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Risky College Enrollment, Dropout, and Student Debt Forgiveness

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Risky College Enrollment, Dropout, and Student Debt Forgiveness

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Abstract

The paper analyzes the effects of two kinds of college education subsidies: unconditional tuition discounts and targeted forgiveness of student loans on student college enrollment and completion or dropout decisions. We focus on students' imperfect knowledge of their academic ability at the time of matriculation and its updating in the course of study as key factors in their responses to funding policies. We find that while unconditional tuition subsidies incentivize both matriculation and continued study even upon the revelation of low ability hence low returns to college, a policy combining such subsidy with partial forgiveness of student debt conditional on dropping out has a doubly efficient effect of risk mitigation: it maintains incentives to matriculate but discourages continued study when low future returns are revealed. It is, moreover, superior in terms of mitigating the "bad debt" held by students, that unrecouped by returns to college. Budget neutral conversion of a part of unconditional tuition subsidy to targeted debt forgiveness reduces the aggregate bad debt held by students.

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1. Introduction

The twin syndromes of American higher education: significant rates of college non-completion (dropout) and growing student debt, are receiving much public attention and have motivated substantial discourse in recent economics literature. According to NCES (2020, Table 303.10) the rates of non-completion of (the first-attended, within six years from matriculation) 4-year colleges in the United States has ranged between 40 and 45 percentage points since 1996.[1] Much public discourse and economics literature are premised on viewing college dropout phenomenon exclusively as a loss to individuals and society[2]. In contrast, Manski (1989) underscored the fact that student decisions whether to matriculate and then persist in college are made under substantially incomplete information about individual abilities and other factors. This underlay his reasoning about the value, to an individual and society, of taking on the risks associated with these decisions. According to it, potential dropout and unrecouped college debt can be seen as akin to business failure, an inevitable part of productive phenomenon of business risk-taking. (We will call such unrecouped student debt "bad debt", to distinguish it from productive debt that pays off through returns to college.)

In such a framework, tuition subsidy and student debt forgiveness should be viewed through the prism of risk-sharing between an individual and the public. Indeed, higher subsidies encourage risk-taking, hence augmented losses as well as gains, both private and social. In particular, the gains can occur through the encouragement effect of tuition subsidies, in terms of enrolling in college, for individuals whose *ex ante* information happened to underestimate their ability and therefore returns to college. Of course, the same encouragement received by those who based their decisions to enroll on upward biased estimate of individual returns to college will likely

[1] The same source relates this to the fact that the expansion of US higher education enrollments, which lasted for almost a century, is reaching the limits of the supply of academically eligible high school graduates (see also a survey and discussion of these trends in Kaganovich et al., 2021). Athreya and Eberly's (2021) model-based analysis reaches the same conclusion along with diagnosing the leading role of enrollment by less-prepared youths in growing dropout rates as well as high-risk borrowing. Bound et al. (2010) affirm this diagnosis while underscoring the additional role of supply-side factors such as congestion, particularly at less selective institutions. Detailed degree completion data is provided by NCES (2019, Table 326.10). Hendricks and Leukhina (2018) further offer an insightful breakdown of the dropout figures.

[2] See, for instance, extensive discussion of the "college completion crisis" and policies to mitigate it by Deming (2017) as well as in the literature referenced therein.

result in *ex post* losses, private and social. It is, indeed, a prevalent verdict of the literature studying college subsidy policies in the US, that the "broad-", rather than merit-based, tuition subsidies, while meeting the goal of expanding access to higher education, tend to predominantly expand the enrollment of academically weaker students (e.g., Caucutt and Kumar, 2003, Garriga and Knightley, 2007) who are at higher risk of dropping out.[3] This conclusion indicates that the post-subsidy tuition faced by students does play a role of higher education's price, in that it acts as a self-sorting device for potential students in their decisions whether to enroll, based on the information they possess regarding future returns.[4] We follow this premise in our model.

In the *op cit.* Manski's framework, since an initial choice to enroll in college is made under incomplete information, potential subsequent decisions by some students to drop out of college without completing the degree is a natural part of the process. Substantial recent empirical literature follows this framework in estimating the contribution of students' information updating to their college persistence or dropping out decisions based on academic performance. For instance, Stinebrickner and Stinebrickner (2012) estimate the contribution of this factor to dropout rate at 40% based on survey data at Berea College, which provides education tuition-free, such that the effect of credit constraints can be presumed small.

To explore the interplay between the imprecise *ex ante* signals of ability received by individuals and tuition subsidy amounts available to students, in their effects on college enrollment, completion, and student debt, we develop a model of two-stage college education. In it, individuals pursue the maximum expected lifetime wage income and decide whether to enroll in college based on the knowledge of college wage premium multiplier and imprecise signal of their ability. Mid-way through college, their true ability is revealed to students through academic

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[3] This conclusion that unconditional, broad-based, tuition subsidies dilute the quality of student body is reinforced by the argument of their further negative effect on student effort made by Blankenau and Camera (2009) as well as Sahin (2004) among others. This effect arises when the signal about a college graduate's skill conveyed by diploma is imperfect, as per Spence (1973), hence an incentive to underinvest in study effort on the part of less prepared students. The lowering of incentive to exert effort spills over to all students. This negative effect strengthens when private cost of education is lowered by higher subsidy, because this draws in more lower quality students.

[4] Gary-Bobo and Trannoy (2008), who also consider incomplete information framework, contrast such demand-driven student self-selection mechanism with one where education is financially accessible to all, but universities set admission barriers based on academic eligibility. They find, in particular, that self-selection is the optimal enrollment mechanism when universities possess no more information about students' characteristics than students themselves. We note that "self-selection" dominates in the majority of American four-year colleges, which are classified as "less selective", and is certainly a feature of the US four-year higher education taken as a whole.

performance in classes up until that point. Thus, upon finishing the first of two stages of college studies, students are faced with an *ex post* fully informed choice whether to persist through graduation or drop out and join the workforce.[5] Government provides a tuition subsidy at each of the two stages of college. Our analysis centers on the comparative effects of these unconditional tuition subsidies as well as a conditional one, provided through a debt forgiveness mechanism to be described in detail shortly, on students' incentives to enroll and subsequently persist in college under initially incomplete information which gets updated in the course of study.

Tuition subsidy and debt forgiveness policy instruments arguably serve government's goals to promote college enrollment and/or enhance subsequent college completion. They also aim to contain college debt, particularly among college dropouts as well as other students whose returns to college won't pay off the tuition cost. The policy goal of *encouraging college enrollment* by able individuals, especially those whose ability is underestimated by its initial signal, is likely served by prioritizing tuition subsidy for the first-stage studies in college. Such generosity, however, also expands the enrollment of those with upwardly biased signal but sub-standard true ability, thus setting more students up for future dropping out with outstanding debt. Therefore, if the government also aims to *contain dropout*, this will require allocating some resources toward subsidizing second-stage tuition. Given limited public resources, there is a tradeoff between these two policy goals. This tradeoff motivates us to introduce an additional tool into the policy mix, whereby unconditional tuition subsidies are combined with targeted forgiveness of debt of underperforming students conditional on their decision to drop out. We show that this conditional debt forgiveness policy has doubly efficient effect on risk mitigation: it continues to encourage matriculation by students receiving promising signals of ability while discouraging their continued investment in education when its returns are revealed to be low. When it comes to the policy's effect on student debt, *conditional debt forgiveness* distinguishes and mitigates the *"bad" debt* accrued by those whose enrollment decision turned out a low return, from the debt

[5] The focus on decisions at half-way point in a typical four-year duration of college studies in the US is common in the literature (see, for instance, Arcidiacono's, 2004, analysis based on NLS72 data) in part because it signifies student transition from lower to upper division classes. By this point, students are also believed to have typically accumulated adequate information about their performance and experience for updating their educational decisions. Besides choosing between persisting or dropping out, students in the US higher education can also decide about switching to a different major concentration of their studies. Our model disregards the choice of major altogether given its focus on higher level decisions about college matriculation and completion or dropout. See Altonji (1993), Kaganovich et al. (2023), and Kaganovich (2023) who do focus on decisions about switching majors as well as dropping out.

taken on by matriculants whose risk-taking has paid off through sufficiently high returns. The policy thus specifically targets the bad debt directly through its full or partial forgiveness and indirectly by reducing the incentives to accrue more of it by persisting in low return education. The comparison of the unconditional tuition subsidies and conditional debt forgiveness in their effects on students' *bad debt* is the main goal of our analysis.

Manski (1989) observed that students' and societal interests may differ when it comes to policies impacting dropout decisions. The same, therefore, concerns student debt which is impacted by such policies. Indeed, the financial losses by some students, when policies encourage taking on the risk of college enrollment, inevitably shadow the positive outcomes for many others that motivated the policies in the first place. This understanding can help shed some new light on the social function of student debt and may offer additional rationale for *dropout-contingent* debt forgiveness. Society may want more people to "try" college to make sure that the potential of able individuals is realized. That is, in Manski's terminology, "experimentation" should be encouraged, hence rationale for subsidizing matriculation. The downside of such subsidy for a subset of individuals, those who are less able but receive an upward biased ability signal, is that they are encouraged to matriculate but then entrapped in a losing activity, with unrecouped college loan debt as a consequence. As mentioned, this issue can be mitigated by encouraging weaker student to drop out rather than continue the inefficient use of society's resources (tuition subsidy for continued college study) by means of student debt forgiveness conditional on dropping out. As we show in the paper, this approach sustains the incentives to "experiment" by "trying college" while mitigating individual downside risk.

It is interesting, from the above perspective, to compare the dropout-contingent debt forgiveness policy we consider to the income contingent college loan programs (ICLP), which operate in some higher education systems and have received considerable attention in the literature (e.g., Eckwert and Zilcha, 2014). Both provide risk sharing in the form of financial relief for those with less favorable outcomes. The important distinction is that ICLPs create additional incentives for students facing lower post-college income to persist through graduation in order to benefit from favorable treatment of college loans, i.e., cross-subsidizing college graduates with less productive *ex post* outcomes. The conditional debt forgiveness policy for students with unfavorable academic realization upon the first stage of college does the opposite: it offers incentives for those choosing to drop out rather than graduate by providing partial relief of debt incurred while taking on a risky educational investment (and thereby also helps save private and public resources). A further, unrelated, argument in favor of conditional debt forgiveness policy that encourages dropout, is that of peer quality effect. Indeed, retention of weak students dilutes the quality of education for the rest.

Given our focus on how tuition subsidy and loan forgiveness policies affect individual decisions based on intellectual ability to succeed, the paper ignores student heterogeneity in the need-based dimension. We effectively presume that all students lack private means to finance their college education other than by borrowing against future wages. For middle class families, i.e., most US students, this is a meaningful benchmark of analysis[6], on which one can build extensions incorporating differential access to private funding of education. This assumption also means that there is no difference, for our purposes, between a contemporaneous tuition subsidy and unconditional forgiveness of student debt. A distinct instrument, in our analysis, is the conditional debt forgiveness.

The rest of the paper is organized as follows. Section 2 lays out the model with two-stage college education and derives student decisions as *ex ante* and *ex post* responses to ability signals and realizations as well as to tuition subsidies. We also categorize groups of students in terms of their *ex post* returns to college, including those with bad debt. Section 3 obtains comparative statics results for student decisions and aggregate outcomes in response to tuition subsidies at each stage of college. Section 4 focuses on the effects of conditional debt forgiveness policy in comparison to those of unconditional tuition subsidies. Section 5 concludes. More technically involved proofs are relegated to the Appendix.

2. Model

The economy is populated by a cohort of mass 2 of individuals of the same age. Each individual i is characterized by academic ability g_i, which is not initially known with certainty. This characteristic is presumed exogenously determined in this model's decision framework. We

[6] Carneiro and Heckman (2002) distinguish short-run credit constraints, which directly affect contemporary college attendance, from the long-term family income factors responsible for youth's academic eligibility for college. They estimate that just up to 8% of US population was credit constrained in the short-run sense.

assume for the sake of tractability that academic ability is distributed uniformly in the population in the interval [0, 2].

At the start of adulthood, time *t=0*, each individual receives imprecise signal of his ability upon taking a standardized test administered by the higher education system. The signal is given by

$$\hat{g}_i = g_i + \varepsilon_i \tag{1}$$

where the error ε_i is *symmetrically* distributed in the interval $[-\overline{\varepsilon}, \overline{\varepsilon}]$ with zero mean, independently across the individuals.[7] We let $F_\varepsilon(x)$ denote the c.d.f. of ε, which characterizes the *testing information system* $\{\varepsilon\}$.[8]

The ability signal $\hat{g}_i$ produced by the testing system serves as the informational basis for an individual's decision at time *t=0* whether to pursue higher education, i.e., to enter college, or not. In the latter case, the individual joins the workforce and receives career earnings

$$I^0 = 1 \tag{2}$$

which thus do not depend on the individual's academic ability.

College education and finance

A full course of college education, for those who decide to enroll at time *t*=0, consists of two periods or stages. The stages may but do not have to be of equal length, which we assume to be negligible compared to the length of a working career. The first stage covers introductory level courses which also help reveal a student's true level of ability by the end of the period. The second stage of college is devoted to producing job market ready human capital. Each student can decide whether to remain in college for this second stage or not, i.e., to drop out, based on the knowledge of his true ability and therefore true human capital attainment potential. We assume, for the sake of simplicity and following most of the literature, that there are no admission restrictions on student enrollment, so student population is determined by self-

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[7] It is natural to require that the distribution of $\hat{g}_i$ were restricted to the same interval [0, 2] as is the true ability g_i. This imposes a modification on the distribution of ε for individuals whose true ability levels g_i are within $\overline{\varepsilon}$ of the end-points of interval [0, 2], skewing it accordingly. We do not, however, devote extra attention to this fact, because, as will soon become obvious, the individuals at these ends of the ability distribution will not face any uncertainty about their best options.

[8] Eckwert and Zilcha (2004) develop a richer model to analyze the role of information systems on students' decisions while focusing on the implications of their improved precision.

selection alone. This corresponds well with the stylized facts of higher education system as an aggregate.

To reiterate, of essence are the following assumptions about the flow of information and the corresponding decision-making process. At the end of the first period in college, i.e., at t=1, each student learns her true ability g_i. Based on this updated information, each student decides whether to persist in college toward graduation, which will occur at t=2, or not, i.e., to drop out of college at t=1. [9]

We posit that college dropouts join unskilled workforce, so their career earnings are as specified by the expression (2). We thus ignore the benefits of the first stage of college for earnings as we do its time cost. Both simplifications afford us clear focus on the main issue of our analysis, the tradeoffs faced by students in their decisions whether to persist in college toward graduation and the implications of these decisions for potential bad debt.

College graduates form skilled workforce, so their career earnings benefit from academic achievement and are given by

$$I_i^C = wg_i \tag{4}$$

where the college premium coefficient $w > 1$. It is important to note the college premium coefficient's distinction from the *individual college premium*, which is given by the ratio of a particular individual's post-college career earnings to the earnings she would have made over the course of her career had she chosen to not enter college. Thus, for individual i, the college premium is wg_i, which obviously increases in one's ability.

We assume that college charges each student tuition τ in each period t=1, 2, which is partly offset by government-provided period-specific tuition subsidy in the form of grants to students in the amount s_t, in periods $t=1, 2$. We can thus denote by

$$d_t = \tau - s_t$$

[9] Such concept of two stages in a college career with student updating of beliefs about their capabilities based on performance at the first stage and the corresponding choices regarding the second is imbedded in some recent empirical analyses. See, for instance, Arcidiacono (2004), Stinebrickner and Stinebrickner (2014), and Kaganovich et al. (2023) for decision-making focused on persistence in or switching across college majors, and Stinebrickner and Stinebrickner (2012) for the analysis of dropout decisions. Manski (1989) laid out a broader seminal theoretical model explaining dropping out of college as an outcome of informational updating. Toward the same goal, Hendricks and Leukhina (2018) build and calibrate a multi-period model of student progress in college.

the amount of debt accrued in periods *t=1,2*, if in college.

We stipulate that individuals are guided solely by maximizing their lifetime expected net income.[10] Then, according to (2) and (4), for a student i who entered college at $t=0$, a decision at $t=1$ to persist until graduation will pay off iff

$$wg_i - d_2 \geq 1 \tag{5}$$

Assumption 1. $w - d_2 \leq 1$.

The assumption imposes an upper bound on the college premium coefficient w (which, as we recall, exceeds 1), but not on the full college premium wg_i, which can be substantial at the high end of ability but will be obviously low for low ability individuals. The assumed upper bound on the college premium coefficient w makes college completion unattractive to individuals of relatively low ability. Parametrically, this will be the case, in particular, for students whose true revealed ability g_i is below 1, the population median ability. Indeed, were such individuals to complete college, their career earnings net of second stage student debt would be $wg_i - d_2$, which according to Assumption 1 is below what they would earn as unskilled workers had they chosen to drop out. The assumption implies, further, that completing college is financially beneficial *ex post* as defined by condition (5), if and only if a student's true revealed ability is at or above the following threshold:

$$g_i \geq g^C = \frac{1+d_2}{w} \tag{6}$$

where $g^C \geq 1$ according to Assumption 1 (although students at the threshold are indifferent between completing college and dropping out, we postulate without loss of generality that the former is pursued by default). Inequality (5) also means that the net earnings of any student who

[10] The model can be easily extended to incorporate student learning effort and choices based on utilitarian comparisons. Specifically, let there be a uniform required level of effort during the first, preparatory, stage of college, and individually chosen level of effort at the second stage. Student decisions about effort level and ultimately about choices among all the educational options are guided by maximizing a strictly concave separable utility function of income and learning effort, increasing in the former, decreasing in the latter. The optimal effort is then uniquely determined by student's ability (and is in complementary relationship with it). Factoring the optimal effort back into expressions for utility levels derived from each of the educational options (no college, dropping out after the first stage, and completing college) makes the model mathematically equivalent to our benchmark version where an individual makes choices between the three options based on net income considerations alone, i.e., without explicitly incorporating the effort cost.

chose to attend college and then, upon revealed ability information, found it rational to complete it, will not do worse, in contemporaneous comparison, than joining the workforce of those who never attempted college and are thus earning the wage $I^0 = 1$.

We summarize the above findings as

<u>Lemma 1</u>. *Under the provisions of Assumption 1, student i who entered college at t=0, will decide at t=1 to persist until graduation, iff her revealed ability clears the threshold* g^C *given by (6). Such student's post-graduation income net of debt accrued during the second stage of college will exceed the wage she would have earned without going to college, that is,* $wg_i - d_2 > 1$.

<u>Remark</u>. To clarify, the applicability of condition (6) per Lemma 1 is predicated on a student's decision to enroll in college at *t=0*. That will only happen, if a student receives sufficiently encouraging testing signal, a condition which we will specifically explore shortly.

As Lemma 1 shows, if an individual whose true ability is at or above g^C happened to have chosen to enter college, she will then find it *ex post* rational to persist through graduation. It is important to note, however, that inequality (5) does not prevent the possibility of regret of having chosen to enter college, since the earnings of the graduate *net of entire student debt*, $wg_i - d_1 - d_2$ may fall below 1. There may be, therefore, a range of ability levels such that some individuals there who are induced to enter college by an upwardly biased signal, would have been better off not doing so, had they received an accurate signal of ability, hence the *ex post* regret. Specifically, it is not hard to see that this will be the case for students whose true ability falls into the interval

$$g^C = \frac{1+d_2}{w} \le g_i \le g^R = \frac{1+d_1+d_2}{w} \tag{7}$$

where g^R is the threshold level of true ability, at which the financial benefits of attending college breaks even with its total cost to student.

We further impose the following parametric condition, obviously consistent with Assumption 1.

<u>Assumption 2</u>. The following inequality holds:

$$w^{-1}d_1 < \bar{\varepsilon} < 1 \tag{8}$$

The left part of inequality (8) requires that the maximum testing error exceed student's tuition cost of first-stage college studies in relation to college premium coefficient. As will become clear shortly, this condition ensures that the set of students who are compelled to matriculate in college by their test outcome and will then find that completing college fully pays off is not empty. The right part of (8) simply means that the testing error is smaller than the range of abilities of potentially college-bound students.

Recall now that the precise level of ability is unknown to a student at *t=0*. Therefore, given the imprecise signal $\hat{g}_i$ of it he receives, the *ex ante* (time *t=0*) conditional probability that if individual *i* goes to college, he will complete it is, according to Lemma 1, given by

$$P_i^1 = P\left(g_i \geq g^C \middle| \hat{g}_i\right) = F_\varepsilon\left(\hat{g}_i - g^C\right) \tag{9}$$

where the threshold ability level g^C is defined according to (6).

The above implies that for an individual who receives testing signal $\hat{g}_i$ the *ex ante* conditional expected net income in case he does choose to go to college is

$$E_0\left(I_i \middle| \hat{g}_i\right) = E_0\left(P_i^1(wg_i - d_2) + (1-P_i^1)I^0 - d_1 \middle| \hat{g}_i\right) = E_0\left(P_i^1(w(\hat{g}_i - \varepsilon_i) - d_2) + (1-P_i^1) - d_1 \middle| \hat{g}_i\right)$$

where subscript in E_0 indicates that the expectations are formed at *t*=0.

Therefore, according to (1), since $E_0\left(\varepsilon_i\right) = 0$ and since ε_i is distributed independently of $\hat{g}_i$,

$$E_0\left(I_i \middle| \hat{g}_i\right) = P_i^1[w(\hat{g}_i - E(\varepsilon_i)) - d_2 - 1] + 1 - d_1 = P_i^1(w\hat{g}_i - d_2 - 1) + 1 - d_1$$

so according to (9),

$$E_0\left(I_i \middle| \hat{g}_i\right) = F_\varepsilon\left(\hat{g}_i - g^C\right)(w\hat{g}_i - d_2 - 1) + 1 - d_1 \tag{10}$$

Note now that individual *i* will decide to enter college at *t=0* iff his expected net income is no less than the opportunity cost, i.e., according to (2), iff the following inequality holds:

$$E_0\left(I_i \middle| \hat{g}_i\right) \geq 1$$

or, according to (10) and (6), iff $F_\varepsilon\left(\hat{g}_i - g^C\right)(w\hat{g}_i - d_2 - 1) - d_1 \geq 0$, i.e.,

$$wF_\varepsilon\left(\hat{g}_i - g^C\right)(\hat{g}_i - g^C) - d_1 \geq 0 \tag{11}$$

Observing that the left-hand side of inequality (11) is a strictly increasing function of $\hat{g}_i$, we define $\hat{g}^C$ as the unique threshold signal value, which satisfies equality $E_0\left(I_i \mid \hat{g}_i\right) = 1$, i.e.,

$$F_\varepsilon\left(\hat{g}^C - g^C\right)(\hat{g}^C - g^C) = w^{-1}d_1 \tag{12}$$

The conclusion about the college attendance decision derived above can therefore be stated as

Lemma 2. (i) *The value* $\hat{g}^C$ *uniquely defined by equation (12) represents the threshold testing outcome, i.e., the cutoff for a signal received by individuals, such that*

- *individuals whose test outcome* $\hat{g}_i$ *is below* $\hat{g}^C$ *will decide against entering college,*
- *those with a higher test result,* $\hat{g}_i \geq \hat{g}^C$, *will decide to enter college.*

(ii) *Furthermore, the following characterization is true (see Appendix for its proof): Under the provision of Assumptions 1 and 2 the value* $\hat{g}^C$ *which solves equation (12) satisfies inequality*

$$g^C < g^R < \hat{g}^C \tag{13}$$

Remark. Although students whose test outcome $\hat{g}_i$ is exactly $\hat{g}^C$ will be indifferent between the above two options, we postulate without reducing generality that these students will choose to enroll in college at *t*=0.

Lemma 2(ii) implies, in particular, that the minimum threshold test outcome (signal of ability) $\hat{g}^C$ which compels students to enroll in college is higher than the minimum threshold level of revealed ability, which compels one to persist through graduation. This automatically ensures the validity of the following common sense outcome: if a student's test result was precise or downward biased, i.e., $\varepsilon_i \leq 0$, and if this student happens to have decided to enroll in college, then she will surely persist *ex post*. In other words, dropout can only occur among students whose test result overestimated their true ability and did so by sufficient margin.

Note that the true ability *g* of individuals receiving test signal $\hat{g}$ is distributed in the interval $[\hat{g} - \bar{\varepsilon},\, \hat{g} + \bar{\varepsilon}]$. Consider the subpopulation of individuals who receive the borderline testing signal $\hat{g}^C$, i.e., on the threshold of deciding to enroll in college at *t*=0 according to Lemma 2. Their decisions at *t*=1 whether to persist or to drop out depend on whether their then revealed true ability $g_i = \hat{g}^C - \varepsilon_i$ exceeds or falls short of the threshold ability level g^C. The decision to persist occurs when the testing error ε_i is either negative, or positive but small, namely such that

$$\varepsilon_i \leq \Delta^C = \hat{g}^C - g^C \tag{14}$$

noting that thus defined value Δ^C is positive according to inequality (13). Thus, Lemma 2(ii) establishes that such persisting individuals certainly exist in this subpopulation. The decision to drop out occurs for the above-described students, if the testing error is positive and sufficiently large, namely such that $\varepsilon_i \geq \Delta^C$. We shall now assume that such students exist, i.e., that testing errors can reach this magnitude. This is equivalent to the following assumption, which is clearly consistent with inequality (13) established by Lemma 2:

<u>Assumption 3</u>. $\overline{\varepsilon} \geq \Delta^C$ where the value Δ^C is defined by expression (14).

Note that this is also consistent with Assumption 2 which requires $\overline{\varepsilon} > w^{-1}d_1$ whereas $\Delta^C = \hat{g}^C - g^C > w^{-1}d_1 = g^R - g^C$ according to inequality (13).

We further observe that the *ex post* net career income of a student of true revealed ability g^C, i.e., who is on the margin between persisting and dropping out, is given by $wg^C - d_1 - d_2 = 1 - d_1$, which is inferior to that of any individual who did not enroll in college, hence the net career income loss. The same *ex post* net career income comparison is true for the dropouts with revealed ability below g^C. According to the discussion following Lemma 1, the fact of *ex post* net income loss is also true for students whose true ability is somewhat above g^C (so these students won't drop out) but falls short of threshold g^R defined in (7). Such students enroll in college because they received a good enough test outcome $\hat{g}_i \geq \hat{g}^C$ but unlike those discussed above find it *ex post* rational to complete it, because their true ability $g_i \geq g^C$ so their wage gain $wg_i - 1$ outweighs the debt d_2 to be incurred at stage 2. However, because their true ability $g_i < g^R$, they end up with an aggregate net financial loss relative to the career income of 1 that would obtain without any college education, with a portion of the first-stage debt d_1 unrecouped. The above reasoning allows us to conclude with the following

Lemma 3. *Under the provisions of Assumptions 1-3, the following outcomes obtain.*

(i) There is a non-empty group $L1 = \{\hat{g}_i \geq \hat{g}^C, g_i < g^C\}$ *of dropouts, i.e., students who find it ex ante individually rational to matriculate but end up dropping out after stage 1; they incur ex post net financial loss (the unrecouped debt) of* d_1*, relative to the no-college option.*

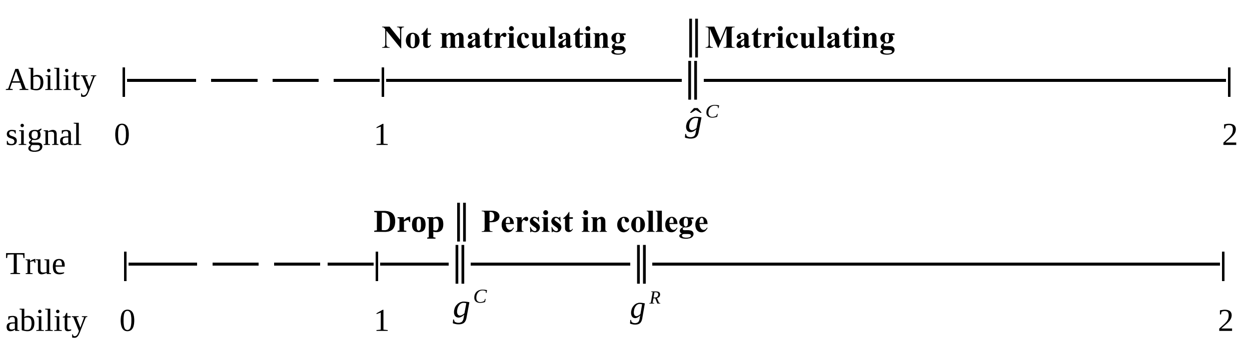
(ii) The group $L2 = \{\hat{g}_i \geq \hat{g}^C, g^C \leq g_i < g^R\}$ *consists of students who enroll in and find it ex post rational to complete college, but with returns not high enough to outweigh the full sunk cost of student debt* d_1 *accrued during the first stage of college, hence incurring net career income loss relative to the no-college option.*

Thus, students in the populations defined in Lemma 3 end up with *ex post* losses as a result of their decisions to attend college. We therefore label the aggregate unrecouped debt accumulated by this group as "bad debt" and will consider, in the rest of the paper, the impact of tuition subsidy policies on this indicator.

Remark. In addition to students ending up on the losing end of risk-taking in college enrollment decisions defined by Lemma 3, one can analogously classify subpopulation of individuals who decided against matriculating because their testing signal significantly underestimated their true ability but would have in fact benefitted from enrolling and graduating. Although the composition of this "losing" group is also affected by government policy, we will not pursue its analysis given our focus on the implications for students' bad debt.

The following figure helps visualize the localizations of the decision thresholds defined in Lemmas 1-3.

Figure 1. Localization of *ex ante* and *ex post* Decision Thresholds.



3. **The Effects of Tuition Subsidies (Unconditional Debt Forgiveness)**

In this section, we apply the decision thresholds derived in Section 2 to analyze the effects of changes in tuition subsidy levels s_1 and s_2 on student flows, including aggregate matriculation *N*, graduation *G*, and dropout *Q*, as well as on the bad debt volume, which we'll denote D^*.

Recall that individuals will enroll in college at *t*=0 iff their test score $\hat{g}_i \geq \hat{g}^C$. This is equivalent to their test's error $\varepsilon = \hat{g} - g_i$ being no smaller than $\hat{g}^C - g_i$, the event whose probability is given by $1 - F_\varepsilon\left(\hat{g}^C - g_i\right)$. Therefore, and due to the assumed uniformity of the distribution of true innate ability on [0, 2] with the total population mass 2, the *aggregate initial matriculation* (enrollment in stage 1 of college) is given by

$$N = \int_{\hat{g}^C - \bar{\varepsilon}}^{2} \left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg$$

It is good to note that $F_\varepsilon\left(\hat{g}^C - g\right) = 0$ for all $g \geq \hat{g}^C + \bar{\varepsilon}$, so we can rewrite the above as

$$N = \int_{\hat{g}^C - \bar{\varepsilon}}^{2} \left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg = \int_{\hat{g}^C - \bar{\varepsilon}}^{\hat{g}^C + \bar{\varepsilon}} \left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg + (2 - \hat{g}^C - \bar{\varepsilon}) \tag{15}$$

Similarly, since a student will drop out if his revealed ability $g_i < g^C$, the *aggregate dropout*, (and thus the mass of group *L*1 defined in Lemma 3) is given by

$$Q = |L1| = \int_{\hat{g}^C - \bar{\varepsilon}}^{g^C} \left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg \tag{16}$$

Indeed, an individual of true ability $g \leq g^C$ will drop out of college upon its revelation. By the same token, the *aggregate completion (graduation)* is:

$$G = \int_{g^C}^{2} \left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg = \int_{g^C}^{\hat{g}^C + \bar{\varepsilon}} \left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg + (2 - \hat{g}^C - \bar{\varepsilon}) \tag{17}$$

The subset of the population of college graduates is the group of *L*2 defined in Lemma 3 of students who though opting to complete college end up with unrecouped student debt. We denote the size of this group G^R:

$$G^R = |L2| == \int_{g^C}^{g^R} \left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg \tag{18}$$

The *aggregate student debt*, the total of tuition charges owed by the entire population of students less the tuition subsidies they receive, is given by

$$D = Nd_1 + Gd_2 = N(d_1 + d_2) - Qd_2$$

A part of the above amount is the bad debt accrued by groups $L1$ and $L2$, the aggregate of individual unrecouped debt, i.e., the tuition debt net of wage gains due to attending college. It is given by

$$D^* = D^Q + D^R = Qd_1 + \int_{g^C}^{g^R} [d_1 + d_2 - wg + 1]\left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg \tag{19}$$

where $D^Q = Qd_1$ is the part of the bad debt accrued by the dropouts while D^R is the part that is unrecouped by college graduates.

We postulate that relieving the bad debt is a key goal of the government tuition subsidy policy. This is consistent with the risk-sharing role of the subsidies as they encourage risky investment in human capital by means of mitigating losses on the downside of risk-taking.

An essential aggregate variable of interest in evaluating the policies is the total cost of the program to the public, i.e., the aggregate volume of tuition subsidies:

$$S = Ns_1 + Gs_2 = N(s_1 + s_2) - Qs_2 \tag{20}$$

In what follows, we focus on exploring and comparing the effects of government subsidy policies on the above outcome variables. The tradeoff between student debt relief provided by alternative policies and their cost to public budget are central to the policy comparison results we aim to obtain.

We'll proceed under an additional parametric assumption. Recall that $F_\varepsilon(\Delta)$ is the probability that the error entailed in a student's test score aimed at measuring her true ability does not exceed Δ. This implies in particular that for a student with test score $\hat{g}^C$, the marginal score for choosing to matriculate in college, the *ex ante* probability of completing college equals

$F_{\varepsilon}\left(\hat{g}^{C}-g^{C}\right)$ since g^{C} is the minimum cutoff for revealed true ability for one's decision to remain in college. By the same token, $1-F_{\varepsilon}\left(\Delta^{C}\right)$, where $\Delta^{C}=\hat{g}^{C}-g^{C}$, is the *ex ante* probability for such marginal matriculating student to drop out of college. We impose the following parametric condition meaning that the *marginal* matriculating student's *ex ante* probability of dropping out is no less than 0.25:

Assumption 4. $1-F_{\varepsilon}\left(\Delta^{C}\right)\geq 0.25$.

This condition is well-justified in the context of the model given that the actual *average* dropout rate (i.e., across the entire student population) from US four-year colleges consistently exceeded 40% in recent decades.

We now proceed to analyzing how the levels of tuition subsidies affect the threshold testing outcome $\hat{g}^{C}$, the ability cut-off for dropping out g^{C}, as well as g^{R}, the threshold level of true ability, at which the financial benefits of attending college break even with its total cost to a student. This will allow us to characterize the effects on the volumes of student enrollment N, dropout Q, and graduation G. Specifically, we consider two distinct policy experiments.

Policy A: marginally increasing first stage tuition subsidy s_1 while keeping the second stage subsidy s_2 fixed.

Policy B: marginally increasing second stage tuition subsidy s_2 while keeping s_1 fixed.

We obtain the following results, all proven in the Appendix.

Lemma 4 (Policy effects on decision thresholds).

The marginal changes to tuition subsidies according to the experiments (policies) A and B will have the following respective effects on the threshold test score for matriculating, the revealed true ability threshold for persisting in college, as well as the ability cutoff for students graduating college with bad debt:

$$\frac{\partial\hat{g}^{C}}{\partial s_1}<-w^{-1}<0,\quad \frac{\partial g^{C}}{\partial s_1}=0,\quad \frac{\partial g^{R}}{\partial s_1}=-w^{-1}<0 \tag{21}$$

$$\frac{\partial \hat{g}^C}{\partial s_2} = \frac{\partial g^C}{\partial s_2} = \frac{\partial g^R}{\partial s_2} = -w^{-1} < 0,$$

Remark. Relationships (21) imply, in particular, that the first stage tuition subsidy has stronger (negative) effect on the test score threshold for matriculation than does the second stage subsidy of the same magnitude: $\frac{\partial \hat{g}^C}{\partial s_1} < \frac{\partial \hat{g}^C}{\partial s_2}$.

Thus, according to Lemma 4, a more generous tuition subsidy *at either stage of college* will lower the test score bar for matriculation, hence expanding it. Increasing stage 1 subsidy will have no effect on the true ability bar for persisting through graduation and thus on the number of dropouts. However, increasing stage 2 subsidy will, not surprisingly, lower this bar, i.e., will reduce impetus to drop out. The relationships (21) imply furthermore that if tuition subsidy were marginally shifted toward the first stage of college at the expense of the second stage, the test margin for enrolling would go down, so matriculation would expand but dropout increase to an even greater extent since the true ability cutoff for persisting would go up.

The results of Lemma 4 lead to the following implications of the respective policies on aggregate student flows in the college system and on student debt outcomes (see Appendix for detailed proofs):

Proposition 1 (Policy effects on the aggregates: student flows and debt outcomes).
The marginal changes to tuition subsidies according to the policies A and B will have the following effects:

(i) Policy A will increase the aggregate matriculation N. The growth in matriculation will result in larger numbers of both college graduates G and dropouts Q (so the growth in dropout Q constitutes a fraction of the growth of matriculation N):

$$\frac{\partial G}{\partial s_1} > 0, \quad \frac{\partial Q}{\partial s_1} > 0, \text{ so } \frac{\partial G}{\partial s_1} < \frac{\partial N}{\partial s_1}$$

(ii) Policy B will likewise increase the aggregate matriculation N and the number of college graduates G, but will have no effect on the aggregate number of dropouts Q:

$$\frac{\partial G}{\partial s_2} > 0, \quad \frac{\partial Q}{\partial s_2} = 0, \text{ so } \frac{\partial G}{\partial s_2} = \frac{\partial N}{\partial s_2}$$

(iii) Comparative effects on student flows: Compared to policy B, policy A will yield higher growth in matriculation as well as in the dropout but will increase graduation less than policy B:

$$\frac{\partial N}{\partial s_1} > \frac{\partial N}{\partial s_2}, \quad \frac{\partial Q}{\partial s_1} > \frac{\partial Q}{\partial s_2} = 0, \quad 0 < \frac{\partial G}{\partial s_1} < \frac{\partial G}{\partial s_2} \tag{22}$$

(iv) Comparative effects on student debt outcomes: Policy A will shrink the number of students who graduate with unrecouped student debt (defined in (18) but will increase, per (22), the number of dropouts, whereas policy B will leave the sizes of these groups unchanged:

$$\frac{\partial G^R}{\partial s_1} < \frac{\partial G^R}{\partial s_2} = 0 \tag{23}$$

The aggregate bad debt held by these groups will increase under policy B:

$$\frac{\partial D^*}{\partial s_2} > 0 \tag{24}$$

Remark 1. As shown in the Proposition's proof, the impact of policy A on bad debt depends, both in its sign and in its comparison to the magnitude of policy B's effect, on the relative size of the dropout group. The same is shown to be true of the relationship between the effects of the two policies (both unequivocally positive) on the aggregate cost of tuition subsidy *S*. Indeed, since policy A unconditionally increases subsidy to students who attend stage one of college but then drop out (equivalently, unconditionally forgives more of bad debt accrued by them), the aggregate bad debt will likely shrink under this policy, if the number of dropouts is high. Under the same condition, the cost increase of policy A is likely to exceed that generated by policy B.

Remark 2. Inequality (24) states that policy B raises bad debt. This may seem puzzling in light of inequalities (22) and (23) which show that this policy leaves the sizes Q and G^R of the two groups holding such debt (respectively, the dropouts and those graduating with net negative college premium) unchanged. The puzzle is easily resolved, however, since the ability composition of these groups changes under the policy: both move down the population's ability distribution as established in relationships (21). Indeed, the fact that increased generosity of unconditional tuition subsidies incentivizes students from lower aptitude hence low returns cohorts to enroll in college, and then persist there more strongly under policy B is a key positive contributor to the bad debt.

The mechanics behind the Proposition's results (i)-(iii) is also transparent. An increased stage 1 subsidy per policy A will attract more students of lower ability to enroll in college (manifested by the reduction of the threshold test outcome $\hat{g}^C$ per Lemma 4), hence increased matriculation, but will have no effect on the dropout margin. Because of the latter fact, increased matriculation leads to expansion of both the group of students completing college and those dropping out. Increased stage 2 subsidy per policy B has a similar effect of reducing enrollment threshold but it also, naturally, lowers the dropout threshold g^C, hence larger matriculation and completion with fewer students dropping out.

An important takeaway from Proposition 1 is that the policies that raise the overall tuition subsidies (no matter during the first or second stage of college), incentivize more students to matriculate, which entails, in the case of policy A, the growth in college graduation by individuals with relatively low post-college earnings. The aggregate student debt D is of course accrued by both "winners" and "losers" of the risky bet on matriculating in college. The former group consists of those whose college debt will pay off in the form of superior college premium. When it comes to the *ex post* "losers" of the risk-taking, both the ones who will end up dropping out and those choosing to persist, it is important to note that it is the component of debt accumulated *during their first stage* of college that can be seen as a part of productive investment, while the part potentially accrued during the second stage by the persisting ones, will at best partially mitigate the losses but not outweigh them. Thus, the bad debt accumulated by individuals in this group is of particular concern for evaluating the efficiency of tuition subsidy policy. This concern is the focus of the alternative policy of *targeted partial debt forgiveness for college dropouts* studied in the next section. It mitigates the debt of students who took on debt due to the expectation of successful outcome but for whom the risk-taking did not pay off upon the realization of true ability. By offering partial forgiveness of first-stage debt, the policy reduces incentives for the members of this group to take on more debt for the sake of graduation with relatively low wage gain.

4. **Targeted Debt Forgiveness**

Proposition 1 has characterized the effects of the policies on aggregate student flows: matriculation, dropout, and graduation, and highlighted the channels of these effects through the incentives created by stage 1and stage 2 subsidies to matriculate in college and to complete it. As the taxonomy of population subgroups at the end of Section 2 indicates, the sets of graduates and dropouts both contain students who lose out, *ex post*, from having matriculated. The policies of raising either subsidy have differential effects on these groups as well as the group of "winners", those graduating with net gains. According to Proposition 1, stage 1 subsidy incentivizes students to matriculate based on their *ex ante* signal of ability. As true ability is revealed to students upon completing stage 1, some students will find persisting through graduation worthwhile, while dropping out will turn out to be the better option for others. The Proposition showed that increased stage 1 subsidy will expand both groups. Indeed, it will reduce the threshold value of ability signal that compels students to matriculate; some of the newly encouraged matriculants are of high true ability, i.e., their ability signal was biased downward, so they will enjoy high *ex post* returns to college, while for others the low signal reflected true low ability, so their *ex post* returns to college will be low. Attracting the former group is clearly socially beneficial. Subsidizing education of the latter group is a social loss; even if they find it individually rational to persistence through graduation given stage 2 subsidy, this will further waste public resources.

It is therefore meaningful from public policy perspective to consider discouraging students with low *revealed* ability from persisting in college. The *conditional forgiveness*, full or partial, of debt accumulated at stage 1 of college for those *who drop out before embarking on stage 2* can fulfill such objective. It partly compensates the risk taken on by students for whom it ended up not paying off, akin to partial forgiveness of debts in bankruptcy process for those on the losing side of business risk, while effectively reducing the incentive to persist in a comparatively lower return endeavor.

We thus posit that if a student matriculating in college chooses to drop out, she will get the amount δ of her stage 1 debt forgiven. Then her net income, according to expression (3), will be $1-d_1+\delta$. If she will instead persist through graduation, her resulting income net of college debt will be $wg_i-d_1-d_2$. The two alternative net income levels equalize when a student's revealed ability is given by:

$$g^F=\frac{1+d_2+\delta}{w} \tag{25}$$

Therefore, students with revealed ability below g^F will drop out subject to the above conditional debt forgiveness policy. Thus, g^F is the threshold value of the true ability (revealed upon completing stage 1 of college) for persisting through graduation under the conditional debt forgiveness policy. Comparing (25) to (6) makes it obvious that

$$g^F > g^C \tag{26}$$

and therefore, *debt forgiveness, conditional on dropping out*, expands dropout by raising the revealed ability cutoff under which students will choose to remain in school through graduation. The following modification of Lemma 1 thus applies:

<u>Lemma 5</u>. *A student i who entered college at t=0, will decide at t=1 to persist through graduation under the conditional debt forgiveness policy, iff his revealed ability clears the threshold* g^F *given by (25). Such student's post-graduation income net of debt accrued during the second stage of college will exceed a dropout's wage income couple with the debt forgiveness "bonus", that is,* $wg_i - d_2 > 1+\delta$.

The conditional debt forgiveness policy also changes the expected cost-benefit calculus for matriculation that led to Lemma 2. We now reassess it below accordingly.

Given the ability signal $\hat{g}_i$ received, the *ex ante* (time *t=0*) conditional probability that if individual *i* goes to college, he will complete it, according to Lemma 5, given by the following expression analogous to expression (9) of Section 2:

$$P_i^{1F} = P\left(g_i \geq g^F \,\middle|\, \hat{g}_i\right) = F_\varepsilon\left(\hat{g}_i - g^F\right) \tag{27}$$

Thus, his *ex ante*, time t=0 conditional expected net income, in case he does choose to go to college under debt forgiveness policy, is

$$E_0\left(I_i^F \,\middle|\, \hat{g}_i\right) = E_0\left(P_i^{1F}(wg_i - d_2) + (1-P_i^{1F})(1+\delta) - d_1 \middle| \hat{g}_i\right) = E_0\left(P_i^{1F}(w(\hat{g}_i - \varepsilon_i) - d_2) + (1-P_i^{1F})(1+\delta) - d_1 \middle| \hat{g}_i\right)$$

Therefore, since $E_0\left(\varepsilon_i\right) = 0$ and ε_i is distributed independently of $\hat{g}_i$,

$$E_0\left(I_i^F \,\middle|\, \hat{g}_i\right) = P_i^{1F}[w(\hat{g}_i - E(\varepsilon_i)) - d_2 - 1 - \delta] + 1 + \delta - d_1 = P_i^{1F}[w\hat{g}_i - d_2 - 1 - \delta] + 1 + \delta - d_1$$

so according to (27),

$$E_0\left(I_i^F \,\middle|\, \hat{g}_i\right) = F_\varepsilon\left(\hat{g}_i - g^F\right)(w\hat{g}_i - d_2 - 1 - \delta) + 1 + \delta - d_1 \tag{28}$$

Recall that individual *i* will decide to enter college at *t=0* iff his expected net income is no less than the opportunity cost, i.e., iff $E_0\left(I_i^F \middle| \hat{g}_i\right) \geq 1$. According to (28) and (25), this is equivalent to condition

$$wF_\varepsilon\left(\hat{g}_i - g^F\right)(\hat{g}_i - g^F) + \delta - d_1 \geq 0$$

Similar to the derivation of equation (12), this leads to the conclusion that there is unique college matriculation threshold signal value $\hat{g}^F$, which satisfies equality

$$wF_\varepsilon\left(\hat{g}^F - g^F\right)(\hat{g}^F - g^F) = d_1 - \delta \tag{29}$$

Analogous to Lemma 2, which applied to equation (12) in Section 2, the following results obtain here:

Lemma 6. *Under the conditional debt forgiveness policy,*

(i) the value $\hat{g}^F$ *uniquely defined by equation (29) represents the threshold testing outcome, i.e., the cutoff for a signal received by individuals, such that individuals whose test outcome* $\hat{g}_i$ *is below* $\hat{g}^F$ *will decide against entering college, while those with a higher test result,* $\hat{g}_i \geq \hat{g}^F$*, will decide to matriculate.*

(ii) $g^F < \hat{g}^F$*, i.e., just as in the case without debt forgiveness, students whose true ability exceeds the received testing signal will certainly persist to graduate from college.*

Proof of the Lemma proceeds by complete analogy with that of Lemma 2.

We now turn to the key part of our analysis, the comparative effects of alternative policies, specifically the effect of the policy of conditional debt forgiveness, which we'll term *Policy F*, vs. unconditional subsidy policies *A* and *B* whose comparative analysis was undertaken in the previous section.

The comparative analysis will proceed within the following scenario. Policy experiments A and B are applied to a benchmark situation where stage 1 and stage 2 per student subsidies s_1 and s_2, respectively, are given (as a convenient *status quo* benchmark, it will be reasonable to posit that $s_1 = s_2$; such assumption will be explicitly stated when utilized), while $\delta = 0$. Policy experiment F proceeds from similarly defined benchmark, except that a starting value of δ can be positive, which will allow us to apply the results of marginal analysis below to characterize the effects of cumulative changes in the policy parameter values. Each policy experiment consists of gradually increasing, from the benchmark situation, the value of its respective policy parameter[11]: $s_1, s_2,$ or δ. We obtain the following result proven in the Appendix:

Lemma 7. *An increase in the amount δ of the debt forgiven per policy F will reduce the threshold test outcome for matriculating at a slower rate than will policies A and B. Namely,*

$$\frac{\partial \hat{g}^C}{\partial s_1} < \frac{\partial \hat{g}^C}{\partial s_2} = -w^{-1} < \frac{\partial \hat{g}^F}{\partial \delta} < 0 \tag{30}$$

Lemma 7 implies that the outcomes, in terms of threshold test signal for decision to matriculation, when applying policy alternatives A, B, F by incrementally increasing individual subsidies s_1, s_2, δ, respectively, in equal monetary amounts will compare as follows:

$$\hat{g}_1^C < \hat{g}_2^C < \hat{g}^F \tag{31}$$

where subscripts 1 and 2 refer to the respective policies of raising s_1 or s_2. Note that these threshold values are all identical when s_1 and s_2 are at their initial levels while $\delta = 0$, whereas the thresholds decline from that common level due to (30). Thus, while all forms of increased tuition subsidy/debt forgiveness encourage more matriculation, policy F does so the most conservatively whereas such impact of policy A is the most significant, owing to its more generous unconditional funding of stage 1 of college study.

To complete the menu of important threshold variables characterizing student choices and outcomes along the lines of Section 2, we note that the level of true ability g^{FR} at which the financial benefits of completing college break even with its total cost to student is unaffected by the debt forgiveness policy because it does not extend to students who complete college, so it remains defined as in expression (7):

[11] The extent of the rise of stage 2 subsidy s_2 per policy B is bound by Assumption 1, which requires that inequality $1 + d_2 \geq w$ remains in effect. This condition merely ensures that a student's cost d_2 of stage 2 of college study remains non-trivial, such that completing college is not automatically attractive to all students of revealed low ability.

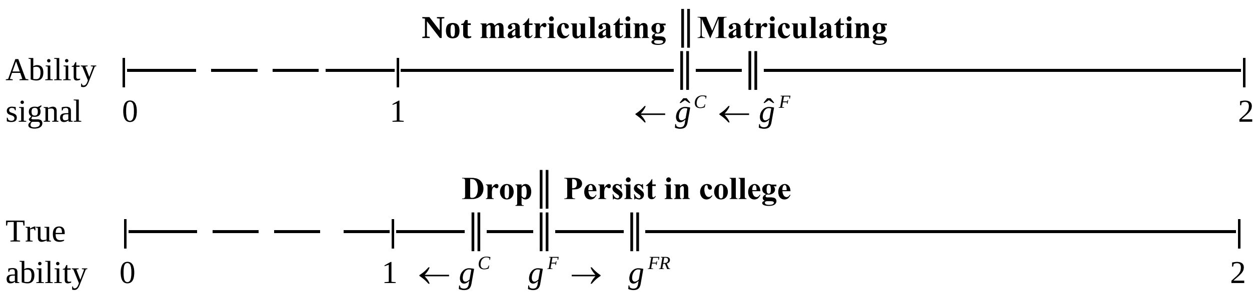
$$g^{FR} = g^{R} = \frac{1 + d_1 + d_2}{w}$$

Then, similar to Lemma 2(ii), the following inequality holds:

$$g^{F} < g^{FR} = g^{R} < \hat{g}^{F}$$

The following Figure 2 helps visualize the localizations of the decision thresholds reflecting the results of Lemmas 6-7.

Figure 2. Decision Thresholds with and without Debt Forgiveness.



The aggregate student flows in terms of matriculation N^F, dropout Q^F, college graduation G^F, as well as the subset of the latter group G^{FR} who will graduate with some unrecouped student debt are given by, here in the framework of conditional partial debt forgiveness policy, similar to expressions (15)-(18):

$$N^{F} = \int_{\hat{g}^{F} - \bar{\varepsilon}}^{\hat{g}^{F} + \bar{\varepsilon}} \left[1 - F_{\varepsilon}\left(\hat{g}^{F} - g\right)\right] dg + (2 - \hat{g}^{F} - \bar{\varepsilon}) \tag{32}$$

$$Q^{F} = \int_{\hat{g}^{F} - \bar{\varepsilon}}^{g^{F}} \left[1 - F_{\varepsilon}\left(\hat{g}^{F} - g\right)\right] dg \tag{33}$$

$$G^{F} = \int_{g^{F}}^{\hat{g}^{F} + \bar{\varepsilon}} \left[1 - F_{\varepsilon}\left(\hat{g}^{F} - g\right)\right] dg + (2 - \hat{g}^{F} - \bar{\varepsilon}) \tag{34}$$

$$G^{FR} = \int_{g^{F}}^{g^{FR}} \left[1 - F_{\varepsilon}\left(\hat{g}^{F} - g\right)\right] dg \tag{35}$$

Based on the above, one can define the *aggregate student debt*

$$D^F = N^F(d_1 + d_2) - Q^F(d_2 + \delta)$$

whereas the bad debt, the aggregate unrecouped tuition debt, that is, net of wage gains due to attending college, is given by (to be compared to (19)):

$$D*^F = Q^F(d_1 - \delta) + \int_{g^F}^{g^{FR}} [d_1 + d_2 - wg + 1]\left[1 - F_\varepsilon\left(\hat{g}^F - g\right)\right] dg \tag{36}$$

The total cost to the public of the tuition subsidy program is given by (to be compared to (20)):

$$S^F = N(s_1 + s_2) - Qs_2 + Q\delta \tag{37}$$

Applying the policy experiments described above, we obtain the following results where we drop superscript *F* for brevity.

<u>Proposition 2.</u> *(i) An increase in the amount* δ *of the debt forgiven per policy F will raise matriculation at a slower rate than will policies A and B:*

$$0 < \frac{\partial N}{\partial \delta} < \frac{\partial N}{\partial s_2} < \frac{\partial N}{\partial s_1} \tag{38}$$

It will raise dropout as much as a commensurate unconditional subsidy increase for stage 1 tuition per policy A:

$$\frac{\partial Q}{\partial \delta} = \frac{\partial Q}{\partial s_1} > \frac{\partial Q}{\partial s_2} = 0 \tag{39}$$

However, unlike policies A and B, it will shrink aggregate college graduation:

$$\frac{\partial G}{\partial \delta} < 0 < \frac{\partial G}{\partial s_1} < \frac{\partial G}{\partial s_2} \tag{40}$$

Furthermore, it will shrink the number of graduates holding bad debt, with the following comparative effects across the policies:

$$\frac{\partial G^R}{\partial \delta} = \frac{\partial G^R}{\partial s_1} < \frac{\partial G^R}{\partial s_2} = 0 \tag{41}$$

Reiterating the results of Proposition 1, policy B of raising the subsidy for stage 2 of college studies leaves unchanged the sizes of groups whose college debt does not fully pay off (the dropouts, group *L*1, as well as some graduates, group *L*2) while lowering, according to Lemma 7, the ability composition of these groups. In contrast, policy A leads to more students trying college but more of them dropping out upon revelation of true ability. According to (39), the latter is equally true of policy F, while it understandably results in fewer students graduating. Relationship (40) shows that both policies A and F also reduce the population of students who decide to graduate with unrecouped college debt. Given the parity in these outcomes between policies A and F, the key to their comparative evaluation is in comparing the resulting magnitudes of bad debt held by these groups, as well as the aggregate costs of the policies to the public. Such comparison leads to the paper's main results below.

Theorem. *(i) An increase in the amount* δ *of the debt forgiven per policy F will have a more favorable effect on the aggregate bad debt (will raise it less or reduce it more) than will a commensurate increase in tuition subsidy rate under either policy A or B. That is, the following inequalities are true:*

$$\frac{\partial D^*}{\partial \delta} < \frac{\partial D^*}{\partial s_1}, \quad \frac{\partial D^*}{\partial \delta} < \frac{\partial D^*}{\partial s_2} \tag{42}$$

(ii) Under a simplifying condition that in the prior status quo tuition subsidy rates don't differ across stages of college: $s_1 = s_2$ *, the aggregate cost of the increase in targeted debt forgiveness per policy F is smaller than the aggregate cost of a commensurate increase in tuition subsidy rate under either policy A or B. That is,*

$$\frac{\partial S}{\partial \delta} < \frac{\partial S}{\partial s_1}, \quad \frac{\partial S}{\partial \delta} < \frac{\partial S}{\partial s_2} \tag{43}$$

Remark. As will be seen in the Theorem's proof in the Appendix, similar to a related issue in Proposition 1, the sign of expression $\frac{\partial D^*}{\partial \delta}$ cannot be determined without additional conditions on the model's parameters. It will be negative if the number of dropouts is sufficiently high: indeed, these are the individuals who reduce their bad debt under policy F of targeted debt forgiveness, unlike students who choose to graduate while holding bad debt and thus do not receive extra subsidy under this policy.

Combining results (42) and (43) leads to the following crucial implication of the Theorem.

Corollary. *Consider the following budget-neutral policy experiments: (i) reducing stage 1 per student subsidy* s_1 *while increasing conditional debt forgiveness in the amount* δ *per student such that the aggregate cost increase due to the latter equals the cost saving due to the former;*

(ii) similar budget-neutral action whereby stage 2 (rather than stage 1) per student subsidy s_2 *is increased. Each of these policy experiments will result in the net reduction of the bad debt held by the students.*

These results imply that the conditional debt forgiveness policy used as a budget-neutral substitute for a fraction of unconditional tuition subsidies leads to the socially desirable outcomes:

- raises the threshold of revealed ability, hence the threshold of returns to college, for students choosing to graduate from college with unrecouped tuition debt;
- reduces these individuals' bad debt while saving public resources that would be otherwise devoted to subsidizing stage 2 tuition of the low returns students;
- since conditional debt forgiveness for low-returns students is funded as budget-neutral substitute for a fraction of unconditional tuition subsidy, it reduces the public funds' crowding out of high private returns to college education for adequately able students.

5. **Concluding Comments**

Our model greatly simplifies students' decisions in college by reducing them to unidimensional questions of whether to matriculate given pre-college signal of ability and then whether to persist through graduation upon the revelation of true ability. In reality, students' decisions involve the choice of major, including potentially switching majors once they get updated information about their ability to pursue the original choice. We note, however, that our simplified model nevertheless captures some essential elements of such more nuanced landscape. Indeed, the initial choice of major is motivated by students' expectation of college premium it can offer. The choice between persisting in the original major, switching to an alternative, or dropping out, entails the same calculus of college premia associated with all these choices upon the update of the information about one's ability. A switch to a less lucrative major, which is a commonly observed adjustment mid-way through college, is assessed against the option to drop out, which is precisely what our model analysis of a decision by students with downward ability update entails: graduate with a lower college premium than initially expected or join the workforce as a worker with "some college" without taking on extra tuition cost. Therefore, our analysis of the effects of alternative tuition subsidy policies on student choices and outcomes, including in terms of "bad debt" can be applied to such richer framework.

Another relevant aspect of student subsidy policy analysis is that institutional incentives on the part of colleges are not necessarily aligned with those of society at large. Indeed, with the exception of selective colleges and universities, institutions' incentives are strongly biased toward "retention" since tuition revenue and some government funding are tied to students' the continued enrollment. This assessment is consistent with the recent study by Denning et al. (2022) who demonstrate that the recent rise in college completion rates was largely driven by grade inflation policy by colleges, which in effect obfuscates the precision of students' information updating about their ability and future earning capacity. We envision an extension of our analytical framework to addressing the effects of diverging tuition subsidy policies, driven by such competing interests, on students' outcomes, including when it comes to student debt, and the implications for potential government regulation of higher education. Such extension, while going beyond the present analysis, appears attractive for future work.

Finally, we note the limitation of our partial equilibrium analysis in that modeling the effect of human capital accumulation on the economy and the skilled and unskilled components of its labor force is beyond the scope of our study (and is a focus of attention of a substantial literature). For instance, our analysis bypasses such plausible general equilibrium effects of college enrollment expansion, including higher retention, as potential positive externality generated by human capital creation, in production and in terms of intergenerational effects at the household level. Such considerations may conceivably shift policy priorities in the direction of increased college enrollment and graduation. Accordingly, they may affect the margins for socially and individually optimal student choices but not the general spirit of our results regarding comparative effects of alternative tuition subsidy policies.

References

Altonji, J.G. (1993). "The Demand for and Return to Education When Education Outcomes Are Uncertain," *Journal of Labor Economics* 11, 48-83.

Arcidiacono, P. (2004). "Ability sorting and the returns to college major," *Journal of Econometrics* 121, 343 – 375.

Athreya, K. and J. Eberly (2021). "Risk, the College Premium, and Aggregate Human Capital Investment," *American Economic Journal: Macroeconomics* 13, 168-213.

Blankenau, W. and G. Camera (2009). "Public Spending on Education and the Incentives for Student Achievement," *Economica* 76, 505-527.

Bound, J., M.F. Lovenheim, and S. Turner (2010). "Why Have College Completion Rates Declined? An Analysis of Changing Student Preparation and Collegiate Resources," *American Economic Journal: Applied Economics* 2, 129–57.

Carneiro, P. and J.J. Heckman (2002), "The Evidence on Credit Constraints in Post-secondary Schooling," *Economic Journal* 112, 989–1018.

Caucutt, E.M. and K.B. Kumar (2003), "Higher education subsidies and heterogeneity: a dynamic analysis," *Journal of Economic Dynamics and Control* 27, 1459-1502.

Deming, D. (2017). "Increasing college completion with a federal higher education matching grant." *The Hamilton Project, Policy Proposal* 3, www.hamiltonproject.org. Brookings, Washington D.C.

Denning, J.T., E.R. Eide, K.J. Mumford, R.W., Patterson, and M. Warnick (2022). "Why Have College Completion Rates Increased?" *American Economic Journal: Applied Economics* 14, 1-29.

Eckwert B. and I. Zilcha (2004). "Economic implications of better information in a dynamic framework," *Economic Theory* 24, 561-581.

Eckwert B. and I. Zilcha (2014). "Higher Education: Subsidizing Tuition versus Subsidizing Student Loans," Journal of Public Economic Theory 16, 835-853.

Garriga, C. and M.P. Keightley (2007). "A General Equilibrium Theory of College with Education Subsidies, In-School Labor Supply, and Borrowing Constraints." Federal Reserve Bank of St. Louis. Working Paper 2007-051A.

Gary-Bobo, R.J. and A. Trannoy (2008) "Efficient Tuition Fees and Examinations," *Journal of European Economic Association* 6, 1211-43

Hendricks, L. and O. Leukhina (2018). "The Return to College: Selection and Dropout Risk," *International Economic Review* 59, 1077-1102.

Kaganovich, M. (2023). "Trade-offs in Choosing a College Major." *CESifo Working Paper* No. 10650.

Kaganovich, M., S. Sarpça, and X. Su (2021). "Competition in Higher Education." In: Brian McCall, ed. The Routledge Handbook of the Economics of Education. Routledge; Taylor & Francis, London.

Kaganovich, M., M. Taylor, and R. Xiao (2023). "Gender Differences in Persistence in a Field of Study: This Isn't All about Grades," *Journal of Human Capital* 17, 503-556.

Manski, C. F., (1989). "Schooling as Experimentation: a Reappraisal of the Postsecondary Dropout Phenomenon," *Economics of Education Review* 8, *305-312.*

NCES (2020). Digest of education statistics 2018. National Center for Education Statistics, December 2019. US Department of Education, NCES 2020–009.

NCES (2019). Digest of education statistics 2017. National Center for Education Statistics, January 2019. US Department of Education, NCES 2018–070.

Sahin, A. (2004). "The Incentive Effects of Higher Education Subsidies on Student Effort." Federal Reserve Bank of New York Staff Reports. Staff Report no 192.

Spence, M. (1973). "Job Market Signaling," *Quarterly Journal of Economics* 87, 355-374.

Stinebrickner, T. and R. Stinebrickner (2012). "Learning about Academic Ability and the College Dropout Decision," *Journal of Labor Economics* 30, 707-748.

Stinebrickner, R. and T.R. Stinebrickner (2014). "A Major in Science? Initial Beliefs and Final Outcomes for College Major and Dropout," *Review of Economic Studies* 81, 426-472.

Appendix

Proof of Lemma 2(ii).

Equation $F_\varepsilon(x)x = w^{-1}d_1$ obviously has unique solution because its left-hand side increases in x as it moves from 0 to 1 and reaches maximum value 1 when x=1, whereas the right-hand side is smaller than 1 according to (8). Thus, equation (12) has unique solution, which is positive, so $\hat{g}^C > g^C$.

To prove that $g^R < \hat{g}^C$, let's replace $\hat{g}^C$ with g^R in the left-hand-side expression of equation (12): $wF_\varepsilon\left(g^R - g^C\right)(g^R - g^C)$. According to the expressions in (7), this equals $wF_\varepsilon\left(w^{-1}d_1\right)(w^{-1}d_1) < d_1$, where the inequality follows from the fact that $F_\varepsilon\left(w^{-1}d_1\right) < 1$, which is due to Assumption 2. Comparing the previous inequality with equation (12), we can conclude (since expression $wF_\varepsilon(x)x$ strictly increases in x) that $g^R - g^C < \hat{g}^C - g^C$, so $g^R < \hat{g}^C$. ■

Proof of Lemma 4.

Recall the relationship $d_t = \tau - s_t$ between government subsidy and student debt incurred in stages t=1, 2 of college study. First, we note that Lemma's results concerning the effects on g^C trivially follow from the expression (6).

We shall now totally differentiate relationship (12) with respect to s_1. Using again expression (6), we obtain:

$$\frac{\partial \hat{g}^C}{\partial s_1} = -\frac{\partial \hat{g}^C}{\partial d_1} = -\left\{F_\varepsilon'\left(\Delta^C\right)(w\hat{g}^C - d_2 - 1) + wF_\varepsilon\left(\Delta^C\right)\right\}^{-1} = -w^{-1}\left\{F_\varepsilon'\left(\Delta^C\right)\Delta^C + F_\varepsilon\left(\Delta^C\right)\right\}^{-1} \qquad \text{(A.1)}$$

where $\Delta^C = \hat{g}^C - g^C$ as defined by expression (14) and $F_\varepsilon'\left(\Delta^C\right) = \dfrac{\partial F_\varepsilon\left(\Delta^C\right)}{\partial \Delta^C}$.

According to relationships (14) and (6), it is clear that expression (A.1) is negative. Moreover,

$$-2w^{-1} < \frac{\partial \hat{g}^C}{\partial s_1} < w^{-1} < 0 \qquad \text{(A.2)}$$

The first of the inequalities in (A.2) is true because $F_\varepsilon\left(\Delta^C\right) > 0.5$ since $\Delta^C > 0$ according to (13). The second one is true because

$$F_\varepsilon'\left(\Delta^C\right)\Delta^C + F_\varepsilon\left(\Delta^C\right) < 1 \tag{A.3}$$

Indeed, recall that testing error ε is distributed symmetrically with mean 0, so $F_\varepsilon\left(0\right) = 0.5$, while $\Delta^C \in (0, \bar{\varepsilon})$. Recall also that according to Assumption 4, $1 - F_\varepsilon\left(\Delta^C\right) \geq 0.25$. Then $F_\varepsilon'\left(\Delta^C\right)\Delta^C < F_\varepsilon\left(\Delta^C\right) - 0.5 \leq 0.25$. This along with the previous inequality yields (A.3).

We now totally differentiate relationship (12) with respect to s_2 and obtain the following using the fact, due to expression (6), that $\dfrac{\partial g^C}{\partial s_2} = -w^{-1}$:

$$\frac{\partial \hat{g}^C}{\partial s_2} = -\frac{\partial \hat{g}^C}{\partial d_2} = -\frac{F_\varepsilon'\left(\Delta^C\right)(w\hat{g}^C - d_2 - 1)w^{-1} + F_\varepsilon\left(\Delta^C\right)}{F_\varepsilon'\left(\Delta^C\right)(w\hat{g}^C - d_2 - 1) + wF_\varepsilon\left(\Delta^C\right)} = -w^{-1} \tag{A.4}$$

It remains to observe that the values $\dfrac{\partial g^C}{\partial s_1} = 0$, $\dfrac{\partial g^R}{\partial s_1} = -w^{-1}$, $\dfrac{\partial g^C}{\partial s_2} = \dfrac{\partial g^R}{\partial s_2} = -w^{-1}$ follow directly from expressions (6) and (7). The proof of Lemma 4 is thus complete. ■

Note. The expression of the form $F_\varepsilon'\left(\Delta\right)\Delta + F_\varepsilon\left(\Delta\right)$ will appear frequently in derivations below, so we'll henceforth use the following notation for brevity:

$$\Phi\left(\Delta\right) = F_\varepsilon'\left(\Delta\right)\Delta + F_\varepsilon\left(\Delta\right) \tag{A.5}$$

Proof of Proposition 1.

Using the definition of the aggregate dropout volume by expression (16), expression (A.1), as well as the facts that $\dfrac{\partial g^C}{\partial s_1} = 0$ and $F_\varepsilon\left(\bar{\varepsilon}\right) = 1$, we can write

$$\begin{aligned}
\frac{\partial Q}{\partial s_1} &= -\int_{\hat{g}^C - \bar{\varepsilon}}^{g^C} F_\varepsilon'\left(\hat{g}^C - g\right)\frac{\partial \hat{g}^C}{\partial s_1} dg + \left(1 - F_\varepsilon\left(\Delta^C\right)\right)\frac{\partial g^C}{\partial s_1} - \left(1 - F_\varepsilon\left(\bar{\varepsilon}\right)\right)\frac{\partial \hat{g}^C}{\partial s_1} = \\
&= -\frac{\partial \hat{g}^C}{\partial s_1}\int_{\hat{g}^C - \bar{\varepsilon}}^{g^C} F_\varepsilon'\left(\hat{g}^C - g\right) dg = \frac{\partial \hat{g}^C}{\partial s_1}\left[F_\varepsilon\left(\Delta^C\right) - F_\varepsilon\left(\bar{\varepsilon}\right)\right] = \\
&= \frac{\partial \hat{g}^C}{\partial s_1}\left[F_\varepsilon\left(\Delta^C\right) - 1\right] = w^{-1}\frac{1 - F_\varepsilon\left(\Delta^C\right)}{\Phi\left(\Delta^C\right)} > 0
\end{aligned} \tag{A.6}$$

Then similarly, also making use of (6) and (A.4),

$$\frac{\partial Q}{\partial s_2} = -\frac{\partial \hat{g}^C}{\partial s_2} \int_{\hat{g}^C-\bar{\varepsilon}}^{g^C} F'_{\varepsilon}\left(\hat{g}^C - g\right)\, dg + \left(1 - F_{\varepsilon}\left(\Delta^C\right)\right)\frac{\partial g^C}{\partial s_2} = \left[F_{\varepsilon}\left(\Delta^C\right) - 1\right]\left(\frac{\partial \hat{g}^C}{\partial s_2} - \frac{\partial g^C}{\partial s_2}\right) = 0 \quad \text{(A.7)}$$

Along the same lines, using expressions (15) and (17), we obtain the marginal effects of changes in student subsidies on the aggregate enrollment and graduation volumes:

$$\begin{aligned}\frac{\partial N}{\partial s_1} &= \left[-\int_{\hat{g}^C-\bar{\varepsilon}}^{\hat{g}^C+\bar{\varepsilon}} F'_{\varepsilon}\left(\hat{g}^C - g\right)\, dg + \left(1 - F_{\varepsilon}\left(-\bar{\varepsilon}\right)\right) - \left(1 - F_{\varepsilon}\left(\bar{\varepsilon}\right)\right)\right]\frac{\partial \hat{g}^C}{\partial s_1} - \frac{\partial \hat{g}^C}{\partial s_1} = \\ &= \left[F_{\varepsilon}\left(-\bar{\varepsilon}\right) - F_{\varepsilon}\left(\bar{\varepsilon}\right) + 1\right]\frac{\partial \hat{g}^C}{\partial s_1} - \frac{\partial \hat{g}^C}{\partial s_1} = -\frac{\partial \hat{g}^C}{\partial s_1} > 0\end{aligned} \quad \text{(A.8)}$$

$$\begin{aligned}\frac{\partial G}{\partial s_1} &= -\frac{\partial \hat{g}^C}{\partial s_1} \int_{g^C}^{\hat{g}^C+\bar{\varepsilon}} F'_{\varepsilon}\left(\hat{g}^C - g\right)\, dg + \frac{\partial \hat{g}^C}{\partial s_1}\left(1 - F_{\varepsilon}\left(-\bar{\varepsilon}\right)\right) - \frac{\partial g^C}{\partial s_1}\left(1 - F_{\varepsilon}\left(\Delta^C\right)\right) - \frac{\partial \hat{g}^C}{\partial s_1} = \\ &= \left[F_{\varepsilon}\left(-\bar{\varepsilon}\right) - F_{\varepsilon}\left(\Delta^C\right) + 1\right]\frac{\partial \hat{g}^C}{\partial s_1} - \frac{\partial \hat{g}^C}{\partial s_1} = -F_{\varepsilon}\left(\Delta^C\right)\frac{\partial \hat{g}^C}{\partial s_1} = w^{-1}\frac{F_{\varepsilon}\left(\Delta^C\right)}{\Phi\left(\Delta^C\right)} > 0\end{aligned} \quad \text{(A.9)}$$

$$\begin{aligned}\frac{\partial N}{\partial s_2} &= \left[-\int_{\hat{g}^C-\bar{\varepsilon}}^{\hat{g}^C+\bar{\varepsilon}} F'_{\varepsilon}\left(\hat{g}^C - g\right)\, dg + \left(1 - F_{\varepsilon}\left(-\bar{\varepsilon}\right)\right) - \left(1 - F_{\varepsilon}\left(\bar{\varepsilon}\right)\right)\right]\frac{\partial \hat{g}^C}{\partial s_2} - \frac{\partial \hat{g}^C}{\partial s_2} = \\ &= \left[F_{\varepsilon}\left(-\bar{\varepsilon}\right) - F_{\varepsilon}\left(\bar{\varepsilon}\right) + 1\right]\frac{\partial \hat{g}^C}{\partial s_2} - \frac{\partial \hat{g}^C}{\partial s_2} = -\frac{\partial \hat{g}^C}{\partial s_2} = w^{-1} > 0\end{aligned} \quad \text{(A.10)}$$

$$\begin{aligned}\frac{\partial G}{\partial s_2} &= \left[-\int_{g^C}^{\hat{g}^C+\bar{\varepsilon}} F'_{\varepsilon}\left(\hat{g}^C - g\right)\, dg + \left(1 - F_{\varepsilon}\left(-\bar{\varepsilon}\right)\right)\right]\frac{\partial \hat{g}^C}{\partial s_2} - \left(1 - F_{\varepsilon}\left(\Delta^C\right)\right)\frac{\partial g^C}{\partial s_2} - \frac{\partial \hat{g}^C}{\partial s_2} = \\ &= -w^{-1}\left[F_{\varepsilon}\left(-\bar{\varepsilon}\right) - F_{\varepsilon}\left(\Delta^C\right) + 1\right] + w^{-1}\left(1 - F_{\varepsilon}\left(\Delta^C\right)\right) + w^{-1} = w^{-1} > 0\end{aligned} \quad \text{(A.11)}$$

where the derivation of (A.11) follows from relationships (6) and (A.4). Likewise, by differentiating (18) and using (A.1), (A.4), as well as (7), we obtain

$$\frac{\partial G^R}{\partial s_1} = -\frac{\partial \hat{g}^C}{\partial s_1} \int_{g^C}^{g^R} F'_\varepsilon\left(\hat{g}^C - g\right) dg + \frac{\partial g^R}{\partial s_1}\left(1 - F_\varepsilon\left(\hat{g}^C - g^R\right)\right) - \frac{\partial g^C}{\partial s_1}\left(1 - F_\varepsilon\left(\Delta^C\right)\right) =$$

$$= \left[F_\varepsilon\left(\Delta^C - w^{-1}d_1\right) - F_\varepsilon\left(\Delta^C\right)\right]\frac{\partial \hat{g}^C}{\partial s_1} - w^{-1}\left[1 - F_\varepsilon\left(\Delta^C - w^{-1}d_1\right)\right] =$$

$$= -w^{-1}\frac{F_\varepsilon\left(\Delta^C - w^{-1}d_1\right) - F_\varepsilon\left(\Delta^C\right) + \left[1 - F_\varepsilon\left(\Delta^C - w^{-1}d_1\right)\right]\left[F'_\varepsilon\left(\Delta^C\right)\Delta^C + F_\varepsilon\left(\Delta^C\right)\right]}{\Phi\left(\Delta^C\right)} = \qquad \text{(A.12)}$$

$$= -w^{-1}\frac{F_\varepsilon\left(\Delta^C - w^{-1}d_1\right)\left[1 - F_\varepsilon\left(\Delta^C\right)\right] + F'_\varepsilon\left(\Delta^C\right)\Delta^C\left[1 - F_\varepsilon\left(\Delta^C - w^{-1}d_1\right)\right]}{\Phi\left(\Delta^C\right)} < 0$$

$$\frac{\partial G^R}{\partial s_2} = -\frac{\partial \hat{g}^C}{\partial s_2} \int_{g^C}^{g^R} F'_\varepsilon\left(\hat{g}^C - g\right) dg + \frac{\partial g^R}{\partial s_2}\left(1 - F_\varepsilon\left(\hat{g}^C - g^R\right)\right) - \frac{\partial g^C}{\partial s_2}\left(1 - F_\varepsilon\left(\Delta^C\right)\right) =$$

$$= \left[F_\varepsilon\left(\Delta^C - w^{-1}d_1\right) - F_\varepsilon\left(\Delta^C\right)\right]\frac{\partial \hat{g}^C}{\partial s_2} - w^{-1}\left(1 - F_\varepsilon\left(\Delta^C - w^{-1}d_1\right)\right) + w^{-1}\left(1 - F_\varepsilon\left(\Delta^C\right)\right) = \qquad \text{(A.13)}$$

$$= -w^{-1}\left[F_\varepsilon\left(\Delta^C - w^{-1}d_1\right) - F_\varepsilon\left(\Delta^C\right)\right] + w^{-1}\left[F_\varepsilon\left(\Delta^C - w^{-1}d_1\right) - F_\varepsilon\left(\Delta^C\right)\right] = 0$$

The above proves parts (i)-(iii) of the Proposition, whereby inequalities (22) follow from direct comparisons of (A.8) to (A.9), (A.6) to (A.7), and (A.10) to (A.11).

The comparison of (A.12) to (A.13) proves expression (23) in part (iv). Proceeding to the rest of part (iv), we differentiate expression (19) to obtain the following, making use of expressions (16), (A.7), (18), (A.4), as well as (7):

$$\frac{\partial D^*}{\partial s_2} = \frac{\partial Q}{\partial s_2}d_1 - \int_{g^C}^{g^R}\left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg - \frac{\partial \hat{g}^C}{\partial s_2}\int_{g^C}^{g^R}[d_1 + d_2 - wg + 1]F'_\varepsilon\left(\hat{g}^C - g\right) dg -$$

$$+ \frac{\partial g^R}{\partial s_2}[d_1 + d_2 - wg^R + 1]\left(1 - F_\varepsilon\left(\hat{g}^C - g^R\right)\right) - \frac{\partial g^C}{\partial s_2}[d_1 + d_2 - wg^C + 1]\left(1 - F_\varepsilon\left(\Delta^C\right)\right) =$$

$$= -G^R + w^{-1}\int_{g^C}^{g^R}[d_1 + d_2 - wg + 1]F'_\varepsilon\left(\hat{g}^C - g\right) dg + w^{-1}d_1\left(1 - F_\varepsilon\left(\Delta^C\right)\right) > \qquad \text{(A.14)}$$

$$> -G^R + w^{-1}d_1\left(1 - F_\varepsilon\left(\Delta^C\right)\right)$$

Then according to expressions in (7), the above implies

$$\frac{\partial D^*}{\partial s_2} > -G^R + w^{-1}d_1 \int_{g^C}^{g^R} F_\varepsilon'\left(\hat{g}^C - g\right)\, dg\; F_\varepsilon\left(\hat{g}^C - g^R\right) + w^{-1}d_1\left(1 - F_\varepsilon\left(\Delta^C\right)\right) =$$
$$> -G^R + w^{-1}d_1\left[-F_\varepsilon\left(\hat{g}^C - g^R\right) + F_\varepsilon\left(\hat{g}^C - g^C\right)\right] + w^{-1}d_1\left(1 - F_\varepsilon\left(\Delta^C\right)\right) =$$
$$= -G^R + w^{-1}d_1\left[-F_\varepsilon\left(\Delta^C - w^{-1}d_1\right) + F_\varepsilon\left(\Delta^C\right)\right] + w^{-1}d_1\left(1 - F_\varepsilon\left(\Delta^C\right)\right) =$$
$$= -G^R + w^{-1}d_1\left(1 - F_\varepsilon\left(\Delta^C - w^{-1}d_1\right)\right) \tag{A.15}$$

Observe now that, according to (18),

$$G^R = \int_{g^C}^{g^R}\left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right]dg < \left[1 - F_\varepsilon\left(\hat{g}^C - g^R\right)\right][g^R - g^C] = w^{-1}d_1\left[1 - F_\varepsilon\left(\Delta^C - w^{-1}d_1\right)\right] \tag{A.16}$$

Combined with (A.15), this implies that $\frac{\partial D^*}{\partial s_2} > 0$, which proves part (iv) of the Proposition.

To demonstrate the ambiguity in the relationship between $\frac{\partial D^*}{\partial s_2}$ and $\frac{\partial D^*}{\partial s_1}$, we differentiate (19) with respect to s_1 using relationships (16), (A.6), (18), and (A.1) to obtain:

$$\frac{\partial D^*}{\partial s_1} = -Q + \frac{\partial Q}{\partial s_1}d_1 - \int_{g^C}^{g^R}\left[1 - F_\varepsilon\left(\hat{g}^C - g\right)\right] dg - \frac{\partial \hat{g}^C}{\partial s_1}\int_{g^C}^{g^R}[d_1 + d_2 - wg + 1]F_\varepsilon'\left(\hat{g}^C - g\right) dg +$$
$$+ \frac{\partial g^R}{\partial s_1}[d_1 + d_2 - wg^R + 1]\left(1 - F_\varepsilon\left(\hat{g}^C - g^R\right)\right) - \frac{\partial g^C}{\partial s_1}[d_1 + d_2 - wg^C + 1]\left(1 - F_\varepsilon\left(\Delta^C\right)\right) =$$
$$= -Q + \frac{\partial \hat{g}^C}{\partial s_1}\left[F_\varepsilon\left(\Delta^C\right) - 1\right]d_1 - G^R - \frac{\partial \hat{g}^C}{\partial s_1}\int_{g^C}^{g^R}[d_1 + d_2 - wg + 1]F_\varepsilon'\left(\hat{g}^C - g\right) dg =$$
$$= -Q + \frac{w^{-1}d_1\left(1 - F_\varepsilon\left(\Delta^C\right)\right)}{\Phi\left(\Delta^C\right)} - G^R + \frac{w^{-1}}{\Phi\left(\Delta^C\right)}\int_{g^C}^{g^R}[d_1 + d_2 - wg + 1]F_\varepsilon'\left(\hat{g}^C - g\right) dg \tag{A.17}$$

and observe that this expression does not render unequivocal comparison to (A.14) and that it, as well as the sign of this derivative depend on the magnitude of dropout Q. Specifically, the result $\frac{\partial D^*}{\partial s_1} > 0$ will obtain if Q is not too large.

Finally, by differentiating expression (20), we obtain the following. Using expressions (A.8), (A.6), and (A.1), we get:

$$\frac{\partial S}{\partial s_1}=\frac{\partial N}{\partial s_1}(s_1+s_2)+N-\frac{\partial Q}{\partial s_1}s_2=-\frac{\partial \hat{g}^C}{\partial s_1}(s_1+s_2)+N+\left(1-F_\varepsilon\left(\Delta^C\right)\right)\frac{\partial \hat{g}^C}{\partial s_1}s_2=$$
$$=\frac{w^{-1}\left(s_1+F_\varepsilon\left(\Delta^C\right)s_2\right)}{\Phi\left(\Delta^C\right)}+N>0 \tag{A.18}$$

Likewise, we use (A.10), (A.7), and (A.4) to write:

$$\frac{\partial S}{\partial s_2}=\frac{\partial N}{\partial s_2}(s_1+s_2)+N-\frac{\partial Q}{\partial s_2}s_2-Q=w^{-1}(s_1+s_2)+N-Q>0 \tag{A.19}$$

The two expressions make it clear that the relationship between $\frac{\partial S}{\partial s_1}$ will and $\frac{\partial S}{\partial s_2}$ depends on parameter values and the magnitude of Q.

This completes the proof of Proposition 1. ■

Proof of Lemma 7.

Compare equations (12) and (29) under benchmark values of tuition subsidies s_1, s_2:

$wF_\varepsilon\left(\Delta^C\right)\Delta^C=d_1$ and $wF_\varepsilon\left(\Delta^F\right)\Delta^F=d_1-\delta$ where $\Delta^F=\hat{g}^F-g^F$. We note that since the left-hand sides of the equations strictly increase in the argument Δ one can conclude that

$$\Delta^F<\Delta^C_{benchmark} \tag{A.20}$$

This implies, in particular, that the modification of relationship (A.3) continues to hold with any $\delta\geq 0$, i.e.,

$$F_\varepsilon'\left(\Delta^F\right)\Delta^F+F_\varepsilon\left(\Delta^F\right)-1<0 \tag{A.21}$$

Differentiating the equation (29) fully with respect to δ we obtain according to (A.20):

$$\frac{\partial \hat{g}^F}{\partial \delta}=w^{-1}\left[1-\frac{1}{\Phi\left(\Delta^F\right)}\right]<0 \tag{A.22}$$

for any $\delta\geq 0$, where we continue to use the notation of (A.5). The above equality implies that $\frac{\partial \hat{g}^F}{\partial \delta}>-w^{-1}$ because $\frac{1}{\Phi\left(\Delta^F\right)}<2$. The latter is true since $F_\varepsilon\left(\Delta^F\right)$ because $\Delta^F>0$, according to Lemma 6(ii). This proves relationships (30), in light of expressions in (21) ■

Proof of Proposition 2.

The result (38) follows directly from inequalities (30) which compare matriculation thresholds across the policies.

Proceeding to (39), we find, using (33), (25), and (A.21):

$$\frac{\partial Q^F}{\partial \delta} = -\int_{\hat{g}^F-\bar{\varepsilon}}^{g^F} F_\varepsilon'\left(\hat{g}^F - g\right)\frac{\partial \hat{g}^F}{\partial \delta}dg + \left(1-F_\varepsilon\left(\Delta^F\right)\right)\frac{\partial g^F}{\partial \delta} - \left(1-F_\varepsilon\left(\bar{\varepsilon}\right)\right)\frac{\partial \hat{g}^F}{\partial \delta} =$$

$$= -\frac{\partial \hat{g}^F}{\partial \delta}\int_{\hat{g}^F-\bar{\varepsilon}}^{g^F} F_\varepsilon'\left(\hat{g}^F - g\right) dg + w^{-1}\left(1-F_\varepsilon\left(\Delta^F\right)\right) = \frac{\partial \hat{g}^F}{\partial \delta}\left[F_\varepsilon\left(\Delta^F\right) - F_\varepsilon\left(\bar{\varepsilon}\right)\right] + \tag{A.23}$$

$$+ w^{-1}\left(1-F_\varepsilon\left(\Delta^F\right)\right) = \left(1-F_\varepsilon\left(\Delta^F\right)\right)\left[w^{-1} - \frac{\partial \hat{g}^F}{\partial \delta}\right] = w^{-1}\left(1-F_\varepsilon\left(\Delta^F\right)\right)\frac{1}{\Phi\left(\Delta^F\right)} > 0$$

Relationships (21), (A.1), (A.4), and (A.22), which characterize the effects of each policy on matriculation and persistence thresholds, enable us to determine the following under the provisions of the policy experiments in the cross-policy comparison:

$$\frac{\partial \Delta^F}{\partial \delta} = -\frac{w^{-1}}{\Phi\left(\Delta^F\right)},\quad \frac{\partial \Delta^C}{\partial s_1} = -\frac{w^{-1}}{\Phi\left(\Delta^C\right)},\quad \frac{\partial \Delta^C}{\partial s_2} = 0$$

This implies that when the policy experiments proceed incrementally from the same benchmark where $\Delta^C = \Delta^C_{benchmark}$, the respective outcomes compare as follows:

$$\Delta^F = \Delta_1^C < \Delta_2^C = \Delta^C_{benchmark} \tag{A.24}$$

where similar to (31) subscripts 1 and 2 refer to the respective policies of raising s_1 or s_2.

Applying (A.24) to the comparison of expressions (A.23), (A.6), and (A.7), we obtain the required relationship (39).

To prove (40), we differentiate expression (34), obtain the following:

$$\frac{\partial G^F}{\partial \delta} = -\frac{\partial \hat{g}^F}{\partial \delta}\int_{g^F}^{\hat{g}^F+\bar{\varepsilon}} F_\varepsilon'\left(\hat{g}^F - g\right) dg + \frac{\partial \hat{g}^F}{\partial \delta}\left(1-F_\varepsilon\left(-\bar{\varepsilon}\right)\right) - \frac{\partial g^F}{\partial \delta}\left(1-F_\varepsilon\left(\Delta^F\right)\right) - \frac{\partial \hat{g}^F}{\partial \delta} =$$

$$= \left[F_\varepsilon\left(-\bar{\varepsilon}\right) - F_\varepsilon\left(\Delta^F\right) + 1 - F_\varepsilon\left(-\bar{\varepsilon}\right)\right]\frac{\partial \hat{g}^F}{\partial \delta} - w^{-1}\left(1-F_\varepsilon\left(\Delta^F\right)\right) - \frac{\partial \hat{g}^F}{\partial \delta} = -w^{-1}\left(1-F_\varepsilon\left(\Delta^F\right)\right) = \tag{A.25}$$

$$= -w^{-1}F_\varepsilon\left(\Delta^F\right)\left[1 - \frac{1}{\Phi\left(\Delta^F\right)}\right] - w^{-1}\left(1-F_\varepsilon\left(\Delta^F\right)\right) = -w^{-1}\left[1 - \frac{F_\varepsilon\left(\Delta^F\right)}{\Phi\left(\Delta^F\right)}\right] < 0$$

and compare this to (A.9) and (A.11).

Likewise, to prove (41), we differentiate (A.20), obtain

$$\frac{\partial G^{FR}}{\partial \delta} = -\frac{\partial \hat{g}^F}{\partial \delta}\int_{g^F}^{g^{FR}} F_\varepsilon'\left(\hat{g}^F - g\right) dg + \frac{\partial g^{FR}}{\partial \delta}\left(1 - F_\varepsilon\left(\hat{g}^F - g^{FR}\right)\right) - \frac{\partial g^F}{\partial \delta}\left(1 - F_\varepsilon\left(\Delta^F\right)\right) =$$

$$= \left[F_\varepsilon\left(\Delta^F - w^{-1}(d_1 - \delta)\right) - F_\varepsilon\left(\Delta^F\right)\right]\frac{\partial \hat{g}^F}{\partial \delta} - w^{-1}\left(1 - F_\varepsilon\left(\Delta^F\right)\right) =$$

$$= w^{-1}\left[1 - \frac{1}{\Phi\left(\Delta^F\right)}\right]\left[F_\varepsilon\left(\Delta^F - w^{-1}(d_1 - \delta)\right) - F_\varepsilon\left(\Delta^F\right)\right] - w^{-1}\left(1 - F_\varepsilon\left(\Delta^F\right)\right) =$$

$$= -w^{-1}\frac{F_\varepsilon\left(\Delta^F - w^{-1}(d_1 - \delta)\right)\left[1 - F_\varepsilon\left(\Delta^F\right)\right] + F_\varepsilon'\left(\Delta^F\right)\Delta^F\left[1 - F_\varepsilon\left(\Delta^F - w^{-1}(d_1 - \delta)\right)\right]}{\Phi\left(\Delta^F\right)} < 0$$

and then compare this to (A.12) and (A.13) while applying relationship (A.24) and noting that the reduction in stage 1 student tuition cost d_1 matches its reduction to $d_1 - \delta$ under policy F. ■

<u>Proof of Proposition 3</u>.

Differentiating expression (36), we obtain

$$\frac{\partial D^*}{\partial \delta} = -Q^F + \frac{\partial Q^F}{\partial \delta}(d_1 - \delta) - \frac{\partial \hat{g}^F}{\partial \delta}\int_{g^F}^{g^{FR}} [d_1 + d_2 - wg + 1]F_\varepsilon'\left(\hat{g}^F - g\right) dg +$$

$$+ \frac{\partial g^{FR}}{\partial \delta}[d_1 + d_2 - wg^{FR} + 1]\left(1 - F_\varepsilon\left(\hat{g}^F - g^{FR}\right)\right) - \frac{\partial g^F}{\partial \delta}[d_1 + d_2 - wg^F + 1]\left(1 - F_\varepsilon\left(\Delta^F\right)\right) =$$

$$= -Q^F + \frac{\partial Q^F}{\partial \delta}(d_1 - \delta) - \frac{\partial \hat{g}^F}{\partial \delta}\int_{g^F}^{g^{FR}} [d_1 + d_2 - wg + 1]F_\varepsilon'\left(\hat{g}^F - g\right) dg - w^{-1}(d_1 - \delta)\left(1 - F_\varepsilon\left(\Delta^F\right)\right) =$$

$$= -Q^F + w^{-1}(d_1 - \delta)\frac{1 - F_\varepsilon\left(\Delta^F\right)}{\Phi\left(\Delta^F\right)} + w^{-1}\left[\frac{1}{\Phi\left(\Delta^F\right)} - 1\right]\int_{g^F}^{g^{FR}} [d_1 + d_2 - wg + 1]F_\varepsilon'\left(\hat{g}^F - g\right) dg -$$

$$-w^{-1}(d_1 - \delta)\left(1 - F_\varepsilon\left(\Delta^F\right)\right)$$

Then, using an approach similar to (A.15), we can write

$$\frac{\partial D^*}{\partial \delta} < -Q^F + w^{-1}(d_1-\delta)\frac{1-F_\varepsilon\left(\Delta^F\right)}{\Phi\left(\Delta^F\right)} + \frac{w^{-1}}{\Phi\left(\Delta^F\right)}\int_{g^F}^{g^{FR}}[d_1+d_2-wg+1]F_\varepsilon'\left(\hat{g}^F-g\right)dg -$$
$$-w^{-1}(d_1-\delta)\left(1-F_\varepsilon\left(\Delta^F\right)\right)-w^{-1}(d_1-\delta)\left[-F_\varepsilon\left(\Delta^F-w^{-1}(d_1-\delta)\right)+F_\varepsilon\left(\Delta^F\right)\right]=$$
$$= -Q^F + w^{-1}(d_1-\delta)\frac{1-F_\varepsilon\left(\Delta^F\right)}{\Phi\left(\Delta^F\right)} + \frac{w^{-1}}{\Phi\left(\Delta^F\right)}\int_{g^F}^{g^{FR}}[d_1+d_2-wg+1]F_\varepsilon'\left(\hat{g}^F-g\right)dg -$$
$$-w^{-1}(d_1-\delta)\left(1-F_\varepsilon\left(\Delta^F-w^{-1}(d_1-\delta)\right)\right) \tag{A.26}$$

Recall now that according to (A.16), $G^R < w^{-1}d_1\left[1-F_\varepsilon\left(\Delta^C-w^{-1}d_1\right)\right]$, so expression (A.17) implies

$$\frac{\partial D^*}{\partial s_1} > -Q + w^{-1}d_1\frac{1-F_\varepsilon\left(\Delta^C\right)}{\Phi\left(\Delta^C\right)} + \frac{w^{-1}}{\Phi\left(\Delta^C\right)}\int_{g^C}^{g^R}[d_1+d_2-wg+1]F_\varepsilon'\left(\hat{g}^C-g\right)dg -$$
$$-w^{-1}d_1\left[1-F_\varepsilon\left(\Delta^C-w^{-1}d_1\right)\right]$$

Comparing this to (A.26) yields $\frac{\partial D^*}{\partial \delta} < \frac{\partial D^*}{\partial s_1}$, as required.

Applying, similar to above, inequality (A.16) to expression (A.14), we can write

$$\frac{\partial D^*}{\partial s_2} = -G^R + w^{-1}\int_{g^C}^{g^R}[d_1+d_2-wg+1]F_\varepsilon'\left(\hat{g}^C-g\right)dg + w^{-1}d_1\left(1-F_\varepsilon\left(\Delta^C\right)\right) >$$
$$> -w^{-1}d_1\left[1-F_\varepsilon\left(\Delta^C-w^{-1}d_1\right)\right] + w^{-1}\int_{g^C}^{g^R}[d_1+d_2-wg+1]F_\varepsilon'\left(\hat{g}^C-g\right)dg + w^{-1}d_1\left(1-F_\varepsilon\left(\Delta^C\right)\right)$$

Comparing this to (A.26) yields $\frac{\partial D^*}{\partial \delta} < \frac{\partial D^*}{\partial s_2}$.

Moving on to the comparative effects on the costs of policies, we differentiate expression (37) to obtain:

$$\frac{\partial S}{\partial \delta} = \frac{\partial N}{\partial \delta}(s_1+s_2) - \frac{\partial Q}{\partial \delta}(s_2-\delta)+Q = -\frac{\partial \hat{g}^F}{\partial \delta}(s_1+s_2) - \frac{w^{-1}\left(1-F_\varepsilon\left(\Delta^F\right)\right)}{\Phi\left(\Delta^F\right)}(s_2-\delta)+Q=$$
$$= w^{-1}\left[\frac{1}{\Phi\left(\Delta^F\right)}-1\right](s_1+s_2) - \frac{w^{-1}\left(1-F_\varepsilon\left(\Delta^F\right)\right)}{\Phi\left(\Delta^F\right)}(s_2-\delta)+Q= \tag{A.27}$$
$$= w^{-1}\left[\frac{1}{\Phi\left(\Delta^F\right)}-1\right]s_1 + \frac{w^{-1}\left(1-F_\varepsilon'\left(\Delta^F\right)\Delta^F\right)}{\Phi\left(\Delta^F\right)}s_2 + \frac{w^{-1}\left(1-F_\varepsilon\left(\Delta^F\right)\right)}{\Phi\left(\Delta^F\right)}\delta + Q$$

Using the Proposition's simplifying premise $s_1 = s_2 = s$, we rewrite the above:

$$\frac{\partial S}{\partial \delta} = w^{-1}\frac{1 - F_\varepsilon\left(\Delta^F\right) - 2F_\varepsilon'\left(\Delta^F\right)\Delta^F}{\Phi\left(\Delta^F\right)}s + \frac{w^{-1}\left(1 - F_\varepsilon\left(\Delta^F\right)\right)}{\Phi\left(\Delta^F\right)}\delta + Q \tag{A.28}$$

Applying the same premise to (A.18) and (A.19), respectively, yields

$$\frac{\partial S}{\partial s_1} = \frac{w^{-1}\left(1 + F_\varepsilon\left(\Delta^C\right)\right)}{\Phi\left(\Delta^C\right)}s + N \tag{A.29}$$

$$\frac{\partial S}{\partial s_2} = 2w^{-1}s + N - Q \tag{A.30}$$

The comparison of (A.28) to (A.29) clearly yields $\frac{\partial S}{\partial \delta} < \frac{\partial S}{\partial s_1}$ keeping in mind that $N > Q$ is obviously the case and that the component of the cost in (A.28) associated with conditional debt forgiveness in amount δ per eligible student under policy F is offset in comparison to expression (A.29) for policy A by increased unconditional per student subsidy at stage 1 of college. Likewise, we obtain $\frac{\partial S}{\partial \delta} < \frac{\partial S}{\partial s_2}$ by comparing (A.28) to (A.30) if we additionally, realistically, assume that the number of dropouts Q is less than half the number of matriculants N. ■