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"Policymaker Preferences, Frictions and Labor Market Outcomes"

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Abstract

The paper examines the effects of endogenously determined labor-market policies on labor market outcomes within a frictional labor market. The policymaker maximizes a social welfare function defined over the present value of worker values and firm values. The labor market features search and matching frictions. The paper explores how asymmetries in policy objectives and in wage bargaining, as well as the degree of frictions, affect labor-market policies and the ensuing outcomes. The paper undertakes a quantitative analysis, implemented for the U.S. It emerges that workers have little power relative to firms in the determination of policy and of wages.

Key Words: policy, labor market, policymaker preferences, frictions, worker value, firm value.

JEL Codes: E24, E61, J64, J68

Policymaker Preferences, Frictions and Labor Market Outcomes[1]

1 Introduction

The paper examines the effects of endogenously determined labor-market policies on labor market outcomes within a frictional labor market. The policymaker maximizes a social welfare function defined over the present value of worker values and firm values. The labor market features search and matching frictions. The paper explores how asymmetries in policy objectives and in wage bargaining, as well as the degree of frictions, affect labor-market policies and the ensuing outcomes. The paper undertakes a quantitative analysis, implemented for the U.S. It emerges that workers have little power relative to firms in the determination of policy and of wages.

The immediate empirical background for this paper is a series of papers showing the effects of policy on EU and U.S. labor market outcomes. Thus, Daveri and Tabellini (2000), report that more than 50 percent of the increase in unemployment in continental Europe since the beginning of the 1970s may have originated in the increase in labor taxation. Ljungqvist and Sargent (1998, 2007a, 2007b) demonstrate the major influence of unemployment benefits on European unemployment. Prescott (2002, 2004) and Pries and Rogerson (2005) argue that differences in taxes are a key source of differences between the European and U.S. labor market experiences. Planas, Werner, and Rossi (2007) support the view that lowering labor taxes may help to reduce unemployment in continental Europe.

The current paper seeks to re-examine these issues by relating policymaker preferences to policy instruments and examining the resulting labor market outcomes in a frictional context. In doing so, the paper relates to two strands in the literature:

Labor markets with frictions. The aggregate labor market model to be used is the Diamond-Mortensen-Pissarides model. The two major frictions in question are costly search by firms and the matching process of vacancies and unemployed workers. The key contributions were made by Diamond (1982a,b), Mortensen (1982), Pissarides (1985) and Mortensen and Pissarides (1994). For recent surveys, see Mortensen and Pissarides (1999), Pissarides (2000), Yashiv (2007), and Rogerson and Shimer (2011).

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Within this literature some papers have looked at the effects of policy, mostly payroll taxes and unemployment benefits; see, for example, Millard and Mortensen (1997), Alvarez and Veracierto (2000), Mortensen and Pissarides (2003), Yashiv (2004) and Zanetti (2011). The current study uses the same basic model but features endogenous policymaking, rather than the usual exogenous policy experiments.

Policymaker Incentives. The model below features a policymaker who maximizes a weighted social welfare function. The approach taken here is to back out the weights used by the policymaker in the Social Welfare Function using data values and steady state relations.

Many of the existing analyses examine the effects of policies without discussing the preferences of policymakers or the key tradeoffs they face. Despite the vast literature on the influence of labor-market institutions and unemployment policies, a critical question has not yet been thoroughly examined: why were labor-market policies, such as high unemployment benefits and labor taxes, set in the first place?

Recently, the political economy literature has addressed this kind of question in different contexts. Acemoglu, Johnson, Querubin and Robinson (2008) claim that in order to understand why reforms do or do not work, it is necessary to investigate the political economy of distortionary policies. Their general argument is that the analysis of whether reforms will improve economic performance must start with an understanding of why distortionary policies were in place to start with. Otherwise, any analysis of the influence of these policies might be misleading. The authors emphasize that policy reform takes place in an environment in which certain policies initially served political purposes, such as redistributing resources to groups that have power and influence. Discussing unemployment insurance, Persson and Tabellini (2002) note: "The view that existing policy choices are not random, but systematically related to the political and economic environment, also has important implications for how to approach the unemployment effects of the alternative labor-market policies and institutions in empirical work. These implications have been neglected so far in the existing empirical literature on the economic causes of unemployment." (pp. 148).

The paper proceeds as follows: Section 2 presents the model. Section 3 the empirical methodology while Section 4 reports the results. Section 5 concludes. Technical derivations are left to appendices.

2 The Model

2.1 The Environment

There are three types of agents in the economy.

Capitalists, who own firms which produce using capital and labor. They open vacancies to recruit workers, a costly process.

Workers: when unemployed they search for jobs and earn income from benefits and from home production; when working, they earn wages and are separated from their jobs at an exogenous rate, returning to the unemployment pool.

These two types of agents are related by two mechanisms: one is a matching function, which captures the random meetings between vacancies and unemployed workers generating hires. The other is a wage bargain, which is made after matching and which splits the job-worker match surplus.

The policymaker is assumed to maximize a Social Welfare Function. This is a weighted average of the representative firm value and the representative worker value, in terms of the expected present value of earnings. The policymaker optimizes over four policy parameters, subject to a budget constraint and the structure of the labor market.

The resulting partial equilibrium model includes the firms' optimization, the matching process and hence employment and unemployment dynamics, the wage bargain and the policymaker optimization problem. We derive and use the non-stochastic steady state of this economy in calibration and simulation.

2.2 Firms

2.2.1 The Optimization Problem

Firms post vacancies and invest in capital in order to maximize profits. The expected discounted present value of profits is given by:

$$\max_{\{V_t,I_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t} \beta_j\right) \left(1 - \tau_{f,t}\right) \left[(F_t - W_t(1 + \tau_{SSC,t})N_t - \Gamma_t - I_t)\right] \quad (1)$$

where $\beta_j = \frac{1}{1+r_j}$ and r_j is the rate of interest, F is output, W is the real wage, τ_{SSC} denotes the employer's social-security contributions, τ_f denotes the corporate income tax rate, N is employment stock, Γ denotes hiring costs and I is investment in capital. Henceforth we shall denote profits as follows:

$$\Pi_t = F_t - W_t(1 + \tau_{SSC,t})N_t - \Gamma_t - I_t \tag{2}$$

Output is produced using capital and labor as follows:

$$F_t = (X_t N_t)^{\alpha} K_t^{1-\alpha} \tag{3}$$

where X is labor-augmenting technology and K is the stock of capital. Hiring costs are given by a convex function of the hiring rate:

$$\Gamma_t = \frac{\Theta}{1+\gamma}(\frac{Q_t V_t}{N_t})^{1+\gamma} F_t \tag{4}$$

Θ is a scale parameter, γ is the degree of convexity, $Q_t V_t$ is the flow of hires

The firm's profit maximization is subject to an employment-dynamics equation and capital-dynamics equation given by:

$$N_{t+1} = N_t(1 - \delta_t) + Q_t V_t \tag{5}$$

where $Q_t = \frac{M_t}{V_t}$ is the probability of filling a vacancy and workers are assumed to be separated from jobs at the exogenous rate δ_t.

Capital evolves according to the equation:

$$K_{t+1} = K_t(1 - d_t) + I_t \tag{6}$$

where d is the depreciation rate.

The F.O.C of problem (1)-(6) are as follows.

For investment, it is given by:

$$1 - \tau_{f,t} = E_t \beta_{t+1} \left(1 - \tau_{f,t+1}\right) \cdot \left[\frac{\partial F(N_{t+1}, K_{t+1})}{\partial K_{t+1}} - \frac{\partial \Gamma(V_{t+1}, N_{t+1}, K_{t+1})}{\partial K_{t+1}} + (1 - d_{t+1})\right] \tag{7}$$

In what follows we shall relate to the non-stochastic steady state, in which case (7) becomes the familiar

$$\begin{aligned} \frac{\partial F}{\partial K} &= r + d + \frac{\partial \Gamma}{\partial K} \\ (1-\alpha)\frac{F}{K} &= r + d + \frac{\Theta(1-\alpha)}{1+\gamma}(\frac{QV}{N})^{1+\gamma}\frac{F}{K} \end{aligned} \tag{8}$$

For vacancies it is given by:

$$\left(1-\tau_{f,t}\right)\cdot\frac{\frac{\partial\Gamma(V_t,N_t,K_t)}{\partial V_t}}{Q_t}=E_t\beta_{t+1}\left(1-\tau_{f,t+1}\right)\cdot\left[\begin{array}{c}\frac{\partial F(N_{t+1},K_{t+1})}{\partial N_{t+1}}-\frac{\partial\Gamma(V_{t+1},N_{t+1},K_{t+1})}{\partial N_{t+1}}\\-\left(1+\tau_{SSC,t+1}\right)\cdot\left\{\frac{\partial W_{t+1}}{\partial N_{t+1}}\cdot N_{t+1}+W_{t+1}\right\}\\+\frac{\partial\Gamma(V_{t+1},N_{t+1},K_{t+1})}{\partial V_{t+1}}\cdot\frac{1}{Q_{t+1}}\cdot\left(1-\delta_{t+1}\right)\end{array}\right] \tag{9}$$

where the firm takes into account the effect of employment on wages via the term $\frac{\partial W_{t+1}}{\partial N_{t+1}}$.

Define the gross rates of labor, capital and labor productivity growth as follows:[2]

$$\begin{aligned}G^L_{t+1} &= \frac{L_{t+1}}{L_t} && (10)\\ G^K_{t+1} &= \frac{K_{t+1}}{K_t} && (11)\\ G^X_{t+1} &\equiv \frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}} && (12)\\ G^X_t-1+G^L_t-1 &= G^K_t-1 && (13)\end{aligned}$$

In the non-stochastic steady state, equation (9) turns into the following equation, using equations (3), (4) and (12) :

$$\Theta\cdot\left(\frac{QV}{N}\right)^{\gamma}=\Omega\cdot\left[\begin{array}{c}\alpha\cdot\left[1-\frac{\Theta}{1+\gamma}\left(\frac{QV}{N}\right)^{1+\gamma}\right]+\Theta\left(\frac{QV}{N}\right)^{1+\gamma}\\-\left(1+\tau_{SSC}\right)\cdot\left\{\frac{\frac{\partial W}{\partial N}N+W}{\frac{F}{N}}\right\}\end{array}\right] \tag{14}$$

where

$$\Omega=\frac{G^X\beta}{1-G^X\beta(1-\delta)}$$

and in what follows we solve for $\frac{W}{F/N}$ and $\frac{\frac{\partial W}{\partial N}}{F/N}$.

[2] Along the steady-state, balanced growth path, these satisfy the relations:
$G^X_t-1+G^L_t-1=G^K_t-1$
whereby G^X captures the growth in productivity X, G^L captures labor force growth, and output and capital grow at the rate G^K, which is the sum of these two rates of growth.

2.2.2 Firm Value

Merz and Yashiv (2007) show that within this set-up the value of the firms is given by:

$$S_t = K_{t+1} Q_t^K + N_{t+1} Q_t^N \tag{15}$$

In the current set-up:

$$\begin{aligned} Q_t^K &= 1 - \tau_{f,t} \\ Q_t^N &= \left(1 - \tau_{f,t}\right) \cdot \frac{\frac{\partial \Gamma_t}{\partial V_t}}{Q_t} \end{aligned}$$

Hence:

$$\begin{aligned} S_t &= K_{t+1}\left(1 - \tau_{f,t}\right) + N_{t+1}\left(\left(1 - \tau_{f,t}\right) \cdot \frac{\frac{\partial \Gamma_t}{\partial V_t}}{Q_t}\right) \\ S_t &= \left(1 - \tau_{f,t}\right) \cdot \left[K_{t+1} + N_{t+1} \cdot \frac{\frac{\partial \Gamma_t}{\partial V_t}}{Q_t}\right] \\ S_t &= \left(1 - \tau_{f,t}\right) \cdot \left[K_{t+1} + \frac{N_{t+1}}{N_t} \cdot \Theta \cdot \left(\frac{Q_t V_t}{N_t}\right)^{\gamma} \cdot F_t\right] \end{aligned}$$

Dividing throughout by F_t :

$$\frac{S_t}{F_t} = \left(1 - \tau_{f,t}\right) \cdot \left[\frac{\frac{K_{t+1}}{K_t}}{\frac{F_t}{K_t}} + \frac{N_{t+1}}{N_t} \cdot \Theta \cdot \left(\frac{H_t V_t}{N_t}\right)^{\gamma}\right]$$

In steady state: $\frac{K_{t+1}}{K_t} = G^K$, $\frac{N_{t+1}}{N_t} = G^L$, $\frac{H_t V_t}{N_t} = \frac{HV}{N}$, $\frac{F_t}{K_t} = \frac{F}{K}$ and so:

$$\frac{S}{F} = \left(1 - \tau_f\right) \cdot \left[\frac{G^K}{\frac{F}{K}} + G^L \cdot \Theta \cdot \left(\frac{HV}{N}\right)^{\gamma}\right] \tag{16}$$

2.3 Matching and Worker Flows

Unemployed workers and job vacancies match according to a CRS matching function:

$$M_t = \mu U_t^{\sigma} V_t^{1-\sigma} \tag{17}$$

The stock of unemployment evolves as follows, with separations and labor force growth increasing it and hires depleting it:

$$U_{t+1} = U_t + N_t\delta_t - M_t + L_{t+1} - L_t \tag{18}$$

Define the job finding rate as follows:

$$P_t \equiv \frac{M_t}{U_t}$$

Hence (18) may be written as:

$$\begin{aligned}
\frac{U_{t+1}}{L_{t+1}}\frac{L_{t+1}}{L_t} &= \frac{U_t}{L_t} + \frac{N_t\delta_t}{L_t} - \frac{M_t}{L_t} + \frac{L_{t+1}}{L_t} - 1 \\
\frac{U_{t+1}}{L_{t+1}}G^L_{t+1} &= \frac{U_t}{L_t} + \frac{(L_t - U_t)\delta_t}{L_t} - \frac{M_t}{U_t}\frac{U_t}{L_t} + G^L_{t+1} - 1
\end{aligned}$$

In steady state all of these rates are constant so:

$$\frac{U}{L} = \frac{\delta + (G^L - 1)}{\delta + (G^L - 1) + P} \tag{19}$$

The last equation is usually known as the Beveridge curve.

2.4 Wage Solution

Nash bargaining is given by:

$$W_t = \arg\max (J^N_t - J^U_t)^{\xi}(J^F_t - J^V_t)^{1-\xi} \tag{20}$$

where J^i are asset values to be defined below and ξ is the worker bargaining power.

2.4.1 Asset Values

Unemployed workers get unemployment income b_t and have the asset value of unemployment given by:

$$J^U_t = b_t + \beta_{t+1}E_t[P_{t+1}J^N_{t+1} + [1 - P_{t+1}]\, J^U_{t+1}]\} \tag{21a}$$

We assume the unemployed engage in home production and get benefits:

$$b_t = z\frac{F_t}{N_t} + \rho_t W_t \tag{22}$$

where z is a parameter and ρ_t is the net replacement ratio.

Employed workers receive gross wage W, pay a wage tax at rate τ_W and have asset values given by:

$$J_t^N = W_t\,(1-\tau_{W,t}) + \beta_{t+1}E_t[(1-\delta_{t+1})J_{t+1}^N + \delta_{t+1}J_{t+1}^U] \tag{23a}$$

Hence:

$$J_t^N - J_t^U = W_t\,(1-\tau_{W,t}) - b_t + \beta_{t+1}E_t(1-\delta_{t+1}-P_{t+1})(J_{t+1}^N - J_{t+1}^U) \tag{24a}$$

The asset value of a filled job – given the above F.O.C. – is:

$$\begin{aligned} J_t^F &= (1-\tau_{f,t})\left[\frac{\partial F_t}{\partial N_t} - \frac{\partial \Gamma_t}{\partial N_t} - W_t\,(1+\tau_{SSC,t}) - (1+\tau_{SSC,t})N_t\frac{\partial W_t}{\partial N_t}\right] \\ &\quad +\beta_{t+1}E_t\left[(1-\delta_{t+1})J_{t+1}^F + \delta_{t+1}J_{t+1}^V\right] \end{aligned} \tag{25}$$

And so:

$$\Lambda_t = \beta_{t+1}J_{t+1}^F$$

Free entry leads to:

$$J_t^V = 0 \tag{26}$$

2.4.2 The Nash Wage Solution

The solution posits:

$$\begin{aligned} \frac{\xi}{J_t^N - J_t^U} &= \frac{1-\xi}{J_t^F - J_t^V} \\ &\rightarrow\ J_t^N - J_t^U = \frac{\xi}{1-\xi}J_t^F \end{aligned} \tag{27}$$

Appendix B shows that the steady state solution is given by (in terms of the wage share, $\frac{W}{F/N}$):

$$\frac{W(N)\cdot N}{F} = \frac{1}{1+\tau_{SSC}}\left\{\frac{\left(z+\Delta\cdot\left[\alpha-\left(\alpha-(\frac{1+\gamma}{\delta})\right)\cdot\frac{\Theta}{1+\gamma}(\delta)^{1+\gamma}\right]\right)}{\Delta}\frac{1}{\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}+\alpha}\right\} \tag{28}$$

where:

$$\Delta = \frac{\xi \cdot (1-\tau_f)}{1-\xi} \cdot \frac{1-\beta(1-\delta-P)G^X}{1-\beta(1-\delta)G^X} \cdot \frac{1-\tau_W-\rho}{(1-\tau_f)(1+\tau_{SSC})}$$

and where the following condition is satisfied:

$$\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta} + \alpha - 1 > 0$$

2.5 The Policymaker Optimization Problem

The policymaker maximizes a weighted social welfare function defined over the representative firm value and the representative worker present value of earnings, as follows:

$$G_t = \max_{\tau_f, \tau_W, \tau_{SSC}, \rho} \begin{pmatrix} \omega \frac{S_t}{F_t} \\ +(1-\omega)\left[\frac{J_t^U}{(F/N)_t} c\left(\frac{U_t}{L_t}\right) + \frac{J_t^N}{(F/N)_t}\left(1-c\left(\frac{U_t}{L_t}\right)\right)\right] \end{pmatrix} \tag{29}$$

where ω, c are parameters, so that $0 \leq \omega \leq 1$ and $0 \leq c\left(\frac{U}{L}\right) \leq 1$.

The weight ω captures the social weight on capitalists/firm owners, with the complementary weight $(1-\omega)$ placed on the representative worker. The function is defined over the relevant values: $\frac{S_t}{F_t}$ for the representative firm and the term in square brackets for the representative worker. This latter term is a weighted average of the present value of earnings J_t^U and J_t^N, where the parameter c allows for the adjustment of the relative weight of the unemployed. The case of $c = 1$ gives the value of unemployment J_t^U a weight equal to the unemployment rate.

The budget constraint is given by:

$$\tau_{f,t}\Pi_t + (\tau_{W,t} + \tau_{SSC,t})\, W_t N_t - \rho_t W_t U_t = T_t \tag{30}$$

Dividing the budget constraint through by F and looking at the steady state, maximization is given by:

$$G = \max_{\tau_f, \tau_W, \tau_{SSC}, \rho} \begin{pmatrix} \omega \frac{S}{F} \\ +(1-\omega)\left[\frac{J^U}{(F/N)} c\left(\frac{U}{L}\right) + \frac{J^N}{(F/N)}\left(1-c\left(\frac{U}{L}\right)\right)\right] \end{pmatrix} \tag{31}$$

subject to the following equation and equations (**??**), (19), (62):

$$\frac{\tau_f \frac{\Pi}{N}}{(F/N)} + (\tau_W + \tau_{SSC}) \frac{W}{(F/N)} - \left[\rho \frac{W}{(F/N)}\right] \frac{U}{N} = \frac{T}{F} \tag{32}$$

The policymaker solves the problem by defining the Lagrangean:

$$\begin{aligned}
L \;=\; & \omega \tfrac{S}{F} \\
& +(1-\omega)\left[\tfrac{J^U}{(F/N)} c\left(\tfrac{U}{N}\right) + \tfrac{J^N}{(F/N)}\left(1 - c\left(\tfrac{U}{N}\right)\right)\right] \\
& +\lambda \left[\frac{\tau_f \frac{\Pi}{N}}{(F/N)} + (\tau_W + \tau_{SSC}) \frac{W}{(F/N)} - \left[\rho \frac{W}{(F/N)}\right] \frac{U}{N} - \frac{T}{F}\right] \\
= \; & \omega\left(1-\tau_f\right) \cdot \left[\frac{G^K}{\frac{F}{K}} + G^L \cdot \frac{\beta \cdot G^X}{1 - \beta \cdot G^X \cdot (1-\delta)} \cdot \left[\begin{array}{c} \alpha \cdot \left(1 - \frac{\Theta}{1+\gamma}\left(\frac{HV}{N}\right)^{1+\gamma}\right) \\ +\Theta \cdot \left(\frac{HV}{N}\right)^{1+\gamma} \\ -(1+\tau_{SSC}) \cdot \left(\frac{\partial W}{\partial N} \cdot \frac{N}{\frac{F}{N}} + \frac{W}{\frac{F}{N}}\right) \end{array}\right]\right] \\
& +(1-\omega)\left[\begin{array}{l} \dfrac{z + \rho \frac{W}{\frac{F}{N}} + \beta G^X \left[P \frac{\frac{W}{\frac{F}{N}}(1-\tau_W-\rho) - z}{1-\beta(1-\delta-P)G^X}\right]}{1-\beta G^X} c\left(\frac{U}{N}\right) \\ + \dfrac{\frac{W}{\frac{F}{N}}(1-\tau^W) - \beta G^X \delta \left[\frac{\frac{W}{\frac{F}{N}}(1-\tau_W-\rho) - z}{1-\beta(1-\delta-P)G^X}\right]}{1-\beta G^X}\left(1 - c\left(\frac{U}{N}\right)\right) \end{array}\right] \\
& +\lambda \left[\frac{\tau_f \frac{\Pi}{N}}{(F/N)} + (\tau_W + \tau_{SSC}) \frac{W}{(F/N)} - \left[\rho \frac{W}{(F/N)}\right] \frac{U}{N} - \frac{T}{F}\right]
\end{aligned} \tag{33}$$

and setting

$$\frac{\partial L}{\partial \tau_f} = \frac{\partial L}{\partial \tau_W} = \frac{\partial L}{\partial \tau_{SSC}} = \frac{\partial L}{\partial \rho} = 0 \tag{34}$$

given the structure of the economy delineated in sub-sections 3.1-3.4.

2.6 Model Solution

The solution is obtained by solving the following equations: the firms' FOC equations (8), (14) and its value (16); the steady state Beveridge curve (19); the wage solution (28); the government budget constraint (30); and the four optimality equations for the policymaker (34). This is done for the following endogenous variables: the capital-labor ratio $\frac{K}{N}$, the key labor market outcomes $\frac{U}{N}\frac{V}{N}$, $\frac{W}{\frac{F}{N}}$, $\frac{\frac{\partial W}{\partial N}N}{\frac{F}{N}}$, the value of the firm $\frac{S}{F}$, the multiplier λ, and the four policy variables $\tau_{SSC}, \tau^f, \tau^{SSC}, \rho$. Implied by them are $\frac{M}{N}, P, \frac{F}{K}$, and $\frac{F}{N}$. All agents take as given the level of technology X, the rates of technological growth G^X, population growth G^L and the ensuing rate of growth of capital G^K, the rate of interest r, the rates of capital deprecation d and worker separation δ, the value of home production z, and the budget constraint $\frac{T}{F}$.

3 Methodology

In this section we explain the empirical strategy and present the calibration values.

3.1 Empirical Strategy

The idea is to first solve the model in steady state for parameter values that are not well known and then to use the emerging set-up to study various variations in parameter values. Hence we proceed in two steps:

In the first stage I use the model equations in steady state and the average sample values of $\frac{V}{N}, \frac{U}{N}, s, \tau_W, \tau_{SSC}, \rho, \tau_f$ to solve for the policymaker parameters ω, c, the wage bargaining parameter ξ, the budget $\frac{T}{F}$, and the parameters defining the frictional labor market Θ, μ, σ and z. Basically in this step we are using the data and the model to "reverse engineer" parameter values.

In the second stage I use the model to solve for the endogenous labor market outcomes $\frac{V}{N}, \frac{U}{N}$ and s and the policy parameters $\tau_W, \tau_{SSC}, \rho, \tau_f$, given $\frac{T}{F}, A, c, \xi, \Theta, \mu, \sigma, z$. We vary the values of these parameters to study the following:

a. Differences in policymaker preferences for the two groups (workers and firms) as expressed by ω. I further examine the role of c in this context.

b. Differences in the budget $\frac{T}{F}$.

c. Differences in the bargaining power of the two groups (workers and firms) in the wage-setting process (ξ) .

d. Differences in the degree of frictions expressed by Θ, μ and σ.

e. Differences in the outside option of workers expressed by z.

In this version I do this for the U.S. In the next version this will be done for four European countries—France and Germany, which have had high unemployment for the past three decades, and the Netherlands and the U.K., which have *relatively low unemployment rates. Doing so I will examine cross-country differences in the underlying parameters, in labor-market policies and in labor-market outcomes. Specifically I will use this framework to examine two labor-market phenomena: the substantial differences in the continental European and U.S. labor-market experiences, especially in unemployment rates, and the cross-European heterogeneity in unemployment rates.*

3.2 Calibration Values

Table 1 reports the benchmark steady-state values of model variables and parameters based on data averages, empirical studies or solved from the steady-state relations.

Table 1
The U.S. Economy
Steady State Values

a. Parameters

symbol	parameter	value
α	production	0.69
γ	hiring costs power	2
σ	matching function power	0.5

b. Variables

symbol	variable	value
r	discount rate	0.012
d	rate of depreciation of capital	0.025
G^K	gross rate of capital growth	1.006
G^L	gross rate of employment growth	1.0033
G^X	gross rate of productivity growth	1.0043
$\frac{S}{F}$	firm value as a fraction of output	$\frac{S}{N}/\frac{F}{F} = 5$
δ	worker separation rate	0.131
$\frac{M}{N}$	hiring rate	$\delta + (G^L - 1) = 0.1343$
$\frac{U}{N}$	unemployment rate	0.15
$\frac{V}{N}$	vacancy rate	0.024
$\frac{W}{\frac{F}{N}}$	wage share	0.65
τ^W	wage tax	0.23
τ^{SSC}	ss contributions	0.08
τ^f	corporate tax	0.42
ρ	replacement ratio	0.3

4 Results

Table 2 shows the solution for what we described above as the first stage.

Table 2
The U.S. Economy
First Stage Solutions

symbol	parameter/variable	solved
ξ	bargaining parameter	0.21
ω	share of firms in policymaker objective	0.88
μ	matching function scale	2.24
Θ	cost function scale	17
z	flow value of unemployment	0.14
$\frac{T}{F}$	budget as % of GDP	0.18
$\frac{F}{K}$	the output-capital ratio	0.12

There are a few noteworthy results:

a. The weight placed by the policymaker on firm values, ω, is big – 0.88.

b. The bargaining power of workers, ξ, is low – 0.21. The flow value of unemployment, $z\frac{F}{N}$, is relatively low with z at 0.14

c. Labor market policy sums up in terms of the budget constraint to 18% of GDP, a sizeable figure.

Points a and b imply that workers have little power relative to firms in the determination of policy and of wages.

Future versions will present second stage results for the U.S. economy as well as for other (European) economies.

5 Conclusions

The paper has shown how to derive and solve for labor market equilibrium with frictions and with endogenous policymaking. Implementing the model on the U.S. economy it emerged that workers have little power relative to firms in the determination of policy and of wages.

References

[1] Acemoglu, D., Johnson, S., Querubin, P., Robinson J. A., 2008. When Does Policy Reform Work? The Case of Central Bank Independence. **Brookings Papers on Economic Activity** 1, 351–418.

[2] Blanchard, O., 2005. A Macroeconomic Survey of Europe. Downloadable at http://econ-www.mit.edu/files/757.

[3] Cahuc, P., F. Marque and E. Wasmer, 2008. "A Theory of Wages and Labor Demand with Intra-Firm Bargaining and Matching Frictions," **International Economic Review** 49,3,943-971.

[4] Cahuc, P., Postel-Vinay, F., Robin J.M., 2006. Wage Bargaining with On-the-Job Search: Theory and Evidence. **Econometrica** 74(2), 323-364.

[5] Chari,V. V. , Kehoe, P.J., 2006. Modern Macroeconomics in Practice: How Theory Is Shaping Policy. **Journal of Economic Perspectives** 20(4), 3-28.

[6] Cooper, R., Haltiwanger, J., Willis J. L., 2007. Implications of Search Frictions: Matching Aggregate and Establishment-level Observations. NBER Working Paper 13115.

[7] Davis, S.J., Faberman, R.J., Haltiwanger, J., 2009. The Establishment-Level Behavior of Vacancies and Hiring. NBER Working Papers 16265.

[8] Diamond, P.A., 1982a. Aggregate Demand Management in Search Equilibrium. **Journal of Political Economy** 90(5), 881–894.

[9] Diamond, P.A., 1982b. Wage Determination and Efficiency in Search Equilibrium. **Review of Economic Studies** 49(2), 217–227.

[10] Hagedorn, M., Manovskii, I., 2008. The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited. **American Economic Review** 98(4), 1692-1706.

[11] Hayashi, F., 1982. Tobin's Marginal q and Average q: A Neoclassical Interpretation. **Econometrica** 50, 213-224.

[12] Hobijn, B., Sahin, A., 2007. Job-Finding and Separation Rates in the OECD. Staff Reports 298, Federal Reserve Bank of New York.

[13] Hornstein, A., Krusell, P., Violante, G.L., 2007. Technology-Policy Interaction in Frictional Labour-Markets. **Review of Economic Studies** 74(4), 1089-1124.

[14] Hosios, A.J., 1990. On the Efficiency of Matching and Related Models of Search and Unemployment. **Review of Economic Studies** 57(2), 279–298.

[15] Ljungqvist, L., Sargent, T.J., 1998. The European Unemployment Dilemma. **Journal of Political Economy** 106(3), 514–550.

[16] Ljungqvist, L., Sargent, T.J., 2007a. Do Taxes Explain European Employment? Indivisible Labour, Human Capital, Lotteries and Savings. CEPR Discussion Papers 6196, C.E.P.R. Discussion Papers.

[17] Ljungqvist, L., Sargent, T.J., 2007b. Understanding European Unemployment with Matching and Search-Island Models. **Journal of Monetary Economics** 54(8), 2139-2179.

[18] Merz, M. and E. Yashiv, 2007. "Labor and the Market Value of the Firm," **American Economic Review,** 97, 4, 1419-1431.

[19] Mortensen, D.T. 1982. The Matching Process as a Non-Cooperative Bargaining Game, in J.J. McCall, 1ed., **The Economics of Information and Uncertainty**, University of Chicago Press, New York.

[20] Mortensen, D.T., Nagypal, E., 2007. More on Unemployment and Vacancy Fluctuations. **Review of Economic Dynamics** 10, 327–347.

[21] Mortensen, D.T., Pissarides, C.A., 1994. Job Creation and Job Destruction in the Theory of Unemployment. **Review of Economic Studies** 61, 397-415.

[22] Mortensen, D.T., Pissarides, C.A., 1999a. Job Reallocation, Employment Fluctuations, and Unemployment. In: Taylor, J.B., Woodford, M. (Eds.), **Handbook of Macroeconomics,** vol. 1B. North-Holland, Amsterdam, pp. 1171–1228 (Chapter 18).

[23] Nash, J., 1950. The Bargaining Problem. **Econometrica** 18(2), 155–162.

[24] OECD, 2009. Chapter 2 in **Employment Outlook**, OECD, Paris.

[25] Persson, T., Tabellini, G., 2002. **Political Economics: Explaining Economic Policy**, first ed. MIT Press, Cambridge.

[26] Petrongolo, B., Pissarides, C.A., 2001. Looking into the Black Box: A Survey of the Matching Function. **Journal of Economic Literature** 39(2), 390–431.

[27] Pissarides, C.A., 1985. Short-Run Dynamics of Unemployment, Vacancies and Real Wages. **American Economic Review** 75, 676-690.

[28] Pissarides, C.A., 2000. **Equilibrium Unemployment Theory**, second ed. MIT Press, Cambridge, MA

[29] Planas, C., R. Werner R., Rossi, A., 2007. How Much Has Labour Taxation Contributed to European Structural Unemployment? **Journal of Economic Dynamics & Control** 31, 1359–1375.

[30] Prescott, E. C., 2002. Prosperity and Depression. **American Economic Review** 92(2), 1-15.

[31] Prescott, E. C., 2004. Why Do Americans Work so Much More than Europeans? NBER Working Papers 10316.

[32] Pries, M. J., Rogerson R., 2005. Hiring Policies, Labor Market Institutions, and Labor Market Flows. **Journal of Political Economy** 113(4), 811–39.

[33] Rogerson, R. and R. Shimer, 2011. "Search in Macroeconomic Models of the Labor Market," in O.Ashenfelter and D.Card (eds.) **Handbook of Labor Economics Vol 4A,** Amsterdam, Elsevier – North Holland.

[34] Shimer, R., 2005. The Cyclical Behavior of Equilibrium Unemployment and Vacancies. **American Economic Review** 95 March, 25–49.

[35] Yashiv, E., 2004. Macroeconomic Policy Lessons of Labor Market Frictions. **European Economic Review** 48, 259–284.

[36] Yashiv, E., 2006. Evaluating the Performance of the Search and Matching Model. **European Economic Review** 50, 909–936.

[37] Yashiv, E., 2007. Search and Matching in Macroeconomics. **European Economic Review** 51, 1859–1895.

6 Appendix A: Model Derivations

6.1 Firm Values

6.1.1 Per Period Profits

Per period profits are given by:

$$\Pi_t = F_t - W_t(1 + \tau_{SSC,t})N_t - \Gamma_t - I_t \tag{35}$$

Dividing througout by N_t we get:

$$\frac{\Pi_t}{N_t} = \frac{F_t}{N_t} - W_t(1 + \tau_{SSC,t}) - \frac{\Gamma_t}{N_t} - \frac{I_t}{K_t}\frac{K_t}{N_t} \tag{36}$$

Dividing througout by $\frac{F_t}{N_t}$ we get:

$$\frac{\Pi_t}{F_t} = 1 - \frac{W_t}{\frac{F_t}{N_t}}(1 + \tau_{SSC,t}) - \frac{\frac{\Gamma_t}{N_t}}{\frac{F_t}{N_t}} - \frac{I_t}{K_t}\frac{K_t}{F_t} \tag{37}$$

In the non-stochastic steady state:

$$\frac{\Pi}{F} = 1 - \frac{W}{\frac{F}{N}}(1 + \tau_{SSC}) - \frac{\Theta}{1+\gamma}(\frac{QV}{N})^{1+\gamma} - \frac{I}{K}\frac{K}{F} \tag{38}$$

6.1.2 Vacancies Optimality Condition

The optimality condition is:

$$(1 - \tau_{f,t}) \cdot \frac{\partial \Gamma(V_t, N_t, K_t)}{\partial V_t} = E_t \beta_{t+1} \left(1 - \tau_{f,t+1}\right) \cdot \left[\begin{array}{c} \frac{\partial F(N_{t+1}, K_{t+1})}{\partial N_{t+1}} \cdot Q_t - \\ (1 + \tau_{SSC,t+1}) \cdot \left\{ \frac{\partial W_{t+1}}{\partial N_{t+1}} \cdot Q_t \cdot N_{t+1} + W_{t+1} \cdot Q \right. \\ + \frac{\partial \Gamma(V_{t+1}, N_{t+1}, K_{t+1})}{\partial V_{t+1}} \cdot \frac{Q_t}{Q_{t+1}} \cdot (1 - \delta_{t+1}) \\ - \frac{\partial \Gamma(V_{t+1}, N_{t+1}, K_{t+1})}{\partial N_{t+1}} \cdot Q_t \end{array} \right.$$

Taking Q_t out of the parentheses on the RHS:

$$\left(1-\tau_{f,t}\right)\cdot\frac{\partial\Gamma\left(V_t,N_t,K_t\right)}{\partial V_t} = E_t\beta_{t+1}\left(1-\tau_{f,t+1}\right)\cdot Q_t\left[\begin{array}{c}\frac{\partial F(N_{t+1},K_{t+1})}{\partial N_{t+1}}-\left(1+\tau_{SSC,t+1}\right)\\ \cdot\left\{\frac{\partial W_{t+1}}{\partial N_{t+1}}\cdot N_{t+1}+W_{t+1}\right\}\\ +\frac{\partial\Gamma(V_{t+1},N_{t+1},K_{t+1})}{\partial V_{t+1}}\cdot\frac{1}{Q_{t+1}}\cdot\left(1-\delta_{t+1}\right)\\ -\frac{\partial\Gamma(V_{t+1},N_{t+1},K_{t+1})}{\partial N_{t+1}}\end{array}\right]$$

$$\left(1-\tau_{f,t}\right)\cdot\frac{\frac{\partial\Gamma(V_t,N_t,K_t)}{\partial V_t}}{Q_t} = E_t\beta_{t+1}\left(1-\tau_{f,t+1}\right)\cdot\left[\begin{array}{c}\frac{\partial F(N_{t+1},K_{t+1})}{\partial N_{t+1}}\\ -\left(1+\tau_{SSC,t+1}\right)\cdot\left\{\frac{\partial W_{t+1}}{\partial N_{t+1}}\cdot N_{t+1}+W_{t+1}\right\}\\ +\frac{\partial\Gamma(V_{t+1},N_{t+1},K_{t+1})}{\partial V_{t+1}}\cdot\frac{1}{Q_{t+1}}\cdot\left(1-\delta_{t+1}\right)\\ -\frac{\partial\Gamma(V_{t+1},N_{t+1},K_{t+1})}{\partial N_{t+1}}\end{array}\right]$$

Denoting:

$$\frac{\frac{\partial\Gamma(V_t,N_t,K_t)}{\partial V_t}}{Q_t}=\Lambda_t$$

Substituting in the FOC:

$$\left(1-\tau_{f,t}\right)\cdot\Lambda_t=E_t\beta_{t+1}\left(1-\tau_{f,t+1}\right)\cdot\left[\begin{array}{c}\frac{\partial F(N_{t+1},K_{t+1})}{\partial N_{t+1}}-\left(1+\tau_{SSC,t+1}\right)\cdot\left\{\frac{\partial W_{t+1}}{\partial N_{t+1}}\cdot N_{t+1}+W_{t+1}\right\}\\ +\Lambda_{t+1}\cdot\left(1-\delta_{t+1}\right)-\frac{\partial\Gamma(V_{t+1},N_{t+1},K_{t+1})}{\partial N_{t+1}}\end{array}\right]$$

In stochastic SS:

$$\Lambda_t=\beta\cdot\left[\begin{array}{c}\frac{\partial F_{t+1}}{\partial N_{t+1}}-\left(1+\tau_{SSC}\right)\cdot\left\{\frac{\partial W_{t+1}}{\partial N_{t+1}}\cdot N_{t+1}+W_{t+1}\right\}\\ +\Lambda_{t+1}\cdot\left(1-\delta\right)-\frac{\partial\Gamma_{t+1}}{\partial N_{t+1}}\end{array}\right] \tag{39}$$

$$\begin{aligned}\Lambda_t &= \frac{\frac{\partial\Gamma_t}{\partial V_t}}{Q_t}=\frac{\Theta}{1+\gamma}\cdot(1+\gamma)\cdot\left(\frac{Q_tV_t}{N_t}\right)^{\gamma}\cdot\frac{Q_t}{N_t}\cdot F_t\cdot\frac{1}{Q_t}=\\ &= \Theta\cdot\left(\frac{Q_tV_t}{N_t}\right)^{\gamma}\cdot\frac{F_t}{N_t}\end{aligned}$$

$$\frac{\partial F_t}{\partial N_t}=\alpha\cdot\left(X_tN_t\right)^{\alpha-1}K_t^{1-\alpha}X_t=\alpha\cdot\frac{F_t}{N_t}$$

$$
\begin{aligned}
\Gamma_t &= \frac{\Theta}{1+\gamma}(Q_t V_t)^{1+\gamma} \cdot (X_t)^{\alpha} K_t^{1-\alpha} N_t^{\alpha-\gamma-1} \\
&\Rightarrow \frac{\partial \Gamma_t}{\partial N_t} = (\alpha-\gamma-1) \cdot \frac{\Theta}{1+\gamma}(Q_t V_t)^{1+\gamma} \cdot (X_t)^{\alpha} K_t^{1-\alpha} \cdot N_t^{\alpha-\gamma-2} = \\
&= (\alpha-\gamma-1) \cdot \frac{\Theta}{1+\gamma} \left(\frac{Q_t V_t}{N_t}\right)^{1+\gamma} \cdot (X_t N_t)^{\alpha} K_t^{1-\alpha} \cdot \frac{1}{N_t} = \\
&= (\alpha-\gamma-1) \cdot \Gamma_t \cdot \frac{1}{N_t}
\end{aligned}
$$

Substituting for the derivatives in 39:

$$
\Lambda_t = \beta \cdot \left[\begin{array}{c} \frac{\partial F_{t+1}}{\partial N_{t+1}} - (1+\tau_{SSC}) \cdot \left\{ \frac{\partial W_{t+1}}{\partial N_{t+1}} \cdot N_{t+1} + W_{t+1} \right\} \\ + \Lambda_{t+1} \cdot (1-\delta) - \frac{\partial \Gamma_{t+1}}{\partial N_{t+1}} \end{array} \right]
$$

$$
\Theta \cdot \left(\frac{Q_t V_t}{N_t}\right)^{\gamma} \cdot \frac{F_t}{N_t} = \beta \cdot \left[\begin{array}{c} \alpha \cdot \frac{F_{t+1}}{N_{t+1}} - (1+\tau_{SSC}) \cdot \left\{ \frac{\partial W_{t+1}}{\partial N_{t+1}} \cdot N_{t+1} + W_{t+1} \right\} \\ + \Theta \cdot \left(\frac{Q_{t+1} V_{t+1}}{N_{t+1}}\right)^{\gamma} \cdot \frac{F_{t+1}}{N_{t+1}} \cdot (1-\delta) - (\alpha-\gamma-1) \cdot \Gamma_{t+1} \cdot \frac{1}{N_{t+1}} \end{array} \right]
$$

Dividing by $\frac{F_{t+1}}{N_{t+1}}$:

$$
\Theta \cdot \left(\frac{Q_t V_t}{N_t}\right)^{\gamma} \cdot \frac{\frac{F_t}{N_t}}{\frac{F_{t+1}}{N_{t+1}}} = \beta \cdot \left[\begin{array}{c} \alpha - (1+\tau_{SSC}) \cdot \left\{ \frac{\partial W_{t+1}}{\partial N_{t+1}} \cdot N_{t+1} \cdot \frac{1}{\frac{F_{t+1}}{N_{t+1}}} + \frac{W_{t+1}}{\frac{F_{t+1}}{N_{t+1}}} \right\} \\ + \Theta \cdot \left(\frac{Q_{t+1} V_{t+1}}{N_{t+1}}\right)^{\gamma} \cdot (1-\delta) - (\alpha-\gamma-1) \cdot \frac{\Gamma_{t+1}}{F_{t+1}} \end{array} \right]
$$

Denoting $\frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}}$ by G_x:

$$
\Theta \cdot \left(\frac{Q_t V_t}{N_t}\right)^{\gamma} \cdot \frac{1}{G_x} = \beta \cdot \left[\begin{array}{c} \alpha - (1+\tau_{SSC}) \cdot \left\{ \frac{\partial W_{t+1}}{\partial N_{t+1}} \cdot N_{t+1} \cdot \frac{1}{\frac{F_{t+1}}{N_{t+1}}} + \frac{W_{t+1}}{\frac{F_{t+1}}{N_{t+1}}} \right\} \\ + \Theta \cdot \left(\frac{Q_{t+1} V_{t+1}}{N_{t+1}}\right)^{\gamma} \cdot (1-\delta) - (\alpha-\gamma-1) \cdot \frac{\Theta}{1+\gamma} (\frac{Q_{t+1} V_{t+1}}{N_{t+1}})^{1+\gamma} \end{array} \right]
$$

In steady state $\left(\frac{Q_t V_t}{N_t}\right) = \left(\frac{Q_{t+1} V_{t+1}}{N_{t+1}}\right) = \left(\frac{QV}{N}\right)$:

$$
\begin{aligned}
\Theta \cdot \left(\frac{QV}{N}\right)^{\gamma} &= G_x \cdot \beta \cdot \\
& \left[\begin{array}{c} \alpha - (1+\tau_{SSC}) \\ \cdot \left\{ \frac{\partial W_{t+1}}{\partial N_{t+1}} \cdot N_{t+1} \cdot \frac{1}{\frac{F_{t+1}}{N_{t+1}}} + \frac{W_{t+1}}{\frac{F_{t+1}}{N_{t+1}}} \right\} \\ +\Theta \cdot \left(\frac{QV}{N}\right)^{\gamma} \cdot (1-\delta) - (\alpha-\gamma-1) \cdot \frac{\Theta}{1+\gamma} (\frac{QV}{N})^{1+\gamma} \end{array} \right] \\
\Theta \cdot \left(\frac{QV}{N}\right)^{\gamma} \cdot [1 - \beta \cdot G_x \cdot (1-\delta)] &= G_x \cdot \beta \cdot \\
& \left[\begin{array}{c} \alpha - (1+\tau_{SSC}) \\ \cdot \left\{ \frac{\partial W_t}{\partial N_t} \cdot N_t \cdot \frac{1}{\frac{F_t}{N_t}} + \frac{W_t}{\frac{F_t}{N_t}} \right\} \\ - (\alpha-\gamma-1) \cdot \frac{\Theta}{1+\gamma} \left(\frac{QV}{N}\right)^{1+\gamma} \end{array} \right] \\
\Theta \cdot \left(\frac{QV}{N}\right)^{\gamma} \cdot [1 - \beta \cdot G_x \cdot (1-\delta)] &= G_x \cdot \beta \cdot \\
& \left[\begin{array}{c} \alpha \cdot \left[1 - \frac{\Theta}{1+\gamma} \left(\frac{QV}{N}\right)^{1+\gamma} \right] + (1+\gamma) \cdot \frac{\Theta}{1+\gamma} \left(\frac{QV}{N}\right)^{1+\gamma} \\ -(1+\tau_{SSC}) \\ \cdot \left\{ \frac{\partial W_t}{\partial N_t} \cdot N_t \cdot \frac{1}{\frac{F_t}{N_t}} + \frac{W_t}{\frac{F_t}{N_t}} \right\} \end{array} \right]
\end{aligned}
$$

Denoting

$$
\Omega = \frac{G_x \cdot \beta}{1 - \beta \cdot G_x \cdot (1-\delta)} \tag{40}
$$

we get:

$$
\Theta \cdot \left(\frac{QV}{N}\right)^{\gamma} = \Omega \cdot \left[\begin{array}{c} \alpha \cdot \left[1 - \frac{\Theta}{1+\gamma} \left(\frac{QV}{N}\right)^{1+\gamma} \right] + \Theta \left(\frac{QV}{N}\right)^{1+\gamma} \\ -(1+\tau_{SSC}) \cdot \left\{ \frac{\frac{\partial W}{\partial N} N + W}{\frac{F}{N}} \right\} \end{array} \right] \tag{41}
$$

6.1.3 Firm Asset Value

Below we show that in steady state:

$$\frac{J^F}{\frac{F}{N}} = \frac{(1-\tau_f)\left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N} - W(1+\tau_{SSC}) - (1+\tau_{SSC})N\frac{\partial W}{\partial N}}{\frac{F_t}{N_t}}\right]}{(1-\beta(1-\delta)G^X)} \tag{42}$$

6.2 Asset Values

Unemployed workers get unemployment income b_t and have the asset value of unemployment given by:

$$J_t^U = b_t + \beta_{t+1} E_t[P_{t+1} J_{t+1}^N + [1 - P_{t+1}]\, J_{t+1}^U]\} \tag{43a}$$

We assume the unemployed engage in home production and get benefits:

$$b_t = z\frac{F_t}{N_t} + \rho_t W_t \tag{44}$$

where z is a parameter and ρ_t is the net replacement ratio.

Inserting (**??**) and dividing througout by $\frac{F_t}{N_t}$ we get:

$$\frac{J_t^U}{\frac{F_t}{N_t}} = z + \rho_t \frac{W_t}{\frac{F_t}{N_t}} + \beta_{t+1} E_t[P_{t+1} \frac{J_{t+1}^N}{\frac{F_{t+1}}{N_{t+1}}} \frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}} + [1 - P_{t+1}]\, \frac{J_{t+1}^U}{\frac{F_{t+1}}{N_{t+1}}} \frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}}]\} \tag{45a}$$

Employed workers receive gross wage W, pay a wage tax at rate τ_W and have asset values given by:

$$J_t^N = W_t\,(1 - \tau_{W,t}) + \beta_{t+1} E_t[(1 - \delta_{t+1}) J_{t+1}^N + \delta_{t+1} J_{t+1}^U] \tag{46a}$$

Dividing througout by $\frac{F_t}{N_t}$ we get:

$$\frac{J_t^N}{\frac{F_t}{N_t}} = \frac{W_t}{\frac{F_t}{N_t}}\,(1 - \tau_{W,t}) + \beta_{t+1} E_t[(1 - \delta_{t+1}) \frac{J_{t+1}^N}{\frac{F_{t+1}}{N_{t+1}}} \frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}} + \delta_{t+1} \frac{J_{t+1}^U}{\frac{F_{t+1}}{N_{t+1}}} \frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}}] \tag{47a}$$

Hence:

$$J_t^N - J_t^U = W_t\,(1 - \tau_{W,t}) - b_t + \beta_{t+1} E_t (1 - \delta_{t+1} - P_{t+1})(J_{t+1}^N - J_{t+1}^U) \tag{48a}$$

and in $\frac{F_t}{N_t}$ terms:

$$\frac{J_t^N}{\frac{F_t}{N_t}} - \frac{J_t^U}{\frac{F_t}{N_t}} = \frac{W_t}{\frac{F_t}{N_t}}(1-\tau_{W,t}) - \left(z + \rho_t \frac{W_t}{\frac{F_t}{N_t}}\right) + \beta_{t+1} E_t (1-\delta_{t+1} - P_{t+1})\left(\frac{J_{t+1}^N}{\frac{F_{t+1}}{N_{t+1}}}\frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}} - \frac{J_{t+1}^U}{\frac{F_{t+1}}{N_{t+1}}}\frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}}\right) \tag{49a}$$

In the non-stochastic steady state:

$$\left(\frac{J^N}{\frac{F}{N}} - \frac{J^U}{\frac{F}{N}}\right)(1-\beta(1-\delta-P)G^X) = \frac{W}{\frac{F}{N}}(1-\tau_W) - \left(z + \rho\frac{W}{\frac{F}{N}}\right) \tag{50}$$

$$\left(\frac{J^N}{\frac{F}{N}} - \frac{J^U}{\frac{F}{N}}\right) = \frac{\frac{W}{\frac{F}{N}}(1-\tau_W-\rho) - z}{1-\beta(1-\delta-P)G^X}$$

The asset value of a filled job – given the above F.O.C. – is:

$$J_t^F = (1-\tau_{f,t})\left[\frac{\partial F_t}{\partial N_t} - \frac{\partial \Gamma_t}{\partial N_t} - W_t(1+\tau_{SSC,t}) - (1+\tau_{SSC,t})N_t\frac{\partial W_t}{\partial N_t}\right] \tag{51}$$

$$+\beta_{t+1}E_t\left[(1-\delta_{t+1})J_{t+1}^F + \delta_{t+1}J_{t+1}^V\right] \tag{52}$$

And so:

$$\Lambda_t = \beta_{t+1}J_{t+1}^F$$

Dividing througout by $\frac{F_t}{N_t}$ we get:

$$\frac{J_t^F}{\frac{F_t}{N_t}} = (1-\tau_{f,t})\left[\frac{\frac{\partial F_t}{\partial N_t} - \frac{\partial \Gamma_t}{\partial N_t} - W_t(1+\tau_{SSC,t}) - (1+\tau_{SSC,t})N_t\frac{\partial W_t}{\partial N_t}}{\frac{F_t}{N_t}}\right] \tag{53}$$

$$+\beta_{t+1}E_t\left[(1-\delta_{t+1})\frac{J_{t+1}^F}{\frac{F_{t+1}}{N_{t+1}}}\frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}} + \delta_{t+1}J\frac{J_{t+1}^V}{\frac{F_{t+1}}{N_{t+1}}}\frac{\frac{F_{t+1}}{N_{t+1}}}{\frac{F_t}{N_t}}\right]$$

Free entry leads to:

$$J_t^V = 0 \tag{54}$$

In the non-stochastic steady state:

$$\frac{J^F}{\frac{F}{N}}(1-\beta(1-\delta)G^X) = (1-\tau_f)\left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N} - W(1+\tau_{SSC}) - (1+\tau_{SSC})N\frac{\partial W}{\partial N}}{\frac{F}{N}}\right] \tag{55}$$

and so:

$$\frac{J^F}{\frac{F}{N}} = \frac{\left(1-\tau_f\right)\left[\frac{\frac{\partial F}{\partial N}-\frac{\partial \Gamma}{\partial N}-W(1+\tau_{SSC})-(1+\tau_{SSC})N\frac{\partial W}{\partial N}}{\frac{F_t}{N_t}}\right]}{(1-\beta(1-\delta)G^X)} \tag{56}$$

7 Appendix B: the Wage Solution[3]

Value of employed minus unemployed:

$$J_t^N - J_t^U = W_t\left(1-\tau_{W,t}\right) - z\frac{F_t}{N_t} - \rho_t W_t + \beta_{t+1} E_t (1-\delta_{t+1} - P_{t+1})(J_{t+1}^N - J_{t+1}^U) \tag{57a}$$

The asset value of a filled job:

$$\begin{aligned} J_t^F &= \left(1-\tau_{f,t}\right)\left[\frac{\partial F_t}{\partial N_t} - \frac{\partial \Gamma_t}{\partial N_t} - W_t\left(1+\tau_{SSC,t}\right) - (1+\tau_{SSC,t})N_t\frac{\partial W_t}{\partial N_t}\right] \\ &\quad + \beta_{t+1} E_t\left[(1-\delta_{t+1})J_{t+1}^F\right] \end{aligned}$$

Nash bargaining is given by:

$$W_t = \arg\max (J_t^N - J_t^U)^{\xi} (J_t^F)^{1-\xi} \tag{58}$$

So, the FOC is:

$$\begin{aligned} \xi (J_t^N - J_t^U)^{\xi-1} (J_t^F)^{1-\xi} \frac{\partial (J_t^N - J_t^U)}{\partial W_t} + (1-\xi)(J_t^N - J_t^U)^{\xi}(J_t^F)^{-\xi}\frac{\partial (J_t^F)}{\partial W_t} &= 0 \\ \frac{J_t^F}{J_t^N - J_t^U}\frac{\xi}{1-\xi} + \frac{\frac{\partial (J_t^F)}{\partial W_t}}{\frac{\partial (J_t^N - J_t^U)}{\partial W_t}} &= 0 \\ \frac{J_t^F}{J_t^N - J_t^U}\frac{\xi}{1-\xi} + \frac{-\left(1-\tau_{f,t}\right)\left(1+\tau_{SSC,t}\right)}{1-\tau_{W,t}-\rho_t} &= 0 \\ \frac{\xi}{1-\xi}\frac{1-\tau_{W,t}-\rho_t}{\left(1-\tau_{f,t}\right)\left(1+\tau_{SSC,t}\right)} J_t^F &= J_t^N - J_t^U \end{aligned}$$

Divide by $\frac{F_t}{N_t}$ throughout:

$$\frac{J_t^N}{\frac{F_t}{N_t}} - \frac{J_t^U}{\frac{F_t}{N_t}} = \frac{\xi}{1-\xi}\frac{1-\tau_{W,t}-\rho_t}{\left(1-\tau_{f,t}\right)\left(1+\tau_{SSC,t}\right)}\frac{J_t^F}{\frac{F_t}{N_t}}$$

In steady state the ratios of asset values to output per worker are constant:

[3] I thank Avihai Lifshitz and Tanya Baron for the derivation.

$$\frac{J^N}{\frac{F}{N}} - \frac{J^U}{\frac{F}{N}} = \frac{\xi}{1-\xi} \frac{1-\tau_W-\rho}{\left(1-\tau_f\right)\left(1+\tau_{SSC}\right)} \frac{J^F}{\frac{F}{N}} \tag{59}$$

From equations (63) and (69) in the main text we know:

$$\left(\frac{J^N}{\frac{F}{N}} - \frac{J^U}{\frac{F}{N}}\right) = \frac{\frac{W}{\frac{F}{N}}\left(1-\tau_W-\rho\right) - z}{1-\beta(1-\delta-P)G^X}$$

and:

$$\frac{J^F}{\frac{F}{N}} = \frac{\left(1-\tau_f\right)\left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N} - W\left(1+\tau_{SSC}\right) - (1+\tau_{SSC})N\frac{\partial W}{\partial N}}{\frac{F}{N}}\right]}{\left(1-\beta(1-\delta)G^X\right)}$$

Inserting into the bargainng solution:

$$\begin{aligned}
\frac{\frac{W}{\frac{F}{N}}\left(1-\tau_W-\rho\right) - z}{1-\beta(1-\delta-P)G^X} &= \frac{\xi}{1-\xi} \cdot \frac{1-\tau_W-\rho}{\left(1-\tau_f\right)\left(1+\tau_{SSC}\right)} \\
&\cdot \left[\frac{\left(1-\tau_f\right)\left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N} - W\left(1+\tau_{SSC}\right) - (1+\tau_{SSC})N\frac{\partial W}{\partial N}}{\frac{F}{N}}\right]}{\left(1-\beta(1-\delta)G^X\right)}\right]
\end{aligned}$$

Denote

$$\Delta = \frac{\xi \cdot \left(1-\tau_f\right)}{1-\xi} \cdot \frac{1-\beta(1-\delta-P)G^X}{1-\beta(1-\delta)G^X} \cdot \frac{1-\tau_W-\rho}{\left(1-\tau_f\right)\left(1+\tau_{SSC}\right)}$$

$$\begin{aligned}
\frac{W}{\frac{F}{N}}\left(1-\tau_W-\rho\right) - z &= \Delta\left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N} - W\left(1+\tau_{SSC}\right) - (1+\tau_{SSC})N\frac{\partial W}{\partial N}}{\frac{F}{N}}\right] \\
\frac{W}{\frac{F}{N}}\left(1-\tau_W-\rho\right) - z &= \Delta \cdot \left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N}}{\frac{F}{N}}\right] \\
&\quad - \left(1+\tau_{SSC}\right) \cdot \Delta \cdot \frac{W}{\frac{F}{N}} - \left(1+\tau_{SSC}\right) \cdot \Delta \cdot N \cdot \frac{\frac{\partial W}{\partial N}}{\frac{F}{N}} \\
\frac{W}{\frac{F}{N}} \cdot \left[1-\tau_W-\rho+\left(1+\tau_{SSC}\right) \cdot \Delta\right] &= z + \Delta \cdot \left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N}}{\frac{F}{N}}\right] - \left(1+\tau_{SSC}\right) \cdot \Delta \cdot N \cdot \frac{\frac{\partial W}{\partial N}}{\frac{F}{N}}
\end{aligned}$$

Multiplying by $\frac{F}{N}$:

$$W \cdot [1 - \tau_W - \rho + (1+\tau_{SSC}) \cdot \Delta] = \left(z + \Delta \cdot \left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N}}{\frac{F}{N}} \right] \right) \cdot \frac{F}{N} - (1+\tau_{SSC}) \cdot \Delta \cdot N \cdot \frac{\partial W}{\partial N}$$

Dividing by N and rearanging:

$$\begin{aligned}
\frac{W}{N} \cdot [1 - \tau_W - \rho + (1+\tau_{SSC}) \cdot \Delta] &= \left(z + \Delta \cdot \left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N}}{\frac{F}{N}} \right] \right) \cdot \frac{F}{N^2} - (1+\tau_{SSC}) \cdot \Delta \cdot \frac{\partial W}{\partial N} \\
(1+\tau_{SSC}) \cdot \Delta \cdot \frac{\partial W}{\partial N} &= \frac{W}{N} \cdot [\tau_W + \rho - (1+\tau_{SSC}) \cdot \Delta - 1] \\
&\quad + \left(z + \Delta \cdot \left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N}}{\frac{F}{N}} \right] \right) \cdot \frac{F}{N^2} \\
\frac{\partial W}{\partial N} &= \frac{W}{N} \cdot \frac{[\tau_W + \rho - (1+\tau_{SSC}) \cdot \Delta - 1]}{(1+\tau_{SSC}) \cdot \Delta} \\
&\quad + \frac{\left(z + \Delta \cdot \left[\frac{\frac{\partial F}{\partial N} - \frac{\partial \Gamma}{\partial N}}{\frac{F}{N}} \right] \right)}{(1+\tau_{SSC}) \cdot \Delta} \cdot \frac{F}{N^2}
\end{aligned}$$

Hiring costs:

$$\begin{aligned}
\Gamma_t &= \frac{\Theta}{1+\gamma} \left(\frac{H_t V_t}{N_t} \right)^{1+\gamma} F_t = \frac{\Theta}{1+\gamma} (\frac{N_{t+1} - (1-\delta_t) N_t}{N_t})^{1+\gamma} \cdot F_t \quad (60) \\
&= \frac{\Theta}{1+\gamma} (\frac{N_{t+1}}{N_t} - (1-\delta_t))^{1+\gamma} \cdot (X_t N_t)^{\alpha} K_t^{1-\alpha} \quad (61)
\end{aligned}$$

So:

$$\frac{\partial \Gamma_t}{\partial N_t} = \Theta (\frac{N_{t+1}}{N_t} - (1-\delta_t))^{\gamma} \frac{-N_{t+1}}{(N_t)^2} F_t + \frac{\Theta}{1+\gamma} (\frac{N_{t+1}}{N_t} - (1-\delta_t))^{1+\gamma} \alpha \frac{F_t}{N_t}$$

In steady state:

$$\frac{\partial \Gamma_t}{\partial N_t} = -\Theta (\delta)^{\gamma} \frac{F}{N} + \frac{\Theta}{1+\gamma} (\delta)^{1+\gamma} \alpha \frac{F}{N} = \left[\alpha - (\frac{1+\gamma}{\delta}) \right] \frac{\Theta}{1+\gamma} (\delta)^{1+\gamma} \frac{F}{N}$$

Using the functional forms for F and Γ:

$$
\frac{\partial W}{\partial N} + \frac{W}{N} \cdot \frac{[1 - \tau_W - \rho + (1 + \tau_{SSC}) \cdot \Delta]}{(1 + \tau_{SSC}) \cdot \Delta} -
$$

$$
\frac{\left(z + \Delta \cdot \left[\alpha - \left(\alpha - (\frac{1+\gamma}{\delta})\right) \cdot \frac{\Theta}{1+\gamma}(\delta)^{1+\gamma}\right]\right)}{(1 + \tau_{SSC}) \cdot \Delta} \cdot \frac{F}{N^2} = 0
$$

Denote:

$$
\begin{aligned}
G(N) &\equiv \frac{\left(z + \Delta \cdot \left[\alpha - \left(\alpha - (\frac{1+\gamma}{\delta})\right) \cdot \frac{\Theta}{1+\gamma}(\delta)^{1+\gamma}\right]\right)}{(1 + \tau_{SSC}) \cdot \Delta} \cdot \frac{F}{N} \\
\chi &= \frac{[1 - \tau_W - \rho + (1 + \tau_{SSC}) \cdot \Delta]}{(1 + \tau_{SSC}) \cdot \Delta} = 1 + \frac{1 - \tau_W - \rho}{(1 + \tau_{SSC}) \cdot \Delta} > 1
\end{aligned}
$$

Then the DE above can be re-written:

$$
\frac{\partial W}{\partial N} + \frac{W}{N} \cdot \chi - \frac{G(N)}{N} = 0
$$

Using the solution method from Cahuc et al. (2008):

- The solution to homogenous DE $\frac{\partial W}{\partial N} + \frac{W}{N} \cdot \chi = 0$ is:

$$
W(N) = C(N) \cdot N^{-\chi}
$$

- Deriving it w.r.t. N :

$$
\frac{dW}{dN} = \frac{dC}{dN} \cdot N^{-\chi} - \chi C(N) N^{-\chi-1}
$$

- Substituting two last equations into the original diff equation:

$$
\begin{aligned}
\frac{dC}{dN} \cdot N^{-\chi} - \chi C(N) N^{-\chi-1} + \frac{C(N) \cdot N^{-\chi}}{N} \cdot \chi - \frac{G(N)}{N} &= 0 \\
\frac{dC}{dN} \cdot N^{-\chi} - \frac{G(N)}{N} &= 0 \\
\frac{dC}{dN} &= N^{\chi-1} \cdot G(N)
\end{aligned}
$$

$$
\Rightarrow C(N) = \int_0^N x^{\chi-1} \cdot G(x) dx + D
$$

- Inserting into the solution of homogenous DE:

$$\begin{aligned} W(N) &= \left(\int_0^N x^{\chi-1} \cdot G(x)dx + D\right) \cdot N^{-\chi} \\ &= N^{-\chi} \cdot \int_0^N x^{\chi-1} \cdot G(x)dx + D \cdot N^{-\chi} \end{aligned}$$

- Following Cahuc et al. (2008) I assume $\lim_{N \to 0} NW(N) = 0$:

$$\lim_{N \to 0} NW(N) = \lim_{N \to 0} N^{1-\chi} \cdot \int_0^N x^{\chi-1} \cdot G(x)dx + D \cdot N^{1-\chi}$$

given that $\chi > 1$, for $\lim_{N \to 0} NW(N) = 0$ to hold, we need $D = 0$. So that the final solution for wage is:

$$W(N) = N^{-\chi} \cdot \int_0^N x^{\chi-1} \cdot G(x)dx$$

Inserting χ and $G(.)$:

$$\begin{aligned} W(N) &= N^{-\frac{[1-\tau_W-\rho+(1+\tau_{SSC})\cdot\Delta]}{(1+\tau_{SSC})\cdot\Delta}} \cdot \int_0^N x^{\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}} \\ &\quad \cdot \frac{\left(z + \Delta \cdot \left[\alpha - \left(\alpha - (\frac{1+\gamma}{\delta})\right) \cdot \frac{\Theta}{1+\gamma}(\delta)^{1+\gamma}\right]\right)}{(1+\tau_{SSC})\cdot\Delta} \cdot \frac{F(x)}{x} \cdot dx \end{aligned}$$

so that:

$$\frac{W(N)\cdot N}{F} = N^{-\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}}\cdot\frac{1}{N^{\alpha}}\int_0^N x^{\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}} \cdot\frac{\left(z+\Delta\cdot\left[\alpha-\left(\alpha-(\frac{1+\gamma}{\delta})\right)\cdot\frac{\Theta}{1+\gamma}(\delta)^{1+\gamma}\right]\right)}{(1+\tau_{SSC})\cdot\Delta}\cdot\frac{(X_t x)^{\alpha}K_t^{1-\alpha}}{x(X_t)^{\alpha}K_t^{1-\alpha}}\cdot dx$$

$$\frac{W(N)\cdot N}{F} = N^{-\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}-\alpha}\cdot\int_0^N x^{\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}} \cdot\frac{\left(z+\Delta\cdot\left[\alpha-\left(\alpha-(\frac{1+\gamma}{\delta})\right)\cdot\frac{\Theta}{1+\gamma}(\delta)^{1+\gamma}\right]\right)}{(1+\tau_{SSC})\cdot\Delta}\cdot x^{\alpha-1}\cdot dx$$

$$\frac{W(N)\cdot N}{F} = N^{-\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}-\alpha}\cdot\int_0^N x^{\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}+\alpha-1} \cdot\frac{\left(z+\Delta\cdot\left[\alpha-\left(\alpha-(\frac{1+\gamma}{\delta})\right)\cdot\frac{\Theta}{1+\gamma}(\delta)^{1+\gamma}\right]\right)}{(1+\tau_{SSC})\cdot\Delta}\cdot dx$$

The solution to the integral is:

$$\int_0^N x^{A-1}\cdot dx = \frac{1}{A}x^{A} \tag{62}$$

As long as $A > 0$. So:

$$\frac{W(N)\cdot N}{F} = \frac{1}{1+\tau_{SSC}}\left\{\frac{\left(z+\Delta\cdot\left[\alpha-\left(\alpha-(\frac{1+\gamma}{\delta})\right)\cdot\frac{\Theta}{1+\gamma}(\delta)^{1+\gamma}\right]\right)}{\Delta}\frac{1}{\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}+\alpha}\right\}$$

With the constraint that:

$$\frac{[1-\tau_W-\rho]}{(1+\tau_{SSC})\cdot\Delta}+\alpha-1>0$$