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Human Capital and Inequality Dynamics: The Role of Education Technology

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Discussion Paper No. 6-2007

June, 2007

The paper can be downloaded from <http://econ.tau.ac.il/sapir>

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ABSTRACT

The paper studies differences in education technology and their effects on income distributions. Our overlapping generations economy has the following features: (1) consumers are heterogeneous with respect to ability and parental human capital; (2) intergenerational transfers take place via parental education and public investments in education financed by taxes (possibly, with a level determined by majority voting). We explore several variations in the production of human capital, some attributed to 'home-education' and others related to 'public-education', and indicate how various types of technological changes affect the intergenerational income inequality along the equilibrium path.

1 Introduction

Statistical offices of international organizations have compiled lists of indicators that compare scholastic achievements across countries. A primary common element of these indicators is that the processes of training and knowledge acquisition differ in various parts of the world. Significant differences between countries arise mainly in the following areas: the level and efficiency of public education, involvement of parents in the education process of their children, the human capital of teachers and the use of existing technologies such as computers and internet. Since human capital formation affects output and the intragenerational distribution of human capital, it is essential to explore how these differences in the provision of education matter. In particular, we explore in this paper the way variations in the education technology affect the distribution of earnings.

Though human capital formation is a complex process theoretical economic models in the literature have assumed various restricted mechanisms governing this process. Due to tractability reasons, these processes have concentrated only on very few parameters [see, e.g., Glomm and Ravikumar (1992), Laitner (1997), Orazem and Tesfatsion (1997), Galor and Omer (2000) and Hanushek (2002)]. The implications of these simplified processes of the human capital production function are far reaching, since the dynamics of the human capital distribution is significantly affected. We shall consider a human capital production process that exhibits two important properties. First, the parental human capital plays an important role in the process of generating the human capital of the offspring. Evidence for that is well established in the literature [see, e.g., Hanushek (1986)]. Glaeser (1994) finds that children from families with educated parents obtain better education. Burnhill et al. (1990) find that parental education influences entry into higher education in Scotland over and above parental social status. Lee and Barro (2001) and Brunello and Checchi (2003) find that family characteristics, such as income and education of parents, enhance student's performance. A reason that is put forward is that parental education elicits more parental involvement (including related private investment) at home. Second, the contribution of public education to human capital formation depends on both the level of provision and the quality of teachers. Individuals from below-average human capital families will have a greater return to investment in public schooling than those from above-average families. In addition, the cost of acquiring human capital will be smaller for societies endowed with relatively higher levels of average human capital.

Income distribution is a key economic issue and a large literature has improved our understanding of its underlying determinants. Besides trade

and technical progress, some believe that social norms are crucial determinants of earnings inequality [e.g., Atkinson (1999), Corneo and Jeanne (2001)]. Others have thoroughly studied the role of human capital accumulation on income distribution in various contexts [see, e.g., Loury (1981), Becker and Tomes (1986), Card and Krueger (1992), Galor and Zeira (1993), Chiu (1998), Fernandez and Rogerson (1998), Tsiddon and Rubinstein (2004)]. However, as the information and communication technology advances and computers are being integrated into the learning process, new issues like the increasing technological contribution to learning arise.

The processes describing the formation of human capital, which are complex processes, has been oversimplified in the theoretical literature. In many cases the education system does not include the role played by parents, the effect of the environment and the quality of public education. In particular, the provision of public education and the determination of its level have not been incorporated properly. For example, Glomm and Ravikumar (1992) establish that majority voting results in a public educational system as long as the income distribution is negatively skewed. Cardak (1999) strengthens this result by considering a voting mechanism where the median preference for education expenditure, rather than median income household, is the decisive voter. In this paper we also consider (in Section 3) an application of the median-voter theorem to generalize these results to our framework.

Our analysis is conducted in an OLG economy in which physical capital and human capital are factors of production. Young individuals in each generation are heterogeneous due to the human capital distribution of parents, as well as (random) innate ability. Education and learning take place via two channels: the time invested by parents at home educating their child (motivated by altruism) and the provision of public education by the government financed by taxing wage incomes. Home education is carried out mainly through parental tutoring, social interaction and the learning devices available at home (such as computer and internet). In this case the human capital of parents and the time dedicated to tutoring are important factors. Public education includes public expenditures related to schooling, in particular, the time children are studying at school, as well as the quality of teachers, size of classes, social interactions, etc. Our framework will generate endogenous growth in human capital, due to investments in education and training, and will allow for a political equilibrium regarding the provision of public education (using the median-voter theorem). Using our general process of human capital formation we derive the following results. Comparing competitive equilibrium paths period by period we obtain: (i) higher provision of public schooling reduces inequality in the distribution of human capital, (ii) Initial endowments matter in the sense that a country starting from a lower level

of human capital has a lower return to public education and, hence, experiences more inequality, (iii) When the provision of public education becomes more efficient intragenerational income inequality declines in all subsequent periods. If, instead, the private provision of education becomes more efficient income inequality increases in all subsequent periods, (iv) If the level of provision of public education is determined by majority voting the above results are strengthened.

The remainder of the paper is organized as follows. Section 2 presents an OLG model with heterogeneous agents and analyzes the properties of this framework. Section 3 studies the effects of variations in the education technology on intragenerational income inequality. Section 4 concludes the paper. We shall relegate some of the proofs to the Appendix, to facilitate the reading.

2 The Dynamic Framework

Consider an overlapping generations economy with a continuum of consumers in each generation, each living for three periods. During the first period each child is engaged in education/training, but takes no economic decisions. Individuals are economically active during the working period which is followed by the retirement period. We assume no population growth, hence population is normalized to unity. At the beginning of the 'working period', each parent gives birth to one offspring. Each household is characterized by a family name $\omega \in [0, 1]$. Denote by $\Omega = [0, 1]$ the set of families in each generation and by μ the Lebesgue measure on Ω .

Agents are endowed with *two units* of time in their working period. One unit is inelastically supplied to labor, while the other is allocated between leisure and self-educating the offspring.¹ Consider generation t , denoted G_t , namely all individuals ω born at the outset of date $t - 1$, and let $h_t(\omega)$ be the level of human capital of $\omega \in G_t$. We assume that the production function for human capital consists of two components: informal education initiated and provided by parents at home and public education provided by the government by hiring 'teachers'. The 'home-education' depends on the time allocated by the parents to this purpose, denoted by $e_t(\omega)$, and the 'quality

¹Though the supply of labor is inelastic, each family's supply of human capital is the result of utility maximization. Also, Munandas (2006) shows that our results hold qualitatively for the case of an inelastic supply of labor. Thus the assumption of inelastic labor supply is less severe since, due to our assumption of no population growth, the time required to raise children is equal at each date and is insensitive to the number of young-age children.

of tutoring' represented by the parent's human capital level $h_t(\omega)$. The time allocated to public schooling (i.e., the level of public education) is denoted by e_{gt} . The human capital of the teachers determine the 'quality' of public education in the formation of the younger generation's human capital. We also assume that the (random) innate ability of individual $\omega \in G_{t+1}$, denoted by $\theta_t(\omega)$, is known when parents make their decision about investment in education. All the random variables $\theta_t(\omega)$ across individuals and across generations are i.i.d., hence, without loss of generality, we take each $\theta_t(\omega)$ to be distributed as some random variable $\tilde{\theta}$. Let $\tilde{\theta}$ assume values in $[\underline{\theta}, \bar{\theta}]$, where $0 < \underline{\theta} < \bar{\theta} < \infty$, and denote its mean by $\hat{\theta}$ where, without loss of generality, $\hat{\theta} = 1$. We assume that for some parameters $\beta_1 > 1$, $\beta_2 > 1$, $v > 0$ and $\eta > 0$, the evolution process of a family's human capital is given as follows. For all $\omega \in G_{t+1}$:

$$h_{t+1}(\omega) = \theta_t(\omega)[\beta_1 e_t(\omega) h_t^v(\omega) + \beta_2 e_{gt} \bar{h}_t^\eta] \quad (1)$$

where the average human capital involved in the public schooling system, denoted \bar{h}_t , is the *average* human capital of generation t. This is justified if we assume that instructors in each generation are chosen randomly from the population of that generation. The parameters v and η measure the externalities derived from parents' and society's human capital respectively. The constants β_1 and β_2 represent how efficiently parental and public education contribute to human capital: β_1 is affected by the home environment while β_2 is affected by facilities, the schooling system, size of classes, neighborhood, social interactions, and so forth².

The production function for human capital given by (1) exhibits the property that public education dampens the family attributes. As it is common to all, individuals from below-average families have, therefore, a greater return to human capital derived from public schooling than those born to above-average human capital families. In addition, the effort of acquiring human capital is smaller in countries endowed with relatively higher levels of human capital. An important difference between our process of generating human capital and most cases discussed in the literature is the representation of the private and the public inputs in the production of human capital via allocation of time.³ Our approach assumes that the *time spent learning*, cou-

²Empirical support for (1) is abundant, but let us point out to Brunello and Checchi (2003) who demonstrate, using Italian data, the importance of both 'home' and 'public' education in human capital formation. The family background in human capital formation has been shown to be empirically significant in the case of East Asia by Woessmann (2003). Card and Krueger (1992) established, using US data, that differences in school quality matters when we consider the rate of return to education. A lower pupil/teacher ratio results in a higher return.

³Home and public education play different roles in the literature. For example, in

pled with the human capital of the instructors, rather than the expenditures on education, are more relevant variables in such a process although there may exist a relationship between the quality of public education and public expenditure on education.⁴

Consider the lifetime income of individual ω , denoted by $y_t(\omega)$. Since the human capital of a worker is observable, it depends on the effective labor supply. Let w_t be the wage rate in period t and τ_t is the tax rate on labor income, then:

$$y_t(\omega) = w_t(1 - \tau_t)h_t(\omega) \quad (2)$$

Under the public education regime the taxes on incomes are used to finance education costs of the young generation. Making use of (1) and (2), balanced government budget means:

$$\int_{\Omega} w_t e_{gt} \bar{h}_t d\mu(\omega) = \int_{\Omega} \tau_t w_t h_t(\omega) d\mu(\omega)$$

or equivalently,

$$e_{gt} = \tau_t \quad (3)$$

that is, the tax rate on labor is equal to the proportion of the economy's effective labor used for public education.⁵

2.1 Dynamic Equilibrium

Production in this economy is carried out by competitive firms that produce a single commodity, using effective labor and physical capital. This commodity is both consumed and used as production input. Physical capital

Eckstein and Zilcha (1994) there is investment in home education on the part of parents in terms of time. In Eicher (1996), young agents must decide whether to enter the private education sector as students or to work in production as unskilled workers. In Orazem and Tesfatsion (1997), there is private investment in terms of effort and in Viaene and Zilcha (2002, 2003) there is a time input for public education only. In Restuccia and Urrutia (2004), children in their first period of life acquire human capital through public education financed by income taxes and through private education via additional personal expenditures.

⁴This is in line with Hanushek (2002) who argues in favor of considering the 'efficiency' in the public education provision rather than 'expenditure' on public education. This distinction is important since in a dynamic framework the cost of financing a particular level of human capital fluctuates with relative factor rewards.

⁵ Under a decentralized system, namely under a fully private education regime, both $\tau_t(\omega)$ and $e_{gt}(\omega)$ are decision variables of each agent, hence the individual's budget constraint on private education is: $\tau_t(\omega)w_t h_t(\omega) = w_t e_{gt}(\omega) \bar{h}_t$, where the level of teachers' instruction $e_{gt}(\omega)$ is chosen freely while their average human capital is the same as their corresponding generation.

fully depreciates and the per-capita effective human capital in date t , \bar{h}_t , is an input in aggregate production. In particular we take the (per-capita) production function to be:

$$q_t = F(k_t, (1 - e_{gt})\bar{h}_t) \quad (4)$$

where k_t is the capital stock and $(1 - e_{gt})\bar{h}_t = (1 - \tau_t)\bar{h}_t$ is the effective human capital used in the production process. $F(\cdot, \cdot)$ is assumed to exhibit constant returns to scale; it is strictly increasing, concave, continuously differentiable and satisfies $F_k(0, (1 - \tau_t)\bar{h}_t) = \infty$, $F_h(k_t, 0) = \infty$, $F(0, (1 - \tau_t)\bar{h}_t) = F(k_t, 0) = 0$.

Given the public education provision and prices, an agent ω at time t maximizes lifetime utility, which depends on consumption, leisure and income of the offspring. Thus:

$$\max_{e_t, s_t} u_t(\omega) = c_{1t}(\omega)^{\alpha_1} c_{2t}(\omega)^{\alpha_2} y_{t+1}(\omega)^{\alpha_3} [1 - e_t(\omega)]^{\alpha_4} \quad (5)$$

subject to

$$c_{1t}(\omega) = y_t(\omega) - s_t(\omega) \geq 0 \quad (6)$$

$$c_{2t}(\omega) = (1 + r_{t+1})s_t(\omega) \quad (7)$$

where $h_{t+1}(\omega)$ and $y_{t+1}(\omega)$ are given by (1) and (2). The α_i 's are known parameters and $\alpha_i > 0$ for $i = 1, 2, 3, 4$; $c_{1t}(\omega)$ and $c_{2t}(\omega)$ denote, respectively, consumption in first and second period of the individual's economically active life; $s_t(\omega)$ represents savings; leisure is given by $(1 - e_t(\omega))$; $(1 + r_{t+1})$ is the interest factor at date t . The offspring's income $y_{t+1}(\omega)$ enters parents' preferences directly and represents the motivation for parents' investment in tutoring and formal education expenditure. Given some tax rates (τ_t), k_0 and the initial distribution of human capital $h_0(\omega)$, a *competitive equilibrium* is $\{e_t(\omega), s_t(\omega), k_t; w_t, r_t\}$ which satisfies: For all t and all individuals $\omega \in G_t$, $\{e_t(\omega), s_t(\omega)\}$ are the optimum to the above problem given $\{w_t, r_t\}$. And, the following market clearing conditions hold:

$$w_t = F_h(k_t, (1 - e_{gt})\bar{h}_t) \quad (8)$$

$$(1 + r_t) = F_k(k_t, (1 - e_{gt})\bar{h}_t) \quad (9)$$

$$k_{t+1} = \int_{\Omega} s_t(\omega) d\mu(\omega) \quad (10)$$

Equations (8) and (9) are the clearing conditions in the factors market. After substituting the constraints, the first-order conditions that lead to the necessary and sufficient conditions for an optimum are:

$$\frac{c_{1t}}{c_{2t}} = \frac{\alpha_1}{\alpha_2(1+r_{t+1})} \quad (11)$$

$$\frac{\alpha_4}{(1-e_t(\omega))} \geq \frac{\beta_1\alpha_3(1-\tau_{t+1})w_{t+1}h_t^v(\omega)\theta_t(\omega)}{y_{t+1}(\omega)}, \quad \text{with } = 0 \text{ if } e_t(\omega) > 0 \quad (12)$$

From (6), (7) and (11) we obtain:

$$c_{1t}(\omega) = \left(\frac{\alpha_1}{\alpha_1 + \alpha_2} \right) y_t(\omega) \quad (13)$$

$$s_t(\omega) = \left(\frac{\alpha_2}{\alpha_1 + \alpha_2} \right) y_t(\omega) \quad (14)$$

Equation (12) allocates the unit of nonworking time between leisure and the time spent on education by the parents. In fact, we find that whenever $e_t(\omega) > 0$:

$$e_t(\omega) = \left(\frac{\alpha_3}{\alpha_3 + \alpha_4} \right) \left[1 - \frac{\alpha_4}{\alpha_3} \frac{\beta_2\tau_t\bar{h}_t^\eta}{\beta_1h_t^v(\omega)} \right]$$

Hence, $e_t(\omega)$ increases with the parents' human capital $h_t(\omega)$ but decreases with the tax rate τ_t . It is also *independent of the ability* of their offspring. By applying (12) and making use of (1), (2) and (3) we obtain the reduced-form solution of the model:

$$y_{t+1}(\omega) = (1-\tau_{t+1})w_{t+1}h_{t+1}(\omega) \quad (15)$$

$$h_{t+1}(\omega) = \left(\frac{\alpha_3}{\alpha_3 + \alpha_4} \right) \theta_t(\omega) \left[\beta_1h_t^v(\omega) + \beta_2\tau_t\bar{h}_t^\eta \right], \quad \text{whenever } e_t(\omega) > 0 \quad (16)$$

$$h_{t+1}(\omega) = \beta_2\theta_t(\omega)\tau_t\bar{h}_t^\eta, \quad \text{whenever } e_t(\omega) = 0 \quad (17)$$

Equations (15)-(17) determine the income at the future date in terms of the net wage at date $t+1$, the parents' human capital, society's level of human

capital at date t , the current education input ($\tau_t = e_{gt}$) and the externalities in education. More importantly, (15) shows that, in our framework, the intragenerational distribution of income is similar to that of human capital.

Non-participation of Parents

The non-participation of parents in the education process is an important characteristic of education systems in some OECD countries like Germany.⁶ This situation, where utility maximization is attained at $e_t(\omega) = 0$, occurs under certain conditions. To derive these recall that (12) establishes a negative relationship between the two types of education, that is, public education *substitutes* for parental tutoring. For each individual there exists a particular tax rate such that $e_t(\omega) = 0$, namely, when the marginal utility of leisure is larger than the marginal utility gained by increasing the offspring's human capital due to parental tutoring. Consider the families which optimally choose $e_t(\omega) = 0$ and denote this set of families in generation t by $A_t \subset G_t = [0, 1]$. In fact, condition (12) holds if:

$$1 - e_t(\omega) < \frac{\alpha_4}{\beta_1 \alpha_3} [\beta_1 e_t(\omega) + \beta_2 e_{gt} \frac{\bar{h}_t^\eta}{h_t^v(\omega)}]$$

Hence, for each individual in G_t we obtain $e_t(\omega) = 0$ and $\omega \in A_t$ if :

$$h_t^v(\omega) < \frac{\alpha_4 \beta_2 e_{gt} \bar{h}_t^\eta}{\alpha_3 \beta_1} \quad (18)$$

Parental and public education being substitutes, inequality (18) shows that the set A_t increases in societies with strong preference for leisure and/or with a high provision of public education e_{gt} . In both cases, families in A_t delegate the task of education to the public sector. It is clear that this set includes individuals with low levels of human capital.

3 Education and Income Inequality

We compare income inequality of any two income distributions according to **second degree** stochastic dominance [see, Atkinson (1970)], i.e., according to the well known Lorentz ordering. The following Lemma will be useful in deriving our results:

⁶See, e.g., Der Spiegel (2001) and DICE Reports (2002) for attempts at explaining the poor performance of German adolescents in the 2000 study of the Programme for International Student Assessment (PISA) of the OECD.

Lemma 1: Let $X(\omega)$ and $Y(\omega)$ be two non-negative random variables which assume values in a compact interval $[a, b]$ and satisfy: $EX = EY$. Let $Z(\omega)$ be a positive random variable independent of X and Y . If $X(\omega)$ is more equal than $Y(\omega)$ (in the SDDS sense), then XZ is more equal than YZ .

The proof is relegated to the Appendix.

3.1 The Role of Initial Endowments

The literature that studies the connection between trade and income inequality provides mixed empirical evidence regarding the sign of this relationship. It depends on the sample of countries, but more importantly, on the definition of income that is used in the computation of inequality (see, Francis and Rojas-Ramagosa (2004)). Also, the relationship is often conditional on factor endowments. For example, Spilimbergo et. al (1999) finds that openness increases inequality but its effect depends on the initial factors endowments. Fischer (2001) finds that labor-abundant countries are more equal. To demonstrate that initial conditions matter in our framework let us consider two economies that differ only in their initial endowments of human capital: one economy has higher *levels* of human capital but the measure of inequality in the *initial* human capital distributions is the same. The reason we start with endowments is to uncover conditions under which international trade based on endowment differences, or differences in educational technology, does not affect income inequality in equilibrium. These conditions provide a justification for our approach that is based on comparing countries educational systems in isolation.

The next proposition compares the equilibrium path of these two countries.

Proposition 1 *Consider two economies which differ only in their initial human capital distributions, $h_0(\omega)$ and $h_0^*(\omega)$. Assume that $h_0^*(\omega) > h_0(\omega)$ for all ω , but the initial distributions have the **same** level of inequality. Then, the equilibrium from $h_0^*(\omega)$ will have lower income inequality than that from $h_0(\omega)$ at all dates.*

The policy implications of this result are: a country that starts with higher levels of human capital, not necessarily more equal, has a higher return to public education and, hence, has a better chance to maintain less inequality in its future income distributions.

Given the different endowments of human capital it is possible to introduce international trade and mobility of physical capital between these two economies, keeping labor immobile internationally. These assumptions about trade and factor mobility guarantee factor price equalization. In this setting, we can show the following:

Proposition 2 *Consider two economies which differ only in their initial conditions. Trade in goods and physical capital mobility will not alter the intragenerational income inequality obtained under autarky.*

Hence, though two economies differ in their initial conditions, introducing trade in goods and capital mobility in our framework will not alter the income inequality measure under autarkic regime. Variations in the equilibrium factor prices do not affect the income distribution since labor incomes vary in the same proportion. In contrast, trade and capital mobility have significant impact on wages, interest rates and outputs of the two countries and, in this regard, affect the *intergenerational* distribution of income as follows: At date t , the income of family ω is given by:

$$q_t(\omega) = c_{2(t-1)}(\omega) + y_t(\omega) \quad (19)$$

where the first term is consumption at date t by the family member who was economically active at date $t-1$ and the second term is the labor income generated by the active member of the family. Using equations (7), (14) and (15) we obtain:

$$q_t(\omega) = (1 + r_t) \left[\frac{\alpha_2}{\alpha_1 + \alpha_2} y_{t-1}(\omega) + \frac{w_t(1 - \tau)}{1 + r_t} h_t(\omega) \right] \quad (20)$$

With $h_0^*(\omega) > h_0(\omega)$ and assuming stocks of physical capital (i.e., $k_0 = k_0^*$) it is clear that, in isolation, $\frac{w_t(1-\tau)}{1+r_t} > \frac{w_t^*(1-\tau^*)}{1+r_t^*}$ for all t . As a result, when capital markets are integrated physical capital will flow from low to high return country until equality in wage-rental ratio is obtained. The implication to intragenerational income distribution is summarized in:

Proposition 3 *Consider two economies which differ only in their initial human capital distributions. Assume that $h_0^*(\omega) > h_0(\omega)$ holds for all ω , but the initial income inequality is the same. Trade in goods and capital mobility results in a lower income inequality at the home country and higher income inequality at the foreign country.*

As in the empirical literature, the above proposition stresses the importance of factor endowments in explaining equilibrium income inequality. In

addition, the last two propositions show that different measures of household income generates different predictions regarding the effect of openness on income inequality. Also, as trade plays no role in explaining intragenerational income inequality in our framework, we can compare countries' education systems separately and ignore how these systems affect the comparative advantage of nations.

3.2 Public Education

Let us consider first a situation in which the government does not contribute to human capital formation. Thus, we take $\tau_t = 0$ for all t . In this case:

$$y_{t+1}(\omega) = w_{t+1}h_{t+1}(\omega)$$

From (18) we know that the set A_t is empty, and from (12) we obtain that:

$$e_t(\omega) = e^*(\omega) = \frac{\alpha_3}{\alpha_3 + \alpha_4} \quad \text{for all } \omega$$

Hence, in the absence of public education the only source of income inequality is the initial distribution of human capital. This is clear from:

$$y_{t+1}(\omega) = [\beta_1 w_{t+1} e^*(\omega) h_t^v(\omega)] \theta_t(\omega)$$

We conclude from these observations that:

Proposition 4 *In the absence of public education : (i) income inequality declines over time under decreasing returns to parental human capital (i.e., if $v < 1$), (ii) income inequality increases over time under increasing returns (i.e., if $v > 1$), and (iii) income inequality remains constant over time under constant returns (i.e., if $v = 1$).*

Our economy generates, in equilibrium, an intragenerational income distribution whose inequality is endogenously determined by the externality in the home-part of education. Inequality may decrease even in the absence of public schooling. When $v > 1$ a family 'poverty trap' arises in that $h_t(\omega)$ goes to zero for some families whose initial endowment of human capital is below a benchmark level. More precisely, this occurs for family ω such that:

$$[h_0(\omega)]^v < \frac{\alpha_3 + \alpha_4}{\beta_1 \alpha_3 \theta_0(\omega)}$$

It segments the population's human capital into two groups: families below this benchmark face a permanent decline in human capital while those to the right of it experience a permanent increase. This result is applicable to China where increasing returns in parents' human capital have been observed [see Knight and Shi (1996)].

Following our demonstration that public education plays an important role in generating human capital, let us now look at its effect on income inequality assuming that its levels are exogenously given. Let us reconsider expression (18): it is clear that as e_{gt} increases more parents may stop educating their children. It is therefore important to further characterize the role of public education, its effect on accumulation of human capital and the distribution of income. We do not choose explicitly the social decision mechanism underlying its determination by the government. The level at date t is e_{gt} and it is financed by taxing labor income at a fixed rate $\tau_t (= e_{gt})$. In the sequel we assume that $v \leq 1$ and that $\eta \leq 1$ and, to simplify our analysis, we also assume that $v \leq \eta$. Does public education reduce inequality in equilibrium?

Proposition 5 *Let $h_0(\omega)$ be any initial human capital distribution and assume that the tax rate that finances public education is constant over time. Increasing this tax rate results in a lower intragenerational income inequality in all subsequent periods. Moreover, income inequality at date $t+1$ is smaller than that in date t .*

This proposition extends similar results in the literature (see, e.g., Glomm and Ravikumar's (1992)) to our setup under active *public* and *private* education. It may not seem surprising since public education in our framework dampens family attributes as it is provided equally to all young individuals (of the same generation), while it is financed by a flat tax rate on wage income. However, its importance lies in the fact that it is proved in *equilibrium* and that it holds for *all periods*. Hence, if one compares two countries which are *similar* in all respects except for the level of public education, the country which invests less in public schooling will face a higher inequality along the equilibrium path.

An improvement in one country (vs. the other) in the production of human capital may result in a more efficient home education or a more efficient public education, or both. We say that the provision of *public* education is *more efficient* if either β_2/β_1 is *larger* (without lowering neither β_1 nor β_2) or η is larger, or both. We say that the *private* provision of education becomes *more efficient* if β_1/β_2 becomes *larger* (while neither β_1 nor β_2 declines) or v becomes larger, or both. It is called *neutral* in the case where both parameters β_1 and β_2 increase while the ratio β_2/β_1 remains unchanged. The

next proposition considers the effect of each type of technological gap on intragenerational income inequality.

Proposition 6 *Consider improvements in the production process of human capital, then: (a) If the public provision of education becomes more efficient the inequality in intragenerational distribution of income declines in all periods; (b) If the private provision of education becomes more efficient then inequality increases in all periods; (c) If the technological improvement is neutral inequality remains unchanged at period 1 but declines for all periods afterwards.*

This result demonstrates the asymmetry between a technological gap that exists primarily in the public schooling system and the one that arises in the home environment of learning. The inequality in human capital distribution increases when the private-component of education/learning becomes more efficient because the family attributes are magnified. In contrast, a more efficient public education reduces inequality because all children are exposed to instructors with the same level of average human capital: below-average families have a greater return to public schooling than above-average families. When the technological gap in education is neutral, then along the 'better' equilibrium inequality declines, except for the first date, since, after the first period, the effectiveness of public schooling outweighs that of home education.

Thus far, Proposition 6 assumes that the tax rates that finances education and, hence, the level of public education, are exogenously given. However, the assumption that the tax rate is independent of the technology parameters is very questionable. The exogeneity of τ_t can be relaxed by introducing a voting scheme into our model. As families are heterogenous, each has a different preference regarding the amount of resources to be invested in public education. The choice of the 'optimal' level of public schooling should then represent a political equilibrium. Another extension of proposition 6 is to examine how inequality relates to economic growth as various parameters in the education process vary. In our framework the sole source of income is generated by the aggregate production which applies both physical capital and human capital. Thus, variations of the parameters tied to educational technology affect growth significantly.

Let us consider the effect that a technological change in the production of human capital has on output in equilibrium. From process (1) we call $\beta_1 e_t(\omega) h_t^\nu(\omega)$ the *home component* and the second term $\beta_2 e_{gt} \bar{h}_t^\eta$ the *public component*. An improvement in the production of human capital which

makes either the public provision more efficient or the private provision more efficient, implies higher human capital stock as of date 1 onwards. Since the initial human capital stock is given it implies higher output and higher capital stock as of date 2. Does such technological progress, which results in higher growth mean more income inequality? Let us combine our results to obtain:

Corollary 1: *Consider improvements in the production process of human capital, then: (a) If the technological progress occurs only in the home component it results in higher growth coupled with higher income inequality in all periods; (b) If the technological progress occurs in the public component of education it results in higher growth accompanied by lower income inequality in all periods.*

The issue of co-movements of economic growth and income inequality has been widely debated in the literature, mainly by using empirical evidence, and this debate is inconclusive [see, e.g., Persson and Tabellini (1994), Barro (2000) and Forbes (2000)]. In our framework this Corollary provides some interpretation to these empirical findings. It establishes conditions on endogenous processes under which growth can be accompanied by more, or less, income inequality.

The political equilibrium we consider here is an application of the median-voter theorem, widely used in economic theory [see, e.g., Persson and Tabellini (2000), Section 3.3]. Let us substitute the conditions (11)-(12) in (5) to obtain an expression for the lifetime utility of agent $\omega \in G_t$ in terms of the tax rate τ_t :

$$U_t(\omega) = B_t[1 - \tau_t]^{\alpha_1 + \alpha_2} [\beta_1 h_t^v(\omega) + \beta_2 \tau_t \bar{h}_t^\eta]^{\alpha_3 + \alpha_4} E[\tilde{\theta}(\omega)]^{\alpha_3} \quad (21)$$

where B_t groups parameters and variables given to this individual at the outset of date t (including τ_{t+1}).⁷ Since $U_t(\omega)$ is concave in τ_t there is a unique maximum for each individual's lifetime utility denoted by $\tau_t(\omega)$. It is obtained directly from the first order (necessary and sufficient) condition:

$$(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)\beta_2 \tau_t(\omega) \bar{h}_t^\eta = (\alpha_3 + \alpha_4)\beta_2 \bar{h}_t^\eta - (\alpha_1 + \alpha_2)\beta_1 h_t^v(\omega)$$

It is clear that the heterogeneity in voter's optimal policy $\tau_t(\omega)$ results from the heterogeneity in their human capital $h_t(\omega)$. In particular, the median voter's choice is:

⁷Self-interested agents vote myopically in this model in that they ignore the effect of current political decision on future political outcomes. Voters may induce the end of public education this period but a constituency for an education policy can regenerate next period. See Hassler et al. (2003) for a model of rational dynamic voting.

$$\tau_t(m) = [\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4]^{-1} [(\alpha_3 + \alpha_4) - (\alpha_1 + \alpha_2) \frac{\beta_1 h_t^v(m)}{\beta_2 \bar{h}_t^\eta}] \quad (22)$$

Some monotonicity results can be verified from the expression in (20):

$$\frac{\partial \tau_t(m)}{\partial \alpha_1} = \frac{\partial \tau_t(m)}{\partial \alpha_2} < 0 \quad , \quad \frac{\partial \tau_t(m)}{\partial \alpha_3} = \frac{\partial \tau_t(m)}{\partial \alpha_4} > 0 \quad \text{and} \quad \frac{\partial \tau_t(m)}{\partial (\frac{\beta_1}{\beta_2})} < 0 \quad (23)$$

Observed cross-country differences in education expenditures can be explained by (22) and (23). For example, as $h_t(m)$ drops relative to \bar{h}_t , $\tau_t(m)$ rises: A below-average median voter favors a higher tax rate than an above-average median voter. Also, an increase in v and β_1/β_2 [or a decrease in η] imply a lower tax rate for financing education.

Given these observations, let us illustrate how using the Median-voter theorem will strengthen our previous results regarding income inequality. Table 1 (see the Appendix) examines how various parameters in our model affect income inequality. The first column contains the result of (23); the second column uses part (ii) of Proposition 5 to infer the effect on income inequality of column one. The third column applies Proposition 6, while the total effect is given in the last column. Consider, for example, a marginal increase in β_1 : by proposition 6 it leads to a higher inequality while majority voting implies a lower tax rate $\tau_t(m)$. In turn, applying part (ii) in proposition 5 leads to even more inequality.

Corrolary 2: When the resources invested in public education are determined by a political equilibrium, applying the median-voter theorem strengthens the results regarding income inequality attained under exogenously given tax rates.

It is important to note that due to majority voting decision making consumer preference parameters become determinant of income inequality⁸. The proof of Corrolary 2 follows directly from the preceding propositions, hence it is omitted.

⁸Likewise, it can be shown that the application of the median-voter theorem increases the likelihood of a negative co-movement between economic growth and income inequality. Consider a marginal increase in β_2 : the higher tax rate $\tau_t(m)$ implied by this increase leads to a higher endogenous growth. Also, the public-component of education becomes more efficient and it enhances growth as well. Thus, all effects on growth are positive and all effects on inequality (see Table 1) are negative.

4 Conclusion

This paper attempts to study, within a general equilibrium framework with human capital accumulation process, the cross-country differences in income distribution. Our framework of analysis is an overlapping-generations economy with heterogeneous households, where heterogeneity results from random innate abilities and the nondegenerate initial distribution of human capital. We derive a number of results which provide explanations for observed cross-country differences in income inequality based on variations in the human capital formation process. In particular, our results suggest hypotheses regarding a cross-country comparison of inequality and : (a) externalities of family's (and society's) human capital; (b) the effective level of public education; (c) the efficiency of public schooling and parental tutoring; (e) initial conditions, represented here by the initial stock of physical capital and distribution of human capital.

The paper illustrates the role of family attributes (assuming altruism) in the formation of human capital. Any education system that elevates the role of a family, such as private education or home education, would lead to increased income inequality. Alternative models, that would include the financing of private education by parents, would magnify the results on income inequality.

Our framework includes some specific assumptions and, therefore, our results are subject to the issue of robustness. First, the selection of our functional forms was strongly motivated by stylized facts. For example, incorporating parental role in the human capital formation process is justified due to the repeatedly reported evidence that it has an empirical relevance in a large number of countries. Second, each agent supplies inelastically one unit of his time to the labor market. The effect of relaxing the assumption that labor supply is inelastic not trivial as each family's supply of human capital is also endogenous. Moreover, due to zero population growth, our assumption seems less severe since it is reasonable to take the time required to raise one child to be equal at each date. Third, the model assumes away taxation of the returns to savings; however, expanding the tax base to include this type of income does not alter the results concerning income inequality. In contrast, this framework can be generalized to include an additional redistributive measure by the government, such as social security. Some of our results may vary in this situation because intergenerational transfers take place via both education and social security.

5 Appendix

Proof of Lemma 1: Denote the cumulative distribution functions of X, Y, Z by $F(\xi), G(\xi)$ and $H(\xi)$ correspondingly. Let $[\alpha, \beta]$ be the support of Z . Define,

$$W(m) = \Pr\{XZ \leq m\} = \Pr\{Z = \rho \text{ and } X \leq \frac{m}{\rho}, \rho \in [\alpha, \beta]\}$$

It is clear that $W(m) = \int_{\alpha}^{\beta} F(\frac{m}{\rho})H(\rho)d\rho$. In the same way we define the c.d.f of YZ as $W^*(m) = \int_{\alpha}^{\beta} G(\frac{m}{\rho})H(\rho)d\rho$. Let the support of W and W^* be $[c, d]$. Now,

$$\Delta(t) = \int_c^t [W(m) - W^*(m)]dm = \int_c^t \int_{\alpha}^{\beta} [F(\frac{m}{\rho}) - G(\frac{m}{\rho})]dH(\rho)dm = \int_{\alpha}^{\beta} \left\{ \int_c^t [F(\frac{m}{\rho}) - G(\frac{m}{\rho})]dm \right\}dH(\rho)$$

Now, by changing variables, for each fixed ρ we obtain that:

$$\int_c^t [F(\frac{m}{\rho}) - G(\frac{m}{\rho})]dm = \rho \int_c^t [F(\frac{m}{\rho}) - G(\frac{m}{\rho})]d(\frac{m}{\rho}) = \rho \int_{\frac{c}{\rho}}^{\frac{t}{\rho}} [F(q) - G(q)]dq \leq 0$$

by our assumption about X and Y . Thus, we obtain that $\Delta(t) \leq 0$ for all t in $[c, d]$ and $\Delta(d) = 0$. This completes the proof. \square

This Lemma allows us to compare inequality in income distributions while ignoring the "mixing" effects of the random ability $\theta_t(\omega)$ since it is independent of the human capital level of the parent or the given individual.

Proof of Proposition 1:

Consider the following two equations attained from (15) and (16) :

$$y_{t+1}(\omega) = C_t [h_t^v(\omega) + \frac{\beta_2}{\beta_1} e_{gt} \bar{h}_t^\eta] \text{ for all } \omega \notin A_t.$$

$$y_{t+1}(\omega) = C_t [\beta_2 e_{gt} \bar{h}_t^\eta] \text{ for all } \omega \in A_t.$$

Similarly,

$$y_{t+1}^*(\omega) = C_t^* [h_t^{*v}(\omega) + \frac{\beta_2}{\beta_1} e_{gt} \bar{h}_t^{*\eta}] \text{ for all } \omega \notin A_t^*.$$

$$y_{t+1}^*(\omega) = C_t^* [\beta_2 e_{gt} \bar{h}_t^{*\eta}] \text{ for all } \omega \in A_t^*.$$

Where C_t and C_t^* are some positive constants. Since h_0 and h_0^* are equally distributed, the same holds for $h_0^v(\omega)$ and $[h_0^*(\omega)]^v$, since $v \leq 1$. Moreover, since $\bar{h}_0 < \bar{h}_0^*$ we obtain that $h_1^*(\omega)$ is more equal than $h_1(\omega)$ [see, Lemma 1 in Karni and Zilcha (1995)]. It is easy to verify from (16) that $h_1(\omega)$ are lower than $h_1^*(\omega)$ for all ω . Note that since $y_1^*(\omega) = C_0^* \beta_2 e_{gt} \bar{h}_t^{*\eta}$ for all $\omega \in A_0$ and $y_1(\omega) = C_0 \beta_2 e_{gt} \bar{h}_t^\eta$ for all $\omega \in A_0^*$ and on these sets $y_1^*(\omega) > y_1(\omega)$ the above argument is not affected by the existence of A_0 and A_0^* with positive measure. In particular we obtain that $[h_1^*(\omega)]^v$ is more equal than $[h_1(\omega)]^v$ [see Theorem 3.A.5 in Shaked and Shanthikumar (1994)]. Also we

have $[\bar{h}_1]^\eta < [\bar{h}_1^*]^\eta$. This implies, using (16), that $h_2^*(\omega)$ is more equal than $h_2(\omega)$. It is easy to see that this process can be continued to generalize this to all periods. \square

Proof of Proposition 2:

Consider the following two equations attained from (15) and (16) :

$$y_{t+1}(\omega) = C_t[h_t^v(\omega) + \frac{\beta_2}{\beta_1}e_{gt}\bar{h}_t^\eta] \quad \text{for all } \omega \notin A_t.$$

$$y_{t+1}(\omega) = C_t[\beta_2e_{gt}\bar{h}_t^\eta] \quad \text{for all } \omega \in A_t.$$

Similarly,

$$y_{t+1}^*(\omega) = C_t^*[h_t^{*v}(\omega) + \frac{\beta_2}{\beta_1}e_{gt}\bar{h}_t^{*\eta}] \quad \text{for all } \omega \notin A_t^*.$$

$$y_{t+1}^*(\omega) = C_t^*[\beta_2e_{gt}\bar{h}_t^{*\eta}] \quad \text{for all } \omega \in A_t^*.$$

Where C_t and C_t^* are some positive constants. Since h_0 and h_0^* are equally distributed, the same holds for $h_0^v(\omega)$ and $[h_0^*(\omega)]^v$, since $v \leq 1$. Moreover, since $\bar{h}_0 < \bar{h}_0^*$ we obtain that $h_1^*(\omega)$ is more equal than $h_1(\omega)$ [again, see Lemma 1 in Karni and Zilcha (1995)]. It is easy to verify from (16) that $h_1(\omega)$ are lower than $h_1^*(\omega)$ for all ω . Note that since $y_1^*(\omega) = C_0^* \beta_2 e_{gt} \bar{h}_t^{*\eta}$ for all $\omega \in A_0$ and $y_1(\omega) = C_0 \beta_2 e_{gt} \bar{h}_t^\eta$ for all $\omega \in A_0^*$ and on these sets $y_1^*(\omega) > y_1(\omega)$ the above argument is not affected by the existence of A_0 and A_0^* with positive measure. In particular we obtain that $[h_1^*(\omega)]^v$ is more equal than $[h_1(\omega)]^v$ [see Theorem 3.A.5 in Shaked and Shanthikumar (1994)]. Also we have $[\bar{h}_1]^\eta < [\bar{h}_1^*]^\eta$. This implies, using (16), that $h_2^*(\omega)$ is more equal than $h_2(\omega)$. It is easy to see that this process can be continued to generalize this to all periods. \square

Proof of Proposition 3: The proof is similar to that of Proposition 6 in Viaene and Zilcha (2003), hence it is omitted.

Proof of Proposition 5: (i) Let us show first that in each generation individuals with a higher level of human capital choose at the optimum higher level of time to be allocated to the private education of their offspring. To see this let us derive from the first order conditions, using some manipulation, the following equation:

$$1 - [1 + \frac{\beta_1 \alpha_4}{\alpha_3}]e_t(\omega) = \frac{\alpha_4 \beta_2}{\alpha_3} e_{gt} \bar{h}_t^\eta [h_t^{-v}(\omega)] \quad \text{for } e_t(\omega) > 0 \quad (24)$$

which demonstrates that higher $h_t(\omega)$ implies higher level of $e_t(\omega)$. Let us show that such a property generates less equality in the distribution of $y_{t+1}(\omega)$ compared to that of $y_t(\omega)$. It is useful however, to apply (16) for this issue. In fact it represents the period $t + 1$ income $y_{t+1}(\omega)$ as a function of the date t income $y_t(\omega)$ via the human capital evolution. Define the function $Q : R \rightarrow R$ such that $Q[h_t(\omega)] = h_{t+1}(\omega)$ using (16) whenever $\omega \notin A_t$, and when $\omega \in A_t$ this function is defined by: $Q[h_t(\omega)] = \beta_2 e_{gt} \bar{h}_t^\eta$. This function

is monotone nondecreasing and satisfies: $Q(x) > 0$ for any $x > 0$ and $\frac{Q(x)}{x}$ is decreasing in x . Therefore [see, Shaked and Shanthikumar (1994)], the human capital distribution $h_{t+1}(\omega)$ is more equal than the distribution in date t , $h_t(\omega)$. This implies that $y_{t+1}(\omega)$ is more equal than $y_t(\omega)$.

(ii) As we saw earlier it is sufficient to prove this result under the assumption that $e_t(\omega) > 0$ for all $\omega \in G_t$. When this is not the case, raising e_{gt} entails higher income for all low income individuals $\omega \in A_t$ which only reinforces the claim. Let us consider (1) for $t = 0$. Since $h_0(\omega)$ is given, $h_0^v(\omega)$ and \bar{h}_0 are fixed. By raising e_{g0} the distribution of the human capital for generation 1, $h_1(\omega)$ becomes more equal. This follows from Lemma 1 in Karni and Zilcha (1995). Moreover, we claim from (16) that the average human capital in generation 1 increases as well. Increasing e_{g0} will result in higher $h_1(\omega)$ for all ω and higher level of \bar{h}_1 . Moreover, it also implies that $h_1^v(\omega)$ will have a *more equal* distribution [see, Shaked and Shanthikumar (1994), Theorem 3.A.5].

Now, let us consider $t = 1$. Increasing e_{g1} will imply the following facts: $h_1^v(\omega)$ becomes more equal and $\beta_2 e_{g1} \bar{h}_1^\eta$ is larger than its value before we increased the level of public education. Using (16) and the same Lemma as before we obtain that $h_2(\omega)$ becomes more equal. This process can be continued for $t = 3, 4, \dots$, which establishes our claim. Now let us consider the set of families with $e_t(\omega) = 0$. To simplify our argument assume that *initially* $e_{g0} = 0$, then as e_{g0} increases $h_1(\omega)$ will be equal or larger than in the private provision case for all $\omega \in G_1$, where $\omega \in A_0$. Namely, we claim that:

$$\beta_2 e_{g0} \bar{h}_0^\eta \geq \beta_1 e_0(\omega) h_0^v(\omega) \quad \text{for all } \omega \in A_0 \quad (25)$$

Let us substitute $e_0(\omega)$ and using the upper bound for $h_0^v(\omega)$ from (18), we see that this inequality always holds since, by assumption, $v \leq \eta$. This fact certainly reinforces the proof of our earlier case since at the lower tail of the distribution of income we *raised* and *equalized* the income for all $\omega \in G_1$, where $\omega \in A_0$. This process can be continued for all generations. \square

Proof of Proposition 6: Let the initial distribution of human capital $h_0(\omega)$ be given. Compare the following two equilibria from the same initial conditions: One with the human capital formation process given by (1) and another with the same process but β_2 is replaced by a larger coefficient $\beta_2^* > \beta_2$. Clearly, we keep β_1 unchanged. Consider again the following expressions for our individual income:

$$\begin{aligned} y_{t+1}(\omega) &= C_t [h_t^v(\omega) + \frac{\beta_2}{\beta_1} e_{gt} \bar{h}_t^\eta] & \text{for all } \omega \notin A_t \\ y_{t+1}(\omega) &= C_t [\frac{\beta_2^*}{\beta_1} e_{gt} \bar{h}_t^\eta] & \text{for all } \omega \in A_t \end{aligned}$$

$$y_{t+1}^*(\omega) = C_t^*[h_t^{*v}(\omega) + \frac{\beta_2^*}{\beta_1^*}e_{gt}\bar{h}_t^{*\eta}] \quad \text{for all } \omega \notin A_t$$

$$y_{t+1}^*(\omega) = C_t^*[\frac{\beta_2^*}{\beta_1^*}e_{gt}\bar{h}_t^{*\eta}] \quad \text{for all } \omega \in A_t$$

Since $h_0(\omega)$ is fixed at date $t = 0$ we find [using once again the Lemma from Karni and Zilcha (1994)] that $\frac{\beta_2^*}{\beta_1^*} > \frac{\beta_2}{\beta_1}$ imply that $y_1^*(\omega)$ is more equal than $y_1(\omega)$. We also derive that $h_1(\omega)$ are lower than $h_1^*(\omega)$ for all ω and, hence, $\bar{h}_1 < \bar{h}_1^*$. This inequality reinforces the result when $\mu(A_0) > 0$. By (16), using the same argument as in the last proof, $h_1^{*v}(\omega)$ is more equal than $h_1^v(\omega)$ and $\frac{\beta_2^*}{\beta_1^*}e_{g1}\bar{h}_1^{*\eta} > \frac{\beta_2}{\beta_1}e_{g1}\bar{h}_1^\eta$, hence $h_2^*(\omega)$ is more equal than $h_2(\omega)$. This same argument can be continued for all dates $t = 3, 4, 5, \dots$. Also note that $A_t \subset A_t^*$ (where A_t^* is the set of families in G_t who choose $e_t(\omega) = 0$) since $\frac{\beta_2^*}{\beta_1^*}e_{gt}\bar{h}_t^{*\eta} > \frac{\beta_2}{\beta_1}e_{gt}\bar{h}_t^\eta$ for all t . This only contributes to the more equal distribution of $y_{t+1}^*(\omega)$ since the left hand tail has been increased and equalized compared to the $y_{t+1}(\omega)$ case.

To complete the proof of part (a) of this Proposition consider the case where we increase η . When we increase the value of η , keeping all other parameters constant, we are basically increasing the second term in (16), $[\bar{h}_0]^\eta$, while $[h_0(\omega)]^v$ remains unchanged. By Lemma 1 in Karni and Zilcha (1995) we obtain that the distribution of $h_1(\omega)$ becomes more equal. Taking into account the families $\omega \in G_1$ who belong to A_0 (i.e., the lower tail of the distribution of income) only reinforces the higher equality since their incomes are uniformly increase to $\beta_2 e_{g1} \bar{h}_0^{*\eta}$, while for all other $\omega \in G_1, \omega \notin A_0$ the proportional raise in their income is smaller. This can be continued for $t = 2$ as well since it is easy to verify that $[\bar{h}_1]^\eta$ increases while $[h_1(\omega)]^v$ becomes more equal. Now, this process can be extended to $t = 2, 3, \dots$, which complete the proof of part (a).

The proof of part (b) follows from the same types of arguments using the fact that if $\beta_1 < \beta_1^*$ then $\frac{\beta_2}{\beta_1} > \frac{\beta_2}{\beta_1^*}$ and, hence, $h_1(\omega)$ is more equal than $h_1^*(\omega)$ and $\bar{h}_1 > \bar{h}_1^*$. This process leads, using similar arguments as before, to $y_t(\omega)$ more equal than $y_t^*(\omega)$ for all periods t . \square

Claim: Compare two economies which differ only in the parameter v . The economy with the higher v will have more inequality in the intragenerational income distribution in all periods.

Since the two economies have the same initial distribution of human capital $h_0(\omega)$ the process that determines $h_1(\omega)$ differs only in the parameter v . Denote by $v^* < v \leq 1$ the parameters, then it is clear that $[h_0(\omega)]^{v^*}$ is more equal than $[h_0(\omega)]^v$ since it is attained by a strictly concave transformation [see, Theorem 3.A.5 in Shaked and Shanthikumar (1994)]. Likewise, the human capital distribution $h_1^*(\omega)$ is more equal than the distribution $h_1(\omega)$. This implies that $y_1^*(\omega)$ is more equal than $y_1(\omega)$. Now we can apply the

same argument to date 1: the distribution of $[h_1^*(\omega)]^{v^*}$ is more equal than that of $[h_1(\omega)]^v$, hence, using (16) and the above reference, we derive that the distribution of $[h_2^*(\omega)]^{v^*}$ is more equal than that of $[h_2(\omega)]^v$. This process can be continued for all t .

Consider now the claim in part (c). From (16) we see that inequality in the distribution of $h_1(\omega)$ remains unchanged even though all levels of $h_1(\omega)$ increase due to this technological improvement. In particular, \bar{h}_1 increases. Now, since inequality of $h_1^v(\omega)$ did not vary but the second term in the RHS of (16) has increased due to the higher value of \bar{h}_1 , we obtain more equal distribution of $h_2(\omega)$. When $\mu(A_0) > 0$ the higher \bar{h}_1 results in higher income to all $\omega \in G_1$ who belong to A_0 , which only reinforces the more equality in $y_2^*(\omega)$. Now, this argument can be used again at dates 3, 4, ..., which completes the proof. \square

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