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Fundamentals, Central Bank Intervention and Exchange Rate Behavior

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Abstract

The paper studies the behavior of fundamentals, central bank intervention and the resulting exchange rate behavior using a structural optimization model and a unique data-set. This is Israeli data of almost 1000 daily observations on a directly-observed fundamental, the amount of central bank intervention and exchange rates at the opening and closing of trading. We characterize the behavior of fundamentals and intervention using various descriptive statistics, propose a model of optimal central bank intervention, and structurally estimate the model.

Key Words: fundamentals, exchange rates, central bank intervention, optimal policy, structural estimation, censored decision variables

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1 Introduction

A major line of research in international monetary economics is the study of the interaction between fundamentals, central bank intervention and exchange rate behavior. However there are major problems hindering a satisfactory characterization of these relationships: typically the relevant fundamentals are unobserved at high frequencies, often their relation to exchange rates is predicated on functional forms (say of asset demand and supply), market micro-structure effects are not well understood, and reliable central bank intervention data are often unavailable. Thus many models *assume* certain processes for fundamentals and intervention - for example the seminal target zone model proposed by Krugman (1991). This paper uses a data set of unique quality with direct observations of the relevant fundamental variable, highly reliable central bank intervention data, a clear market micro-structure and data on exchange rates at the opening and closing of each trading day. The data are taken from the Israeli exchange rate market and include 989 daily observations in the years 1990-1994.

We proceed as follows: Section 2 briefly discusses the exchange rate regime in Israel and central bank policy in terms of goals and instruments, relying on official Bank of Israel publications and on key studies. Section 3 delineates the microstructure of the market and discusses the data. Section 4 presents a set of descriptive statistics and tests of the behavior of fundamentals and intervention policy. Based on the preceding sections, Section 5 proposes a theoretical model that fits the Israeli experience and the econometric methodology to structurally estimate the model. Section 6 presents the results and their implications. Section 7 concludes.

2 Exchange Rate Policy in Israel

In this section we first briefly describe exchange rate policy in Israel, focusing on the bands regime (2.1). We then review policy goals, instruments and modes of operation, relying on official publications and on previous research (2.2).

2.1 The Exchange Rate Regime

The foreign exchange regime in Israel underwent many changes, and included a fixed peg (1948-1975), a crawling peg (1975-1977), a quasi-flexible rate (1977-1985) and a fixed peg again (from the July 1985 inflation stabilization plan till early 1989). Beginning January 3, 1989 the Bank of Israel declared a target zone of the New Israeli Shekel (NIS) vis a vis a basket of five currencies.[1] The band regime began with a ±3% range and later (March 1, 1990) was widened to ±5%. On December 17, 1991 the band was turned into a "crawling" band, with a pre-announced crawl of the mid-band and the boundaries. The crawl was initially set to 9% in annual terms.[2]

[1]These include the U.S. dollar, the German Mark, the British Pound, the French Franc and the Japanese Yen.

[2]There were several realignments in both the fixed band and crawling band periods. Within the sample period (1990-1994) there were two further changes: on November 9, 1992 the crawl was reduced to 8%, and the mid-band was devalued 3%; on July 26,1993 the crawl was reduced to 6% and the mid-band was devalued 2%. After the sample period two changes occured: on May 31, 1995 the central parity was devalued 0.8% and the band was widened to ±7% with no change in the rate of crawl; on June 18, 1997 the band was widened to ±14% with the rate of crawl of the lower band being lowered to 4% and that of the upper band remaining at 6%.

2.2 Policy Goals and Instruments

Monetary and exchange rate policy objectives and instruments were frequently discussed and analyzed in the period under study. The brief discussion that follows draws on the annual reports of the Bank of Israel [see in particular Chapter 7 of each report in the period 1990-1994] and on several key studies cited below.

The high inflation experience of Israel in the years 1979-1985 (185% on average in the period, with 191% in the year 1983 and 445% in 1984) left a profound mark on policymakers. A key ingredient of the July 1985 stabilization plan was the fixing of the nominal exchange rate with the purpose of anchoring inflationary expectations.[3] This inflation objective led to a desire to limit nominal exchange rate changes also within the band. Additionally, there was an aversion to interest rate and exchange rate volatility. In order to achieve these aims, there was the currency band itself (rather than a free float) and daily intra-marginal intervention. Interventions clearly did not conform the marginal intervention assumption embedded in the Krugman (1991) model (as amply demonstrated by the empirical analysis below).

At the same time, policymakers were concerned with the level and volatility of the real exchange rate and its effects on current account flows and on GDP growth [Helpman, Leiderman and Bufman (1994), Ben Bassat (1995)]. They seemed particularly averse to protracted periods of real appreciation, as an inflation differential persisted between domestic and foreign prices. As a consequence, realignments devaluing the currency and, later, the institution of the crawling central parity in 1991, were designed to accommodate the inflation differential. All the while policymakers realized the limits of using the nominal exchange rate

[3] A specific, numerical inflation target emerged only in late 1991 - the rate of crawl was set to be the difference between this target and expected foreign inflation. However it was not widely perceived as the Bank's target. Only in 1994 did the government begin to explicitly set an inflation target [see Sokoler (1997) for a discussion of the gradual emergence of the target].

to affect the real rate for two reasons: a real depreciation following a nominal one is often short-lived due to indexation, which was wide-spread in the Israeli economy, and ultimately, the real exchange rate is an endogenous variable [Ben Bassat (1995)].

In an effort to serve both aims, exchange rate fluctuations were allowed within a ±5% band (twice as wide as the pre-1993 ERM bands in the EMS). The introduction of the crawling mid-band was an attempt to achieve a better balance between the different aims. The pre-announcement of the rate of crawl, and later, its linkage with explicit inflation targets (the rate of crawl was set to be the difference between the inflation target and the foreign inflation forecast), was intended to enhance credibility and allow for long-term planning.

The so-called "operational targets" of the Bank were the interest rate on domestic credit to banking institutions and the exchange rate. Two actual policy tools were used to attain these targets: loans to banks and intervention in the foreign exchange market. The former came in place of open market operations, which are an underdeveloped tool in the economy, and were directed at commercial banks. The Bank set the interest rate and varied domestic credit creation to achieve this targeted rate. Credit creation came through a combination of a step-function supply of loans (defined by 'quotas'), a weekly auction and, since late December 1990, a daily auction. In addition, a direct weekly auction to the non-bank private sector was used since June 1990 as well as short-term bond issuance on the Tel Aviv Stock Exchange. The operation of foreign exchange intervention is described in detail in Section 3 below.

The frequency of policy decision-making is of importance for the model below and can be divided into two levels: medium-term and long-term goals were served at discrete points in time through changes in interest rates and through adjustment of the exchange rate band parameters (location of the mid-band, its rate of crawl and band width). Short-term goals were served by domestic credit creation and foreign exchange intervention on a daily basis.

The former assured that there would be minimal deviations from the low-frequency interest rate goal, typically set for a month. Thus, within the month, interest rate fluctuations were insignificant.

3 The Market Micro-Structure and the Data

In this section we present the micro-structure of the foreign exchange market (3.1) and the data set to be used (3.2).

3.1 The Market Micro-Structure

The foreign exchange market in Israel is essentially composed of two segments: trading of commercial banks with their clients and with other banks and trading between the banks and the central bank, the Bank of Israel. It is in the latter segment that the intervention takes place and the exchange rate determined there serves as a benchmark for all foreign currency transactions. In the sample period, May 24, 1990 - July 3, 1994, an auction governed the trade between the Bank of Israel and the commercial banks. This trading system had the following micro-structure:

(i) A "leader" stage lasting from 8 A.M to 12 P.M. whereby the commercial banks transmit their orders to the Bank of Israel. These orders are either "best" orders (buy and sell with no limit) or "limit" orders. Only at this stage of trading can the traders cancel previous orders or add to them.

(ii) A data processing stage lasting from 12 P.M to 12:15 P.M whereby the Bank of Israel sums the "best" orders in order to determine the opening net supply or demand and then announces - via the computerized public trading system - this total amount.

Here one should note three features of trading: official policy (the band and the parity) is defined in terms of the Basket/NIS rate; actual trading is conducted mostly in U.S. dollars and thus the rates mentioned above are U.S Dollar/NIS rates; the opening rate - released at 11.30 A.M - is the previous day's closing rate in terms of Basket/NIS multiplied by the U.S Dollar/Basket rate as of 11:15 A.M. based on global trading cross rates. The latter rate is in effect throughout the trading day.

(iii) An "auction" stage lasting from 12:15 P.M till 1:15 P.M (at the latest) whereby traders are permitted to transmit only "best" type orders that serve to reduce the opening net excess supply or demand. They may do so anonymously vis a vis other (commercial) traders if they wish. Limit orders are executed automatically at the appropriate rate. The Bank of Israel serves as an auctioneer changing the exchange rate in small increments (usually of 0.001 NIS per U.S Dollar) as the auction proceeds. It may intervene at any time but usually it is a one-time intervention at the end of trading. The Bank then declares the closing rates. Doing so it uses the same U.S Dollar/ Basket rate that was used at stage (ii). Summer (1993) computed the average time for the auction stage to be 35 minutes.

On July 4, 1994 this trading system was replaced by a different, inter-bank system.

3.2 The Data

We use daily data on the foreign exchange market provided by the Foreign Currency Department of the Bank of Israel. The sample covers the entire period of operation of the system just described. There are 989 daily observations in the data sample. The data set includes the following series:

(i) Exchange rates - the NIS/basket, NIS/dollar at the opening and closing of trading.

(ii) Net demand by the private sector at the start of trading.

(iii) The amount of intervention by the Bank of Israel (sales or purchases).

Table 1 presents summary statistics of the data, for the full sample and for the six sub-periods in which there was no realignment.

Table 1

Note that the sample may also be sub-divided into two sub-periods according to the slope of the mid-band (fixed or crawling). This sub-division turns out to be important below. Panel (a) reports an average daily depreciation of 0.035%. Evidently the average is lower in the first sub-period relative to the second. Note that the exchange rate tended to spend more time below the mid-band, with an average deviation of 1.5%. Panel (b) shows that the Bank intervened frequently: 84% of trading days.

4 Fundamentals and Intervention

In this section we characterize the behavior of fundamentals and intervention using descriptive statistics and various tests. The aim is twofold: as data on the fundamentals affecting exchange rate behavior is often unavailable at such high frequency there is interest in a statistical description of these data; to serve as a further basis for the build-up of the model beyond the stylized facts on policy objectives given in Section 2.

First, note the following features of this market:

(i) The commercial banks were acting on behalf of numerous customers, for both current and capital account transactions. Israel is a very open economy with exports and imports each accounting for 40%-45% of GDP. Thus it is reasonable to assume that there was no strategic behavior on the part of the private sector.

(ii) The timing structure was such that intervention took place after the initial net demand and the rate at the opening of trading were known.

Table 2 presents a multinomial logit analysis of the probability of intervention. In what follows we refer to "net demand" as the net excess demand of the private sector at the opening of trading. The dependent variable is the ratio of the log odds of intervention (sales in the left column and purchases in the right column) to the log odds of no intervention. The independent variables differ across specifications and include net demand (linear and quadratic), the interaction of net demand and a dummy variable indicating the position of the exchange rate relative to the mid-band, and dummy variables for realignment dates.

Table 2

The main conclusion from the table is that net demand plays the key role. In Sections 5 and 6 below it will be shown that it is a major determinant of expected depreciation. The significance of the quadratic term is an expression of the non-linear relationship between expected depreciation and net demand (see Section 5.3 below). The position of the exchange rate within the band matters only in interaction with net demand; when we added the position in the band independently it was usually insignificant and when it served as the only explanatory variable it generated very poor predictions. Thus the decision whether to intervene or not seemed to depend on the strength of the expected change in the exchange rate and not so much on the position within the band per se. There is some evidence of asymmetry with respect to the strength of the coefficient on demand for positive and negative interventions.

The next step is to ask what are the determinants of the amount of intervention. We run OLS regressions of the rate of intervention on the position of the exchange rate at the opening of trading and on net demand interacted with two dummy variables: position in the band (above or below the mid-band) and the sign of net demand (positive or negative). Table 3 reports the results for a linear and a linear-quadratic specification.

market. We discuss the model and the methodology of estimating it using structural estimation.

5.1 The Set-Up

Every trading day, the NIS/Basket exchange rate at the opening is the previous day closing to be denoted x_t^o (in logs). The private sector then submits net demand D_t for dollars. Denote the demand for dollars at date t by B_t^d and the supply of foreign currency at date t by B_t^s , then:

$$D_t = B_t^d - B_t^s \tag{1}$$

Throughout the trading day the same basket/dollar rate is in effect (see Section 3.1 above). Following the observation of D_t the amount of intervention S_t is determined. S_t represents the amount of dollars sold by the central bank. These quantities of net demand and sales lead at the end of the trading day to the determination of the closing rate in NIS/basket to be denoted x_t^c. Each day there are boundaries of the band $\overline{x}_t$ and $\underline{x}_t$; in the period of crawling bands these are time-varying, hence the time sub-script. They are all expressed in NIS/basket. In addition we shall use the notation x_t^{mid} to denote the mid-band (in logs).

5.2 The Objective Function

The objective function needs to reflect the variety of aims that the central bank was trying to pursue: one aim was to limit exchange rate volatility. Beyond stabilizing the price of foreign exchange, this was intended to serve the nominal anchoring, inflation-stabilization target. Thus we include exchange rate volatility in the objective function. A second argument of this function reflects the existence of a target zone, modelling the central bank's loss as decreasing in the distance of the actual rate from the boundaries. We take as given the band's width and the level and slope of its central parity. A third aim has to do with losses from intervention: the greater the intervention, the wider was the gap between the actual rate and the free equilibrium rate and the higher are the borrowing costs associated with the reserves needed to carry out the intervention.

Thus, the daily loss function is defined as follows:

$$L(x_t^o, x_t^c, S_t, D_t, \overline{x_t}, \underline{x}_t) = U(x_t^o, x_t^c, \overline{x_t}, \underline{x}_t) + IC_(S_t) \tag{2}$$

$$\begin{aligned} U(x_t^o, x_t^c, \overline{x_t}, \underline{x}_t) &= \frac{1}{2}(x_t^c - x_t^o)^2 + \beta_1 \overline{Z}(x_t^c, \overline{x_t}) + \beta_2 \underline{Z}(x_t^c, \underline{x}_t) \\ IC(S_t) &= IC_+(S_t)I(S_t > 0) + IC_-(S_t)I(S_t < 0) \end{aligned}$$

The first two aims are formalized in the function U while the third is expressed in the intervention cost function IC.

U is the one-trading-day loss function of the central bank which we assume to be a function of the exchange rate and the boundaries. Its first term expresses losses from volatility. The second and third terms include the functions $\overline{Z}, \underline{Z}$ representing the cost of the exchange rate approaching the boundary. We assume $\overline{Z}(x^c)$ is positive and increasing in x^c, that $\underline{Z}(x^c)$ is positive and decreasing in x^c,[4] and that $\overline{Z}(x^c)+\underline{Z}(x^c)$ has a unique minimum at $x^c = x^{mid}$. This formulation captures the possible asymmetry in the loss function due to the position of the exchange rate within the band. When $\beta_1 = \beta_2$ then (all else equal) the minimum of the loss function is attained for $x_t^c = x_t^{mid}$, but if for example $\beta_1 > \beta_2$ then the minimum will be attained for a value of $x_t^c < x_t^{mid}$ indicating that it is more costly for the

[4]We assume both functions are positive to avoid "compensation" of one by the other.

central bank to observe high exchange rates (relative to the mid-band). Moreover to capture the fact that large deviations from the central parity are more costly than small ones we assume that both $\overline{Z}(\cdot)$ and $\underline{Z}(\cdot)$ are strictly convex.

IC are the intervention costs associated with the sales or purchases of dollars by the central bank. Intervention costs are modeled as a convex function of the amount of intervention S. We allow it to be asymmetric depending on whether it consists of selling foreign currency ($S > 0$) or buying foreign currency ($S < 0$).

We summarize the above discussion of our assumptions about the one-period loss function of the central bank in the following:

Assumption 1 *Both $\overline{Z}(\cdot)$ and $\underline{Z}(\cdot)$ are non-negative, continuous, twice differentiable and strictly convex. $\overline{Z}(\cdot)$ is increasing, $\underline{Z}(\cdot)$ is decreasing, and $\overline{Z}(x) + \underline{Z}(x)$ admits a unique minimum at $x = x^{mid}$. Thus the daily loss function $U(x_t^o, x_t^c)$ is continuous and twice differentiable on its admissible range. It is strictly convex in both its arguments. The adjustment cost functions $IC_{\pm}(S)$ are continuous, strictly convex and differentiable. The time preference parameter verifies: $0 < \delta < 1$.*

We assume that the central bank minimizes the expected, present value of future daily loss functions, i.e. that it chooses an intervention policy $\{S_t\}_{t=0,1,...}$ such that:

$$\min_{\{S_t\}_{t=0,1,...}} \mathrm{E}_0 \left[\sum_{t=1}^{\infty} \delta^t L(x_t^o, x_t^c, S_t, D_t, \overline{x}_t, \underline{x}_t) \right] \tag{3}$$

where δ is the time preference parameter of the central bank.

The exchange rate x_t is the endogenous state variable, related to the control variable, the intervention S_t and the exogenous state variables. We discuss the transition equation for the endogenous state variable next.

5.3 The Transition Equation

The afore-going loss function is minimized subject to the daily evolution of the exchange rate. We posit the following transition equation:

$$x_t^c = x_t^o + \sum_{i=1}^{4} \psi_i(t) g_i(S_t, D_t) + z_t \quad (4)$$

$$x_{t+1}^o = x_t^c \quad (5)$$

The g_i functions depend on net demand D and on the intervention S. The z term captures random effects. We allow for asymmetries in this process across net demand positions and across interventions. We define the family of indicator functions ψ such that:

$$\begin{aligned} \psi_1(t) &= I(D_t > 0, S_t > 0) \quad (6) \\ \psi_2(t) &= I(D_t < 0, S_t < 0) \\ \psi_3(t) &= I(D_t > 0, S_t = 0) \\ \psi_4(t) &= I(D_t < 0, S_t = 0) \end{aligned}$$

The g_i functions are defined, continuous and differentiable on $\mathcal{R} \times \mathcal{R}^*$. While the variable z is an unobservable state variable, it is important to note that z_t is known at time t. In other words, we realistically assume that the central bank can pick the closing exchange rate *exactly* when it intervenes by supplying S_t dollars. We make the following technical assumptions on the processes of the exogenous state variables D and z.

Assumption 2 *The private sector demand D and the noise term z follow independent univariate Markov processes with continuous, twice differentiable transition densities, $P_D(D_{t+1}|D_t)$ and $P_z(z_{t+1}|z_t)$.*

The following technical assumption will insure the existence of an optimal intervention policy with censoring.

Assumption 3 *We assume that the function $g_i(\cdot,\cdot)$ are continuous and twice differentiable in all its arguments. Moreover, define the functions $F_i(x, S, D, z) = U(x, x + g_i(S, D) + z)$. We assume that F is continuous, twice differentiable in all its arguments, and strictly convex in (x, S).*

This assumption on the function F insures that the value function is well defined and convex so that there is a well-defined solution to the stochastic programming problem (see Stokey and Lucas (1989) theorem 9.8 page 265, it is essentially their assumption 9.10). Given assumption 1 on the daily loss function U it is easily seen that sufficient conditions for the latter assumptions on the function F to hold are:

Assumption 4 (Sufficient conditions) *Given assumption 1, sufficient conditions for Assumption 3 to hold are, for example:*

- $\forall i$ $g_i(S, D)$ *is affine in* S, *or*
- $\forall i$ $g_i(S, D)$ *is continuous and twice differentiable in all its arguments, strictly convex in* S, *and* $1 + \beta_1 \overline{Z}'(\cdot) + \beta_2 \underline{Z}'(\cdot) > 0$

The sufficient condition above will be sufficient for our empirical implementation below (i.e. our estimated parameter for the given parametrization of the g and Z functions verify the sufficient conditions stated above). To motivate the choice of our formulation of the transition equation above, we provide below an argument that relates the coefficients to explicit demand and supply curves.

5.4 Motivation of the transition equation

One justification for the formulation (4)-(6) is obtained by considering the underlying demand and supply functions. At the opening of trading the following equation describes the market:

$$D_t = B_t^D(x_t^o, y_t^o) - B_t^S(x_t^o, y_t^o) \tag{7}$$

At the prevailing exchange rate (x_t^o), pre-determined by the previous day trading, there is a certain amount of demand for foreign exchange B^D a certain supply B^S, given all other relevant variables summarized by the process y that will affect demand and supply curves and thus change the excess demand of dollars of the private sector at the opening. Such relevant variables could be related to the basket-dollar rate, interest rates and quantity variables such as GDP, export and import flows etc. Suppose none of these variables were to change from day to day, then demand and supply function would not change, and the excess demand at the opening would be the same as at the close of the previous day. In fact, excess demand would be equal to zero if there was no intervention by the central bank the previous day.

As we describe above, the second stage is an "auction process" where the central bank moves the rate by small increment and allows order that reduce the excess demand of the private sector. During that "tatonnement process" it is likely that both demand and supply curves shift slightly in response to changes in information on the relevant state variables. At the end of the second stage we may expect the following relation to hold:

$$D_t^a = B_t^D(x_t^a, y_t^a) - B_t^S(x_t^a, y_t^a) \tag{8}$$

At the end of the trading day, the central bank steps in and picks the closing rate x_t^c, effectively setting the total excess demand of the market to achieve the desired exchange rate:

$$D_t^a - S_t = B_t^D(x_t^c, y_t^a) - B_t^S(x_t^c, y_t^a) \tag{9}$$

Evidently if $S = 0$ the closing rate x_t^c is the free market rate and differs from the opening rate only because of the demand and supply shocks, i.e. $y_t^a - y_t^o$.

Assuming that the demand and supply functions are sufficiently well behaved C^1 functions, the implicit function theorem guarantees that there exists a C^1 function $J(\cdot,\cdot)$ such that:

$$\begin{aligned} x_t^o &= J(D_t, y_t^o) \\ x_t^c &= J(D_t^a - S_t, y_t^a) \end{aligned}$$

Since the auction process is very short, we may assume that the sufficient statistic y_t exhibits only small changes, i.e. $y_t^a = y_t^0 + \epsilon_t^1$, where ϵ^1 is a stationary i.i.d. process of small magnitude. The same must hold for the excess demand after the auction process $D_t^a = D_t(1-\epsilon^2)$, where $\epsilon^2 \in [0,1]$, and we may assume that it is more or less chosen by the central bank (since it controls the auction process and the change $x_t^a - x_t^o$). Using a simple Taylor expansion we obtain the following expression:

$$x_t^c - x_t^o = J(D_t - S_t, y_t^o) - J(D_t, y_t^o) - J_1(D_t - S_t, y_t^o)\epsilon^2 D_t + J_2(D_t - S_t, y_t^o)\epsilon_t^1 \tag{10}$$

where J_i denotes the partial derivative with respect to the ith argument. Note also that the coefficient affecting the noise term ϵ is given explicitly by the implicit function theorem in terms of derivative of the demand and supply functions. The analysis above may be used to justify many form of transition equations depending on the assumed functional form for the demand and supply functions. For example if demand and supply functions are linear $(B^d(x,y) = a^d - b^d x - c^d y$ and $B^s(x,y) = b^s x + c^s y)$ then $J(D,y) = \frac{a^d}{b^d+b^s} - \frac{1}{b^d+b^s} D - \frac{c^d+c^s}{b^d+b^s} y$

and the resulting transition equation is simply $x_t^c - x_t^o = \frac{1}{b^d+b^s}S - \frac{\epsilon_2}{b^d+b^s}D_t + z_t$ where the noise term $z_t = -\frac{c^d+c^s}{b^d+b^s}\epsilon_t^1$.

In general, this equation represents the "tatonnement" process in the market. Intuitively it says that the rate changes as a function of excess demand, that this may be a linear or non-linear function depending upon the slopes of the relevant demand and supply functions, and that there are random effects due to the effects of additional variables on demand and supply.

5.5 Form of the optimal intervention policy

In the following proposition we give the form of the optimal intervention policy in the present model. It turns out that with the assumption made above the optimal policy is a censored decision type as can be found in for example Pakes (1994) and Aguirregabiria (1996).

Proposition 1 *Given assumptions 1 and 2, the form of the optimal intervention policy in the present model is:*

$$S_t = \begin{cases} S_t^u & \text{if} \quad S_t^u > 0 \\ S_t^d & \text{if} \quad S_t^d < 0 \\ 0 & \quad \text{otherwise} \end{cases} \tag{11}$$

where S^d and S^u are the optimal interior solutions corresponding to a negative and positive intervention respectively and are implicitly defined by:

$$\begin{aligned} U_S(x_t^o, x_t^c) + \delta \mathrm{E}V_S(H_t, S_t^d) + IC_-(S_t^d) &= 0 \\ U_S(x_t^o, x_t^c) + \delta \mathrm{E}V_S(H_t, S_t^u) + IC_+(S_t^u) &= 0 \end{aligned} \tag{12}$$

where $EV(\cdot)$ is the expected conditional value function, subscripts of functions denote partial derivatives, and the state vector $H_t = (D_t, z_t, x_t^o)$.

Proof: et us define the value function $V(H_t) = \min_{\{S_t\}_{t=0,1,\ldots}} \mathrm{E}_t \left[\sum_{s=1}^{\infty} \delta^{s-t} L(x_s^o, x_s^c, S_s, D_s)\right]$ where we define the state vector $H_t = (D_t, z_t, x_t^o)$. By a standard argument we have:

$$V(H_t) = \min_{S_t} \left[U(x_t^o, x_t^c) + IC_(S_t) + \delta \int V(D', z', x^{o\prime}) P_z(z'|z_t) dz' P_D(D'|D_t) dD'\right]$$

where we recall that $x^{o\prime} = x_t^o + + \sum_{i=1}^{4} \psi_i(t) g_i(S_t, D_t) + z_t$.

Let us define $EV(H_t, S_t) = \int V(D', z', x^{o\prime}) P_z(z'|z_t) dz' P_D(D'|D_t) dD'$ and $G(H_t, S_t) = U(x_t^o, x_t^c) + \delta\, EV(H_t, S_t)$. We have $V(H_t) = \min_{S_t} \{G(H_t, S_t) + IC(S_t)\}$. It is easy to see that given assumption 1, 2 and 3 G is strictly convex in S. Indeed,

$$G_{SS}(H_t, S_t) = \sum_i \psi_i \frac{\partial^2 F_i(x_t^o, S_t, D_t, z_t)}{\partial S^2} + \delta \int V_{SS}(D', z', x^{o\prime}) P_z(z') dz' P_D(D') dD'$$

Given assumption 3 the functions F_i are strictly convex in S $\forall i$, and Lucas and Stokey (1989) theorem 9.8 guarantees that under the assumptions 1 to 3 the value function is convex in S. Thus $G_{SS} > 0$.

Given assumption 1, the functions $G(H_t, S_t) + IC_+(S_t)$ and $G(H_t, S_t) + IC_-(S_t)$ are strictly convex in S and there is a unique global minimum as defined in the above proposition.

5.6 The Euler Equations

We use GMM estimation; the basic procedure is due to Hansen (1982); here we follow a methodology proposed by Pakes (1994) and Aguirregabiria (1996). This methodology allows for the censoring of the decision variable when formulating the Euler equation. In the current context censoring occurs as the central bank did not intervene (i.e. $S = 0$) in about 20% of the sample observations. The formulation of the Euler equation in this case follows a variational principle. Suppose that there is a non-zero intervention at time t and that the next non-zero intervention will occur at time $t+\tau_t$. Perturb the intervention policy along the optimal path by α, i.e. suppose that at time t the optimal policy S_t is changed to $S_t+\alpha$. We suppose that given the continuity of the value function, in the limit (as α tends to zero) the discrete choice is unchanged (i.e. τ_t is independent of α). This one time deviation from the optimal policy triggers a change in the value of the endogenous state variable. The dynamics of the exchange rate given in (4) and (5) imply that along the modified path:

$$\begin{aligned} \widetilde{x}^o_{t+u} &= x^{o*}_{t+u} + \Delta(\alpha) \;\; \forall u \in [1, \tau_t) \\ \widetilde{x}^c_{t+u} &= x^{c*}_{t+u} + \Delta(\alpha) \;\; \forall u \in [0, \tau_t - 1) \end{aligned}$$

where x^*_{t+u} and $\widetilde{x}_{t+u}$ denote the value of the endogenous state variable along the optimal path and along the perturbed path respectively, and

$$\Delta(\alpha) = \sum_{i=1}^{4} \psi_i(t) \left\{ g_i(S_t + \alpha, D_t) - g_i(S_t, D_t) \right\}$$

Furthermore, assume that intervention is changed at $t + \tau_t$ to $S_{t+\tau_t} - h(\alpha)$ where $S_{t+\tau_t}$ is the optimal policy and $h(\alpha)$ is chosen such that the path of the endogenous state variable x_u remains unchanged relative to its path under the optimal policy after $t + \tau_t$ (i.e. $x^o_u = x^{o*}_u \; \forall u > t + \tau_t$). Let us first determine $h(\alpha)$. Since we have perturbed the path of the exchange rate then

$$\widetilde{x}^o_{t+\tau_t} = x^{o*}_{t+\tau_t} + \Delta(\alpha). \tag{13}$$

By definition of the dynamics of the exchange rate we also have:

$$\begin{aligned}\widetilde{x}^{c}_{t+\tau_t} &= \widetilde{x}^{o}_{t+\tau_t} + \sum_{i=1}^{4}\psi_i(t+\tau_t)g_i(S_{t+\tau_t}-h(\alpha),D_{t+\tau_t}) + z_{t+\tau_t} \\ &= x^{o*}_{t+\tau_t} + \Delta(\alpha) + \sum_{i=1}^{4}\psi_i(t+\tau_t)g_i(S_{t+\tau_t}-h(\alpha),D_{t+\tau_t}) + z_{t+\tau_t}\end{aligned}$$

Hence, it is clear that in order to obtain

$$\widetilde{x}^{c}_{t+\tau_t} = x^{c*}_{t+\tau_t} = x^{o*}_{t+\tau_t} + \sum_{i=1}^{4}\psi_i(t+\tau_t)g_i(S_{t+\tau_t},D_{t+\tau_t}) + z_{t+\tau_t}$$

we need to choose $h(\alpha)$ such that for each $i \in \{1,\dots,4\}$:

$$\Delta(\alpha) + g_i(S_{t+\tau_t}-h(\alpha),D_{t+\tau_t}) - g_i(S_{t+\tau_t},D_{t+\tau_t}) = 0 \tag{14}$$

Since for each i, g_i is continuous on $\mathcal{R}\times\mathcal{R}^*$ and has continuous derivatives with respect to all its arguments, the implicit function theorem guarantees the existence of a solution to the above equation. Moreover, the implicit function theorem guarantees that the derivative of $h(\cdot)$ is well-defined. We have now established that it is possible to change the optimal policy by α at time t and by $h(\alpha)$ at time $t+\tau_t$ so that the path of the endogenous state variable is going to be modified only between t and $t+\tau_t$. Obviously the paths of the exogenous state variables are not affected by the change in policy. A classic variational argument leads to the optimality condition which states that an infinitesimal variation of the policy along the optimal path should leave the value function unchanged. In other words the difference between the optimal value function and the value function obtained by following the perturbed policy described above should have a minimum at $\alpha = 0$. Computing the difference between the two paths yields:

$$
\begin{aligned}
DIF(\alpha) &= U(x_t^{o*}, \widetilde{x}_t^c) + IC(S_t + \alpha) - (U(x_t^{o*}, x_t^{c*}) + IC(S_t)) \\
&\quad + \mathrm{E}_t \left[\sum_{k=1}^{\tau_t - 1} \delta^k \left[U(\widetilde{x}_{t+k}^o, \widetilde{x}_{t+k}^c) - U(x_{t+k}^{o*}, x_{t+k}^{c*}) \right] \right. \\
&\quad + \delta^{\tau_t} \left[U(\widetilde{x}_{t+\tau_t}^o, x_{t+\tau_t}^{c*}) + IC(S_{t+\tau_t} - h(\alpha)) \right] \\
&\quad \left. - \delta^{\tau_t} \left[U(x_{t+\tau_t}^{o*}, x_{t+\tau_t}^{c*}) + IC(S_{t+\tau_t}) \right] \right]
\end{aligned} \tag{15}
$$

Using the fact that $\widetilde{x} = x^* + \Delta(\alpha)$, we obtain:

$$
\begin{aligned}
\left. \frac{\partial DIF(\alpha)}{\partial \alpha} \right|_{\alpha=0} &= U_2(x_t^{o*}, x_t^{c*}) \Delta'(0) + IC_S(S_t) \\
&\quad + \mathrm{E}_t \left[\sum_{k=1}^{\tau_t - 1} \delta^k \left[U_1(x_{t+k}^{o*}, x_{t+k}^{c*}) + U_2(x_{t+k}^{o*}, x_{t+k}^{c*}) \right] \Delta'(0) \right. \\
&\quad \left. + \delta^{\tau_t} \left[U_1(x_{t+\tau_t}^{o*}, x_{t+\tau_t}^{c*}) \Delta'(0) - IC_S(S_{t+\tau_t}) h'(0) \right] \right]
\end{aligned} \tag{16}
$$

where we define $IC_s(s) = \frac{\partial IC_+(s)}{\partial s} I(s > 0) + \frac{\partial IC_-(s)}{\partial s} I(s < 0)$ and have used the notation $\Delta'(0) \equiv \frac{\partial \Delta}{\partial \alpha} |_{\alpha=0}$ and $h'(0) \equiv \frac{\partial h}{\partial \alpha} |_{\alpha=0}$ and $U_i(\cdot, \cdot)$ denotes the derivative of U with respect to its ith argument.

Some calculations give:

$$
\Delta'(0) = \sum_{i=1}^{4} \psi_i(t) \frac{\partial g_i(S_t, D_t)}{\partial S}
$$

Using the implicit function theorem in equation (14) we obtain:

$$
h'(0) = \sum_{i=1}^{4} \sum_{j=1}^{4} \psi_i(t) \psi_j(t + \tau_t) \frac{\partial g_i(S_t, D_t)/\partial S}{\partial g_j(S_{t+\tau_t}, D_{t+\tau_t})/\partial S}
$$

Setting (16) equal to zero and using the expressions for $\Delta'(0)$ and $h'(0)$ we get:

$$
\begin{aligned}
0 &= \sum_{i=1}^{4} \psi_i(t) \left\{ U_2(x_t^{o*}, x_t^{c*}) \frac{\partial g_i(S_t, D_t)}{\partial S} + IC_1(S_t, D_t) \right. \qquad (17) \\
&+ \mathrm{E}_t \left[\sum_{k=1}^{\tau_t - 1} \delta^k \left[U_1(x_{t+k}^{o*}, x_{t+k}^{c*}) + U_2(x_{t+k}^{o*}, x_{t+k}^{c*}) \right] \right) \frac{\partial g_i(S_t, D_t)}{\partial S} \\
&+ \left. \delta^{\tau_t} \left[U_1(x_{t+\tau_t}^{o*}, x_{t+\tau_t}^{c*}) \frac{\partial g_i(S_t, D_t)}{\partial S} - IC_1(S_{t+\tau_t}, D_{t+\tau_t}) \sum_{j=1}^{4} \psi_j(t+\tau_t) \frac{\partial g_i(S_t, D_t)/\partial S}{\partial g_j(S_{t+\tau_t}, D_{t+\tau_t})/\partial S} \right] \right\}
\end{aligned}
$$

where:

$$
\begin{aligned}
U_1(x_t^o, x_t^c) &= -(x_t^c - x_t^o) \qquad (18) \\
U_2(x_t^o, x_t^c) &= (x_t^c - x_t^o) + \beta_1 \overline{Z}'(x_t^c) + \beta_2 \underline{Z}'(x_t^c)
\end{aligned}
$$

The intuition underlying this equation is similar to the standard Euler equation that appears for example when maximizing expected utility problems. It states that any time along the optimal intervention path the central bank should be indifferent between the reduction in the one period loss function resulting from a marginally increased intervention at t and incurring the future repercussions on the loss function resulting from that intervention. The difference between this Euler equation and more standard ones is that the future impact of a time-t decision affects the endogenous processes and hence the value function for a random number of time periods, i.e. until the next optimal intervention time $t + \tau_t$.

6 Estimation

We jointly estimate the F.O.C (17) and the transition equation (4) using the GMM methodology described above.

6.1 Parameterization

In order to do so we must parameterize the functions $\overline{Z}(x_t^c)$ and $\underline{Z}(x_t^c)$ which describe the loss related to the position of the exchange rate within the band, the intervention costs function $IC(S_t)$, and the transition function $g(S_t, D_t)$. We experimented with several specifications.

(i) For the $\overline{Z}(x_t^c)$ and $\underline{Z}(x_t^c)$ functions we use three formulations:

Logarithmic:

$$\overline{Z}(x_t^c) = -\ln(\overline{x}_t - x_t^c) \tag{19}$$

$$\underline{Z}(x_t^c) = -\ln\left(x_t^c - \underline{x}_t\right) \tag{20}$$

Square root:

$$\overline{Z}(x_t^c) = \frac{1}{\sqrt{(\overline{x}_t - x_t^c)}} \tag{21}$$

$$\underline{Z}(x_t^c) = \frac{1}{\sqrt{\left(x_t^c - \underline{x}_t\right)}} \tag{22}$$

In the above two specifications when the rate reaches the boundary the loss is infinite. We also try a parabolic function around the width of the band (10%) which has increasing but finite losses outside the boundaries:

$$\overline{Z}(x_t^c) = (\overline{x}_t - x_t^c - 0.1)^2 \tag{23}$$

$$\underline{Z}(x_t^c) = \left(x_t^c - \underline{x}_t - 0.1\right)^2 \tag{24}$$

(ii) We adopt the following parametrization of intervention costs:

$$\begin{aligned} IC(S_t, D_t) &= \left(\gamma_1 S_t + \frac{\gamma_3}{2} S_t{}^2 + \frac{\gamma_5}{3} S_t^3\right) I(S_t > 0) \\ &+ \left(\gamma_2 S_t + \frac{\gamma_4}{2} S_t{}^2 + \frac{\gamma_6}{3} S_t^3\right) I(S_t < 0) \end{aligned}$$

IC_s is defined by:

$$IC_s(S,D) = \left(\gamma_1 + \gamma_3 S_t + \gamma_5 S_t^2\right) I(S_t > 0) + \left(\gamma_2 + \gamma_4 S_t + \gamma_6 S_t^2\right) I(S_t < 0)$$

We then tried three cases: the third-order case presented above, the linear-quadratic case ($\gamma_5 = \gamma_6 = 0$) and the linear case ($\gamma_3 = \gamma_4 = \gamma_5 = \gamma_6 = 0$).

(iii) We also try three formulations for $g(S_t, D_t)$ each of which corresponds, at least theoretically, to a certain demand and supply curve for foreign currency (see our precious discussion 5.4) :

A linear formulation in terms of excess net demand i.e. $D - S$. This would hold true if in (8) and in (9) $\frac{\partial B^S}{\partial x_t^o} - \frac{\partial B^D}{\partial x_t^o}$ is a constant:

$$g_i(S_t, D_t) = a_i + b_i(D_t - S_t) + z_t \tag{25}$$

A linear-quadratic formulation in $D - S$:

$$g_i(S_t, D_t) = a_i + b_i(D_t - S_t) + c_i(D_t - S_t)^2 + z_t \tag{26}$$

A second-order approximation of $g(S_t, D_t)$:

$$g_i(S_t, D_t) = a_i + b_i D_t + c_i {D_t}^2 + d_i S_t + e_i S_t^2 + z_t \tag{27}$$

6.2 Estimation Results

It turns out that there is little difference in the results whichever $\overline{Z}(x_t^c)$ and $\underline{Z}(x_t^c)$ functions are used, that a third-order polynomial works best for the $IC(S_t)$ function and that a linear-quadratic formulation in $D - S$ works best for the transition equation. Table 4 reports the results for the first and second sub-periods, using different instrument sets.

Table 4

To further illustrate the results, Figure 1 plots the $g(S, D)$ function and Figure 2 plots the $IC(S)$ function as reported in Table 4 for the relevant range of $D - S$ in the former case and S in the latter case.

Figures 1 and 2

The picture that emerges from the table and figures is the following:

(i) Estimation is relatively robust across instrument sets in the second sub-period, but not as robust in the first one. In all cases the J-statistics do not reject the null hypothesis.

(ii) There are differences between periods of positive net demand ($D > 0$) and thus central bank sales ($S > 0$) and periods of negative net demand ($D < 0$) and central bank purchases ($S < 0$). For the former, the transition function ($g(D, S)$ is almost linear while it is concave in the latter case. The IC function is convex in both cases.[5]

(iii) The coefficients of the position in the band functions are insignificant.

(iv) Intervention costs are higher in the second-sub period relative to the first one, suggesting that the central bank became more averse to intervention.

In the next version of the paper we intend to further explore the differences across sub-periods and to further examine robustness issues in the first sub-period.

[5]The exceptions are that in one specification in the first sub-period the g function is convex in negative demand and the IC function is linear in positive sales. *We shall further explore these exceptions in the next version.*

7 Conclusions

The paper proposed a model of optimal intervention within a currency band. The central bank trades off the loss from exchange rate volatility and intervention costs. The methodology used – structural estimation taking into account the option of no intervention – yielded results that are consistent with the model's formulations. These indicate that intervention costs are convex in the intervention amount and thus that intervention may be described as the solution to a convex optimization problem, with due adjustment for "kinks." The results do not produce a clear picture as to how the position in the band affects intervention, as the relevant estimates are often insignificant. The empirical results also characterized the relationship between daily exchange rate changes, the fundamental and the amount of intervention as a linear or concave relationship.

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Table 1

a. Exchange Rate Behavior

Period	No. Obs.	Mid-band Rate of Crawl	Average Exchange Rate Level (NIS/basket)	Average Rate of Daily Change (%)	Average Deviation from Mid-band (%)
Full Sample 24.5.1990 27.6.1994	989	–	2.742 (0.337)	0.035 (0.242)	-1.5 (2.2)
first sub-period					
24.5.1990 9.9.1990	72	0	2.235 (0.031)	0.036 (0.496)	2.0 (1.4)
11.9.1990 10.3.1991	120	0	2.292 (0.008)	-0.003 (0.190)	-4.9 (0.3)
12.3.1991 16.12.1991	188	0	2.523 (0.059)	0.027 (0.333)	-1.2 (2.4)
second sub-period					
18.12.1991 8.11.1992	210	9%	2.693 (0.102)	0.049 (0.133)	-1.5 (1.7)
10.11.1992 25.7.1993	170	8%	2.994 (0.036)	0.029 (0.110)	-0.4 (1.2)
27.7.1993 27.6.1994	224	6%	3.187 (0.068)	0.041 (0.151)	-1.6 (0.8)

Notes:

1. Standard deviations are given in parentheses.

b. Net Demand and Intervention

Period	No. Obs.	Days with Intervention (%)	Average Net Demand (thousands of US dollars)	Average Intervention[2] (thousands of US dollars)	Average Intervention Rate[2] (%)
Full Sample 24.5.1990 27.6.1994	989	84	-234.62 (23,113)	-726.03 (18,133)	59 (35)
first sub-period					
24.5.1990 9.9.1990	72	69	3651.1 (18,664)	2698.1 (14,239)	33 (35)
11.9.1990 10.3.1991	120	88	-8931.2 (13,288)	-7941.9 (11,598)	69 (34)
12.3.1991 16.12.1991	188	86	413.67 (28,377)	420.48 (24,747)	62 (35)
second sub-period					
18.12.1991 8.11.1992	210	88	3317.8 (18,795)	2300.7 (15,223)	63 (32)
10.11.1992 25.7.1993	170	92	-1483.2 (22,410)	-1276.5 (18,554)	67 (32)
27.7.1993 27.6.1994	224	77	-494.51 (22,090)	-1265.5 (15,218)	51 (36)

Notes:

1. Standard deviations are given in parentheses.
2. Including days with no intervention.

Table 2: Probability of Intervention (Multinomial Logit Analysis)

	Fixed Band Period 382 obs.				Crawling Band Period 606 obs.			
	demand specification		*demand * position − dummy specification*		*demand specification*		*demand * position − dummy specification*	
	$\ln\left(\frac{\Pr(S>0)}{\Pr(S=0)}\right)$	$\ln\left(\frac{\Pr(S<0)}{\Pr(S=0)}\right)$	$\ln\left(\frac{\Pr(S>0)}{\Pr(S=0)}\right)$	$\ln\left(\frac{\Pr(S<0)}{\Pr(S=0)}\right)$	$\ln\left(\frac{\Pr(S>0)}{\Pr(S=0)}\right)$	$\ln\left(\frac{\Pr(S<0)}{\Pr(S=0)}\right)$	$\ln\left(\frac{\Pr(S>0)}{\Pr(S=0)}\right)$	$\ln\left(\frac{\Pr(S<0)}{\Pr(S=0)}\right)$
const	-0.982^a	-1.14^a	-1.04^a	-1.16^a	-0.928^a	-1.31^a	-0.930^a	-1.35^a
	(0.314)	(0.319)	(0.329)	(0.318)	(0.314)	(0.294)	(0.264)	(0.301)
d_100990	-44.7	-23.5	-42.0	-27.3				
	(8.09e+07)	(7.22e+07)	(6.01e+07)	(5.37e+07)				
d_110391	60.6	30.7	75.7	30.2				
	(3.82e+07)	(3.13e+07)	(2.88e+07)	(2.37e+07)				
d_091192					32.5	15.5	31.6	17.0
					(1.06e+08)	(1.37e+08)	(1.10e+08)	(1.22e+08)
d_260793					186	174	188	209
					(1.06e+08)	(1.37e+08)	(1.10e+08)	(1.22e+08)
demand	3.99e-04^a	−3.49e-04^a			3.61e-04^a	−3.34e-04^a		
	(6.10e-05)	(5.37e-05)			(4.09e-05)	(4.32e-05)		
$demand^2$	-1.99e-09^a	-3.03e-09^a			-5.58e-09^a	-2.71e-09^a		
	(4.25e-10)	(7.79e-10)			(8.26e-10)	(5.36e-10)		
$demand*P_+$			3.41e-04^a	−2.62e-04^a			3.64e-04^a	−3.03e-04^a
			(7.00e-05)	(9.96e-05)			(6.74e-05)	(6.88e-05)
$demand^2*P_+$			-2.67e-09^a	-9.25e-11			-5.14e-09^b	-2.26e-09^b
			(8.83e-10)	(7.57e-09)			(2.02e-09)	(1.14e-09)
$demand*P_-$			5.01e-04^a	-3.85e-04^a			3.59e-04^a	−3.55e-04^a
			(9.38e-05)	(6.28e-05)			(4.31e-05)	(4.89e-05)
$demand^2*P_-$			-2.51e-09^a	-3.38e-09^a			-5.66e-09^a	-3.49e-09^a
			(6.94e-10)	(8.15e-10)			(8.96e-10)	(7.64e-10)
χ^2	508		514		800		802	
Pseudo R^2	0.65		0.66		0.66		0.66	

Notes:
1. Standard deviation are given in parentheses.
2. a – coefficient is significant at 1% confidence level.
3. b – coefficient is significant at 5% confidence level.
4. $P_+=\begin{cases} 1 & \text{if open > midband} \\ 0 & \text{otherwise} \end{cases}$, $P_-=\begin{cases} 1 & \text{if open < midband} \\ 0 & \text{otherwise} \end{cases}$

Table 3
Intervention Rates (OLS)

Dependent Variable is: Intervention	Linear Demand		Quadratic Demand	
	Fixed Band 382 obs.	Crawling Band 606 obs.	Fixed Band 382 obs.	Crawling Band 606 obs.
const	0.276[a] (0.039)	0.398[a] (0.030)	0.150[a] (0.043)	0.269[a] (0.034)
D_100990	-0.642[b] (0.324)	–	-0.799[b] (0.310)	–
D_110391	-0.333 (0.342)	–	-0.164 (0.380)	–
D_091192	–	1.29e-04 (0.324)	–	-0.012 (0.312)
D_260793	–	-2.24[a] (0.428)	–	12.8[a] (3.09)
position$*P_+$	1.07 (2.37)	5.42 (4.76)	0.132 (2.35)	3.70 (5.07)
position$*P_-$	7.19[a] (1.06)	3.13[b] (1.30)	8.06[a] (1.08)	2.68[b] (1.30)
demand$*D_+*P_+$	1.19e-05[a] (1.88e-06)	9.93e-06[a] (2.63e-06)	3.02e-05[a] (3.80e-06)	3.04e-05[a] (6.08e-06)
demand$*D_+*P_-$	4.86e-06[a] (1.39e-06)	9.76e-06[a] (1.50e-06)	1.51e-05[a] (3.02e-06)	2.99e-05[a] (3.77e-06)
demand$*D_-*P_+$	-1.37e-05[a] (2.78e-06)	-5.12e-06[b] (2.24e-06)	-3.40e-05[a] (7.21e-06)	-1.60e-05[a] (4.60e-06)
demand$*D_-*P_-$	-6.91e-06[a] (1.50e-06)	-8.38e-06[a] (1.13e-06)	-1.63e-05[a] (3.14e-06)	-2.38e-05[a] (2.81e-06)
demand$^2*D_+*P_+$			-2.33e-10[a] (4.39e-11)	-4.68e-10[a] (1.44e-10)
demand$^2*D_+*P_-$			-6.38e-11[a] (1.92e-11)	-4.82e-10[a] (9.51e-11)
demand$^2*D_-*P_+$			-4.45e-10[b] (1.84e-10)	-1.04e-10[b] (5.23e-11)
demand$^2*D_-*P_-$			-1.38e-10[a] (4.60e-11)	-2.51e-10[a] (4.68e-11)
R^2	0.28	0.13	0.36	0.20
D.W.	1.57	1.84	1.59	1.87

Notes:

1. Standard deviation are given in parenthesis.

2. a – coefficient is significant for 1% confidence level.

3. b – coefficient is significant for 5% confidence level.

4. c – coefficient is significant for 10% confidence level.

5. $D_+=\begin{cases} 1 & \text{if demand} > 0 \\ 0 & \text{otherwise} \end{cases}$, $D_-=\begin{cases} 1 & \text{if demand} < 0 \\ 0 & \text{otherwise} \end{cases}$

6. $P_+=\begin{cases} 1 & \text{if open} > \text{midband} \\ 0 & \text{otherwise} \end{cases}$, $P_-=\begin{cases} 1 & \text{if open} < \text{midband} \\ 0 & \text{otherwise} \end{cases}$

Table 4
F.O.C and Transition Equation - GMM Estimation

a. First Sub-Period

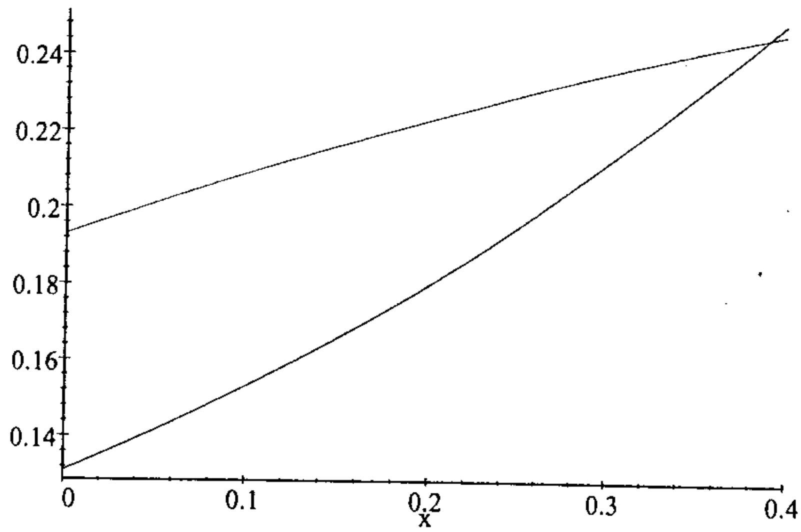
specification	$D(30), S(20)$	$D(30), S(0)$
$\beta_1(\times 10^5)$	55.62 (64.46)	-40.91 (149.63)
$\beta_2(\times 10^5)$	0.98 (22.06)	-100.14 (121.08)
$\gamma_1(\times 10^3)$	2.00 (1.26)	4.09 (8.08)
$\gamma_2(\times 10^3)$	-0.57 (0.43)	-0.89 (2.50)
$\gamma_3(\times 10^3)$	-2.81 (2.48)	-0.29 (0.45)
$\gamma_4(\times 10^3)$	3.81 (2.82)	-2.76 (14.04)
$\gamma_5(\times 10^3)$	2.46 (1.99)	2.39 (3.45)
$\gamma_6(\times 10^3)$	-4.58 (4.12)	8.16 (32.44)
$a_1(\times 10^3)$	1.31 (0.31)	2.05 (0.53)
$b_1(\times 10^3)$	1.97 (1.51)	2.67 (0.26)
$c_1(\times 10^3)$	2.42 (2.18)	-0.52 (2.95)
$a_2(\times 10^3)$	-0.37 (0.19)	-0.34 (0.30)
$b_2(\times 10^3)$	0.40 (0.47)	0.009 (0.001)
$c_2(\times 10^3)$	-21.06 (6.45)	-0.10 (0.81)
J-statistic	49.5	22.08
p-value	0.998	0.995

b. Second Sub-Period

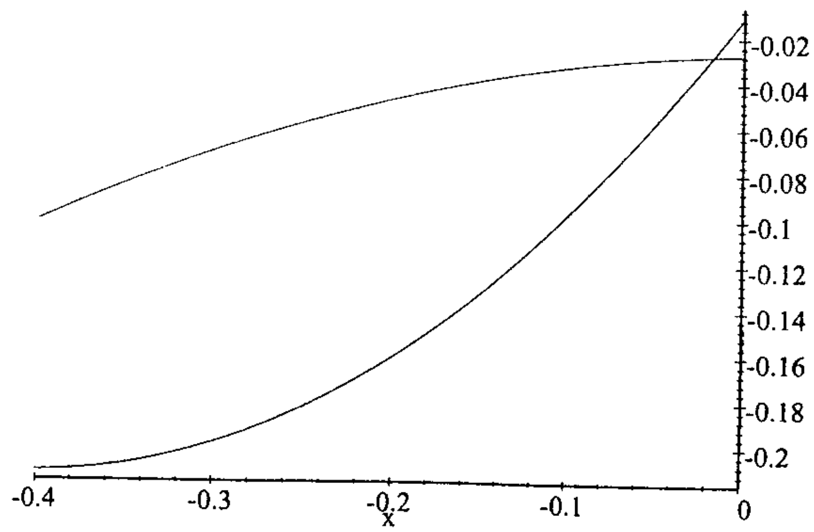
specification	$D(30), S(20)$	$D(30), S(0)$
$\beta_1(\times 10^5)$	10.69 (52.16)	11.51 (58.01)
$\beta_2(\times 10^5)$	4.95 (26.41)	6.51 (30.39)
$\gamma_1(\times 10^3)$	1.41 (1.01)	0.79 (2.44)
$\gamma_2(\times 10^3)$	-0.81 (0.71)	-0.63 (1.72)
$\gamma_3(\times 10^3)$	16.67 (7.18)	16.74 (10.76)
$\gamma_4(\times 10^3)$	4.82 (3.01)	7.29 (4.98)
$\gamma_5(\times 10^3)$	-9.04 (20.79)	-10.44 (28.27)
$\gamma_6(\times 10^3)$	-2.62 (4.33)	-5.51 (6.51)
$a_1(\times 10^3)$	0.47 (0.15)	0.31 (0.23)
$b_1(\times 10^3)$	13.68 (2.32)	14.30 (4.25)
$c_1(\times 10^3)$	-3.40 (0.73)	-3.73 (1.28)
$a_2(\times 10^3)$	-0.11 (0.19)	-0.21 (0.33)
$b_2(\times 10^3)$	9.98 (3.96)	10.28 (6.23)
$c_2(\times 10^3)$	12.40 (5.39)	11.49 (8.95)
J-statistic	52.86	23.99
p-value	0.995	0.988

Figure 1
The g Function

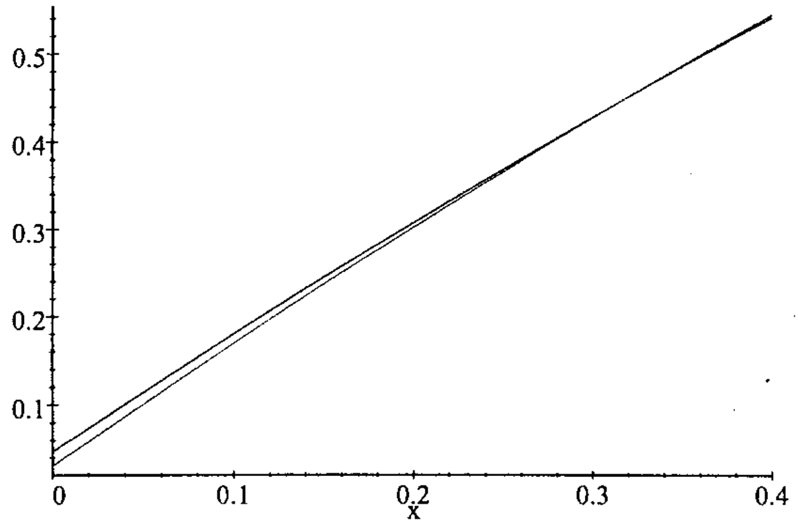
a. First Sub-Period
positive net demand



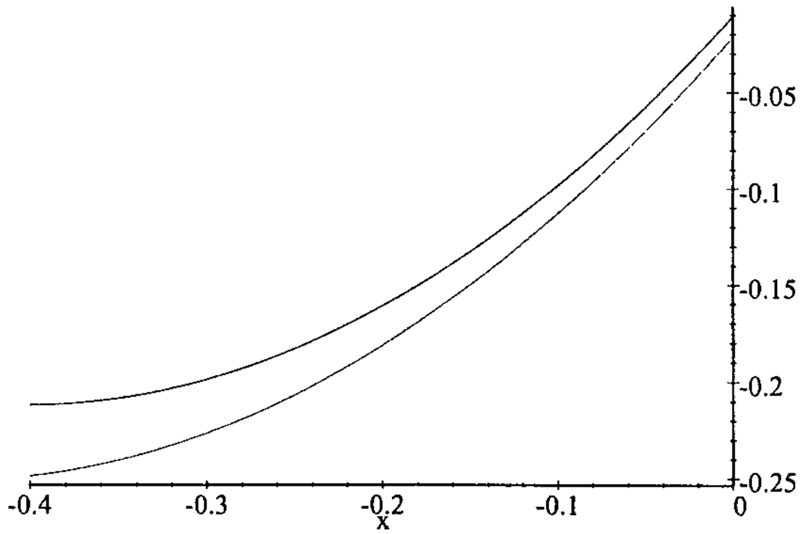
negative net demand



b. Second Sub-Period
positive net demand



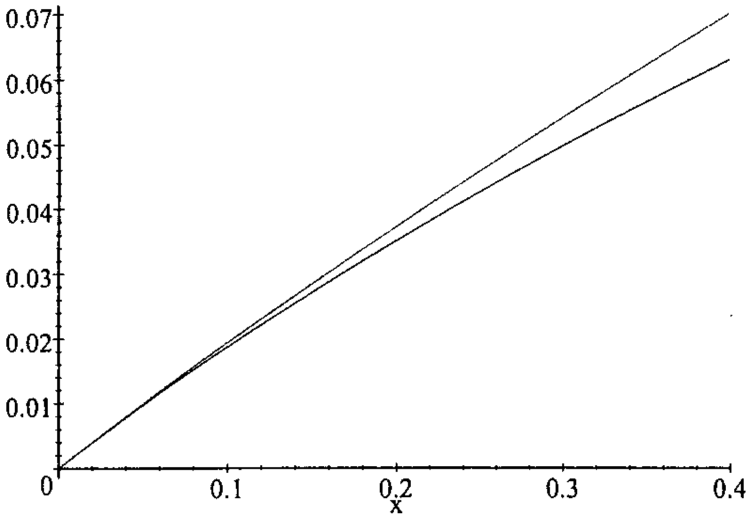
negative net demand



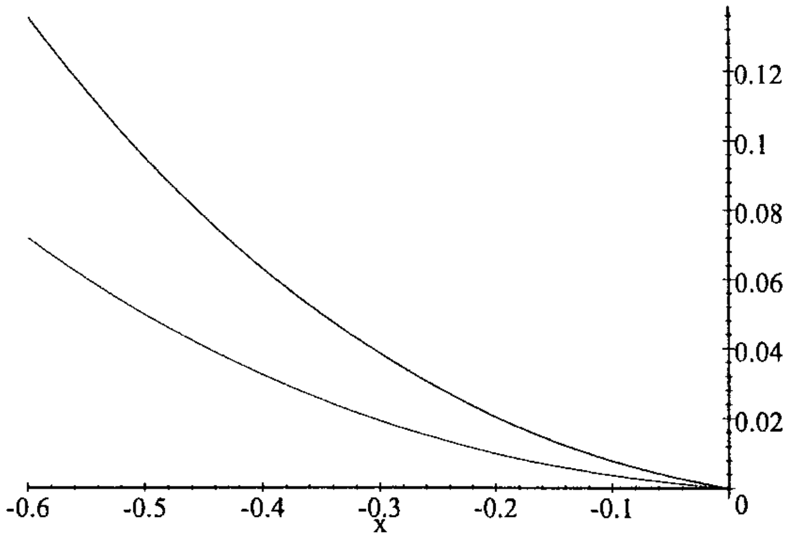
Notes:solid line - demand 30, sales 20; dashed line - demand 30, sales 0

Figure 3
The *IC* Function

a. First Sub-Period
positive sales

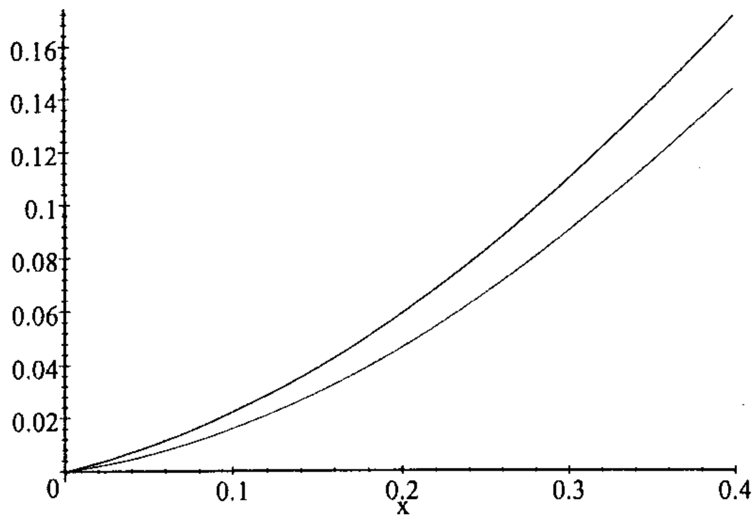


negative sales

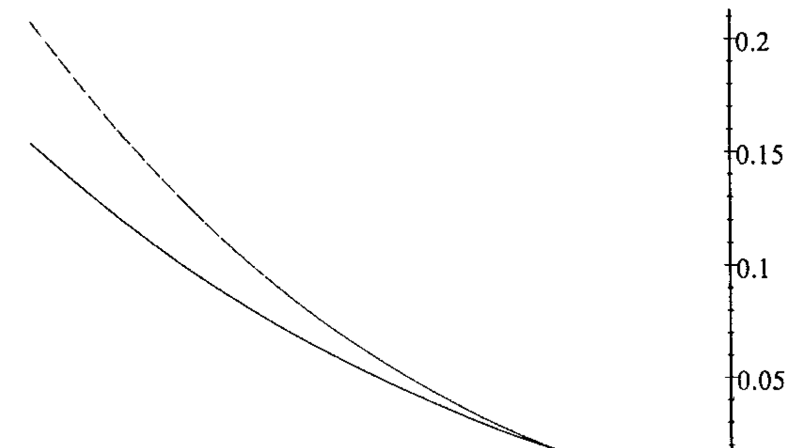


b. Second Sub-Period

positive sales



negative sales



Notes:solid line - demand 30, sales 20; dashed line - demand 30, sales 0