

# IV and GMM Inference in Endogenous Stochastic Unit Root Models

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Lieberman and Phillips (2017; LP) introduced a multivariate stochastic unit root (STUR) model, which allows for random, time varying local departures from a unit root (UR) model, where nonlinear least squares (NLLS) may be used for estimation and inference on the STUR coefficient. In a structural version of this model where the driver variables of the STUR coefficient are endogenous, the NLLS estimate of the STUR parameter is inconsistent, as are the corresponding estimates of the associated covariance parameters. This paper develops a nonlinear instrumental variable (NLIV) as well as GMM estimators of the STUR parameter which conveniently addresses endogeneity. We derive the asymptotic distributions of the NLIV and GMM estimators and establish consistency under similar orthogonality and relevance conditions to those used in the linear model. An over-identification test and its asymptotic distribution are also developed. The results enable inference about structural STUR models and a mechanism for testing the local STUR model against a simple UR null, which complements usual UR tests. Simulations reveal that the asymptotic distributions of the NLIV and GMM estimators of the STUR parameter as well as the test for over-identifying restrictions perform well in small samples and that the distribution of the NLIV estimator is heavily leptokurtic with a limit theory which has Cauchy-like tails. Comparisons of STUR coefficient and a standard UR coefficient test show that the one-sided UR test performs poorly against the one-sided STUR coefficient test both as the sample size and departures from the null rise. The results are applied to study the relationships between stock returns and bond spread changes.