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PAYGO or Funded Social Security? A General Equilibrium Comparison

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Abstract

Social security systems in most developed economies are facing sustainability challenge due to demographic trends. Therefore social security reform proposals are on the agenda in many countries. This paper demonstrates that the analysis of fiscal sustainability of social security must include an additional dimension of public policy, namely education funding, which affects the productivity growth of future workers. This fact is true under both pay-as-you-go (PAYG) and fully funded (FF) social security system. While FF systems are known to be more conducive of national saving, some economists have conjectured that PAYG system have an advantage of an “intergenerational compact” leading to a relatively stronger political support for public education funding. This paper demonstrates that the latter conjecture may not be true in a closed economy framework. We consider an OLG economy where government, in addition to running social security, also funds education of future workers by means of taxes collected from the current ones. The education tax rates are chosen, in each period, by a majoritarian rule among the relevant constituents. We demonstrate, under conditions plausible for developed economies, that while the FF system produces political support for relatively higher (compared to PAYG) education funding, hence higher rate of human capital accumulation, as well as relatively higher rates of physical capital accumulation and economic growth, Furthermore, we show that under some meaningful conditions the FF system also results in lower degree of agents’ lifetime income inequality.

1. Introduction

Social security systems exist in all developed economies and, moreover, it represents the largest public program in most. The demographic trends in all these economies lead to a decreasing dependency ratio in the public pension systems and are therefore rightly viewed as the signs of looming crisis of public pension systems, raising doubts about future solvency of existing programs. Hence the social security reform proposals on the immediate public policy agenda. The public policy debate has stimulated and has relied on empirical and theoretical analyses that evaluate the prospects of sustainability of public social security programs and those of some fully or partially funded alternatives, as well as their implications for economic growth (Diamond (1999), Feldstein (2005)).

While this literature focuses on the demographic trend of the dependency ratio for its predictions, a large part of it treats dynamics of future productivity as exogenously given. Many other papers, e.g., Diamond and Orszag (2005), Hines and Taylor (2005), Sinn (2000), similarly unequivocal about the looming insolvency, rightly observe that dynamics of future productivity is itself negatively affected by the growth of social security program's obligations, due to the depressing effect on private saving. The effect of introducing a pay-as-you-go (PAYG) social security on savings and welfare has been widely discussed in the literature going back to Feldstein (1974).[2] Most of this literature, however, tended to overlook the fact that social security system may affect future productivity also through the channel of human capital accumulation. A notable exception, in direct response to Feldstein (1974), was the paper by Pogue and Sgontz (1977). They pointed out that a PAYG social security system creates incentives (both individual and collective) for investment in younger generation's education.

[2] Karni and Zilcha (1989) provided a general equilibrium analysis of this problem within an OLG framework.

Furthermore, they argued that a portion of the consumption increase in the data, interpreted by Feldstein (1974) as an evidence of dissaving triggered by the expected pension transfers, in fact consisted of such human capital investment. Thus, they conjectured that "the introduction of [PAYG] social security has led to a substitution of human for physical capital" and pointed to the post-1938 trends consistent with this claim. Becker and Murphy (1988) also argued that PAYG social security strengthens political support among the current workers for public investment in the education of the future ones; they furthermore pointed out that this incentive is stronger for the middle and lower income populations.

Our intended contribution in this paper is to provide a rigorous theoretical analysis of the above conjecture in a general equilibrium framework and moreover to explore the comparative effects of alternative social security systems on human capital accumulation and through it on labor productivity and economic growth. Indeed, we note that the incentives to to enhance productivity of future workforce is present not only under PAYG pension system, where benefits are directly dependent on the stream of payroll taxes paid by future workers, but also in the case of fully funded social security where investment return on social security's funds too depends on future labor productivity. Therefore the human capital dimension is essential for a comparative analysis of PAYG and fully funded social security systems, which is the focus of this paper.

The role of both public and private investment in education in the relationship between social security funding and economic growth was noted and analyzed for the case of PAYG social security by Kaganovich and Zilcha (1999), Glomm and Kaganovich (2003) and Köthenbürger and Poutvaara (2006).[3] A branch of recent literature which includes Bellettini and

[3] Zhang and Zhang (1998) and Pecchenino and Utendorf (1999) have analyzed the impact of PAYG social security system on <u>privately</u> funded education and through it on growth. Docquier and Paddison (2003) compare the implications of PAYG and fully funded social security regimes for growth via individual incentives to invest (privately) in education. Under some conditions

Ceroni (1999), Kemnitz (2000), Boldrin and Montes (2005), Poutvaara (2006) and Soares (2006), examines the relationship between public provision of education and PAYG social security as an issue of political economy and intergenerational contract. The present paper is the first to our knowledge to focus, in a dynamic general equilibrium framework, on the comparison of PAYG and fully funded social security systems in terms of the incentives they generate for public funding of education, which plays the dominant role in education systems of most developed countries.[4] In particular, we consider the implications of the alternative pension regimes for political determination of public education funding levels, hence for growth in a general equilibrium framework.

In most cases major social security reform proposals discussed in the US and Europe entail a transition from PAYG to a fully funded system. The predominance of public funding of education is also a common feature of these developed economies. We therefore undertake a comparative analysis of PAYG and fully funded social security systems in an economy where government, in addition to running social security trust fund, also finances education of future workers by means of taxes collected from the current workers. We consider a closed overlapping generations economy where individuals differ in the levels of human capital they attain and thereby in the levels of income. In this framework, public funding of education is provided by the government uniformly to all young agents, whereas inequality of attainment arises due to unequal innate abilities and parental inputs. Public funds are budgeted through a dedicated education tax on working adults. We further assume that education tax rate is determined in each

they find that the effect of a fully funded system may be positive, while the effect of a PAYG system is negative. Lambrecht et al (2005) qualify that the above conclusion about the negative effect of PAYG systems may be overturned in economies where private bequest motives are inoperative.

[4] Iturbe-Ormaetxe and Valera (2004) raise a similar question for the case of small open economy; their analysis is limited to numerical examples in a static (two-period) model..

period through a majoritarian political mechanism. Contrary to Pogue and Sgontz (1977) conjecture we demonstrate, within a dynamic political equilibrium in a closed economy, that under meaningful conditions (among which the assumption that median voter's earnings do not exceed the average is of essence) the fully funded regime dominates the PAYG regime in terms of human capital accumulation. Specifically, we show (in our Theorem 1) that when the education tax rates are politically determined, the rates will be relatively higher under the fully-funded social security regime.

The intuition for this somewhat surprising result can be derived from the fact that an agent's choice of the education tax level is driven by two motives: (i) the effect of the tax on the present value of his social security benefits, and (ii) its effect on the rate of return on his private savings (the details of this analysis are provided in Section 4 of the paper). Under the fully funded system, future pensions of the current generation of workers are financed by the returns on investment of the social security trust fund which is formed by the payroll tax contributions by those current workers. Therefore, under the fully funded social security the chosen tax rate has no effect on the present value of social security benefits (note that the cost of the tax obviously rises with one's wage income). In contrast, we show that under the PAYG system while the nominal future value of social security benefits will be enhanced by greater investment in education, this effect will be counteracted by an increase in the interest rate ("the factor price effect"), such that moreover the total effect on the present value of social security benefits will be negative. As a result, the first motive ends up negative under PAYG, while being neutral in the fully funded regime. While the second motive (the rate of return) is present and positive under both social security regimes, we find that the advantage of the fully funded regime in terms of the first motive dominates, which results in the choice of a relatively higher education tax rate (compared to PAYG scenario) by below average wage earners for whom the cost of the tax is lower. Thus the factor price effect on the present value of social security benefits in a closed economy is responsible for the breakdown of the conjecture that the "intergenerational compact" inherent in a PAYG scheme creates stronger incentives to fund public education than a funded pension system.

Further, we combine the established dominance of the fully funded system in terms of the political support for education funding with the well known advantage of the funded system in terms of physical capital accumulation and show (see our Theorem 2) that under plausible conditions fully funded social security regime, compared to PAYG, indeed generates consistently higher levels of aggregate physical and human capital stocks, hence higher output levels along the dynamic political equilibrium path.[5] Next we turn to the comparison of the alternative social security regimes on inequality and find that under some conditions which appear plausible for developed economies, fully funded system will results in lower degree of agents' lifetime income inequality along the dynamic political equilibrium program.

The paper is organized as follows. Section 2 sets up of the model, defines the alternative social security regimes, as well as the dynamic general equilibrium. Dynamic general equilibrium solutions are derived recursively in Section 3. Section 4 introduces political process, defines dynamic political equilibria and obtains the main results (Theorems 1 and 2) comparing the effects of the alternative social security systems on human and physical capital accumulation and economic growth. Section 5 addresses the income inequality dimension. Section 6 concludes.

[5] In a precursor to the present paper, Kaganovich and Zilcha (2008), we showed that the dominance of the fully funded regime over PAYG in terms of human capital accumulation (but not in terms of physical capital capital accumulation and the overall economic growth) can be overturned under a stringent assumption that the median voter's earnings are substantially above the average.

2. The Model

We study an overlapping generations economy populated by heterogeneous family dynasties, indexed by the family name $\omega \in \Omega$. The only sources of heterogeneity are the differences of human capital levels of the members of the initial generation in period t=0 and the (random) innate ability. Each generation consists of agents whose adult life has two periods of equal lengths: the young adult age during which each agent inelastically supplies one unit of labor time to work and raises one offspring, and, subject to survival, the old age spent in retirement. Since each young adult produces one offspring, the population remains constant in every generation. Let μ be the Lebesgue measure on Ω. Without loss of generality we set the measure of individuals born in each generation $\mu(\Omega) = 1$. Thus in each time period there is measure 1 of workers and measure 1 of children. At the end of the working period, everyone faces a lottery: dying immediately, or living throughout the entire retirement period. Implicitly, we assume a 'childhood period' in which education is attained and no economic decisions are made. The probability of survival p is identical for all individuals. Since the measure of working population is always 1, this means that the measure of retired population is always equal to p. We label the generation whose young adult age occurs in time period t as "generation t". An individual ω who belongs to generation t is endowed when entering the adulthood period with the stock of human capital $h_t(\omega)$ which also defines his effective labor capacity.

Education Sector

Human capital $h_{t+1}(\omega)$ of a young individual in generation t+1 is produced by using a uniform public education input, but the individual's attainment depends on his parent's level of human capital $h_t(\omega)$, which reflects a well established factual such relationship. The public input at date t is given by uniform and universal public expenditure X_t on educating each student of generation t+1. We assume the following form of the human capital production function:

(1) $$h_{t+1}(\omega) = b_{t+1}(\omega)(h_t(\omega))^{\sigma} X_t^{1-\sigma}$$

where $0 < \sigma < 1$ while $b_{t+1}(\omega)$ is a random ability parameter which is distributed on Ω, identically at all times with probability measure P(.) independently of $h_t(\omega)$. Furthermore, realizations of $b_{t+1}(\omega)$ are uniformly bounded above and below by positive numbers.

According to the expression (1) human capital formation is affected by a public component represented by the public spending on education X_t (in both per student and aggregate terms due to our convention that population measure is 1 at all times) as well as a private "home" education component which we assume to be proportionate to human capital level of a student's parent. Thus we can interpret the coefficient σ as a measure of efficiency of parental factor in education while $1-\sigma$ characterizes the role of the public component. These parameters may vary across countries reflecting cultural differences in the relative roles of home and public schooling in educating a child.

Decisions of Individual Agents

Working adults of generation t allocate their after-tax wage income $(1-\tau_t-\theta_t)w_t h_t(\omega)$ between current consumption $c_{t,t}$ and saving s_t, thus they face individual budget constraints

(2) $$c_{t,t}(\omega) + s_t(\omega) = (1-\tau_t-\theta_t)w_t h_t(\omega)$$

where w_t is the current competitive wage rate per unit of the effective labor while τ_t and θ_t stand for current (uniform and flat) tax rates earmarked for social security and public education expenditures, respectively, which will be discussed in more detail later.

We assume that public pension benefits T_t are uniform across (living) retirees in a given period t. In view of the agents' uncertain survival to retirement, and the lack of bequest motive we assume that they make their private savings in the form of actuarially fair annuities. Individuals are also allowed to borrow against their future assets, in which case the variable $s_t(\omega)$ has a negative value. The same type of actuarial notes (see Yaari (1965)) can be used to borrow from future (random) income (i.e., buying life insurance). Thus, negative savings are allowed at higher interest rates that reflects the possibility that payback may not occur (in case of death). This corresponds to negative savings which is equivalent to buying life insurance. Let R_{t+1} denote the gross rate of return on private savings of generation t. Then the retirement period budget constraint faced by a surviving member of generation t is given by:

(3) $$c_{t,t+1}(\omega) = R_{t+1}s_t(\omega)/p + T_{t+1}$$

We assume that fair actuarial notes can be traded in the market hence actuarially fair annuities and life insurance policies are traded via this single financial instrument (see Yaari (1965) for a detailed discussion).

Each individual in generation t determines the values of his decision variables $c_{t,t}(\omega), c_{t,t+1}(\omega), s_t(\omega)$ so as to maximize the expected utility of his life-time consumption[6]:

(4) $$\max EU(c_{t,t}(\omega), c_{t,t+1}(\omega)) = \ell n c_{t,t}(\omega) + p\beta \ell n c_{t,t+1}(\omega)$$

[6] This expression obviously ignores the additional altruistic motivation which is typically factors in an individual's choices affecting education attainment of his child. The inclusion of corresponding components of individual preferences is common in the literature. Bearing in mind that the focus of this study is the comparison of outcomes under alternative pension system arrangements, we have found that if the parental altruism component is added on to preferences, this will have a identical effect, ceteris paribus, on the outcomes under either of the pension systems in question. Therefore this would add nothing to the essence of our analysis and would not justify the associated greater complexity. The logarithmic form of preferences is likewise chosen for the sake of simplicity: we have been able to verify that our conclusions remain intact under CRRA preferences.

subject to the budget constrains (2)-(3), where the intertemporal discount factor $\beta \in (0,1)$, and the economy's variables w_t, R_{t+1}, T_t, τ_t, θ_t are taken as given.

Government Education and Social Security Budgets

As stated earlier, public education expenditure X_t is funded by a dedicated uniform tax on current wage income, thereby the education budget, assumed balanced at all times, is given by the equation

$$X_t = \theta_t w_t H_t \tag{5}$$

where

$$H_t = \int_{\Omega} h_t(\omega) d\mu(\omega) \tag{6}$$

is the aggregate supply of effective labor, i.e., the human capital, so that $w_t H_t$ is the aggregate (as well as per capita, since population measure is normalized to one) labor income in period t. We will initially assume that the education tax rates θ_t are exogenously given, but later, in Section 4 of the paper, we will incorporate it in a political economy model where in each time period θ_t is determined by a majoritarian rule among the relevant constituents.

In this paper we analyze two alternative funding arrangements for a defined contribution public pension system, both the focal points of the public debates on social security: a *fully funded* (FF) system and a *pay-as-you-go* (PAYG) system. Consistent with this focus, our purpose is to compare economic outcomes (in terms of human and physical capital accumulation, economic growth and income distribution) in the parallel universes characterized by the alternative pension systems whose magnitudes are exogenously given and identical. More specifically, we assume that the social security payroll tax is given by a fixed flat rate τ for either system.

Under the FF social security system, the pension benefits received in period t+1 by all surviving retired individuals of generation t are funded by the proceeds from the payroll tax collected at the rate τ in period t from all workers of this same generation. Thereby, the balance of the social security fund, maintained at all times, is given by the equation

(7) $$T_{t+1} = R_{t+1}\tau w_t H_t /p$$

Thus the social security tax revenue collected from generation t workers is invested in the economy; the gross returns on this investment are then redistributed uniformly in period t+1 to surviving retirees.

Under the PAYG system, the pension benefits received by the generation t retirees are paid for by the payroll tax, at the flat rate τ , on the contemporary workers, i.e., the young adults of generation t+1. Then the PAYG social security benefit received by each surviving member of generation t (retiree) is given by

(8) $$T_{t+1} = \tau w_{t+1} H_{t+1} /\mathrm{p}$$

Thus unlike the fully funded system where social security fund is a part of national savings, under the PAYG system the payroll tax collected from generation t+1 workers passes through directly to the pension beneficiaries, i.e. they are redistributed among the contemporary retirees in this same time period t+1.

Production Economy and Public Finance

We assume the standard form of the aggregate production function:

(9) $$Y_t = AK_t^{\delta} H_t^{1-\delta}$$

where parameters satisfy $A > 0$, $0 < \delta < 1$, Y_t is total output, K_t is the aggregate stock of physical capital, which is financed by the savings of the previous generation.

In the case of FF public pension system, the aggregate savings include, in addition to the private component, also the savings in the social security trust. Thus one can write

(10) $$K_{t+1} = \int_{\Omega} s_t(\omega) d\mu(\omega) + \tau_t w_t H_t$$

When the social security system is PAYG, i.e., the social security payroll tax is spent in the period when it is collected, there are only private savings in the economy, so that the aggregate physical capital in period t+1 is given by

(11) $$K_{t+1} = \int_{\Omega} s_t(\omega) d\mu(\omega)$$

Based on the above descriptions we can now define the dynamic competitive equilibria in this overlapping generations model.

Definition. Given the initial stock of physical capital K_0, the initial distribution of human capital $h_0(\omega)$ and the sequence of education tax rates $\{\theta_t\}_{t=0}^{\infty}$, a *dynamic competitive equilibrium (DCE)* is a collection of sequences of distributions of individual household decisions $\{c_{t,t}(\omega), c_{t,t+1}(\omega), s_t(\omega)\}_{t=0}^{\infty}$, sequences of distributions of individual levels of human capital $\{h_{t+1}(\omega)\}_{t=0}^{\infty}$, and of aggregate amounts of physical capital and effective labor $\{K_t, H_t\}_{t=0}^{\infty}$, sequences of factor prices $\{w_t, R_t\}_{t=0}^{\infty}$, as well as the sequences of government expenditures $\{T_t,\ X_t\}_{t=0}^{\infty}$ such that:

(i) For each $\omega \in \Omega$ and t=0,1,…, the collection $c_{t,t}(\omega)$, $c_{t,t+1}(\omega)$, $s_t(\omega)$ solves the individual household's problem (2)-(4) where factor prices w_t, R_{t+1}, social security transfers T_{t+1} and the current tax rates are taken as given;

(ii) Labor markets clear, i.e. given the individual human capital attainments evolve according to (1), the aggregate amount of effective labor H_t employed in period t is determined by formula (6);

(iii) Physical capital market clears, i.e., the aggregate stock of physical capital K_{t+1} employed in period t+1 equals to aggregate investment (saving) made in period t. Under the <u>fully funded</u> arrangement of the social security system this means that K_{t+1} is determined by the relationship (10), whereas under the <u>PAYG</u> system the relationship (11) applies instead.

(iv) Factor markets are competitive, hence according to the economy's production function (9) the factor prices are determined by their marginal products:

(12) $$R_{t+1} = \delta Y_{t+1} / K_{t+1} = \delta A K_{t+1}^{\delta-1} H_{t+1}^{1-\delta}$$

(13) $$w_t = (1-\delta) Y_t / H_t = (1-\delta) A K_t^{\delta} H_t^{-\delta}$$

(v) Government expenditures on education X_t are determined according to formula (5), where the aggregate wage income as well as the education tax rate are given;

(vi) Social security benefits received by generation t retirees are always defined by the formula (7) or (8) – respectively under the FF or PAYG arrangement.

3. Benchmark Solutions

In this section we will solve the model and derive recursive dynamic relationships that define the DCE under each of the pension systems and exogenously given corresponding sequences of education tax rates $\left\{\theta_t^F\right\}_{t=0}^{\infty}$ and $\left\{\theta_t^G\right\}_{t=0}^{\infty}$. In some instances dictated by the need of reference or clarification we will mark DCE parameters, variables and value functions corresponding to the

FF system with a superscript ^{F}; likewise the superscript ^{G} will be used in such instances in the PAYG case. When redundant, we'll drop the superscripts for simplicity.

Fully Funded Social Security System

Since pension benefit under the fully funded system is defined by the expression (7), the old-age budget constraint (3) has the form

(14) $$c_{t,t+1}(\omega) = R_{t+1}s_t(\omega)/p + R_{t+1}\tau w_t H_t/p$$

Therefore the first-order necessary (as well as sufficient, due to concavity of the logarithmic utility function) condition of optimum in the household problem (2)-(4) is given by the equation

$$\frac{p\beta}{s_t + \tau w_t H_t} = \frac{1}{(1-\tau-\theta_t)w_t h_t - s_t}$$

(note that no negativity constraints were imposed on variable s_t). Solving this system we obtain

(15) $$s_t(\omega) = \frac{1}{1+p\beta}[p\beta(1-\tau-\theta_t)w_t h_t(\omega) - \tau w_t H_t]$$

Substitution of this in (2) yields

(16) $$c_{t,t}(\omega) = \frac{1}{1+p\beta}[(1-\tau-\theta_t)w_t h_t(\omega) + \tau w_t H_t]$$

Similarly, substituting (15) in equation (14) and then applying formula (12) we obtain

(17) $$\begin{aligned} c_{t,t+1}(\omega) &= R_{t+1}\frac{\beta}{1+p\beta}[(1-\tau-\theta_t)w_t h_t(\omega) + \tau w_t H_t] = \\ &= \delta A H_{t+1}^{1-\delta} K_{t+1}^{\delta-1}\frac{\beta}{1+p\beta}[(1-\tau-\theta_t)w_t h_t(\omega) + \tau w_t H_t] \end{aligned}$$

We now integrate equation (15) to obtain

$$\int s_t(\omega)d\mu(\omega) = \frac{1}{1+p\beta}[p\beta(1-\tau-\theta_t) - \tau]w_t H_t$$

Combining this with the relationship (10) yields

(18) $$K_{t+1}^F = \frac{1}{1+p\beta} p\beta(1-\theta_t^F)w_t H_t^F$$

Integrating the expression (1) and using formula (5) and the assumption that child's ability $b_{t+1}(\omega)$ is distributed independently of his parents' human capital $h_t(\omega)$ we get the following expression for the dynamics of the aggregate human capital:

(19) $$H_{t+1}^F = B[\theta_t^F w_t H_t^F]^{1-\sigma} \int [h_t(\omega)]^\sigma d\mu(\omega)$$

where $B = \int b(\omega)dP(\omega)$.

Relationships (18) and (19) show that both K_{t+1} and H_{t+1} are uniquely determined by the choice of the education tax rate θ_t, as long as the prior period's economic variables are given. These aggregate stocks, in turn determine equilibrium wage and interest rates in period t+1, according to competitive relationships (12)-(13).

The above analysis shows that the dynamic competitive equilibrium has Markov property, i.e. current DCE variables can be determined recursively based on their values in the previous period as well as the education tax rate given for that period. Indeed, we have shown that given the DCE fundamentals at time t, namely the stocks of physical capital K_t and human capital H_t, the distribution of individual levels of human capital $\{h_t(\omega)\}$, and the education tax rate θ_t, one can uniquely determine the next period's fundamentals K_{t+1}, H_{t+1}, the distribution of individual household consumption and saving decisions characterized by (15) and (using formulae (1) and (5)) the distribution of human capital $\{h_{t+1}(\omega)\}$. Note that DCE wage and interest rates are also uniquely determined by the economy's contemporary fundamentals according to (12)-(13).

The recursive determination of DCE also implies that welfare of households in generation t attained in DCE, i.e. the maximum value function in (4), are uniquely determined by the education tax rate θ_t levied from them. This justifies their notation as $V_t(\omega, \theta_t)$ for $\omega \in \Omega$. Substituting the optimal solution given by (16) and (17) into the utility function (4) and then applying relationships (18) and (19) we obtain:

$$\begin{aligned} V_t^F(\omega,\theta_t) &= (1+p\beta)\ln[(1-\tau-\theta_t)h_t(\omega)+\tau H_t] + p\beta \ln R_{t+1} + D_t = \\ &= (1+p\beta)\ln[(1-\tau-\theta_t)h_t(\omega)+\tau H_t] + p\beta(1-\delta)[\ln(H_{t+1}) - \ln(K_{t+1})] + D_t \end{aligned}$$

where D_t is an expression that does not depend on individual decision variables or θ_t. We transform the argument of the first logarithmic function in the above expression:

$$(1-\tau-\theta_t)h_t(\omega)+\tau H_t = h_t(\omega)[1-\theta_t-\tau(1-H_t/h_t(\omega))]$$

and denote

(20) $$g_t(\omega) = \tau(1-H_t/h_t(\omega))$$

-- a variable which corresponds to deviation of individual human capital and income from respective current averages. Substituting this expression into the above welfare function we can rewrite as

(21) $$V_t^F(\omega,\theta_t) = (1+p\beta)\ln[1-\theta_t-g_t(\omega)] - p\beta(1-\delta)\ln(1-\theta_t) + p\beta(1-\delta)(1-\sigma)\ln\theta_t + D_t^1$$

where D_t^1 is again an expression that does not depend on individual decision variables or θ_t.

This is clearly a strictly concave function of parameter θ_t, as a composition of a linear and a strictly concave function.

Pay-as-you-go Social Security System

Derivations here are similar to those obtained for the case of fully funded system, with differences due to the fact that relationships (8) and (11) need to be used in place of (7) and (10), respectively. According to (8), the old-age budget constraint (3) under the PAYG system has the form

(22) $$c_{t,t+1}(\omega) = R_{t+1}s_t(\omega)/p + \tau w_{t+1}H_{t+1}/p$$

Therefore the first-order necessary and sufficient condition of optimum in the household problem (2)-(4) is given by the equation

$$\frac{p\beta}{s_t + \tau w_{t+1}H_{t+1}/R_{t+1}} = \frac{1}{(1-\tau-\theta_t)w_t h_t - s_t}$$

Solving this system and using relationships (12) and (13) we obtain

(23) $$(1+p\beta)s_t(\omega) + \tau(1-\delta)\delta^{-1}K_{t+1} = p\beta(1-\tau-\theta_t)w_t h_t(\omega)$$

so that

(24) $$s_t(\omega) = \frac{1}{1+p\beta}[p\beta(1-\tau-\theta_t)w_t h_t(\omega) - \tau(1-\delta)\delta^{-1}K_{t+1}]$$

Substituting this in relationship (2) we obtain

(25) $$c_{t,t}(\omega) = \frac{1}{1+p\beta}[(1-\tau-\theta_t)w_t h_t(\omega) + \tau(1-\delta)\delta^{-1}K_{t+1}]$$

Similarly, substituting (24) in (22) and then using relationships (12)-(13) results in

(26) $$\begin{aligned} c_{t,t+1}(\omega) &= \frac{R_{t+1}}{p}\left[\frac{p\beta}{1+p\beta}(1-\tau-\theta_t)w_t h_t(\omega) - \frac{1}{1+p\beta}\tau(1-\delta)\delta^{-1}K_{t+1}\right] + \frac{R_{t+1}}{p}\frac{\tau w_{t+1}H_{t+1}}{R_{t+1}} \\ &= \frac{R_{t+1}}{p}\frac{p\beta}{1+p\beta}\left[(1-\tau-\theta_t)w_t h_t(\omega) + \tau(1-\delta)\delta^{-1}K_{t+1}\right] \end{aligned}$$

Further, according to (6) and (11) the integration of (23) yields

$$[1+p\beta+\tau(1-\delta)\delta^{-1}]K_{t+1} = p\beta(1-\tau-\theta_t)w_t H_t$$

which leads to

(27) $$K_{t+1}^G = \frac{p\beta(1-\tau-\theta_t^G)w_t H_t}{1+p\beta+\tau(1-\delta)\delta^{-1}}$$

Note also that relationship (19) applies here without change, so we rewrite for future reference:

(28) $$H_{t+1}^G = B[\theta_t^G w_t H_t^G]^{1-\sigma}\int [h_t(\omega)]^\sigma d\mu(\omega)$$

The above analysis confirms (see a similar explanation in the case of fully funded system) a recursive determination of the dynamic competitive equilibrium: given the stocks K_t and H_t , the distribution $h_t(\omega)$, as well as the education tax rate θ_t , the above relationships (27), (28) and (1) along with (5) will uniquely determine the next period's fundamentals K_{t+1} , H_{t+1} , distribution of human capital $h_{t+1}(\omega)$.

To obtain the maximum welfare value function $V_t(\omega, \theta_t)$ of a generation t agent ω we first substitute the expressions for the young- and old-age consumption given by formulae (25) and (26), respectively, into the utility function (4), then apply relationships (12), (13) :

(29) $$\begin{aligned} V_t^G(\omega,\theta_t) &= (1+p\beta)\ln\left[(1-\tau-\theta_t)w_t h_t(\omega)+\tau(1-\delta)\delta^{-1}K_{t+1}\right]+p\beta\ln R_{t+1}+G_t \\ &= (1+p\beta)\ln\left[(1-\tau-\theta_t)w_t h_t(\omega)+\tau(1-\delta)\delta^{-1}K_{t+1}\right]+p\beta(1-\delta)[\ln(H_{t+1})-\ln(K_{t+1})]\ +G_t \end{aligned}$$

where G_t is an expression that does not depend on decision variables or θ_t . Now we use expressions (27) and (28) to further derive:

(30) $$\begin{aligned} V_t^G(\omega,\theta_t) &= (1+p\beta)\ln(1-\tau-\theta_t)+p\beta(1-\delta)(1-\sigma)\ln\theta_t - p\beta(1-\delta)\ln(1-\tau-\theta_t)+G_1 \\ &= (1+p\beta\delta)\ln(1-\tau-\theta_t)+p\beta(1-\delta)(1-\sigma)\ln\theta_t + G_t^1 \end{aligned}$$

where G_t^1 is an expression that does not depend on decision variables or θ_t . This is a strictly concave function of parameter θ_t .

4. Comparative Analysis of Dynamic Political Equilibrium

We now define the political economy mechanism that determines the sequence of education tax rates θ_t. Recall that the dynamic competitive equilibrium (DCE) (see its definition in Section 2) corresponds to a sequence of education tax rates $\{\theta_t\}_{t=0}^{\infty}$ assumed to be exogenously given. As demonstrated in Section 3, the DCE has Markov property, i.e., it can be defined recursively: given the DCE variables up to time t, the choice of tax rate θ_t uniquely determines the relevant batch of individual, aggregate, and policy variables of the DCE for the current period. In particular, the choice of θ_t determines the solution $\{c_{t,t}(\omega), c_{t,t+1}(\omega), s_t(\omega)\}$ of each household's problem (2)-(4) in DCE as a function of θ_t, and thereby the levels of welfare (4) attained by the households. Thus we have denoted these maximum value levels for each household $\omega \in \Omega$ explicitly as functions of θ_t, namely $V_t(\omega, \theta_t)$. It is important to observe that a choice of tax level θ_t affects welfare of generation t individuals in two ways. The first effect is on the expenditure side: the tax obviously reduces their disposable income as seen from the budget equation (2). Secondly, it affects both their private retirement savings and public pension benefits expressed in formula (3). Indeed, it contributes to the next generation's aggregate human capital H_{t+1} and thereby enhances return on the generation t retiree's private investment; furthermore, formulae (7) and (8) show that under each of the alternative social security systems, a higher level of H_{t+1} also enhances public pensions received by generation t agents. Also note that the welfare of generation t-1 retirees who are alive in period t is unaffected by the choice of tax rate θ_t. Therefore we assume that only generation t individuals will be involved in the political process of determining the choice of θ_t.

Definition: Given the initial stock of physical capital K_0 and the initial distribution of human capital $h_0(\omega)$ for $\omega\in\Omega$, a *dynamic political equilibrium (DPE)* is a sequence of education tax rates $\{\theta_t\}_{t=0}^{\infty}$ along with the corresponding *dynamic competitive equilibrium (DCE)* such that in any time period t the level of education tax rate θ_t is the most preferred by a *majority of generation t individuals*. In other words, any change in the value of θ_t accompanied by the corresponding change in DCE would reduce welfare levels for a majority of generation t individuals.

We will furthermore assume that the income distribution among workers is right-skewed, which is of course consistent with evidence. Specifically, given the initial human capital distribution $h_0(\omega)$ we impose:

Condition 1. The median human capital h_0^{med} of generation t=0 does not exceed its mean H_0.

We note that according to the evolution of human capital distribution defined by expression (1) if the above condition is valid at t=0, it will be also true for t=1,2,… , i.e. it implies that in every generation t = 0, 1, … the median voter's wage income does not exceed the economy's average, i.e. $h_t^{med} \le H_t$ for t=0,1,… . Further, it will become evident that Condition 1 can be relaxed: our analysis will show that our results remain valid if median income exceeds the mean by a certain limited factor.

We will now compare the dynamic political equilibria resulting under the PAYG and Fully Funded pension systems, given the identical rates of payroll social security, i.e. within the policy experiment stated in

Condition 2. A social security system (FF or PAYG) is instituted (announced) in the economy at t = 0 in the sense that generation 0 is its first beneficiary under either regime, i.e., first social security payments are issued at t=1. (Note that in the case of PAYG system this means that generation 0 is in the exceptional position of receiving the benefit without having to pay the social security tax.) Furthermore, we assume for the purposes of the comparative analysis the social security tax rate τ is exogenously given and identical under the FF and PAYG systems in question.

Pay-as-you-go Social Security System

The first order necessary and sufficient condition of maximum of the strictly concave welfare function $V_t^G(\omega,\theta_t)$ defined by the expression (29) is:

(31) $$\frac{\partial}{\partial\theta}V_t^G(\omega,\theta_t) \equiv -\frac{1+p\beta\delta}{1-\tau-\theta}+\frac{p\beta(1-\delta)(1-\sigma)}{\theta}=0$$

Solving this equation we obtain the value of the optimal tax rate:

(32) $$\theta_t^G \equiv \theta^G = \frac{p\beta(1-\tau)(1-\delta)(1-\sigma)}{1+p\beta(1-\sigma+\delta\sigma)}$$

Thus under the PAYG regime the welfare maximizing education tax rate is the same for all households $\omega\in\Omega$, moreover it is constant over time. Thus the DPE sequence of education tax rates under PAYG social security is given by the stationary optimal rate θ^G .

Note that according to expression (32) the optimal education tax rate θ^G is increasing with longevity, namely, that

(33) $$\frac{\partial\theta^G}{\partial p} > 0$$

This is due to two effects of increasing longevity: it increases the weight of utility of the old-age consumption in individuals' welfare function; on the other hand the actuarial value of lifetime income becomes more dependent on the social security tax revenues from future workers. We also observe that $\frac{\partial \theta^G}{\partial \sigma} < 0$, i.e. the optimal level of education tax is lower the higher is the relative effectiveness of the home component in educating a child.

Fully Funded Social Security System

Optimal values of education tax rates are defined here by maximizing welfare function $V_t^F(\omega, \theta_t)$ given by expression (21). Denote by $\theta_t^F(\omega)$ the optimal choice of education tax by agent ω at date t. It is clear that, unlike the PAYG case, individually optimal tax rates $\theta_t^F(\omega)$ differ across households and time.

The first order necessary and sufficient condition of maximum of function $V_t^F(\omega, \theta_t)$ is:

(34) $$\frac{\partial}{\partial \theta} V_t^F(\omega, \theta_t) = -\frac{1 + p\beta}{1 - g_t(\omega) - \theta} + \frac{p\beta(1-\delta)}{1-\theta} + \frac{p\beta(1-\delta)(1-\sigma)}{\theta} = 0$$

where function $g_t(\omega)$ is as defined in (20). Note that according to relationship (20) the value of $g_t(\omega)$ is positive if and only if $h_t(\omega) > H_t$, i.e., for households with above average income.

Since the welfare function $V_t^F(\omega, \theta_t)$ is strictly concave most preferred education tax rate $\theta_t^F(\omega)$ is uniquely defined for each household. By differentiating the expression (34) with respect to household-specific value $g_t(\omega)$ one can see that $\frac{\partial^2}{\partial \theta \partial g} V_t^F(\omega, \theta_t) < 0$, which means that the most preferred education tax rate is a declining function of household income. We summarize this analysis as the following

Lemma 1: *Under fully funded social security, the most preferred education tax rates* $\theta_t^F(\omega)$ *of households are uniquely defined and satisfy equation (34). Furthermore, the most preferred rate* $\theta_t^F(\omega)$ *is a decreasing function of household income.*

By directly comparing expressions (34) and (31) we see that if $g_t(\omega) \leq 0$, i.e. for a household ω with income at or below the economy's average $\frac{\partial}{\partial\theta} V_t^F(\omega,\theta) > \frac{\partial}{\partial\theta} V_t^G(\omega,\theta)$ for any given value of $\theta \in (0,1)$. Since $\frac{\partial}{\partial\theta} V_t^G(\omega,\theta^G) = 0$ the above inequality implies that $\frac{\partial}{\partial\theta} V_t^F(\omega,\theta^G) > 0$ is for all households with income at or below the average. Thus all such households would always prefer that the education tax rate was set above θ^G, the DPE value corresponding to the PAYG regime. We note that the comparison of relationships (34) and (31) will also lead to the same conclusion when expression $g_t(\omega)$, which is defined by (20), is positive but not too close to τ , which means that household's ω income is not too far above the mean in period t.

We now recall that Condition 1 implies that the wage income of the majority of voters in each generation t = 0, 1, … is below the average. Thus we can summarize the result of the above analysis:

Theorem 1: *Under the provisions of Conditions 1 and 2*

$$\theta_t^F > \theta^G \quad for \;\; t=0,1,\ldots$$

i.e, the DPE education tax rates chosen at t=0,1,… under the fully funded social security regime always exceed the DPE tax rate chosen under the PAYG regime.

Discussion. The intuition for the Theorem's result can be obtained by comparing the effects of the choice of an education tax rate θ_t on individual households' intertemporal allocation decisions under the alternative social security systems. Consider the relationships (16)-(17) and (25)-(26) which derive the intertemporal allocation of the agents' consumption under the FF and PAYG systems, respectively, which determine individual welfare according to (4) [7]

Firstly, (16)-(17) show that the lifetime benefit an individual derives from the FF system incorporates two components:

(A) The present value of *social security benefit* $\tau w_t H_t$ which amounts (by its design) to the redistribution of wage income during the agent's working period. The significance of this component obviously increases with the decline of one's relative position in the income distribution. We note, that this social security benefit of an individual is obviously unaffected by the choice of education funding level (tax rate) θ_t^F.

(B) The *rate of return on savings*, through its direct effect on income, proportionately across income levels.

Similarly, according to (25)-(26), the lifetime benefit an individual derives from the PAYG system incorporates the following two components:

(A) The present value of *social security benefit* $\tau w_{t+1} H_{t+1} / R_{t+1}$, i.e. the present value of the share of the next period's aggregate wage bill, according to the PAYG setup. According to the formulas (12)-(13), $w_{t+1} H_{t+1} / R_{t+1} = (1-\delta)\delta^{-1} K_{t+1}$, i.e. this present value is proportionate to the current period's aggregate savings.

[7] We note again that the following analysis will remain qualitatively unchanged if the household utility functions were of CRRA form and also included a parental altruistic motive to invest in education.

(B) The *rate of return on savings*, through its direct effect on income, proportionately across all income levels, which is similar to the FF case.

The expression $w_{t+1}H_{t+1}/R_{t+1} = (1-\delta)\delta^{-1}K_{t+1}$ (component (A)) for the present value of social security benefits in the PAYG regime shows that while the future wage bill is affected by the politically chosen education funding level through its effect on the next generation's human capital, this effect is cancelled out by the effect of education funding on future factor prices. Thus the *price effect* neutralizes this particular expected incentive to support education funding: *education funding has no direct positive effect on present value of social security benefits under the PAYG system*. Furthermore, formula (27) shows that unlike in the case of FF system, education tax reduces the economy's future aggregate capital stock due to the negative income effect of the education tax on the present workers' their savings.. Thus according to the expression $w_{t+1}H_{t+1}/R_{t+1} = (1-\delta)\delta^{-1}K_{t+1}$ education tax has the overall negative effect on the present value of social security benefits under PAYG regime. The effect on the present value of an agent's overall life-time income is proportionate to one's gross wage income, hence the unanimous choice of the education tax rate $\theta_t = \theta^G$ across income groups. By comparison, under the FF regime, education tax has no effect on pension benefit, but it obviously does reduce net wage income. Thus, according to expressions (16)-(17), the lower one's relative position in the income distribution, the disproportionately lower is the cost of the education tax in terms of its effect on the present value of the overall life-time income. As a result, the social security benefit component (A) makes individuals with low and moderate incomes prefer a relatively higher education tax rate under FF, in comparison to the corresponding component under the PAYG system. This difference is the stronger the lower one's position in the income distribution.

Consider now the comparative effects of the chosen education tax rate on the rate of return of savings component of an agent's welfare (component (B)) under the alternative pension systems. The comparison of formulas (18) and (27) shows that education tax has a somewhat stronger negative marginal effect on the aggregate capital (hence a stronger positive effect on the interest rate) under the PAYG system than under FF. Therefore, based on this factor, PAYG system has an advantage over FF in terms of incentives to support education tax. However, our analysis has shown that this effect is outweighed by the relative advantage of the FF regime in terms the present value of social security benefits (component (A)), as discussed above, for all individuals whose income is at or below the mean (and indeed others except for those in a certain relatively higher income tail).

The main insight of this analysis is the demonstrated breakdown of the conjecture proposed in the earlier literature, which we discussed in the Introduction, that a PAYG system could be expected to generate relatively stronger incentives for public funding of education. We have shown that in a closed economy framework the intuition behind this conjecture fails due to the *factor price effect*: the incentive to invest in education through its positive effect on the aggregate human capital of the future workforce (as the tax base for the future social security benefits under the PAYG system) is cancelled by its negative effect, through factor prices, on the present value of the future pension benefits.

To conclude the above analysis, we have found that the FF system generates majority support for a higher rate of education tax than under PAYG system when income distribution is right-skewed (in fact, under all income distributions except for those strongly left-skewed). We will now explore whether this result translates into the dominance of the fully funded system over PAYG, in terms of the overall economic growth. The answer will be certainly affirmative, if the FF system is also shown to generate relatively higher national savings. While this is immediately true *ceteris paribus*, according to the expressions (18) and (27), it is not guaranteed in the comparison at hand. Indeed, (18) and (27) show that the fact of higher education tax rates chosen under FF regime may counteract its dominance in terms of aggregate physical capital accumulation. However, we obtain the following fact (see Appendix for its proof).

Lemma 2: *Under the provisions of Conditions 1 and 2 the DPE education tax rates* θ_t^F *and* θ^G *chosen, respectively, under the FF and PAYG social security regimes satisfy the following inequality at all t = 0, 1, …*

$$\theta_t^F - \theta^G < \frac{\tau\theta^G}{1-\tau}\frac{H_t}{h_t^{med}}$$

In order to refine the above estimate we impose the following parametric requirements:

Condition 3. $\theta^G < 0.5(1-\tau)$

Condition 4. $h_0^{med} \geq 0.5H_0$

The only technical purpose of these requirements is to ensure that the inequality $\frac{\theta^G}{1-\tau}\frac{H_t}{h_t^{med}} < 1$ holds at all times, so for example, Condition 4 can be relaxed if an accordingly stricter bound is imposed in Condition 3.

Condition 3 constitutes a more than reasonable constraint on the overall taxation burden. Moreover, its parametrization according to formula (31) is clearly satisfied under meaningful conditions that capital income share δ is at least 0.25 and the intertemporal discount factor $p\beta$ in the household utility function (4) does not exceed 1.

Condition 4 stating that the median income in the initial generation is not below half of the mean is also empirically valid in all developed economies. We note again that in our model imposing this requirement for generation t = 0 guarantees that it will hold in all subsequent generations.

It is obvious that Lemma 2 combined with Conditions 3 and 4 implies that the inequality

(35) $$\theta_t^F - \theta^G < \tau$$

holds at all times.

This leads to our main result:

Theorem 2: *Under the provisions of Conditions 1 - 4 the dynamic political equilibria obtained under the fully funded regime strictly dominates the one attained under the PAYG social security regime in terms of both human and physical capital accumulation and thereby in terms of aggregate output at all times from the inception of the systems. In particular, inequalities* K_t^F > K_t^G *and* Y_t^F > Y_t^G *hold for* t = 1,2,..., *while* $h_t^F(\omega)$ > $h_t^G(\omega)$ *are true for all* $\omega \in \Omega$ *and* t = 2,3,... .[8]

5. Social Security Funding and Income Inequality

We shall consider now the equilibrium intragenerational income inequality under the two regimes of the social security program. The comparison of any two income distributions with

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[8] Note that according to the relationships (1) and (5) $h_1^F(\omega) \equiv h_1^G(\omega)$ for the initial young generation.

respect to inequality will be made in terms of the partial ordering of the second degree stochastic dominance (see, Atkinson (1970)):

Definition: Income distribution $Y(\omega)$ is *more unequal* than income distribution $Y^*(\omega)$ if $\frac{Y^*(\omega)}{E[Y^*(\omega)]}$ stochastically dominates $\frac{Y(\omega)}{E[Y(\omega)]}$ in second-degree, i.e. the Lorenz curve of $Y^*(\omega)$ lies strictly above that of $Y(\omega)$.

Starting from a given initial human capital distribution $h_0(\omega)$ and K_0, H_0, let us define the *present actuarial values of lifetime income* for each generation t =1,2,… under the alternative social security regimes.

Given the DPE tax rates $\left\{\theta_t^F\right\}_{t=0}^{\infty}$ chosen at each date by majority rule under the FF regime, the present actuarial value of lifetime income of generation t under the fully funded regime is:

$$I_t^F(\omega) = (1-\tau-\theta_t^F)w_t h_t(\omega) + p[\frac{\tau w_t H_t^F}{p}]$$

This can be rewritten as:

$$I_t^F(\omega) = B_t\left\{\frac{h_t^F(\omega)}{H_t^F} + \frac{\tau}{1-\tau-\theta_t^F}\right\} \tag{36}$$

where B_t is positive and independent of ω,

Now we derive a similar expression for the present actuarial value of lifetime income under PAYG social security regime given the DPE and its education tax rate θ^G chosen by the majority in all dates:

(37) $$I_t^G(\omega) = (1-\tau-\theta_t^G)w_t h_t(\omega) + \frac{\tau w_{t+1}H_{t+1}^G}{R_{t+1}} = M_t\left\{\frac{h_t^G(\omega)}{H_t^G} + \frac{\tau}{1-\tau-\theta^G}\frac{w_{t+1}H_{t+1}^G}{w_t H_t^G}\frac{1}{R_{t+1}}\right\}$$

where M_t is positive and independent of ω.

According to the definition of the production function in (9) we obtain that,

(38) $$\frac{1}{R_{t+1}}\frac{w_{t+1}H_{t+1}^G}{w_t H_t^G} = \frac{1}{\delta Y_{t+1}^G \big/ K_{t+1}^G}\left[\frac{(1-\delta)Y_{t+1}^G}{(1-\delta)Y_t^G}\right] = \frac{K_{t+1}^G}{\delta Y_t^G}$$

Using equation (27) we can write:

$$\frac{K_{t+1}^G}{\delta Y_t^G} = \frac{1}{\delta Y_{t+1}^G}\frac{p\beta(1-\tau-\theta^G)w_t H_t^G}{1+p\beta+\tau(1-\delta)\delta^{-1}} = \frac{p\beta(1-\tau-\theta^G)}{(1+p\beta)(1-\delta)^{-1}\delta+\tau}$$

Combining this with equation (38) we rewrite equation (37):

(39) $$I_t^G = M_t\left\{\frac{h_t^G(\omega)}{H_t^G} + \frac{\tau p\beta}{\tau+(1+p\beta)\delta(1-\delta)^{-1}}\right\}$$

To proceed with our comparison exercise we note that by using the human capital accumulation process given by equation (1) recursively for each DPE path, and the fact that the given initial human capital distribution $h_0(\omega)$ is the same for dynamic equilibria under both social security regimes, we obtain that $\frac{h_t^G(\omega)}{H_t^G} = \frac{h_t^F(\omega)}{H_t^F}$ for all ω and all t, i.e., the choice of a social security system, other things being equal, while differently affecting income levels of individuals will have identical effects on their relative positions in the distribution of incomes.

Comparing the expressions in (36) and (39) and referring to the well known sufficient conditions of Lorentz dominance (see Lemma 1 in Karni and Zilcha (1995)), we can state:

Fact: $I_t^G(\omega)$ is more unequal than $I_t^F(\omega)$ if and only if the following inequality is valid:

$$\frac{\tau}{1-\tau-\theta_t^F} > \frac{\tau\, p\, \beta}{\tau+(1+p\beta)\delta(1-\delta)^{-1}}$$

which is equivalent to

(40) $$p\beta(1-\tau-\theta_t^F) < \tau+(1+p\beta)\delta(1-\delta)^{-1}$$

We obtain the following result which compares the effects of the alternative pension systems on inequality.

Proposition: *Let the provisions of Theorem 1 be satisfied. Then the inequality ranking of DPE intragenerational income distributions under the alternative social security regimes satisfy:*

(i) If condition (40) is satisfied in period t, then the income distribution attained in this dynamic equilibrium in period t under the pay-as-you-go regime is more unequal than the corresponding income distribution under the fully funded regime.

(ii) Furthermore, if the following stronger condition

(41) $$p\beta(1-\tau-\theta^G) \le \tau+(1+p\beta)\delta(1-\delta)^{-1}$$

holds (which According to Theorem 1 is stronger than condition (40)), then income distribution in DPE under the PAYG regime is <u>at all times</u> more unequal than the corresponding income distribution under the FF regime.

(iii) If inequality (40) is reversed in period t, i.e.,

$$p\beta(1-\tau-\theta_t^F) > \tau+(1+p\beta)\delta(1-\delta)^{-1}$$

then the income distribution in period t under the FF regime is more unequal than the respective income distribution under the PAYG regime.

Note that conditions (40) and (41) certainly hold under parameter values typical of developed economies: for example, both are by far warranted if the capital income share δ is around 0.3 while the social security tax rate τ is 0.07 or higher. Therefore combining part (ii) of the Proposition and Theorem 2 we can state the following results characterizing the comparison of the alternative pension funding regimes, which to the best of our knowledge are novel to the literature:

Corollary. *Consider the comparison of the dynamic political equilibria resulting under the PAYG and FF pension systems, as stated in Condition 1. If the relative size of a pension system is 'not too small', i.e., τ satisfies condition (41), then under the provisions of Theorem 2 the FF system will result in higher rates of economic growth, physical and human capital accumulation as well as lower income inequality compared to those resulting in respective time periods under the PAYG alternative.*

6. Concluding Remarks

Given the ongoing debate in the US and in Europe regarding the desirability of transition from pay-as-you-go social security regime to a fully-funded one it became important to examine within an equilibrium framework whether such a transition has economic advantages. This is the main purpose of this theoretical work: we compare dynamic equilibria under these two social security systems. Moreover, in analyzing such important issue over time it is essential to take into account the productivity of future workers since it is determined endogenously via the governmental investments in education. Thus, in our framework human capital formation is a contributing factor to economic growth, while the government provides public education. Since in most countries the two major programs that are run by each government are the provision of education and social security, we feel comfortable with these assumptions. In this sense, we depart from the main models used in the literature to evaluate these two social security systems, since we examine this issue in a wider framework: under the circumstances where the productivity of future workers can be endogenously affected. We focus on comparing the dynamic equilibria, from a given initial conditions, under the PAYG and FF social security regimes with identical defined contribution rates. Our study has also abstracted from the determination of '*optimal* social security tax rate', due to the complexity of the political process under heterogeneous population (Sheshinski and Weiss (1981) have analyzed this issue within *partial equilibrium* in an economy with *homogeneous* population). We have also ignored technological change over time and its implications to social security (Karni and Zilcha (1989) examined this issue for the FF social security case in a model with no explicit production of human capital). The emphasis of our study is on the linkage between human capital formation and the social security benefits, since the effects of the alternative programs on the productivity of future workers are essential for determining their overall comparative outcomes.

As far as we know, this paper presents the first analytical study which compares economic growth under pay-as-you-go and fully funded social security systems while examining their implications for future workers' productivity as well as income inequality. We show that there is a natural link between the provision of defined contribution social security, investment in human capital and economic growth. In our view the evaluation of the transition from PAYG social security to the FF system, must include the relevant measures taken to increase the productivity of the coming generations.

Since in the vast majority of countries certain versions of PAYG social security system prevail (the Netherlands and Chile are among the very few exceptions), the results obtained in our theoretical framework cannot be readily subjected to empirical verification. Most of these countries are facing increasing challenges to sustainability of their social security programs due to demographic changes (increasing longevity combined with falling fertility). Our study suggests that when considering the alternative of fully funded system we should take into account the important role it plays in neutralizing adverse effects of falling dependency ratios by enhancing labor productivity.

It is important to note that the factor price (interest rate) effect has played a critical role in our results overturning the conjecture stated in the literature about the advantage of PAYG social security in generating support for public education funding. This general equilibrium effect is essential in the case of a closed economy which we have focused on. However, this effect is obviously absent in the case of a small open economy where factor prices are exogenous. Indeed, in a follow-up to this study, Kaganovich and Meier (2008) have demonstrated for a small open economy the aforementioned conjecture is indeed valid: under most plausible conditions, including the below average earning median voter, PAYG regime ensures political support for higher education taxes, hence speedier human capital accumulation and economic growth.

Appendix

Proof of Lemma 2:

According to the expression (20) the function $g_t(\omega)$ is non-positive when $h_t(\omega) \le H_t$, therefore based on Condition 1 we can state that $g_t(\omega^{med}) \le 0$ for the median voter ω^{med}. Combining this fact with the first order condition (34) we can write

$$0 = \frac{\partial}{\partial \theta} V_t^F(\omega, \theta_t^F) \le -\frac{1+p\beta}{1-g_t(\omega^{med})-\theta_t^F} + \frac{p\beta(1-\delta)}{1-\theta_t^F - g_t(\omega^{med})} + \frac{p\beta(1-\delta)(1-\sigma)}{\theta_t^F}$$

which implies that

$$\theta_t^F \le \frac{p\beta(1-g_t(\omega^{med}))(1-\delta)(1-\sigma)}{1+p\beta(1-\sigma+\delta\sigma)}$$

Combining the above inequality with formula (32), we obtain

$$\theta_t^F - \theta^G \le \left(\tau - g_t(\omega^{med})\right)\frac{\theta^G}{1-\tau}$$

which according to (20) proves the inequality stated in the Lemma. ■

<u>Proof of Theorem 2.</u>

Since the two DPE's start from the same initial K_0 and distribution $h_0(\omega)$, we have $[w_0 H_0]^F = [w_0 H_0]^G$. Then according to expressions (18) and (27) and inequality (35) we can write:

(A1) $$K_1^G < \frac{p\beta(1-\tau-\theta^G)(w_0 H_0)^G}{1+p\beta} < \frac{p\beta(1-\theta_0^F)(w_0 H_0)^F}{1+p\beta} = K_1^F$$

At the same time, due to Theorem 1, we have $X_0^G = \theta^G (w_0 H_0)^G \le X_0^F = \theta_0^F (w_0 H_0)^F$, so the relationship (1) yields $h_1^G(\omega) \le h_1^F(\omega)$ for all ω and therefore $H_1^G \le H_1^F$ according to (6). The last inequality combined with (A1) and (9) means that $Y_1^G < Y_1^F$ and therefore according to (12) we obtain $[w_1 H_1]^G < [w_1 H_1]^F$.

We now apply the induction argument. Indeed, provided that $[w_t H_t]^G < [w_t H_t]^F$ we can write similarly to (A1):

(A2) $$K_{t+1}^G < \frac{p\beta(1-\tau-\theta^G)(w_t H_t)^G}{(1+p\beta)} < \frac{p\beta(1-\theta_t^F)(w_t H_t)^F}{1+p\beta} = K_{t+1}^F$$

Since $X_t^G = \theta^G (w_t H_t)^G < X_t^F = \theta_t^F (w_t H_t)^F$ we conclude from (1) and (6) that $h_{t+1}^G(\omega) < h_{t+1}^F(\omega)$ for all ω and $H_{t+1}^G < H_{t+1}^F$. Then due to (A2) similarly to the above logic $Y_{t+1}^G < Y_{t+1}^F$ and thereby $[w_{t+1} H_{t+1}]^G < [w_{t+1} H_{t+1}]^F$, which completes the induction proof. ■

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