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"Precautionary Saving and Labor Supply"

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Abstract

Saving for a rainy day is driven by precaution. Because, in principle, leisure affects utility similarly as consumption, it also should respond to the saving motive. Here, we study the role of precautionary saving for labor supply, and focus on the wage elasticity of labor supply. We study this link in the context of a calibrated quantitative model in which households are impatient and use durable goods, thought of as including real estate, as a collateral for borrowing. In this model, households save by holding more equity on durable goods than they are required, which is here identical to holding interest earning assets. The buffer stock of assets makes possible to smoothen better consumption and leisure, and this enhances the elasticity of labor supply to wage changes

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1 Introduction

In a recent working paper, Campbell and Hercowitz (2009) document the co-movement of hours worked and debt using data from the Panel Study of Income Dynamics (PSID) data. Denominating debt by the work hours that are required to repay it, they find that every additional hour of debt corresponds to between 2.5 and 5 minutes of additional labor every year. To analyze the link between debt and hours worked, they employ a partial equilibrium model in which impatient households enjoy consumption, leisure and services from a durable goods stock which serves as a collateral for borrowing. A persistent positive wage shock increases the desired stock of durable goods but households must pay a down payment on new purchases. This drives households to increase labor supply.

Campbell and Hercowitz (2009) explores the co-movement of debt and labor supply by studying it under a rolling certainty equivalence setup. This means that although the environment is stochastic, households make decisions assuming that the future evolution of the wage is certain at the mean level in each period. Hence, that setup does not include precautionary saving behavior that arises when agents incorporate the stochastic nature of the income stream in to their decisions.

Usually, precautionary saving is studied in the context of consumption as in the work by Carroll (1997). Here, we focus on labor supply, which, to the best of our knowledge, was not studied yet in this context. For precautionary saving, the household should face a borrowing constraint. Otherwise, it can take a loan when necessary and there is no need for saving.

For realism, as in Campbell and Hercowitz (2009), we focus on a collateral constraint on durable goods. In the US, 90% of household debt is collateralized by houses and vehicles, and the borrowing constraint takes the form of an equity requirement on those durable goods. In the model, households can save by holding equity on durable goods at a higher rate than required, or alternatively by saving. Because we assume no interest rate spread, the household is indifferent between these two saving routes.

In terms of methodology, we calibrate the model to actual U.S. data, and then simulate it to gauge the quantitative importance of precautionary labor supply behavior. The questions we wish to address in this project are: (a) How wage uncertainty affects the elasticity of labor supply and (b) the dynamics after a wage shock.

2 The Model

This is the problem of an impatient household, in the usual sense that the subjective discount factor is smaller than the financial discount factor. Denoting $0 < \beta < 1$ as the subjective weight households assign to next period utility, and $1/R$ as the financial discount factor (where R is the gross real interest rate), then impatience implies that $\beta R < 1$.

The household faces a the constant interest rate R, at which can borrow up to a borrowing constraint specified below. The household also faces an exogenous wage following the process $W_t/\chi - 1 = \rho\left(W_{t-1}/\chi - 1\right) + \varepsilon_t$, $0 < \rho < 1$, $\varepsilon_t \sim N(0, \sigma)$, where χ is the long-run wage—or it's unconditional mean.

We also adopt the following notation:
C_t: nondurable consumption,
S_t: consumer durable stock (including housing)
N_t: fraction of time working
B_t: debt at the beginning of period t

Budget constraint (always binding):

$$B_{t+1} + W_t N_t - C_t - S_{t+1} + (1-\delta)S_t - RB_t = 0,$$

where $0 < \delta < 1$ is the depreciation rate.

Borrowing is collateralized by durable goods, and it cannot exceed a fraction of their value. Specifically, the household faces the following borrowing constraint, which is occasionally binding:

$$B_{t+1} \leq \frac{(1-\delta)(1-\pi)}{R} S_{t+1}.$$

Here, π is the required equity next period from the depreciated stock. The division by R turns the expression $(1-\delta)(1-\pi)/R$ into the current loan to value ratio. Hence $1 - (1-\delta)(1-\pi)/R$ is the minimum required down payment fraction of durable goods purchases.

Given the initial stocks of durable goods and debt S_0 and B_0, the problem to be solved by the household can be expressed by the Lagrangean:

$$£ = \max E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} (1-\theta)\ln C_t + \theta \ln S_t + \omega \frac{(1-N_t)^{1-\eta}}{1-\eta} \\ +\lambda_t \left(B_{t+1} + W_t N_t - C_t - S_{t+1} + (1-\delta) S_t - R B_t \right) \\ +\Gamma_t \left(\frac{(1-\delta)(1-\pi)}{R} S_{t+1} - B_{t+1} \right) \end{array} \right],$$

where $0 < \theta < 1$, $\omega > 0$, $\eta > 0$, and λ_t, Γ_t are the Lagrange multipliers associated with the budget and the borrowing constraints.

The first-order conditions to this problem are:

$$0 = \frac{\partial £}{\partial C_t} = \frac{1-\theta}{C_t} - \lambda_t, \tag{1}$$

$$0 = \frac{\partial £}{\partial S_{t+1}} = \left\{ -\lambda_t + \frac{(1-\delta)(1-\pi)}{R} \Gamma_t + \beta \frac{\theta}{S_{t+1}} + \beta(1-\delta) E_t \left[\lambda_{t+1} \right] \right\}, \tag{2}$$

$$0 = \frac{\partial £}{\partial B_{t+1}} = \lambda_t - \Gamma_t - \beta R E_t \left[\lambda_{t+1} \right], \tag{3}$$

$$0 = \frac{\partial £}{\partial N_t} = -\omega \left(1 - N_t \right)^{-\eta} + \lambda_t W_t, \tag{4}$$

and the constraints are:

$$0 = \frac{\partial £}{\partial \lambda_t} = B_{t+1} + W_t N_t - C_t - S_{t+1} + (1-\delta) S_t - R B_t, \tag{5}$$

$$0 \le \frac{\partial £}{\partial \Gamma_t} = \frac{(1-\delta)(1-\pi)}{R} S_{t+1} - B_{t+1}. \tag{6}$$

In any period t, the state variables are stocks of durable goods and wage S_t and B_t, and the current wage W_t.

For convenience of the analysis, let us define F_t to be the excess collateral the borrower chooses to hold:

$$F_{t+1} = \frac{(1-\delta)(1-\pi)}{R} S_{t+1} - B_{t+1}.$$

Then the borrowing constraint can be expressed as:

$$B_{t+1} = \frac{(1-\delta)(1-\pi)}{R} S_{t+1} - F_{t+1}.$$

The first-order conditions are not affected by this definition, but the budget constraint now becomes:

$$0 = -\left[1 - \frac{(1-\delta)(1-\pi)}{R} \right] S_{t+1} - F_{t+1} + W_t N_t - C_t + \pi(1-\delta) S_t + R F_t,$$

and the borrowing constraint is now:

$$F_{t+1} \ge 0.$$

3 Calibration

The calibration is based on Cambpell and Hercowitz (2009). The unit of time is chosen to be one year.

The financial parameters are: Required equity share π is 0.16 and the gross interest rate R is 1.04.

The values chosen for parameters of the utility function are the following. First, the subjective discount rate is $\beta = 1/1.06$, i.e., the rate of impatience is about 2 percent per year. The weight of durable goods in utility $\theta = 0.37$, which is derived from the expenditure share of durable goods. The parameters of leisure in utility are $\eta = 0.9$ and 1.93. Note that $\eta = 1$ corresponds to the logarithmic form, i.e., we adopt a slightly more elastic response of labor supply to the wage relative to the logarithmic form. These parameter values are set so that hours worked are 0.3 of the household's available time in steady state.

The depreciation rate of the durable stock $\delta = 0.04$. The wage process is determined as follows. The autoregressive coefficient is $\rho = 0.9$, the standard deviation is $\sigma = 0.185$, and the steady-state wage value is normalized to $\chi = 1$. Note that the standard deviation is much higher than used in macro models because here they are based on individual households data—the Panel Study of Economy Dynamics (PSID). The volatility of wages at the aggregate level is significantly lower because much of the individual variation averages out.

4 Results

Precautionary saving affects the response in ways that are absent in the usual labor supply model. To understand the implications of precautionary savings for labor supply we analyze the responses of labor supply and other variables to wage shocks along three aspects: (1) perfect foresight versus stochastic setups, i.e., a certainty equivalent approach and incorporating uncertainty into the household's decisions, (2) positive versus negative shocks, (2) small versus large shocks. To provide an insight on the results, we address the interaction between these dimensions. The key aspect is, or course, the perfect foresight/stochastic setups. The interaction between these aspects is important, for example, because the buffer stock can be depleted with a negative shock that is sufficiently large. Then, the borrowing constraint becomes binding.

Consider a transitory negative wage shock of one percent as in Figure 1. In the "F" panel, we can see that assets do not change in the perfect foresight setup, i.e., the borrowing constraint keeps binding following the shock. However, assets decline in the stochastic setup. Hence, because assets are used to mitigate the effects of the negative shock, nondurable consumption declines less and labor supply declines more in the stochastic setup than under perfect foresight. This means that the elasticity of labor supply is higher with uncertainty than under prefect foresight. Once most of the shock had passed, labor supply with uncertainty is slightly higher in order to rebuild the buffer stock of assets. The stock of durable goods declines less than under perfect foresight

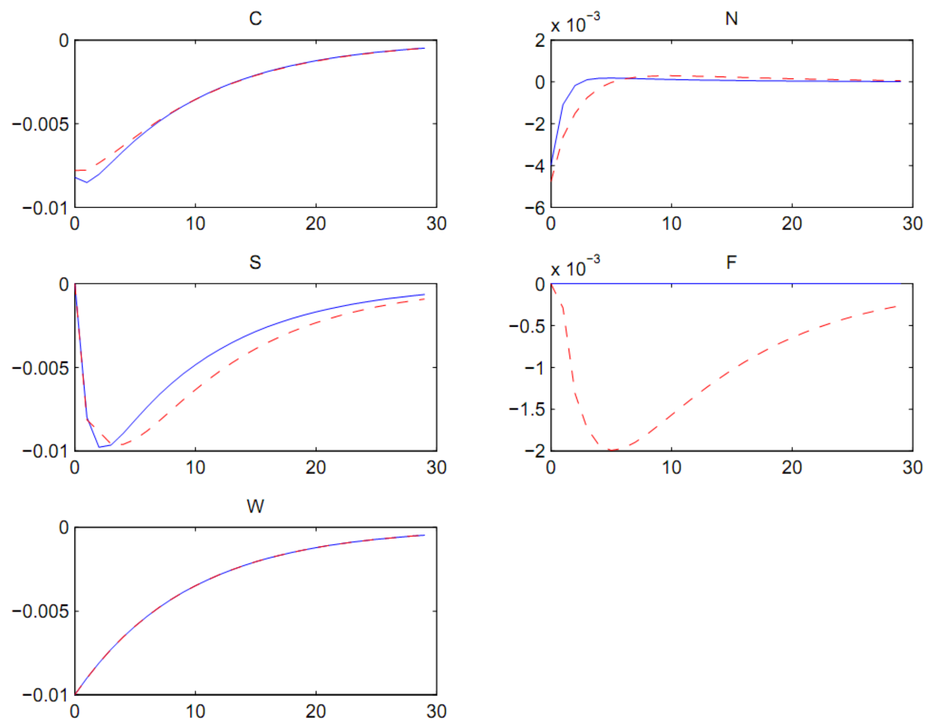

Figure 1: The dynamic response to a 1% negative wage shock. Solid Blue - prefect foresight; Dashed Red - full stochastic solution. Y axis units are log differences from respective steady states. Note that the full stochastic steady state includes a buffer stock in F so it is possible for it to become negative (in relation to its steady state).

in the first periods following the shock, similarly as nondurable consumption, but, then, rebuilding the stock is delayed given the household's desire to rebuild the buffer stock of assets.

Figure 2 shows the responses to a positive one percent shock, which are in general quite symmetric to those under a negative shock. As long as there is still some buffer stock left the response to a negative shock mirrors the response to a positive shock.

Figure 3 shows the responses to a negative shock of 18 percent, i.e., one standard deviation of the wage shocks. The response of the buffer stock of assets—the "F" panel—displays strikingly different behavior under perfect foresight and with uncertainty. A negative shock brings the buffer stock all the way down to zero for a number of periods, which correspond to the horizontal straight portion of the dashed line. Given that the borrowing constraint binds during these periods, there is then no more buffer to cushion additional wage

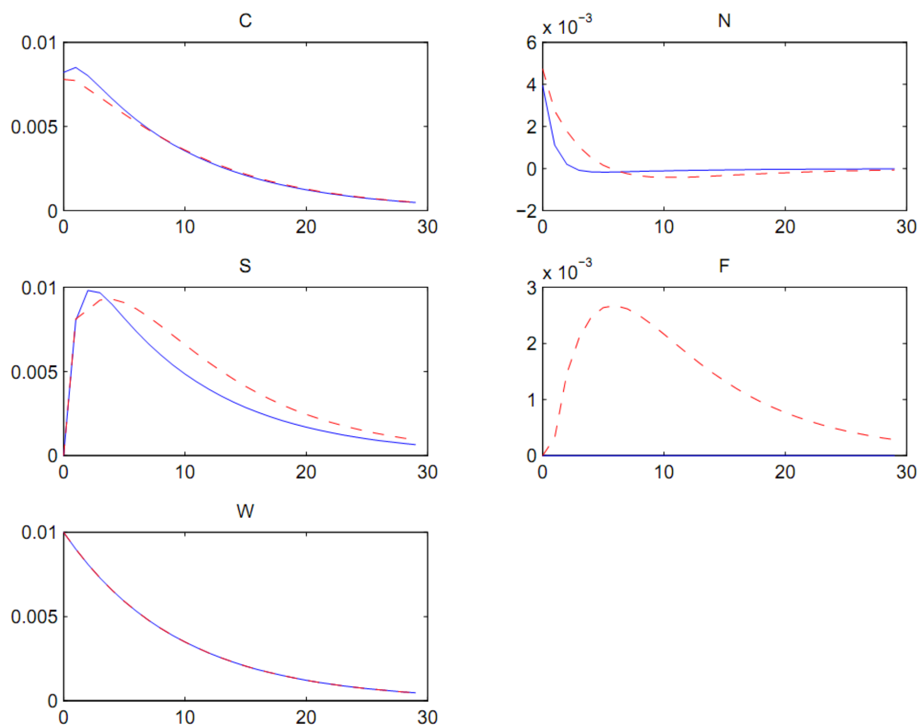

Figure 2: The dynamic response to a 1% positive wage shock. Solid Blue - prefect foresight; Dashed Red - full stochastic solution.

shocks. Under perfect foresight, in contrast, the buffer stock of assets increase, although slightly. Hence, the negative shock relaxes the borrowing constraint. This can be seen as a quite surprising result, given the basic intuition is that assets decline in bad times. However, this result is part of the mechanism generated by the realistic borrowing constraint based on durable goods as collateral for debt accompanied by a down payment. The reason for this is that the sharp decline of the affordable durable goods stock releases funds from the required equity which was held on those durable goods. Hence, assets including durable goods do decline in bad times, but the composition of assets change.

Figure 4 displays the reaction to a positive shock of 18 percent. Let us compare these responses to those to a negative shock in Figure 3. In the stochastic setup, the stock of assets responds in general symmetrically the reaction in Figure 3. They increase, consistently with the basic intuition about an optimizing consumer when the borrowing constraint does not bind. However, unlike the reaction of assets to a negative shock in Figure 3, where the borrowing constraint binds for a while, here the household moves further away from that situation. Under perfect foresight, the basic intuition that assets respond positively to a wage shock does not apply because the borrowing constraint plays an important role. The reaction to a positive shock is not symmetric to the reaction to a negative shock in Figure 3. There, the constraint is relaxed because required equity funds are released. Here, the large increase in the stock of durable goods generates a shortage of funds to finance the down payments; hence, the borrowing constraint keeps binding.

5 Conclusion

In this paper we added an explicit treatment of uncertainty to the model that Campbell and Hercowitz (2009) explore in a rolling certainty equivalent environment. That is, there is uncertainty in that model, but households' decisions are made ignoring it. In that model, the borrowing constraint binds in a large range around the steady state. Movements in labor supply are affected by equity requirements on durable goods. Persistent wage shocks affect the desired stock of durable goods, and hence a positive wage shock generates a shortage of funds to finance the down payment on the additional durable goods. This shortage of funds motivates the household to increase hours worked.

Here, the incorporation of uncertainty in households' decisions generates demand for a buffer stock of assets, or excess equity above the required amount as precautionary saving. This paper addresses the case of persistent but not permanent shocks. The borrowing constraint binds only in the case of negative shocks. The effects of uncertainty on labor supply behavior compared to the behavior under perfect foresight are small, specially for negative shocks. Negative shocks drive the borrowing constraint to bind in several periods during the response, and this makes the reactions in the two setups similar. For positive shocks the effect of uncertainty is larger, but the labor supply response is still similar.

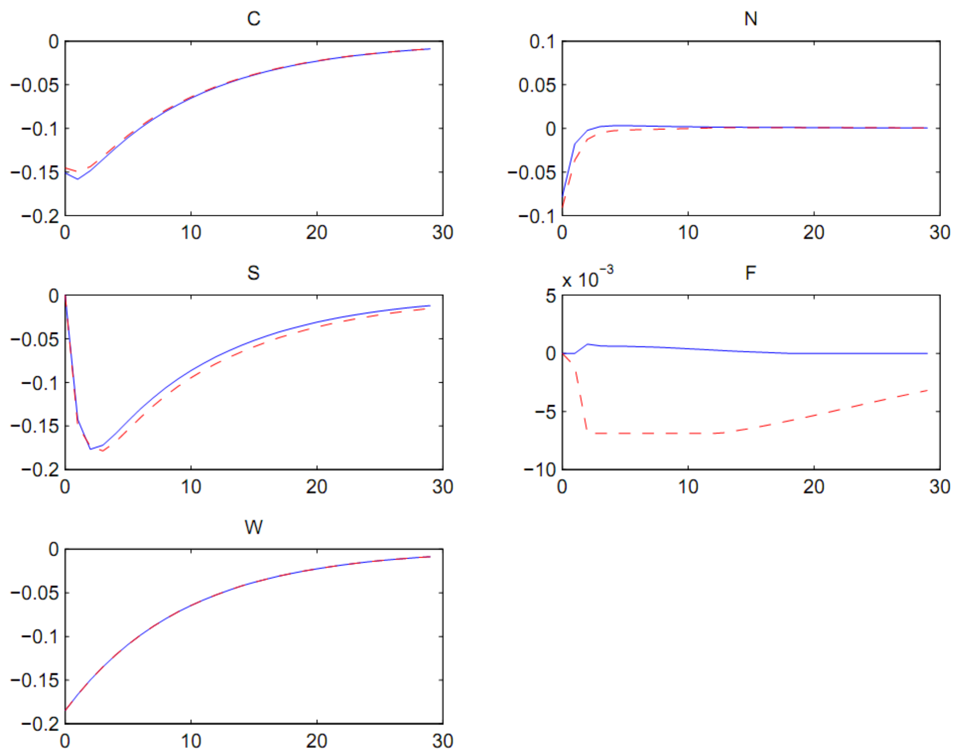

Figure 3: The dynamic response to a 18% negative wage shock. Solid Blue - prefect foresight; Dashed Red - full stochastic solution.

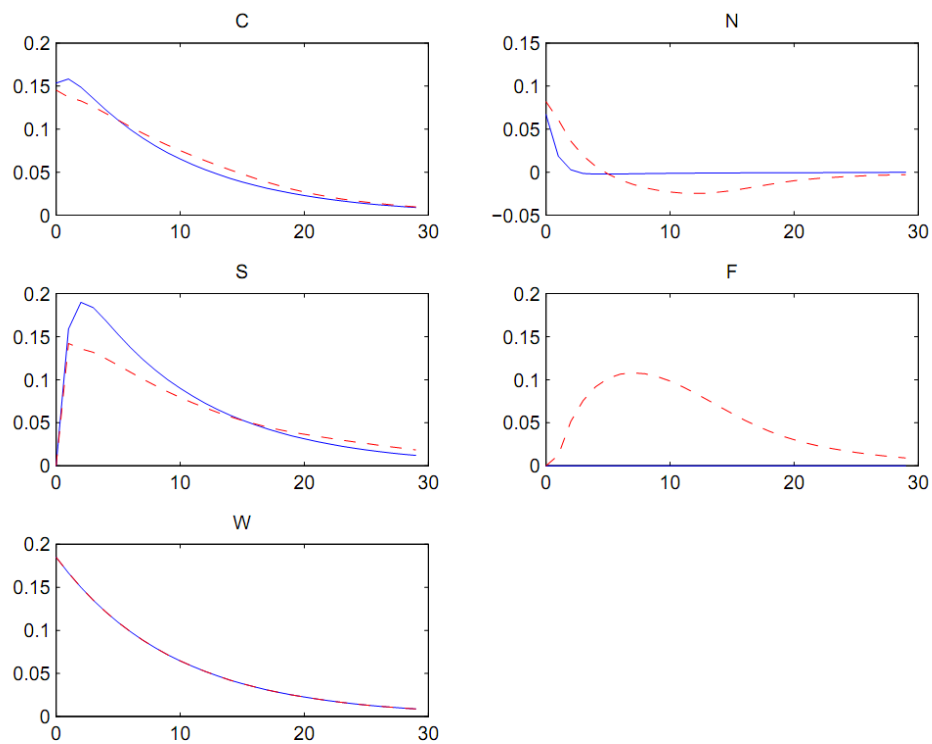

Figure 4: The dynamic response to a 18% positive wage shock. Solid Blue - prefect foresight; Dashed Red - full stochastic solution.

We suspect that the small effect of uncertainty is related to the logarithmic utility used. We hypothesize that a CRRA utility with a risk aversion parameter grater than 1 would provide results of grater quantitative significance, but qualitatively similar. Our attempts to numerically solve such models have so far failed.

References

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Appendix

A Grid-Based Solution to the Borrower's Problem

A.1 Policy Notation

Let say that $X_t = \{W_t, S_t, F_t\}$ is the state of the world and that there is a policy function $g(X_t)$ that gives $\{C_t, N_t, S_{t+1}, F_{t+1}\}$. Denote the different outputs of the policy function by $g_C(X_t)$, $g_N(X_t)$, $g_S(X_t)$ and $g_F(X_t)$ respectively.

A.2 Discretization

We discretize the state space by a grid $X^{m,n,l} = \left\{W^m, S^n, F^l\right\}$ where m, n and l denote discrete levels of this period wage, durable stock, debt to borrowing value and borrowing value to housing value respectively. Let $X^{k,j,o} = \left\{W^k, S^j, F^o\right\}$ denote the next period state in the same way. Define $p^{m,k}$ to be the transition probability of the Markov process approximating the process $W_t/W_{ss} - 1 = \rho\left(W_{t-1}/W_{ss} - 1\right) + \varepsilon_t$, $0 < \rho < 1$, $\varepsilon_t \sim N(0, \sigma)$:

$$p^{m,k} = P\left(W_{t+1} = W^k | W_t = W^m\right)$$

For brevity note the policy on the grid point $X^{m,n,l}$ as $g_C^{m,n,l}$, $g_N^{m,n,l}$, $g_S^{m,n,l}$ and $g_F^{m,n,l}$ and assume the policy is solved only for such points.

A.3 Approximating the Expectation

Define

$$\lambda^{m,n,l}(g) = \frac{1-\theta}{g_C^{m,n,l}} \tag{7}$$

Summing over all next-period wage levels k we can approximate the expected next-period marginal utility of consumption depending on this period wage level m and the *next-period state* levels j and o:

$$\tilde{\lambda}_{+1}^{m,j,o}(g) = \sum_k p^{m,k} \lambda^{k,j,o}(g)$$

Next, we use this to compute the expected marginal utility of consumption depending on the *current state* levels n and l. The connection between the current state and the future state is governed by the policy function g. However, the decisions from the policy function g on the grid point $X^{m,n,l}$ need not lay also on a grid point. Namely, there is no reason to expect that there are integers j and o such that $g_S^{m,n,l} = S^j$, $g_F^{m,n,l} = F^o$.

Generally, as long as $g_S^{m,n,l}$ and $g_F^{m,n,l}$ fall within the range of the grid, there will be integers j and o such that $S^j \leq g_S^{m,n,l} < S^{j+1}$ and $F^o \leq g_F^{m,n,l} < F^{o+1}$.

Once we find those j and o we can approximate the expected marginal utility of consumption by interpolating the grid points $\tilde{\lambda}_{+1}^{m,j+a,o+a}(g)|a=0,1$. Denote the interpolation of $\tilde{\lambda}_{+1}(g)$ values by the future state g directs us to as $\bar{\lambda}_{+1}^{m,n,l}(g)$.

A.4 Discretized and Interpolated System

Define

$$\Gamma^{m,n,l}(g)=\lambda^{m,n,l}(g)-\beta R\bar{\lambda}_{+1}^{m,n,l}(g) \tag{8}$$

Restating the first order conditions and constraints in the terms of the grid, we are looking for a policy g such that for every grid point state $X^{m,n,l}$ we will have:

$$\begin{aligned}
0 &= \left\{ \begin{array}{c} -\lambda^{m,n,l}(g)+\frac{(1-\delta)(1-\pi)}{R}\Gamma^{m,n,l}(g) \\ +\beta\frac{\theta}{g_S^{m,n,l}}+\beta(1-\delta)\bar{\lambda}_{+1}^{m,n,l}(g) \end{array} \right\} \\
0 &= -\omega\left(1-g_N^{m,n.l}\right)^{-\eta}+\lambda^{m,n.l}(g)\,W^m \\
0 &= g_F^{m,n.l}\Gamma^{m,n,l}(g) \\
0 &= \left\{ \begin{array}{c} -\left[1-\frac{(1-\delta)(1-\pi)}{R}\right]g_S^{m,n.l}-g_F^{m,n.l}+W^m g_N^{m,n,l} \\ -g_C^{m,n,l}+\pi(1-\delta)S^n+RF^l \end{array} \right\}
\end{aligned}$$

Note that the time index is now unnecessary.

A.5 Policy Improvement

The first guess for the policy is generated by solving the convergence path from each point on the grid using certainty equivalence and taking the decisions from the first period. We need to make sure all policy grid points we will be solving for have a policy that directs us to a state inside the grid range. To do that we run a simulation using the above certainty equivalence grid and mark off the 0.5% least used grid points. If the grid is chosen to be large enough then this means we mark most of the grid points as points not to updated and are left with a subset of policy grid points that share the property that they direct us to states within the range of the grid.

We would now like a numeric search procedure to find values for g that minimizes the deviation from the model's equations. The input to the numeric search procedure is a function from the four element vector

$$\left(g_C^{m,n,l},g_N^{m,n,l},g_S^{m,n,l},g_F^{m,n,l}\right)$$

to a four element vector of deviations

$$\left(d_1^{m,n,l},d_2^{m,n,l},d_3^{m,n,l},d_4^{m,n,l}\right).$$

We define the following function to serve as an input to the numeric search:

$$d_1^{m,n,l} = \left\{ \begin{array}{c} \left(g_F^{m,n.l} < 0\right) g_F^{m,n.l} + \left(\Gamma^{m,n,l}(g) < 0\right) \Gamma^{m,n,l}(g) \\ + \left(g_F^{m,n.l} \geq 0 \wedge \Gamma^{m,n,l}(g) \geq 0\right) g_F^{m,n.l} \Gamma^{m,n,l}(g) \end{array} \right\}$$

$$d_2^{m,n,l} = \left\{ \begin{array}{c} -\lambda^{m,n,l}(g) + \frac{(1-\delta)(1-\pi)}{R} \Gamma^{m,n,l}(g) \\ +\beta\theta / g_S^{m,n,l} + \beta(1-\delta)\bar{\lambda}_{+1}^{m,n,l}(g) \end{array} \right\}$$

$$d_3^{m,n,l} = -\omega\left(1 - g_N^{m,n.l}\right)^{-\eta} + \lambda^{m,n.l}(g) W^m$$

$$d_4^{m,n,l} = \left\{ \begin{array}{c} -\left[1 - \frac{(1-\delta)(1-\pi)}{R}\right] g_S^{m,n.l} - g_F^{m,n.l} + W^m g_N^{m,n,l} \\ -g_C^{m,n,l} + \pi(1-\delta)S^n + RF^l \end{array} \right\}$$

B The Deterministic Steady State

At the deterministic steady state the Euler equation is not satisfied. (Substituting $C_{t+1} = C_t = C$ yields $0 \leq 1 - \beta R$ while $\beta R < 1$ by assumption). Hence, the Euler equation is replaced with the binding liquidity constraint. The deterministic steady state has:

$$F = 0,$$

Rearranging the first-order conditions we can obtain $\lambda(C), \Gamma(C), S(C)$ and $N(C)$:

$$\lambda(C) = \frac{1-\theta}{C}$$

$$\Gamma(C) = \lambda(C)(1 - \beta R)$$

$$S(C) = \beta\theta / \left\{ [1 - \beta(1-\delta)] \lambda(C) - \frac{(1-\delta)(1-\pi)}{R} \Gamma(C) \right\}$$

$$N(C) = 1 - \left(\frac{\omega}{1-\theta} \frac{C}{\chi}\right)^{\frac{1}{\eta}}$$

We can now numerically search for a C value such that the budget constraint holds:

$$C = \chi N(C) - \left[1 - (1-\delta)\left(\frac{1}{R}(1-\pi) + \pi\right)\right] S(C) \tag{9}$$

Arbitrarily we start the numeric search from $C = 1$.