of particular interest because assortative mating based on schooling is a common feature of marriage patterns in modern societies. However, schooling is an acquired trait that responds to economic incentives. We mentioned two interrelated causes that may diminish the incentives of women to invest in schooling: lower market wages and larger amount of household work. Although we did not fully specify the sources of discrimination against women in the market, we noted that such discrimination tends to decline with schooling and therefore increases the incentive to invest. This is a possible explanation for the slightly higher investment in schooling

by women that we observe today. We do not view this outcome as a permanent phenomenon but rather as a part of an adjustment process, whereby women who now enter the labor market in increasing numbers, following technological changes at home and in the market that favor women, must be "armed" with additional schooling to overcome norms and beliefs that originate in the past.

We should add that there are other possible reasons for why women may invest in schooling more than men. One reason is that there are more women than men in the marriage market at the relatively young ages at which schooling is chosen, because women marry younger. Iyigun and Walsh (forthcoming) have shown, using a similar model to the one discussed here, that in such a case women will be induced to invest more than men in competition for the scarce males. Another reason is that divorce is more harmful to women, because men are more likely to initiate divorce when the quality of match is revealed to be low. This asymmetry is due to the higher income of men and the usual custody arrangements (see Chiappori and Weiss, 2006). In such a case, women may use schooling as an insurance device that mitigates their costs from unwanted divorce.

References

[1] **Aguiar, M. and E. Hurst (2006)**. "Measuring Trends in Leisure: The Allocation of Time over Five Decades," Federal Reserve Bank of Boston Working Paper No. 06-2.

[2] **Albanesi, S. and C. Olivetti.** (2005). "Home Production, Market Production and the Gender Wage Gap: Incentives and Expectations," CEPR Discussion Paper No: 4984, December.

[3] **Albanesi, S. and C. Olivetti.** (2006). "Gender Roles and Technological Progress," Columbia University, unpublished manuscript.

[4] **Baker, M. and J. Jacobsen.** (2005). "Marriage, Specialization, and the Gender Division of Labor," United States Naval Academy.

[5] **Becker G.** (1991). *Treatise on the Family*, Harvard University Press.

[6] **Behrman, J. R.** (1997). "Mother's Schooling and Child Education: A Survey" PIER Working Paper No: 97-024.

[7] **Borda, M., Hamermesh, D. and P. Weil.** (2006). "The Distribution of Total Work in the EU and the US," University of Texas, unpublished manuscript.

[8] **Browning, M., P. A. Chiappori, and Y. Weiss.** (in progress). *Family Economics,* Tel Aviv University, unpublished textbook manuscript, , unpublished manuscript..

[9] **Chiappori, P. and Y. Weiss.** (2006), "Divorce, Remarriage and Child Support," *Journal of Labor Economics*, forthcoming.

[10] **Cole, H., G. Mailath and A. Postlewaite.** (2001). "Efficient Non Contractible Investments in Large Economies," *Journal of Economic Theory*, 101, 333-373.

[11] **Chichilnisky, G.** (2005). "The Gender Gap," Columbia University.

[12] **Dougherty, C.** (2005). "Why is the Return to Schooling Higher for Woman Than For Men?" *Journal of Human Resources*, 40, 969-988.

[13] **Felli, L. and K. Roberts.** (2002). "Does Competition Solve the Hold-Up Problem?" CEPR Discussion Paper 3535.

[14] **Glewwe, P.** (1999). "Why does Mother's Schooling Raise Child Health in Developing Countries? Evidence from Morocco" *Journal of Human Resources*, 34, 124-159.

[15] **Goldin, C. L. Katz and I. Kuziemko.** (2006). "The Homecoming of American College Women: A Reversal of the College Gender Gap," NBER Working Paper 12139.

[16] **Goldin, C.** (1997). "Career and Family: College Women Look at the Past,": in F. Blau and R. Ehrenberg (eds.) *Gender and Family Issues in the Workplace*, New York: Russel Sage Press.

[17] **Greenwood, J. A. Seshadri, and M. Yorukoglu.** (2005). "Engines of Liberation," *Review of Economic Studies*, 72, 109-33.

[18] **Gronau, R.** (1988). "Sex Related Wage Differences and Women Interrupted Labor Careers-the Chicken or the Egg," *Journal of Labor Economics*, 6. 277-301.

[19] **Hadfield, G.** (1999). "A Coordination Model of Sexual Division of Labor," *Journal of Economic Behavior and Organization*, 40, 125-53.

[20] **Iyigun, M. F.** (2004). "Bargaining and Specialization in Marriage" IZA DP No. 1744, August.

[21] **Iyigun, M. and R. Walsh.** (forthcoming). "Building the Family Nest: Pre-Marital Investments, Marriage Markets and Spousal Allocations," *Review of Economic Studies*.

[22] **Lewis, S. K. and V. K. Oppenheimer.** (2000). "Educational Assortative Mating across Marriage Markets: Non-Hispanic Whites in the United States," *Demography*, February, 37, 29-40.

[23] **Lommerud, K. and E. Vagstad.** (2000). "Mommy Tracks and Public Policy; On Self Fulfilling Prophecies and Gender Gaps in Promotion," CEPR Discussion Paper 2378.

[24] **Lundberg, S. and R. A. Pollak.** (1993). "Separate Spheres Bargaining and the Marriage Market," *Journal of Political Economy,* 101 (6), 988-1010.

[25] **Lundberg, S.** (2005). "Gender and Household Decisions," Lecture Notes University of Sienna.

[26] **O'Neill J. and D. O'Neill.** (2006). "What do Wage Differentials Tell about Labor Market Discrimination?" in Polachek, S., Chiswick, C. and H. Rapoport (eds.) *Research in Labor Economic*s, 24, 293-360.

[27] **Mincer, J and S. Polachek.** (1974). "Family Investments in Human Capital: Earnings of Women," *Journal of Political Economy*, 82, s76-s R108.

[28] **Mulligan, C. and Y. Rubinstein.** (2006). "Selection, Investment, and Women Relative Wages Since 1975," NBER Working Paper 11159.

[29] **Peters, M. and A. Siow.** (2002). "Competing Premarital Investments," *Journal of Political Economy*, 110, 592-208.

[30] **Peters, M.** (2005). "The Pre-marital Investment Game," Department of Economics, University of British Columbia.

[31] **Shapely, L. and M. Shubik.** (1972). "The Assignment Game I; the Core," *International Journal of Game Theory*, 1, 111-130.

[32] **Weiss, Y. and R. Gronau.** (1981). "Expected Interruptions in Labor Force Participation and Sex-Related Differences in Earning Growth," *Review of Economic Studies*,. 48, 607-619.

6 Appendix

We provide here *sufficient* conditions that ensure that if any of the 3 equilibrium types (strict or mixed) exists then the equilibrium shares are all strictly positive. The conditions are based on the monotonicity of the line that represents the equilibrium condition (21) that ensures the clearing of the marriage market. We have already shown that this condition defines a line with a negative slope in the $V_{1,}$ V_2 space. We now want to state conditions that ensure that, for any V_1 satisfying $0 \leq V_1 \leq z_{11}$, equation (21) has a solution that satisfies $0 < V_2 < z_{22}$.

Consider, first, the solution for V_2 in equation (21) when we set $V_1 = z_{11}$ (and $U_1 = 0$)

$$F(z_{11}) + \int_{z_{11}}^{V_2} G(R^m + V_2 - \theta) f(\theta) d\theta = F(0) + \int_{0}^{z_{22}-V_2} G(R^w + z_{22} - V_2 - \theta) f(\theta) d\theta. \quad \text{(A1)}$$

We want to show that this solution which we denote by V_2^l is positive. Now because the LHS of (A1) rises with V_2 and the RHS declines with V_2, it is sufficient that the RHS exceeds the LHS when we evaluate them at $V_2 = 0$. That is,

$$F(z_{11}) - \int_{0}^{z_{11}} G(R^m - \theta) f(\theta) d\theta < F(0) + \int_{0}^{z_{22}} G(R^w + z_{22} - \theta) f(\theta) d\theta. \quad \text{(A2)}$$

The economic meaning of condition (A2) is that if under the situation in which uneducated men get all the surplus when married to uneducated women but educated men get none of the surplus when married to educated women, there is excess supply of women who wish to marry and the market adjusts by a rise in V_2, because V_1 is already at its maximal value, z_{11}.

Consider, next, the solution for V_2 in equation (21) when we set $V_1 = 0$ (and $U_1 = z_{11}$);

$$F(0) + \int_{0}^{V_2} G(R^m + V_2 - \theta) f(\theta) d\theta = F(z_{11}) + \int_{z_{11}}^{z_{22}-V_2} G(R^w + z_{22} - V_2 - \theta) f(\theta) d\theta. \quad \text{(A3)}$$

We want to show that this solution which we denote by V_2^h is below z_{22}. Now, because the LHS of (A3) rises in V_2 and the RHS of (A3) declines with V_2, it is sufficient that the LHS exceeds the RHS when we try the value, $V_2 = z_{22}$. That is,

$$F(0) + \int_{0}^{z_{22}} G(R^m + z_{22} - \theta) f(\theta) d\theta > F(z_{11}) - \int_{0}^{z_{11}} G(R^w - \theta) f(\theta) d\theta. \quad \text{(A4)}$$

The economic meaning of condition (A4) is that if under the situation in which uneducated men get none of the surplus when married to uneducated women but educated men get all of the surplus when married to educated women, there is excess supply of men who wish to marry and the market adjusts by a decrease in V_2, because V_1 is already at its minimal value, 0.

Because the equilibrium line (21) has a negative slope, conditions (A2) and (A4) together ensure that, for any V_1 satisfying $0 \leq V_1 \leq z_{11}$, we have $0 \leq V_2 \leq z_{22}$. Using symmetry, conditions A2 and A4 can be rewritten more compactly as

$$\int_0^{z_{11}} G(\theta - R^m) f(\theta) d\theta < \int_0^{z_{22}} G(R^w + z_{22} - \theta) f(\theta) d\theta, \tag{A2'}$$

$$\int_0^{z_{22}} G(R^m + z_{22} - \theta) f(\theta) d\theta > \int_0^{z_{11}} G(\theta - R^w) f(\theta) d\theta. \tag{A4'}$$

We can make two observations about these conditions. For *any* distributions $F(.)$ and $G(.)$, conditions (A2) and (A4) hold if $z_{22} + R^m + R^w > 2z_{11}$, because then $G(R^w + z_{22} - \theta) > G(\theta - R^m)$ and $G(R^m + z_{22} - \theta) > G(\theta - R^w)$ for all θ such that $\theta \leq z_{11}$. When $2z_{11} > z_{22} > z_{11}$, conditions (A2) and (A4) hold if the distributions $F(.)$ and $G(.)$ are sufficiently widely spread, because then there will be a relatively large mass in the range $z_{22} > \theta > z_{11}$.

To ensure that at least one equilibrium exist in which $0 < V_1 < z_{11}$, we need to impose one of the following:

In a mixed equilibrium at the lower bound, the equilibrium shares are all strictly positive if $V_2^l < z_{21}, V_2^h > z_{21} - z_{11}$.

In a mixed equilibrium at the upper bound, the equilibrium shares are all strictly positive if $V_2^h > z_{22} - z_{12}$, $V_2^l < z_{11} + z_{22} - z_{12}$.

In a strict assortative equilibrium, we shall have $0 < V_1 < z_{11}$ if $\bar{V}_2^h > V_2^l$ and $\bar{V}_2^l < V_2^h$, where $\bar{V}_2^h$ and $\bar{V}_2^l$ are the solutions for V_2 of the equilibrium condition (22)

$$F(V_1)G(-R^m + V_1 - V_2) = F(U_1)G(-R^w + U_1 - U_2). \tag{22}$$

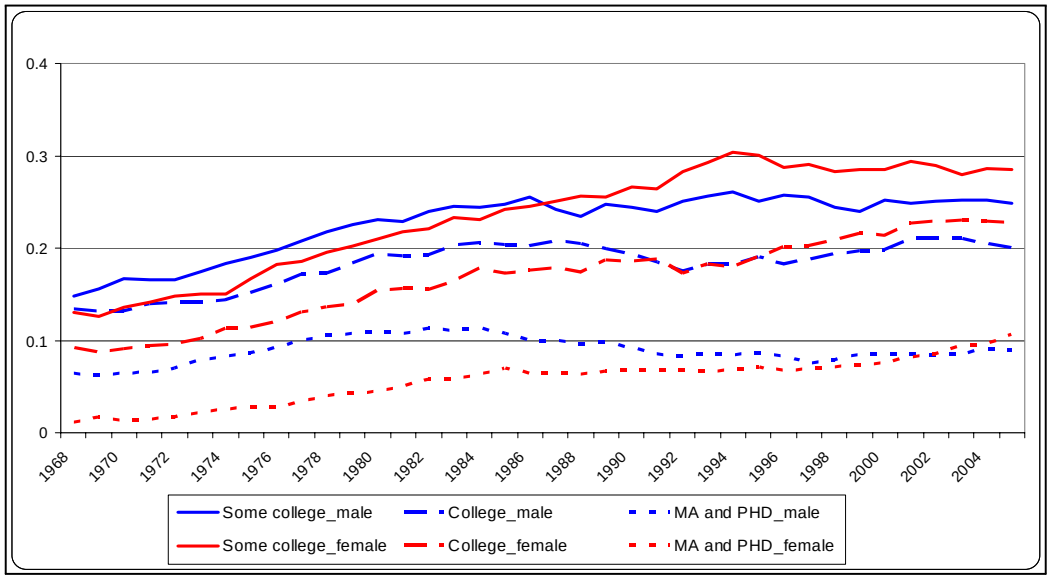
when V_1 is set at z_{11} and 0, respectively.

Table 1: Time Use (hours per day) of US Married Men and Women with Children

	1975		2003	
	Women	Men	Women	Men
Paid work				
Child <5	1.55	6.98	2.81	6.39
Child 5-17	2.71	7.17	3.68	6.40
Household Work				
Child <5	3.67	1.10	2.64	1.38
Child 5-17	3.63	1.18	2.83	1.52
Child Care				
Child <5	1.63	0.40	2.67	1.24
Child 5-17	0.65	0.20	1.13	0.57
Shopping				
Child <5	0.50	0.28	0.60	0.39
Child 5-17	0.59	0.24	0.61	0.34
Leisure				
Child <5	5.98	5.43	5.01	4.93
Child 5-17	6.14	5.38	5.61	5.49

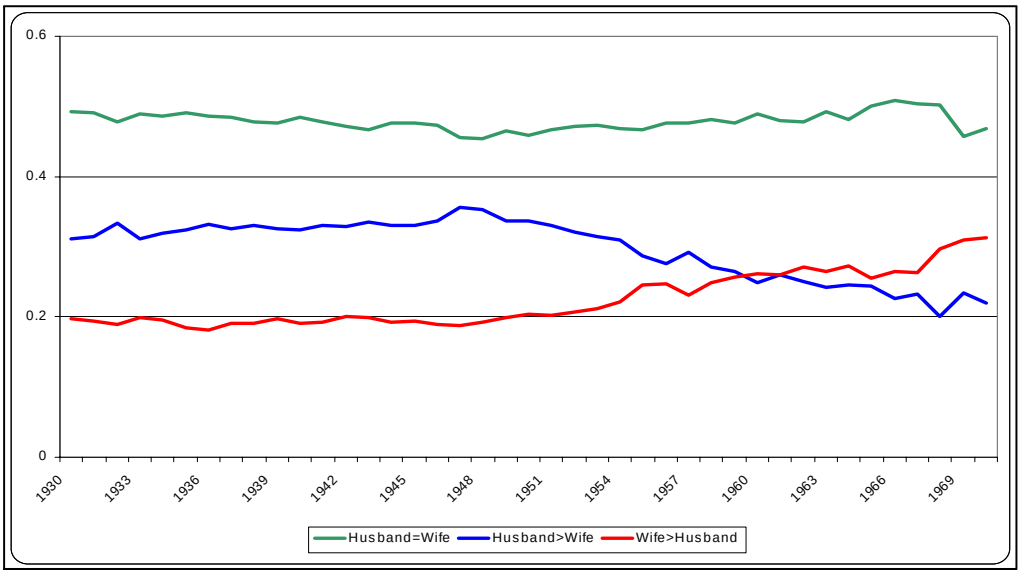
Source: American's Use of Time (1975) & Time Use Survey (2003).

Figure 1: Completed Education by Sex, 30-40 years old, US 1968-2005 (CPS)



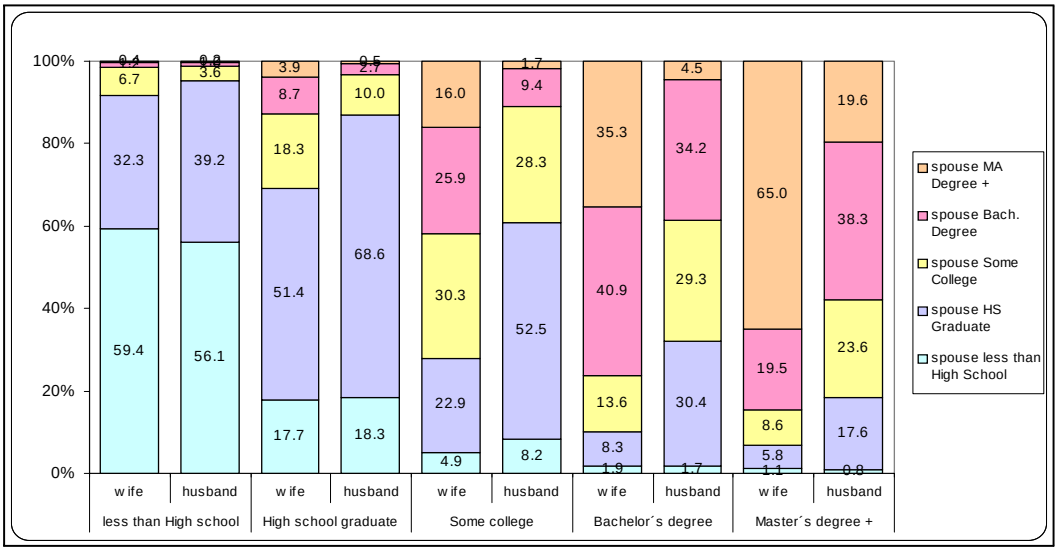
Source: Current Population Surveys.

Figure 2: Educational Attainment of Spouses by Husbands' Year of Birth (United States)



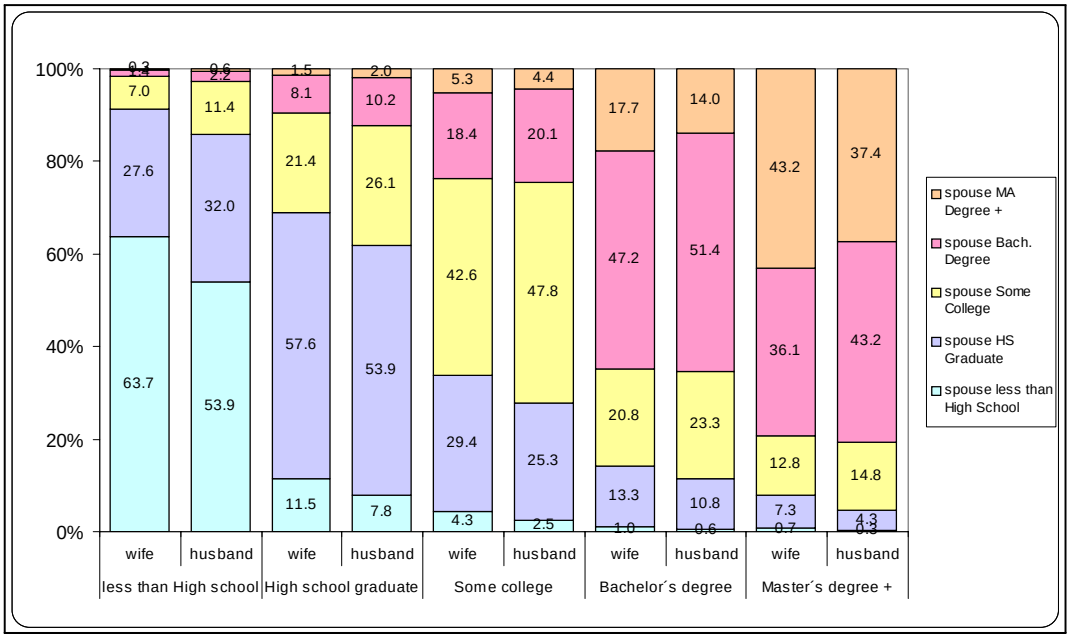
Source: Current Population Surveys.

Figure 3.a: Spousal Education by Own Education, Ages 30-40, U. S., 1970-1979



Source: Current Population Surveys.

Figure 3.b: Spousal Education by Own Education, Ages 30-40, U. S., 1996-2005



Source: Current Population Surveys.

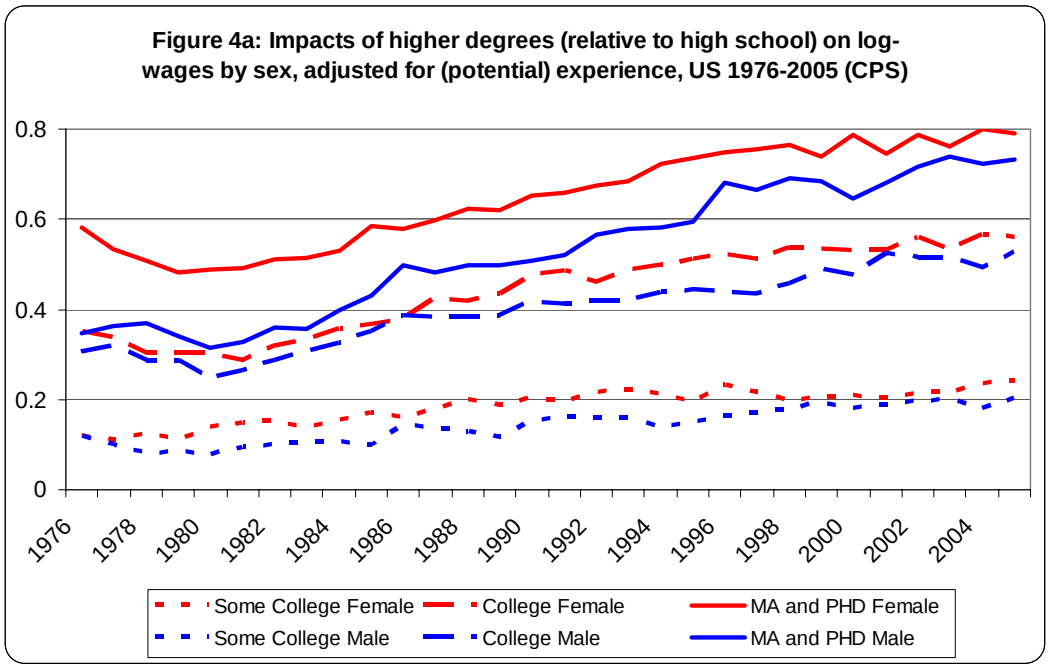


Figure 1:

Figure 5: Regions for Marriage and Investment

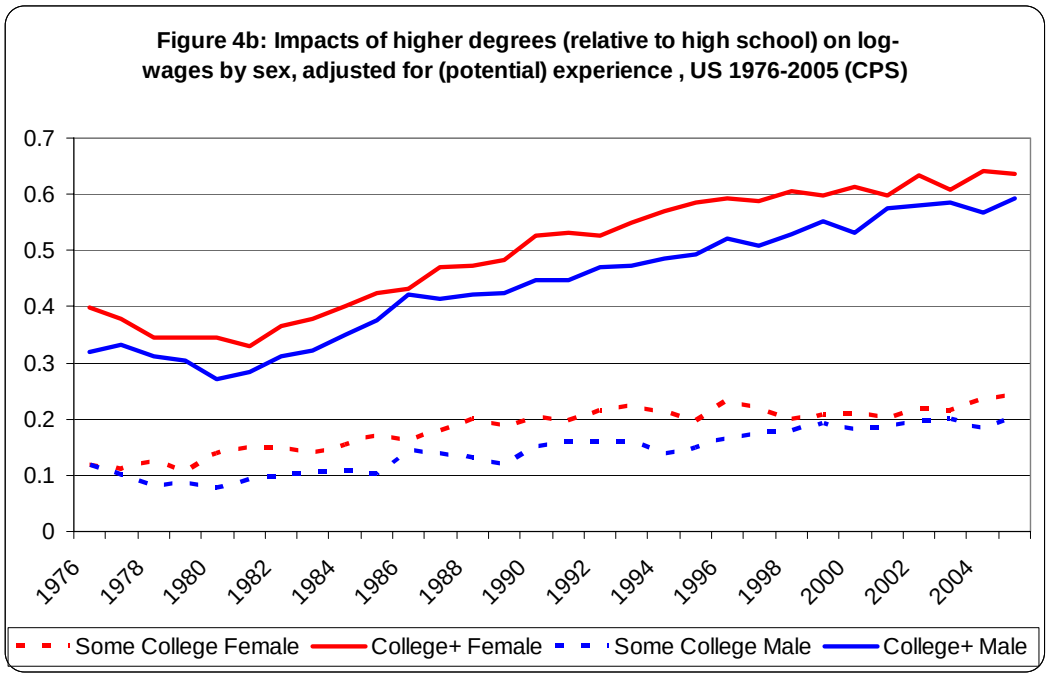


Figure 4b: Impacts of higher degrees (relative to high school) on log-wages by sex, adjusted for (potential) experience , US 1976-2005 (CPS)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
Some College Female
College+ Female
Some College Male
College+ Male

Figure 2:

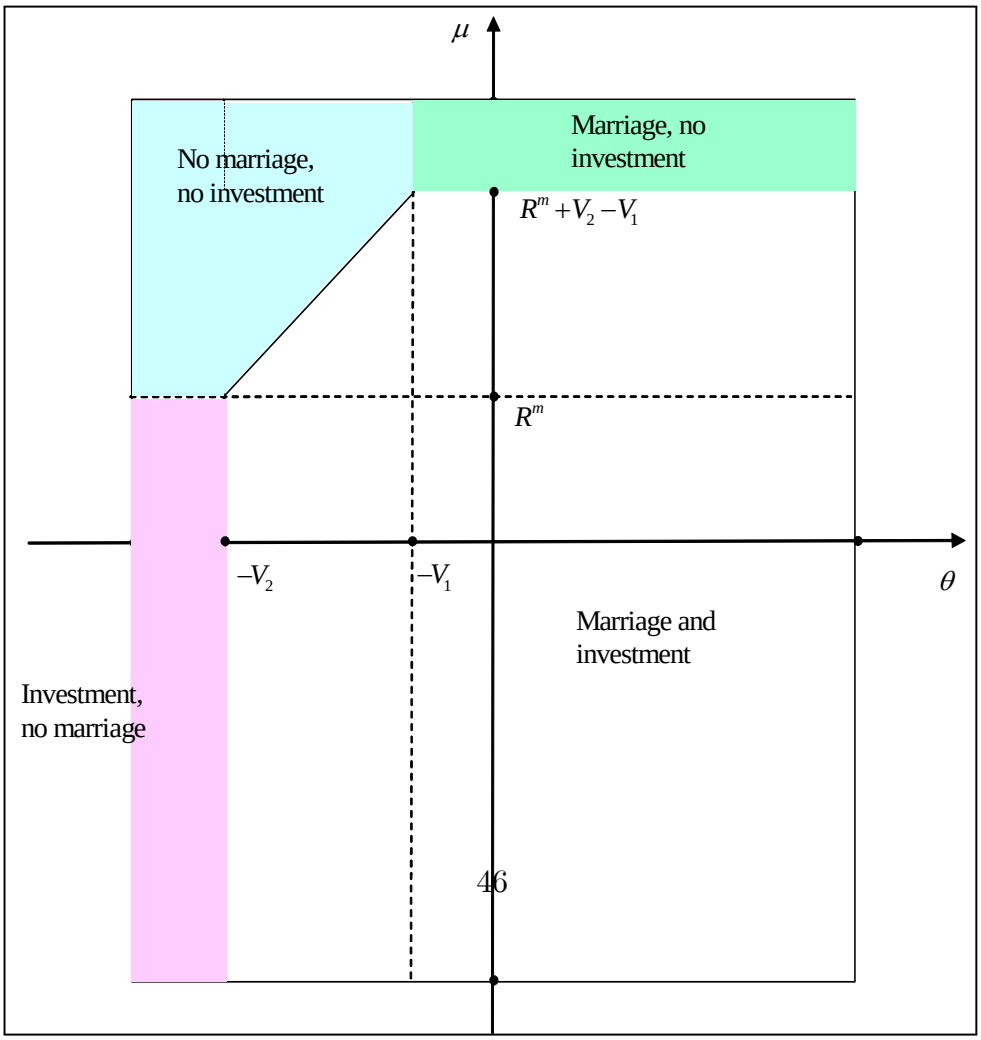

μ
No marriage, no investment
Marriage, no investment
$R^m + V_2 - V_1$
R^m
$-V_2$
$-V_1$
θ
Marriage and investment
Investment, no marriage

Figure 6: Equilibrium with Strictly Positive Assortative Matching

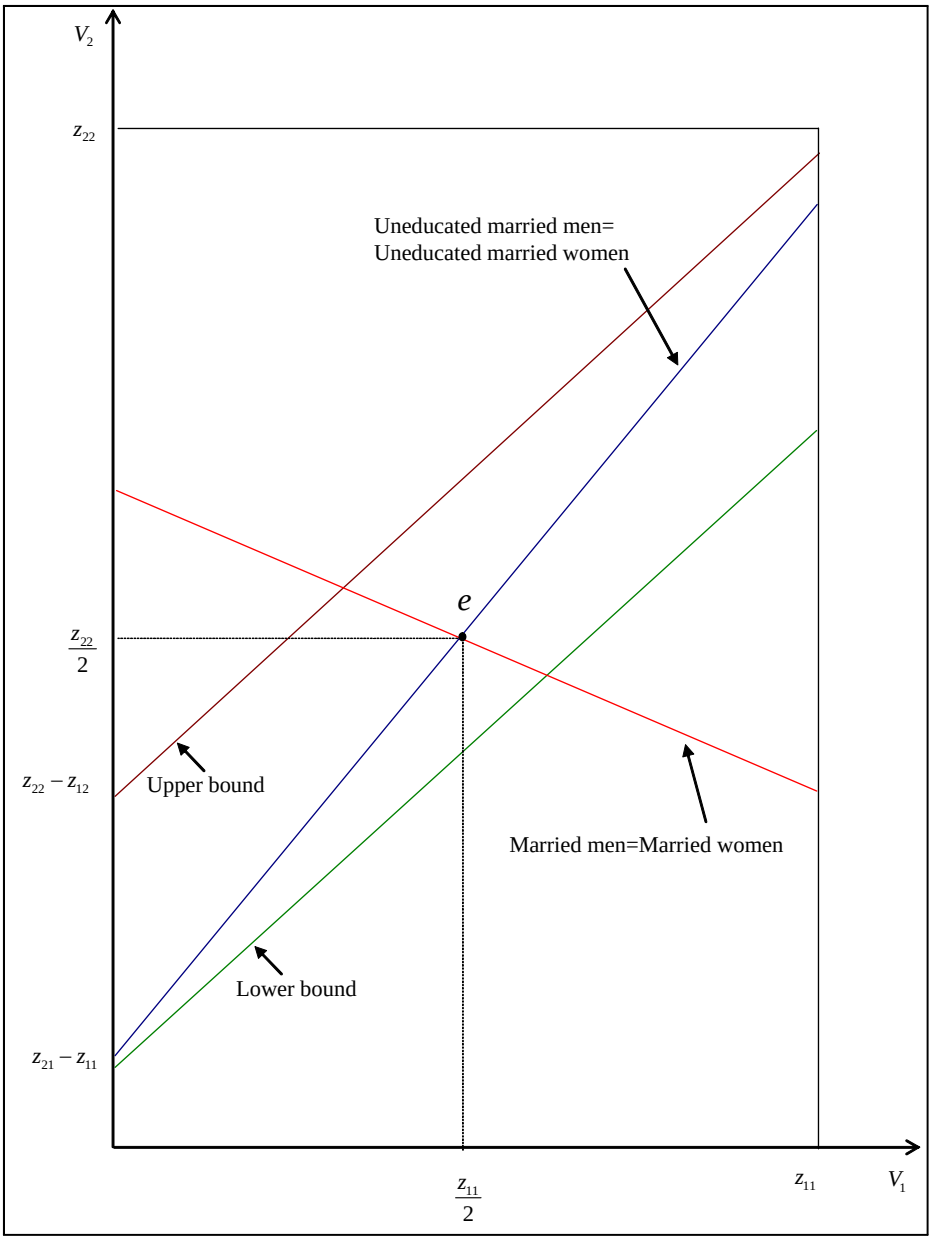


Figure 7: Mixed Equilibrium with More Educated Men than Educated Women

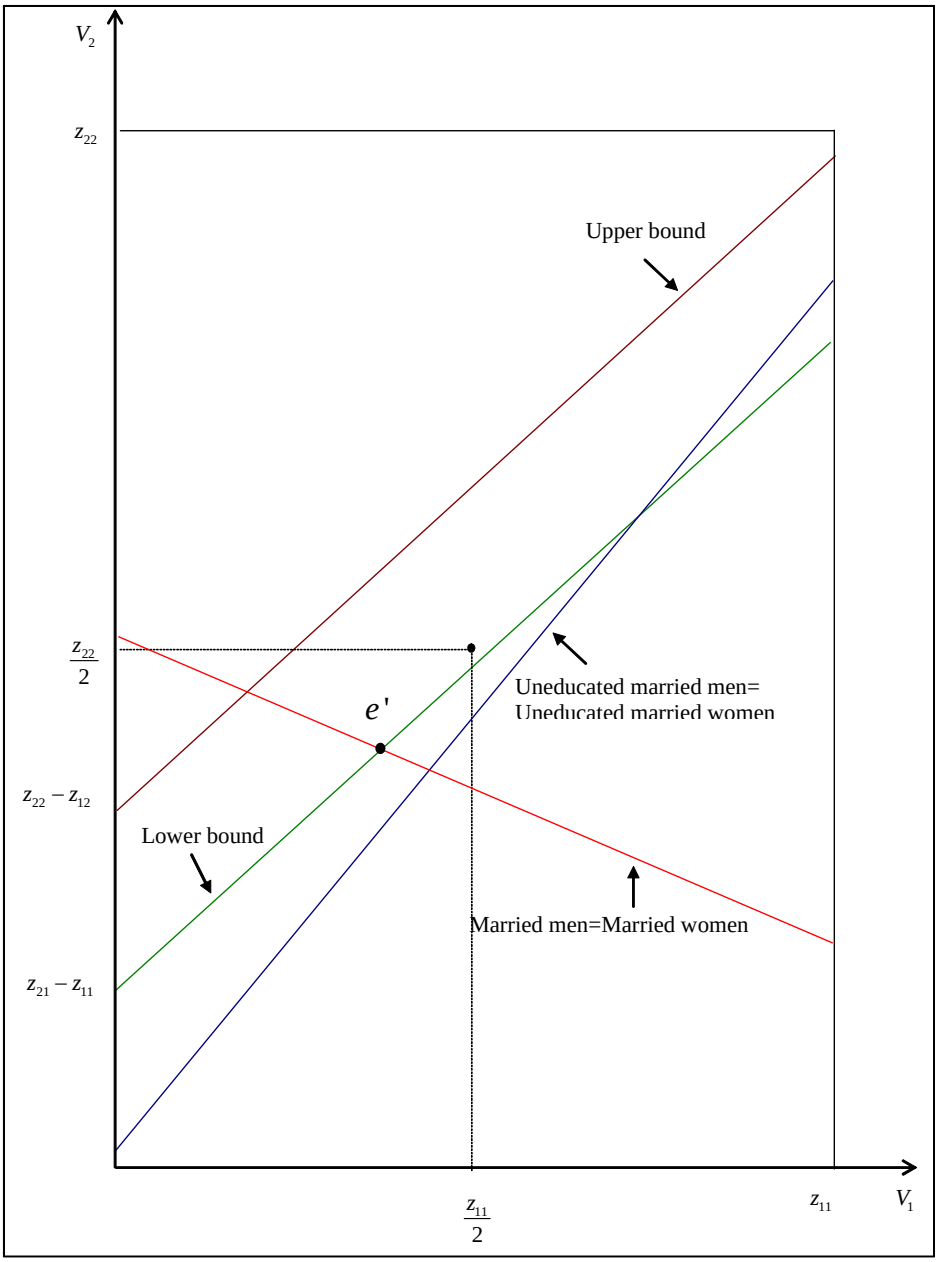


Figure 8: The Impact of an Increase in the Wage of Educated Men Combined with a Reduction in the Wage of Educated Women

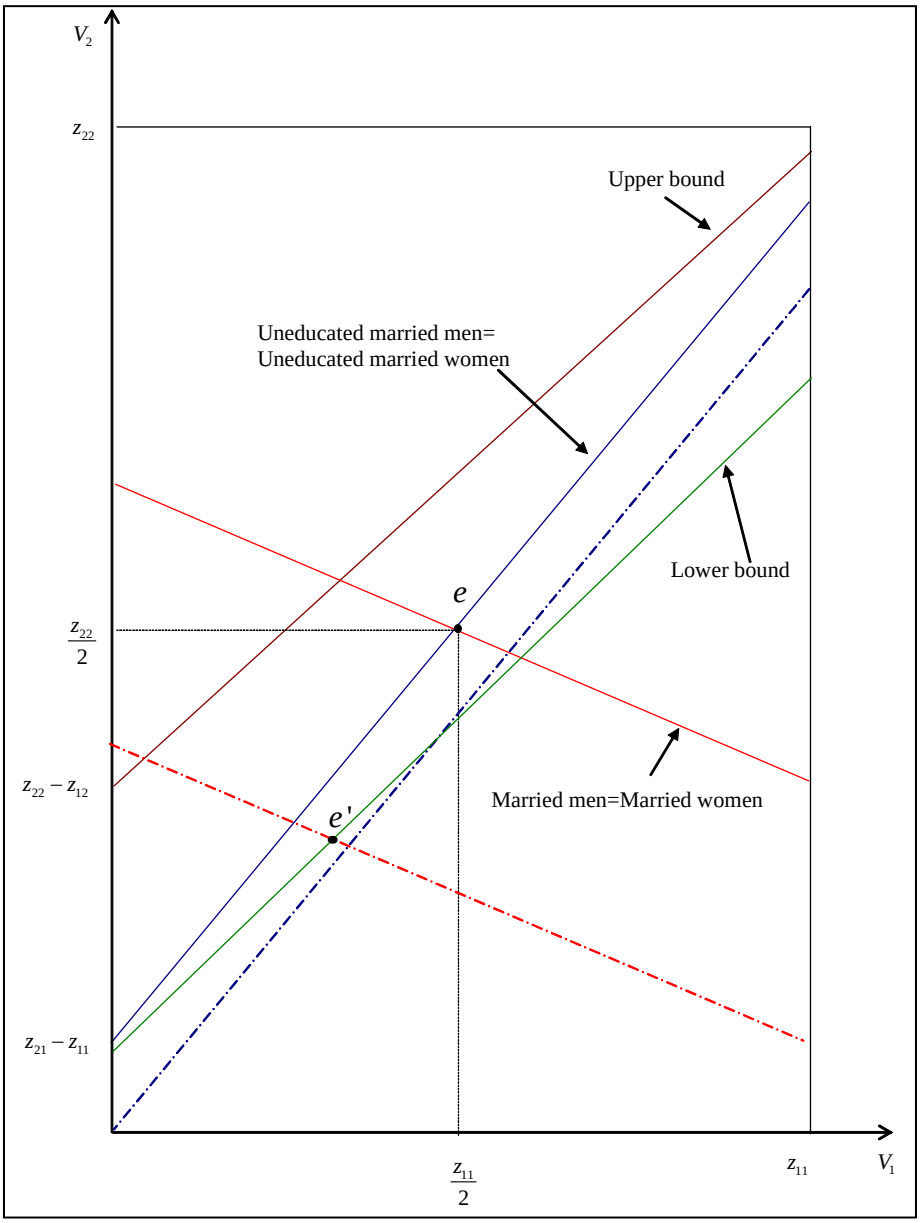


Figure 9: The Impact of an Increase in the Wife's Work at Home

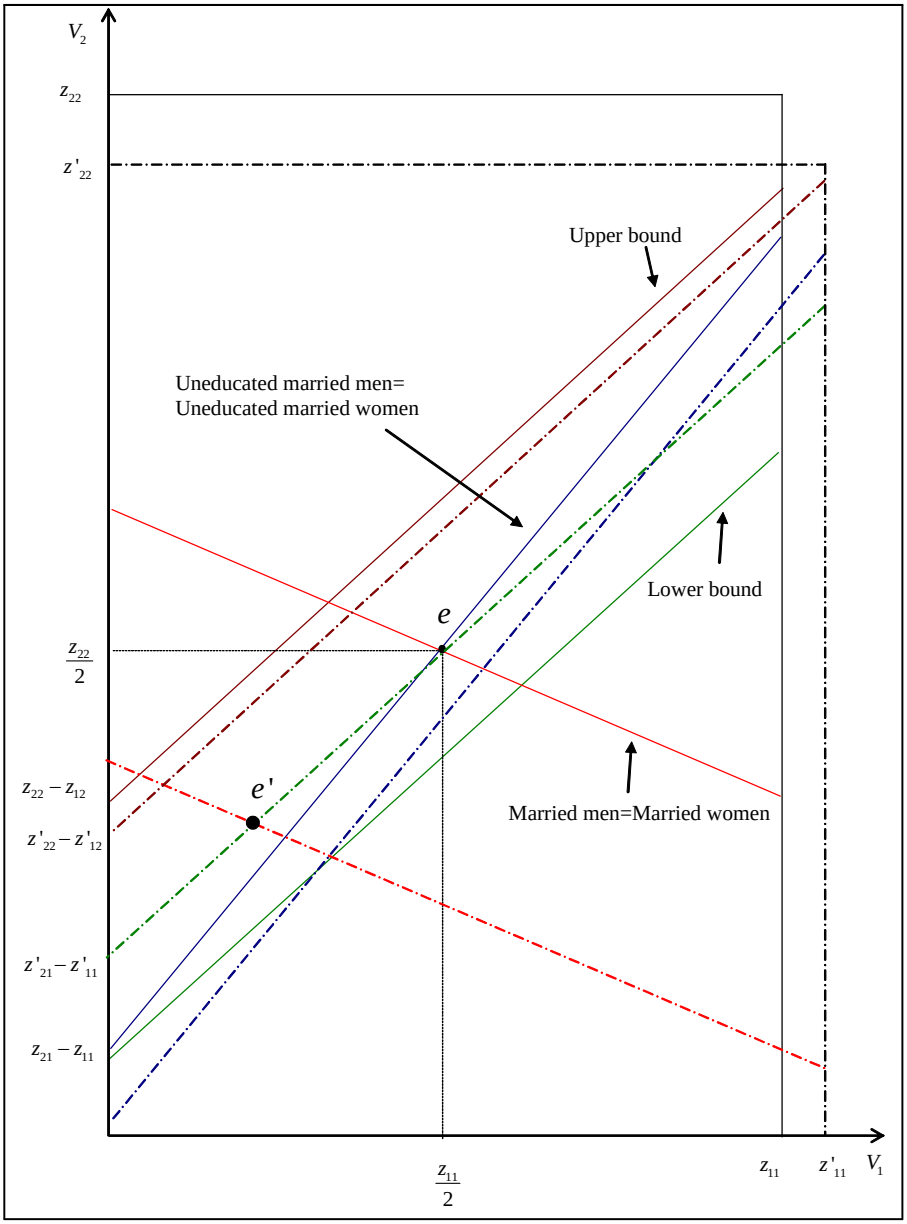


Figure 10: The Impact of a Decrease in the Wife's Work at Home Combined with an Increase in the Wage of Educated Women