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Dynamic Disclosure with Uncertain Disclosure Costs

Daniel Bird*

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Abstract

This paper extends the canonical single-period model of voluntary costly disclosure to a dynamic setting with asymmetric information over disclosure costs. I show that dynamic incentives have an ambiguous effect on firms' disclosure decisions and that the cost of disclosure no longer prevents full disclosure. My main results show that (1) dynamic incentives increase the probability of disclosure for low-cost firms and decrease the probability of disclosure for high-cost firms and (2) when the variance of the disclosure-cost distribution is high and the expected disclosure cost is low, dynamic incentives lead firms to use a strategy of full disclosure.

Keywords: Dynamic voluntary disclosure; Uncertain disclosure costs.
JEL Classification: D82, D83, M41

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1 Introduction

This paper offers a new model to explore the incentives guiding voluntary disclosure by firms functioning in a complex economic environment, with multiple periods and uncertain disclosure costs. As such, the model departs from the stylized assumptions of a single period and common knowledge of disclosure costs that date back to Jovanovic (1982) and Verrecchia (1983), who redressed the unrealistic unraveling prediction of Grossman and Hart (1980), Grossman (1981), and Milgrom (1981). Grossman, Hart and Milgrom all showed that when firms are known to have verifiable information that they can disclose at no cost, adverse selection leads to full disclosure of the information in equilibrium, a prediction contrary to real-world experience. Jovanovic and Verrecchia added a known disclosure cost into this setting; their assumption resolves this conundrum and has since been a staple assumption in the study of voluntary disclosure,[1] even though it conceals other economic forces relevant to understanding firms' disclosure decisions.

The model I construct extends Verrecchia's (1983) model of costly disclosure in two directions: it adds asymmetric information about the cost of disclosure, and it considers a multi-period environment. With asymmetric information about the cost of disclosure, the market's expectation of the firm's value depends on its belief about disclosure costs. If the firm is believed to have a low cost, its expected disclosure expenditure is likely to be low and, consequently, its expected value is high. However, if such a firm fails to disclose, it receives a harsh evaluation of its earnings and suffers a large decrease in value. Alternatively, when the firm's disclosure cost is believed to be high, the expected disclosure expenditure is high and the firm's value is low. However, such a belief also implies that nondisclosure has a small effect on the firm's value. As the cost (and probability) of disclosure varies among firms, different firms may prefer to shift the market's belief in different directions. Thus, equilibria may contain rich communication structures wherein disclosure aims to increase the firm's future valuation by altering the market's belief, in addition to its traditional goal of revealing a high profit.

[1] For a comprehensive review of this literature, see Dye (2001), Healy and Palepu (2001), and Verrecchia (2001).

In some cases, the firm's attempts to alter the market's belief can lead to an unraveling result similar to that of Milgrom (1981), albeit for different reasons. Whereas in Milgrom's work the unraveling sequence unfolds on the axis of current profit, in my model it does so on the axis of disclosure cost. That is, firms disclose a low profit in an attempt to signal a low cost of disclosure (and a high value to investors), rather than to avoid the unfavorable interpretation that nondisclosure brings to bear on current profit.

Signaling a low cost via disclosure is costly for the firm. In addition to the direct cost of disclosing a low profit, the combination of asymmetric information and a dynamic model generates multiple indirect costs that are incurred by disclosure. The first indirect cost is an implicit commitment to continue disclosing once the firm begins doing so. Disclosing the firm's current profit leads the market to believe that the firm has a low disclosure cost, which, in turn, leads to a harsher judgment of future nondisclosure and, consequently, increases future disclosure expenditure. Note that the cost of this "commitment" is determined in equilibrium and depends on the firm's actual cost of disclosure.

The other indirect costs are related to the time at which disclosure costs are realized. Many, if not most, disclosure costs have a long-lasting effect of the firm's profitability (e.g., by diverting managerial time from profit-generating activities to communicating with investors or by revealing proprietary information). Therefore, disclosing information may reduce the firm's earnings in the future rather than create a one-time expense at present. This implies that disclosure reduces not only the firm's current price, but also its future price. Naturally, the effective cost of disclosure for a firm that maximizes a discounted sum of its stock prices is thus greater, and disclosure is less likely.

Furthermore, when disclosure has a postponed effect on the firm's earnings and there is uncertainty about the magnitude of this effect, a change in the market's belief about the firm's cost alters its estimate of past disclosure expenditure. Therefore, low-cost firms are inclined to continue disclosing in order to reduce the market's estimate of their previous disclosure expenditure. And, therefore, high-cost firms may refrain from early disclosure in order to avoid the drop in their liquidation value when the market learns their true disclosure cost.

In this paper, I show that when the variance of the disclosure-cost distribution is high, the expected disclosure cost is low, and the firms' discount rate is higher than the market's discount rate, full disclosure by all firms is the unique equilibrium. Moreover, I show that even if there is no difference in the discount rates, low-cost firms may still use a strategy of full disclosure in equilibrium.

I next show that even when no firm uses a strategy of full disclosure, dynamic incentives generally increase the probability of disclosure for low-cost firms due to their attempts to signal a low disclosure cost. On the other hand, the probability of disclosure by high-cost firms is reduced, as the indirect costs of disclosure by such firms significantly increase the effective cost of doing so.

The model I construct in this paper also shows some noteworthy effects that asymmetric information over disclosure costs creates a single-period model that have not been previously considered. When the market does not know the firm's cost of disclosure, it does not know if nondisclosure is the result of a high disclosure cost or a low profit. This increases the ability of low-cost firms to conceal low profits and increases their value at the expense of high-cost firms. Moreover, I show that asymmetric information about disclosure costs reduces the disclosure probability of each firm, and that the magnitude of this effect is increasing in the expectation and variance of the cost distribution. In particular, this finding implies that small disclosure costs may have a larger effect on disclosure decisions than was previously thought possible.

1.1 Related Literature

The literature on disclosure of verifiable information originated in reaction to Grossman, Hart and Milgrom's controversial prediction that adverse selection leads firms to voluntarily disclose all their private information. This unrealistic result hinged on an unraveling argument, namely, that among those firms that do not disclose information, the one with the best information has an incentive to separate itself from the others. Subsequently, Jovanovic (1982) and Verrecchia (1983) precluded full unraveling by assuming a cost associated with disclosure of private information, which implies that the unraveling stops once the value of separation from the pool of non-disclosing firms drops below the cost of disclosure. Dye (1985) and (n.d.) further showed that full unraveling can also be prevented by assuming that the firm is not always informed about its profit. Given uncertainty about the information endowment, the market cannot know whether nondisclosure is a strategic choice and therefore evaluates it less harshly. Dye (2001) and Verrecchia (2001) comprehensively reviewed the use of these two approaches.

In this broad body of literature, two papers (Beyer and Dye 2012 and Einhorn and Ziv 2008) also study disclosure in a repeated interaction and examine the incentives created by asymmetric information about the firm's type. These two papers assume that firms differ in their objectives and information endowments, respectively, while I assume that the difference is in their cost of disclosure. Beyer and Dye (2012) combine the ideas of Dye (1985) and (n.d.) with a reputation-based model à la Kreps and Wilson (1982) and analyze a setting in which some managers are uninformed and some are nonstrategic (honest) and disclose their information regardless of its content. Beyer and Dye show that a reputation for being honest is valuable, and that a strategic manager tries to manipulate her reputation by (partially) mimicking an honest manager's behavior, that is, by disclosing unfavorable information. My paper develops an alternative model with uncertainty about disclosure costs, showing that dynamic incentives have an ambiguous effect on the level of disclosure when firms are assumed to be fully rational. Moreover, the richer type space of firms in my model allows me to show that dynamic incentives may work in opposite directions for different firms.

Einhorn and Ziv (2008) consider a dynamic model in which all firms are profit-maximizing but there is uncertainty about the probability that the firm is informed. They assume a positive intertemporal correlation between the probabilities that the firm is informed in subsequent periods, and thus disclosure creates an implicit commitment to continue disclosing in the future. In their model the cost of disclosure is common knowledge; thus this commitment reduces the firm's future profits without providing any benefit. Therefore, in their model, dynamic incentives reduce disclosure.[2] By contrast, in my model

[2]The commitment to continue disclosing in their model is the result of a mechanism similar to the one that generates the same commitment in my model.

the cost of disclosure is private information and investors attach a higher value to firms with low disclosure costs. Since the cost of committing to disclose is lower for a firm with low disclosure costs, it follows that if the cost of committing to disclose is not too high, a low-cost firm can use this commitment as a means of signaling its type to investors. Therefore, dynamic incentives may increase the probability of disclosure as opposed to the unambiguous prediction of reduced disclosure given by Einhorn and Ziv (2008).

Additional related papers include Shin (2003, 2006), Acharya, DeMarzo and Kremer (2011) and Guttman, Kremer and Skrzypacz (2014), all of which study dynamic disclosure models in which the dynamic element originates from a combination of gradual learning by the firm about its profit and discretion about when to disclose information. Shin analyzes the effect of gradual learning when the firm immediately discloses any positive information it receives. He derives how the uncertainty about the firm's intrinsic values evolves over time (under this strategy) and demonstrates that this leads to short-term momentum and a long-term reversal of the firm's price. The latter two papers allow for discretion in the timing of disclosure. Acharya, DeMarzo and Kremer (2011) focus on the effect of exogenous news in such a setting. They show that the arrival of an exogenous news event (that is informative of the firm's value) can alter the firm's disclosure policy and lead to a clustering of voluntary disclosure. Guttman, Kremer and Skrzypacz (2014) analyze the effect of multiple signals on the disclosure strategy and pricing functions. In particular, they show that the market's interpretation of the same piece of disclosed information becomes more favorable if the information was disclosed at a later date.

This paper proceeds as follows. Section 2 introduces the model and defines the equilibrium concepts. Section 3 characterizes the equilibrium of a single-period model with asymmetric information about disclosure costs. Section 4 builds on this characterization and analyzes the equilibrium in a two-period model, highlighting the effect of dynamic incentives on the probability of disclosure. Section 5 then considers a variant of the model in which the cost of disclosure reduces the utility of the firm's manager without affecting investors. The final section offers concluding remarks. All proofs are relegated to the appendix.

2 Model

The model I construct builds on Verrecchia's (1983) canonical disclosure model, with two major additions: uncertainty about the cost of disclosure and a two-period setting. Formally, I assume that a firm is active for two periods, in each of which it generates a profit of q_t. The profit generated in each period is distributed i.i.d. according to a distribution $G(q)$ with a support of $[\underline{q}, \bar{q}]$ for $0 \leq \underline{q} < \bar{q} \leq \infty$. At the beginning of each period, the firm learns its current profit and decides whether to verifiably disclose this information to the market at a cost of[3] $\tilde{c}$. The cost of disclosure is the firm's private information, does not change over time, and is distributed according to $F(c)$ with a support of $[0, \bar{c}]$. I further assume that the firm cannot credibly reveal its disclosure cost, and thus the market ignores any claim the firm makes regarding its cost.

The literature on voluntary disclosure for the most part does not define the exact source of disclosure costs, which is valid in a single-period model. In a two-period model with uncertainty about disclosure costs, however, expected disclosure expenses in period 2 may affect the firm's value in period 1, and thus the source of disclosure costs must be considered. In the main part of this paper, I assume that disclosure costs are paid by the firm. That is, the firm's liquidation value is reduced by the disclosure costs it incurs. Examples of such costs include the cost of disclosing propriety information, and the cost of managerial time that is diverted from profit-generating activities to communicating with investors. Notably, these costs affect future (not current) profit. In the final part of this paper I consider the opposite case where disclosure costs affect the manager's personal decision to reveal information and yet, for one reason or another, they do not affect the firm's liquidation value. I show that this alternative source of disclosure costs has a different quantitative effect on the firm's disclosure decision.

In addition to assuming that disclosure costs affect the firm's value, in line with the above examples of disclosure costs I assume that *all* disclosure costs are paid in the second period. That is, the firm distributes its entire first-period

[3]I later assume that the firm is evaluated by a risk-neutral and competitive market. Thus, were I to assume that the firm learns (and may disclose) an unbiased estimate of its profit instead of its actual profit, the results presented in this paper would remain unchanged.

profit to its shareholders (irrespectively of whether or not disclosure costs were paid), and is liquidated at the end of the second-period. Furthermore, I assume that shareholders receive the first-period dividend only at the end of the second period. Therefore, unless the firm discloses its first-period profit, the market cannot use this information to update its beliefs about the firm's disclosure cost.[4] These assumptions are equivalent to assuming that if the firm discloses information in the first period, the distribution of its second-period profit is shifted down by $\tilde{c}$. In some parts of the paper I use this interpretation to help clarify the narrative.

2.1 The Firm's Disclosure Decision

In each period, the firm has a simple choice: to disclose its information – or not. I denote these actions by d and nd, respectively, and the firm's chosen action in period t by a_t. However, the information that can be disclosed in each period is different. Whereas in the first period the firm decides whether or not to disclose its first-period profit (but not its disclosure costs), in the second period the firm discloses its liquidation value (the difference between the second-period profit and its disclosure expenditure in both periods). Based on the firm's decision, the market updates its belief about the firm's value, and prices the firm accordingly. The information relevant for pricing the firm at time t is thus the beliefs about the firm's disclosure cost and current profit, and the firm's actions to date. I denote the market's information at time t by $\mathcal{I}_t$. The firm's equilibrium price in the first period is denoted by $p(\mathcal{I}_1)$, and the firm's expected liquidation value by $L(\mathcal{I}_2)$. I assume that the firm's goal is to maximize the sum of its current price and discounted expected liquidation value. The fact that the firm does not know its second-period profit, and hence its liquidation value, implies that in the first period the firm maximizes

$$\max_{d,nd} p(\mathcal{I}_1) + \delta \mathbb{E}(L(\mathcal{I}_2)),$$

[4]An alternative model where the dividend is received at the end of the first period yields the same qualitative results.

where $\delta \in (0, 1]$ is the firm's discount factor.[5] Whereas in the second period the firm maximizes the market's expectation of its liquidation value

$$\max_{d,nd} L(\mathcal{I}_2)$$

2.2 The Pricing Function

In order to focus attention on the firm's disclosure decision I make the standard assumptions that the market is competitive and risk-neutral. Moreover, I assume that the market does not discount future payments; this assumption implies that the delay in the payment of disclosure costs affects the analysis via an informational channel and not by altering the real cost of disclosure. The firm's liquidation value is the difference between its second-period profit and its expenditure on disclosure in both periods. Therefore, the firm's expected liquidation value is given by

$$L(\mathcal{I}_2) = \begin{cases} q_2 - 2\tilde{c} & \text{if } a_2 = d, a_1 = d \\ q_2 - \tilde{c} & \text{if } a_2 = d, a_1 = nd \\ \mathbb{E}(q_2|\mathcal{I}_2) - \mathbb{E}(c|\mathcal{I}_2) & \text{if } a_2 = nd, a_1 = d \\ \mathbb{E}(q_2|\mathcal{I}_2) & \text{if } a_2 = nd, a_1 = nd \end{cases}$$

The price of the firm, then, is the sum of its expected first-period dividend and expected liquidation value:[6]

$$p(\mathcal{I}_1) = \mathbb{E}(q|\mathcal{I}_1) + \mathbb{E}(L(\mathcal{I}_2)|\mathcal{I}_1).$$

2.3 Equilibrium Concept and Time Line

The solution concept used in this analysis is (weak) Perfect Bayesian Equilibrium. This concept requires that each player chooses an optimal action with regard to his beliefs at all decision nodes, and that beliefs are formed by Bayesian updating whenever possible. As the firms are always fully informed and on-path beliefs are pinned down by the firms' strategies, I need only specify explicitly the market's belief at decision nodes that are not along the path of play.

[5]Due to the delayed impact of disclosure expenditure, myopic firms are affected not by their actual disclosure cost, but only by the market's belief about this cost. Thus, I assume that firms are non-myopic to avoid the degenerate case where the firm's true cost is irrelevant.

[6]Recall that first-period disclosure expenditure reduces the firm's liquidation value.

At the beginning of the game the firm learns its type $\tilde{c}$. The time line of events in each period is depicted in Figure 1 below.

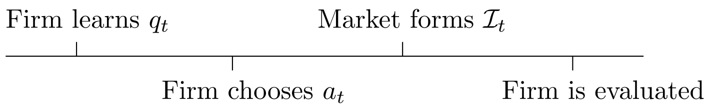

Figure 1: Time line of events in each period

3 Uncertain Disclosure Costs: Single Period

To understand the effect of asymmetric information about the cost of disclosure in a dynamic setting it is necessary to understand it in a static setting as well. In this section I analyze a single-period model and derive results that I use in the multi-period analysis that follows. In this section I omit the time indexes from all variables.

In a single-period model, the firm's expected liquidation value after nondisclosure is independent of its type; thus, I can denote this value by v_{nd}. As the firm has no incentive to alter the market's belief about its cost, it discloses its profit if and only if

$$q - \tilde{c} \geq v_{nd};$$

that is, firms use a threshold strategy. Furthermore, each firm is indifferent between disclosure and nondisclosure at its threshold, and nondisclosure provides the same payoff to all firms. This, in turn, implies that disclosure thresholds are linear in the cost of disclosure and that the threshold for a firm with no disclosure cost equals v_{nd}.

Firms with a profit of less than $\underline{q} + \tilde{c}$ obviously do not disclose their profit. Therefore, the market's belief after nondisclosure is the result of Bayesian updating:

$$v_{nd} \equiv \int_0^{\bar{c}} \frac{Pr(nd|c)}{Pr(nd)} \mathbb{E}(q|nd, c) dF(c). \tag{1}$$

In equilibrium, a firm discloses information if and only if $q > v_{nd} + c$; thus (1) simplifies to

$$v_{nd} = \frac{1}{\int_0^{\bar{c}} G(v_{nd}+c)dF(c)} \int_0^{\bar{c}} G(v_{nd}+c)\mathbb{E}(q|q < v_{nd}+c)dF(c) \qquad (2)$$

In addition to providing a simple characterization of equilibrium, equation (2) has two important implications listed in the following proposition.

Proposition 1.
Asymmetric information about disclosure costs:

1. *Reduces disclosure by firms with low costs and, conversely, increases disclosure by firms with high costs.*
2. *Increases the value of low-cost firms and, conversely, reduces the value of high-cost firms.*

The intuition behind this proposition is the same idea Dye (1985) used to show that uncertainty about information endowment can prevent unraveling. He argued that a firm with no disclosure cost that would only conceal the lowest profit if its type were known finds nondisclosure more attractive when it is pooled together with firms that cannot disclose information. In my setting, a nondisclosing low-cost firm is pooled together with nondisclosing high-cost firms. As the latter firms do not disclose high profits the way the former firm would if its type were known, this makes nondisclosure more attractive for the low-cost firm. Thus, the expected value of a low-cost firm is greater under asymmetric information, which in turn implies that the expected value of high-cost firms must decrease due to the market's rationality. Similarly, high-cost firms disclose more as the pool of nondisclosing firms under asymmetric information has lower expected profits than what it would have if all firms had a high cost. Unfortunately, asymmetric information may increase or decrease the aggregate probability of disclosure depending on the exact form of the distribution functions.

3.1 Additional Assumptions

The equilibrium characterization provided by equation (2) is sufficient for deriving basic insights into the effect of uncertainty about the cost of disclosure, but is unsatisfactory in two ways. First, this condition suggests that for some distributions of profit there may be multiple equilibria. Second, this equation does not provide a tractable way to calculate a firm's expected value.

To circumvent these problems I make the strong assumption that profit is distributed uniformly[7] and normalize the support of the profit distribution to[8] $[\bar{c}, 1+\bar{c}]$.

An additional factor that can complicate the analysis is the non-monotone effect of the disclosure cost on the firm's expected value. If a firm is known to have no disclosure cost, it always discloses its profit and its expected value equals its expected profit. Analogously, if the firm has a prohibitively high disclosure cost it uses a strategy of no disclosure and its expected value also equals its expected profit. However, a firm with an intermediate cost uses a partial disclosure strategy, and pays its (strictly positive) disclosure cost with positive probability. Thus, its expected value is strictly less than its expected profit. To simplify the analysis, I focus on the natural case where a higher disclosure cost reduces the firm's expected value. In particular, when profits are distributed uniformly and the cost of disclosure is not prohibitive, the expected value of a firm with a known disclosure cost of c is $\mathbb{E}(q) - c(1-2c)$ – a function that is minimized at $c = \frac{1}{4}$. Therefore, I assume that $\bar{c} < \frac{1}{4}$.

3.2 The Value Function

When the market's belief about the cost of disclosure is μ and the equilibrium strategy is represented by v_{nd}, the expected liquidation value of a type-$\tilde{c}$ firm can be represented by the value function

$$V(\mu, c) \equiv \int_0^{\bar{c}} \Big(G(v_{nd}+c)v_{nd} + (1 - G(v_{nd}+c))\mathbb{E}(q-c|q > v_{nd}+c) \Big) d\mu(c) \quad (3)$$

[7] Beyer and Dye (2012) also use the assumption that profit is distributed uniformly in order to obtain a tractable model.

[8] I normalize the lower bound of the support to $\bar{c}$ in order to maintain the interpretation that second-period profit is positive regardless of the firm's first disclosure decision.

The first element of this integrand is the value of a firm that does not disclose its profit, while the second element is the expected value when profits are disclosed. When profits are distributed according to $U[\bar{c}, 1+\bar{c}]$, not only is there a unique equilibrium, but the value function and disclosure thresholds also have a simple closed-form representation

Lemma 1.

When $G(q) \sim U[\bar{c}, 1+\bar{c}]$

$$V(\mu, c) = \frac{1}{2}\left(c\left(c + 2\sqrt{k} - 2\right) + 2\bar{c} + k + 1\right) \quad (4)$$

$$v_{nd} = \bar{c} + \sqrt{k}$$

$$\textit{where } k = \mathbb{E}(c^2)$$

Lemma 1 establishes that a sufficient statistic for the market's belief is the expectation of the squared disclosure cost. Moreover, the fact that $\mathbb{E}(c^2) = var(c) + (\mathbb{E}(c))^2$ allows us to decompose the effect of disclosure-cost uncertainty on liquidation value into two distinct channels. First, a higher average disclosure cost increases the market's valuation after nondisclosure. This is because higher disclosure costs for other firms make disclosure by those firms less likely. Therefore, for each firm, an increase in the average disclosure cost (of other firms) reduces its probability of disclosure and increases its value. Second, a firm's liquidation value is increasing in the variance of the cost distribution. When profit is distributed uniformly, the expected disclosure expenditure of a type c firm is $c - c^2$, which is a concave function. Therefore, asymmetric information about disclosure costs tends to reduce the aggregate disclosure costs paid by all firms.[9] This implies that an increase in the variance of disclosure costs decreases the aggregate disclosure expenditure and increases firm value. Formally,

Corollary 1.

1. *A firm's liquidation value is increasing in the expectation and the variance of the disclosure cost distribution.*

2. *The probability of disclosure is decreasing in the expectation and the variance of the disclosure cost distribution.*

Henceforth, with a slight abuse of notation, I use the sufficient statistic $k(\mu) = \mathbb{E}(c^2|\mu)$, instead of the entire distribution μ, as the argument for the

[9] See the proof of Corollary 2 for a formal proof.

value function. Moreover, it will be convenient to denote the value function for a firm with a known disclosure cost of c by $V(c)$. From the above representation of the value function it follows that

$$V(c) = \bar{c} + \frac{1}{2} - c(1 - 2c)$$

4 Uncertain Disclosure Costs: Multiple Periods

Armed with the simple representation of the second period of the game that equation (4) provides, I now turn to the analysis of the strategic interaction between the firm and the market during the first period. As in most dynamic games, different off-path beliefs can be used to sustain multiple equilibria; however, in this paper I focus on equilibria that use threshold strategies. The primary reason for doing so is twofold. First, threshold strategies are simple and intuitive. Second, to support non-threshold equilibria, the market's belief about disclosure costs must be non-monotonic and discontinuous in the firm's disclosed profit; a feature that does not seem plausible.[10]

A more formal justification for focusing on such equilibria is that they are the unique type of equilibrium that satisfies condition $D.1$ of Cho and Kreps (1987), a selection criteria with a strong intuitive appeal that is used in a similar setting by Einhorn and Ziv (2012).

An additional attractive feature of the threshold equilibrium is that it maximizes the disclosure of information. In non-threshold equilibria, firms avoid disclosing intermediate levels of profit (while disclosing higher and lower profits) as the market's arbitrarily chosen off-path beliefs state that only firms with high disclosure costs disclose such profit. To satisfy the $D.1$ condition, after unexpected disclosure the market must believe that the firm has the lowest possible disclosure cost – a belief that maximizes the firm's expected liquidation value after off-path disclosure and thus maximizes the probability of disclosure.

[10] In related papers such as Einhorn and Ziv (2008) and Beyer and Dye (2012) there is no multiplicity of equilibria due to arbitrary specification of off-path beliefs. The existence of uninformed firms implies that there are no off-path beliefs about profits, while the certainty about the cost disclosure negates the need to specify beliefs about costs.

4.1 The Second Period

In the multi-period setting, the profit distribution in the second period depends on the firm's action in the first period. If the firm did not disclose, then there is no postponed disclosure expenditure and profit is distributed according to $G(q)$. However, if the profit was disclosed in the first period, the postponed disclosure expenditure shifts down the second-period profit distribution. Furthermore, as the market does not know the firm's disclosure cost, it does not know the correct distribution of the second period-profit.

The second-period profit distribution of a type-c firm following disclosure in the first period is $U[\bar{c} - c, 1 + \bar{c} - c]$; thus low-cost firms have a better profit distribution than high-cost firms. Therefore, it is no longer clear how asymmetric information affects disclosure decisions.[11] While the existence of firms with a higher cost of disclosure enables a firm to hide low profits more easily, doing so also signals that the firm's profit is drawn from a distribution that is worse (FOSD dominated) than it actually is. Due to the linear effect of the firm's type on both its disclosure cost and its expected profit distribution, these opposing forces cancel out and the firm's second-period disclosure decision is not affected by the market's uncertainty about disclosure costs. Formally,

Lemma 2. *There is a unique equilibrium following first-period disclosure. Moreover, in this equilibrium each firm acts as if its type were known.*

Disclosure in the first period implies that in the second period firms act as if their type and liquidation value distribution were known. However, if no disclosure was made in the first period, asymmetric information affects the firm's second-period decision. Therefore, disclosure in the first period increases aggregate disclosure in the second period if and only if asymmetric information reduces aggregate disclosure.[12] When liquidation value is distributed uniformly asymmetric information about costs decreases the probability of disclosure; thus

[11] Proposition 1 was established under the assumption that the profit distribution is identical for all firms.

[12] When firms act as if their type were known, the probability of disclosure does not depend on the level of possible liquidation values, but only on the distribution of the liquidation values in excess of their minimal possible value. Thus, the fact that the minimal level of profit is different for different types of firms is not important for calculating the probability of disclosure.

any incentive increasing disclosure in the first period increases disclosure in the second period as well. This simple observation has an important implication as it states that there is no meaningful trade-off between first- and second-period disclosures.

Corollary 2. *The probability of disclosure in the second period is increasing in the probability of first-period disclosure.*

4.2 The First Period

Lemma 2 also enables us to clarify the expressions for the firm's price in the first period. Following disclosure in the first period, firms act as if their type were known, and thus the expected liquidation value of a type-c firm is[13] $V(c) - c$. Thus, when the market's belief about the firm's disclosure cost is μ the expected liquidation value is

$$\mathbb{E}(L_d(\mu)) \equiv \int_0^{\bar{c}} (V(c) - c)d\mu(c) = \int_0^{\bar{c}} (\frac{1}{2} + \bar{c} - 2c(1-c))d\mu(c)$$

Following nondisclosure in the first period, the liquidation value of a type-c firm is given by $V(k_{nd}, c)$, where k_{nd} is the market's expectation of the squared disclosure cost conditional on nondisclosure. Therefore, the expected liquidation value of a firm after nondisclosure is given by

$$\begin{aligned}\mathbb{E}(L_{nd}(\mu)) \equiv& \int_0^{\bar{c}} (V(k_{nd}, c)d\mu(c) = \int_0^{\bar{c}} (\frac{1}{2}\left(c^2 + 2c\left(\sqrt{k_{nd}} - 1\right) + 2\bar{c} + k_{nd} + 1\right))d\mu(c) \\ =& \frac{1}{2} + \bar{c} + k_{nd} + (\sqrt{k_{nd}} - 1)\mathbb{E}(c|nd)\end{aligned}$$

The firm's price in the first period is the sum of its expected first-period profit and liquidation value. Thus, if a firm discloses that its first-period profit is q, its price is given by

$$p_d(q) = q + \mathbb{E}(L_d(\mu_q)),$$

where μ_q is the market's beliefs about the firm's disclosure cost following the disclosure of a first-period profit equal to q.

[13] Recall that disclosure in the first period decreases the liquidation value by c.

If the firm does not disclose in the first period, the market prices the firm according to the expected profit conditional on nondisclosure. Denote the disclosure threshold of a type c firm by q_c and the expected profit conditional on nondisclosure by q_{nd}. Then, when q_{nd} is the result of Bayesian updating we must have that[14]

$$q_{nd} = \int_0^{\bar{c}} \mathbb{E}(q|q < q_c) d\mu_{nd}(c) = \int_0^{\bar{c}} \frac{\bar{c} + q_c}{2} d\mu_{nd}(c),$$

where μ_{nd} is the market's beliefs about the firm's disclosure cost following nondisclosure. Therefore, the firm's price following nondisclosure is

$$p_{nd} = q_{nd} + \mathbb{E}(L_{nd}(\mu_{nd}))$$

Combining the preceding analysis shows that the expected payoff of a type-c firm that discloses a first-period profit of q is

$$p_d(q) + \delta(V(c) - c) = q + \mathbb{E}(L_d(\mu_q)) + \delta(V(c) - c) \tag{5}$$

while its expected payoff from nondisclosure is

$$p_{nd} + \delta V(k_{nd}, c) = q_{nd} + \mathbb{E}(L_{nd}(\mu_{nd})) + \delta V(k_{nd}, c) \tag{6}$$

From the explicit representation of the firm's payoff from each action we can establish three important properties. First, we can see that a strategy of no disclosure is sub-optimal for all firms. This result, intuitive as it may seem, is in fact dependent on the assumptions that profit is distributed uniformly and that the cost of disclosure is small enough. When the profits are distributed uniformly on an interval of measure one, and the cost of disclosure is less than $\frac{1}{4}$, disclosing the maximal profit will increase the market's expectation of the profit by at least $\frac{1}{2}$ relative to nondisclosure. Clearly, disclosure decreases the firm's liquidation value. However, the maximal decrease in liquidation value would occur if the market were to assume that the firm will disclose with probability one in the second period, a belief under which the firm's total disclosure expenditure is less than $\frac{1}{2}$. Thus, it is optimal for all firms to disclose the maximal level of profits. Formally,

Lemma 3. *In the first period all firms disclose high enough profits.*

[14] If all firms use a strategy of full disclosure, then the $D.1$ condition demands that $q_{nd} = \bar{c}$.

Second, these equations highlight how a firm's incentives are affected by its disclosure cost. The firm's price following disclosure in the first period is not affected by its real disclosure cost, but only by the market's belief about this cost. However, firms still have different incentives in the first-period, as they maximize the sum of their first period price and their discounted expected liquidation value, which depends on the true cost. Thus, a firm's cost affects its expected payoff from disclosing and withholding information, and hence different types of firms use different disclosure strategies in the first period.

At its disclosure threshold a firm is indifferent between paying its disclosure cost and revealing its type, to entering the second period with uncertainty over the cost of disclosure. Moreover, it is straightforward to see that the benefit from maintaining uncertainty is increasing in the firm's type. Therefore, if a high-cost firm is willing to pay its its disclosure cost and reveal its threshold profit and type, it must be the case that a firm with a lower disclosure cost is willing to reveal the same level of profit as well. Formally,

Lemma 4. *The first-period disclosure threshold is weakly increasing in the firm's cost of disclosure. Moreover, when the threshold is interior it is strictly increasing in the cost.*

The intuition behind this lemma is twofold. First, consider how the market's estimation of a (nondisclosing) firm's profit depends on its belief about the cost of disclosure. Clearly, a higher disclosure cost increases the market's estimate of this profit. Disclosure in the first period leads the market to believe that the firm has lower disclosure costs, which, in turn leads the market to evaluate nondisclosure in the second period more harshly. Therefore, disclosure in the first period reduces the payoff associated with nondisclosure in the second period, and creates a de-facto commitment to disclose in the second period. Since the cost of disclosing in the second period depends on the firm's true type, the cost of this commitment is higher for high-cost firms, and so high-cost firms are less likely to disclose in the first period.

Second, the firms liquidation value is reduced by the expenditure (unknown to the market) on first-period disclosure. By disclosing in the second period, low-cost firms can show that their expenditure was low, whereas high-cost firms can either prove that expenditure was high by disclosing their liquidation value, or let the market infer the same thing by withholding information. Thus, high-cost firms avoid first-period disclosure as they know that the market will learn in the second period that the disclosure expenditure was high and hence will reduce its estimate for the liquidation value.

Finally, these equations show that increasing the discount factor makes disclosure less likely. The intuition for this result follows from the indirect costs of first-period disclosure that are incurred in the second period. In particular, as these costs are paid in the second period, decreasing the firm's discount factor decreases the indirect (discounted) cost of first-period disclosure, which, in turn, makes disclosure in the first period more profitable. Formally,

Lemma 5. *Disclosure thresholds are weakly increasing and continuous in δ. When a disclosure threshold is strictly interior it is strictly increasing in δ.*

4.3 Full Disclosure

When the firm's discount factor is low the indirect costs of first-period disclosure are low as well. Therefore, it may be the case that the firm decides to disclose its profit in order to signal a low cost, and considers the hard information this signal contains about its first-period profit as unimportant. If disclosure decisions are indeed used to convey information (only) about costs, then a standard unraveling argument shows that all firms use a strategy of full disclosure. To understand why this is the case, recall that the firm's first-period price is not affected by the firm's true disclosure cost, but only by the market's belief about this cost. Furthermore, as disclosure thresholds are weakly increasing in the firm's cost, disclosing a lower level of profit must (weakly) improve the market's beliefs about the firm's disclosure costs. Thus, in equilibrium firms disclose all profits in an attempt to better the market's belief about their type. The following proposition states that when the discount factor is low enough, the mechanism described above will indeed lead to full disclosure by all firms.

Proposition 2. *Full revelation by all firms is the unique first-period equilibrium if and only if $\delta \leq \delta_1 = (1 - 2\bar{c}) - \frac{2}{\bar{c}}\left(\mathbb{E}(c) - \mathbb{E}(c^2)\right)$.*

The explicit expression of the critical discount factor required for full disclosure to be an equilibrium suggests numerous economic insights. First, as $\delta_1 < 1$ it shows that, as expected, full disclosure by all firms can only occur when there is a difference between the firm's discount factor and the market's discount factor. Second, if we recall that

$$\mathbb{E}(c) - \mathbb{E}(c^2) \equiv \mathbb{E}(c) - (\mathbb{E}(c))^2 - var(c)$$

we see that reducing expected disclosure costs makes full disclosure more likely in the first period of a multi-period interaction. This result stands in sharp contrast to the positive connection between the expected disclosure cost and the probability of disclosure established by Proposition 1 in the static setting. Moreover, it shows that increasing the variance of the disclosure cost distribution also makes full disclosure less likely as a result of effect of the variance of a firm's expected liquidation value, as discussed in Corollary 1.

From Proposition 2 and Lemma 5 it follows that as the discount factor increases, the number of firms that use a strategy of full disclosure decreases. Even when the discount factor is equal to one, it is possible to find examples in which some firms use a strategy of full disclosure. A simple example of this is when the cost of disclosure has only two possible values, $\{0, c\}$, and the probability of the cost being zero is given by η. In this example, if $\eta \in (\frac{1}{2}, 1)$ and $c \in (0, \frac{4\eta-2}{5+4\eta})$, then the firm with a disclosure cost of zero uses a full-disclosure strategy while the firm with a disclosure cost of c uses a partial disclosure strategy. Naturally, in many other cases, when the discount factor is equal to one, all firms use a partial disclosure strategy in the first period. In particular, this occurs when the cost distribution is uniform.[15]

4.4 Affect of Dynamic Incentives

In the previous subsection I highlighted how a firm's desire to signal its low-cost creates a channel through which dynamic incentives increase the firm's voluntary disclosure of information. However, in this model there exists an additional, opposing force that makes disclosure less likely. Namely, disclosure in the first period reduces the firm's liquidation value, which, in turn, reduces the market's evaluation of the firm in *both* periods. That is, the effective cost of first-period disclosure is multiplied in the dynamic environment. Clearly, both the firm's benefit from signaling it has a low cost and the enhancement of its effective cost of disclosure depend on its true type. Thus, a natural question arises as to

[15] The calculations behind both examples are straightforward and are thus omitted.

what is the qualitative effect of dynamic incentives on the disclosure decision of different types of firms. To conduct a valid comparison between the dynamic and static models, in this subsection I assume that the firm does not discount the future.[16]

Intuitively, one might expect that dynamic incentives lead low-cost firms to increase disclosure. Since such firms incur a low cost for signaling their type as with high probability they will disclose in the second period and in so doing mitigate the increase in the effective disclosure cost. Unfortunately, without further assumptions this intuition is false.

The market's evaluation of a firm in the first period depends on the market's belief about the disclosure expenditure in the second period. In particular, when a type c firm discloses its threshold profit, the market learns that the firm's cost is at most c. Thus, the market uses the truncated expectation of disclosure costs to determine the firm's expected disclosure expenditure in the second period. The expected disclosure expenditure (in both periods) for a type c firm that disclosed in the first period is $2(c - c^2)$. Thus, the curvature of $L(x) \equiv \mathbb{E}(c - c^2 | c \leq x)$ determines the sensitivity of the marginal disclosing firm's valuation to small changes in the market's information set. For this analysis, I assume that this conditional expectation is concave in x; this assumption has the natural interpretation that the marginal effect of increasing the set of possible disclosure costs is greater when that set is small.

In addition to the previous "local" restriction on the shape of the cost distribution function, I must also add a global restriction that relates the size of the cost distribution's support to its moments. Roughly speaking, this condition states that both the expectation and the variance of the disclosure cost are large, yet not too large relative to the maximal possible cost of disclosure. Formally, I refer to a cost distribution function as regular if the following two conditions are satisfied:

Regularity Conditions.

[16] Were the firm to discount the future, the direct cost of disclosure in the first period of the dynamic mode, would be less than the cost of disclosure in the static model.

1. $L(x) = \mathbb{E}(c - c^2 | c \leq x)$ is a concave function.
2. $\mathbb{E}(c) - \mathbb{E}(c^2) - \frac{\sqrt{\mathbb{E}(c^2)}}{2} \in (\frac{3}{4}\bar{c}^2 - \frac{\bar{c}}{2}\sqrt{\mathbb{E}(c^2)}, \frac{\bar{c}}{2} - \frac{9}{4}\bar{c}^2)$.

When the cost distribution satisfies the above regularity conditions, the hypothesis that dynamic incentives increase the probability of low-cost firms is in fact true.

Proposition 3. *Assume that $\delta = 1$ and that $G(c)$ satisfies the regularity conditions. There exists $c^* \in (0, \bar{c})$ such that dynamic incentives increase the probability of disclosure if and only if $c < c^*$.*

The results presented in the last two subsections constitute the main insight of this paper. In words, that disclosure costs have an ambiguous effect on disclosure decisions in dynamic environments under asymmetric information and, moreover, their effect may be qualitatively different for different firms. This result is contrary to the previous results of Einhorn and Ziv (2008). The conflict between these results is due to the difference in the firm's type space in the two models. In Einhorn and Ziv (2008) the firm's type is associated with its information endowment. Thus, the only valuable reputation is a reputation for being uninformed – a reputation that allows a firm to conceal unfavorable information more easily. In my model, however, the firm's type is associated with the cost of disclosure. Therefore, a firm may have two types of valuable reputation: 1) a reputation for having a low disclosure cost that increases the market's expectation of the firm's future dividend stream and 2) a reputation for having a high disclosure cost that increases the market's evaluation of nondisclosure in the future. As the former reputation is acquired by disclosure and the latter by nondisclosure, dynamic incentives have an ambiguous and firm-specific effect.

5 Disclosure Costs Borne by the Manager

So far I have explored the effect of uncertainty about the cost of disclosure under the plausible assumption that when the cost is significant enough to affect the firm's disclosure decision it also affects the firm's value. Yet Bamber, Jiang and Wang (2010) have empirically shown that a manager's personal history is relevant in predicting the firm's disclosure decisions. In light of this research, in this section and thus I consider the alternative assumption that the cost of disclosure is borne by the manager, who makes the disclosure decision, but does not affect the firm's value. In addition, for ease of exposition I modify the assumption on the timing of the disclosure expenditure. In particular, I assume that the manager's disutility from disclosure is incurred immediately and not in the following period.[17]

The source of disclosure costs is not important in a static setting, so the analysis of the final period provided in Section 3 remains valid. Moreover, as the cost of disclosure does not affect shareholders, the market's expectation of the firm's liquidation value in the first period is independent of previous disclosure decisions and the manager's type (the disclosure cost). Thus, the firm's price in the first period is

$$p(\mathcal{I}_1) = \mathbb{E}(q|\mathcal{I}_1) + \mathbb{E}(q)$$

The utility for a manager of type c from disclosure is

$$q - c + \mathbb{E}(q) + \delta V(k_q, c)$$

where $k_q = \mathbb{E}(c^2|d, q)$. Her utility from nondisclosure is

$$q_{nd} + \mathbb{E}(q) + \delta V(k_{nd}, c)$$

In this alternative, it is impossible to get equilibrium with full disclosure as a manager's continuation value is increasing in k, and unexpected nondisclosure increases k due to condition $D.1$. Thus, equilibrium is characterized by the indifference conditions

$$q_c - c + \delta V(k_{q_c}, c) = q_{nd} + \delta V(k_{nd}, c)$$

Under this alternative assumption, it is clear that dynamic incentives decrease disclosure as all managers prefer to be perceived as managers with high disclosure costs in order to avoid paying these costs in the second period. Managers whose actual disclosure cost is high are less likely to disclose, and so all managers will imitate their behavior and disclose less information. Formally,

Proposition 4. *When the cost of disclosure does not affect investors, dynamic incentives reduce disclosure.*

[17]This modification is immaterial as any future cost, c, which is the certain result of the current disclosure, is equivalent to an immediate cost of δc.

6 Conclusion

Although asymmetric information and intertemporal considerations are commonplace in many financial settings, the existing literature says little about how these features jointly affect disclosure decisions. This is even more surprising due to the fundamental importance of disclosure costs in explaining why firms refrain from disclosure. In this paper, I tried to rectify this oversight of the literature and offered a model that sheds light on the complex dynamic incentives created by asymmetric information about disclosures costs.

Firstly, I showed that in addition to affecting the disclosure decision of a single firm, uncertainty about disclosure costs has general equilibrium implications. Namely, it leads to a transfer of value between firms and increases the value of low-cost firms at the expense of high-cost ones. Furthermore, as asymmetric information may decrease aggregate disclosure, my work suggests that a small number of firms with high disclosure costs can account for a larger decrease in aggregate disclosure than was previously thought possible.

Secondly, I demonstrated that in a two-period setting, disclosure costs may fail to prevent full disclosure as nondisclosure acts as a signal for low value in the future. This insight relies on the existence of a known final period, which prevents a firm with high disclosure costs from credibly committing to not disclose in the future. However, even without a terminal date, the mechanism studied in this paper may lead to full disclosure by some firms. Moreover, it raises an important question about the conditions under which a firm can credibly commit to avoid disclosure in the future. Such a commitment, when credible, can massively enhance the affect of disclosure costs on disclosure decisions.

Thirdly, I highlighted how different firms may use disclosure as a means to convey different types of information to the market. In particular, low-cost firms may disclose their profit in order to signal their cost, whereas high-cost firms may disclose their profit to convey information about their profit. More generally, this observation suggests that when the firm is limited in the type of information it can credibly provide, the hard information it discloses may act merely as a vehicle to convey the information it cannot reveal directly. Furthermore, as the firm's ability to use such indirect communication is determined in equilibrium, particular care must be taken when conducting inference based on the firm's voluntary disclosure (or lack thereof).

Clearly, this paper is a first step in understanding the mechanisms by which asymmetric information about disclosure costs affects disclosure decisions. However, it shows the importance of pursuing this idea, and indicates subsequent questions we need to pursue. For example, models with an infinite horizon in which the firm's type may change over time need to be investigated in future research in order to understand the full effect of asymmetric information.

Appendix: Proofs

Proof of Proposition 1

For a firm with a known disclosure cost of c, the disclosure threshold, $\hat{q}(c)$, solves the equation

$$\hat{q}(c) - c = \mathbb{E}(q|q < \hat{q}(c))$$

When there is a unique solution to this equation, it is immediate that $\hat{q}(c) - c$ is a continuous and increasing function in c. Thus, there is a unique value of c^* for which $v_{nd} + c^* = \hat{q}(c^*)$.

In equilibrium the market's belief about $E(q)$ conditional on nondisclosure is at least $\underline{q}$. Therefore, no firm will disclose a profit of less than $\underline{q} + c$, which, in turn, implies that $v_{nd} > \underline{q}$. This implies that the disclosure threshold for a firm with $c = 0$ is strictly greater than $\hat{q}(0) = \underline{q}$. Combining the previous two points shows that firms with $c < c^*$ $(c > c^*)$ disclose less (more) due to uncertainty. The second part of the proposition follows from the fact that a firm's value is monotone in its payoff conditional on nondisclosure. □

Proof of Lemma 1

Denote the disclosure threshold of a firm of type c by q_c. Since disclosure thresholds are linear in c, $q_c = q_0 + c$. The probability of nondisclosure for a firm of type c is $q_0 + c - \bar{c}$ and the ex-ante probability of nondisclosure is

$$Pr(nd) = \int_0^c (q_c - \bar{c})dc = \int_0^c dF(c)(q_0 + c - \bar{c})dc = q_0 - \bar{c} + \mathbb{E}(c)$$

The expected profit conditional on nondisclosure for a firm of type c is

$$\mathbb{E}(q|q < q_c) = \frac{q_c + \bar{c}}{2} = \frac{q_0 + c + \bar{c}}{2}$$

Plugging these expressions into equation (2), we get

$$\begin{aligned} v_{nd} =& \frac{1}{q_0 - \bar{c} + \mathbb{E}(c)} \int_0^{\bar{c}} (q_0 + c - \bar{c}) \frac{q_0 + c + \bar{c}}{2} dF(c) \\ =& \frac{1}{2(q_0 - \bar{c} + \mathbb{E}(c))} (q_0^2 - \bar{c}^2 + 2q_0\mathbb{E}(c) + k) \end{aligned}$$

Recall that $v_{nd} = q_0$. Plugging this into (7) and solving for q_0 gives that

$$q_0 = \bar{c} + \sqrt{k}$$

Plugging $v_{nd} = q_0 = \bar{c} + \sqrt{k}$ into equation (3), gives the desired result. □

Proof of Lemma 2

After disclosure in the first period a firm of type c has a liquidation value of $q_2 - c$. Denote the liquidation value by $\hat{q}$ and recall that for a firm of type c, $\hat{q} \sim U[\bar{c} - c, 1 + \bar{c} - c]$. By the same argument used in Section 3, disclosure thresholds for the liquidation value exist and are linear in c when they are interior.

I will first show that following disclosure in the first period, only a firm with $c = 0$ uses a strategy of full disclosure.

If there is a firm with a strictly positive disclosure cost that uses a strategy of full disclosure, i.e., discloses a liquidation value of $\bar{c} - c$, there is also a firm with strictly positive costs that is indifferent to disclosing its minimal liquidation value. Denote the disclosure cost of the latter firm by x and observe that the firm's payoff from disclosing its minimal value is $\bar{c} - 2x$. A firm with $c > x$ discloses liquidation values greater than $\bar{c} - 2x + c$ and has an expected value conditional on nondisclosure of $\bar{c} - x$. Moreover, the ex-ante probability of nondisclosure is $\int_x^{\bar{c}} 2(c - x)d\mu(c)$, where $\mu(c)$ are the market's beliefs about the cost of disclosure. The expected value of a firm conditional on nondisclosure is

$$\int_x^{\bar{c}} \frac{Pr(nd|c)}{Pr(nd)} \mathbb{E}(q|nd, c)d\mu(c) = \int_x^{\bar{c}} \frac{2(c - x)}{\int_x^{\bar{c}} 2(c - x)\mu(c)dc} (\bar{c} - x)d\mu(c) = \bar{c} - x$$

This implies that a firm of type x prefers not to disclose its minimal profit.

Since only a firm with no disclosure costs may find it optimal to disclose its minimal profits, disclosure thresholds are given by $q_c = q_0 + c$. Therefore,

$$Pr(nd|c) = Pr(\hat{q} < q_0 + c) = q_0 + 2c - \bar{c}$$
$$Pr(nd) = \int_0^c Pr(nd|c)d\mu(c) = q_0 - \bar{c} + 2\mathbb{E}^{\mu}(c)$$
$$\mathbb{E}(q|nd, c) = \frac{(q_0 + c) + (\bar{c} - c)}{2} = \frac{q_0 + \bar{c}}{2}$$

Plugging the above expressions into equation (2) we get

$$v_{nd} = q_0 = \frac{q_0 + \bar{c}}{2} \Rightarrow q_c = \bar{c} + c$$

If a firm is known to have a disclosure cost of c and its profit is distributed according to $U[\bar{c} - c, 1 + \bar{c} - c]$, its disclosure threshold is the $\bar{q}_c$ that solves

$$\bar{q}_c - c = \frac{\bar{c} - c + \bar{q}_c}{2} \Rightarrow \bar{q}_c = \bar{c} + c$$

□

Proof of Corollary 2

For a firm with a known cost of c the probability of nondisclosure is $2c$. Thus, when costs are distributed according to μ the probability of nondisclosure is $2\mathbb{E}(c|\mu)$.

When there is asymmetric information about disclosure costs and liquidation value has the same distribution for all firms, the disclosure threshold for a firm of type c is $\bar{c} + c + \sqrt{\mathbb{E}(c^2|\mu)}$ and it does not disclose information with probability $c + \sqrt{\mathbb{E}(c^2|\mu)}$. Therefore, when costs are distributed according to μ the probability of nondisclosure is $\mathbb{E}(c|\mu) + \sqrt{\mathbb{E}(c^2|\mu)}$.

Since c^2 is a convex function, $\sqrt{\mathbb{E}(c^2|\mu)} > \mathbb{E}(c|\mu)$ and the probability of nondisclosure is greater when there is asymmetric information about disclosure costs.

□

Proof of Lemma 3

If a strategy of nondisclosure is profitable for any type of firm, then it is also profitable for a firm of type $\bar{c}$. Not disclosing profits of $\bar{c} + 1$ is a best response for such a firm if

$$1 + \bar{c} + \mathbb{E}(L_d(\mu_{1+\bar{c}}) + \delta(V(\bar{c}) - \bar{c}) \leq q_{nd} + L_{nd} + \delta V(k_{nd}, \bar{c})$$

Plugging the explicit expression for liquidation values into the above inequality we get

$$1+\bar{c}+\int_0^{\bar{c}}(V(c)-c)d\mu_{1+\bar{c}}(c)+\delta(V(\bar{c})-\bar{c})$$
$$\leq q_{nd}+\int_0^{\bar{c}}(V(k_{nd},c))d\mu_{nd}(c)+\delta V(k_{nd},\bar{c})$$

Plugging in the expressions for the value functions into the above inequality we get

$$\bar{c}+1+\frac{1}{2}+\bar{c}-2\mathbb{E}(c|d,q=1+\bar{c})+2\mathbb{E}(c^2|d,q=1+\bar{c})\leq$$
$$q_{nd}+\frac{1}{2}+\bar{c}+\mathbb{E}(c^2|nd)+E(c|nd)(\sqrt{\mathbb{E}(c^2|nd)}-1)+\delta(\bar{c}(1+\sqrt{\mathbb{E}(c^2|nd)})-\frac{3}{2}(\bar{c})^2+\frac{\mathbb{E}(c^2|nd)}{2})$$

Since $\bar{c}\leq\frac{1}{4}$ it follows that the term multiplying δ is positive, and thus the strategy of no disclosure is most profitable when $\delta=1$. Therefore, a necessary condition for no disclosure to be an optimal strategy is

$$1+2\mathbb{E}(c^2|d,q=1+\bar{c})-2\mathbb{E}(c|d,q=1+\bar{c})\leq$$
$$q_{nd}-\frac{3}{2}(\bar{c})^2+\bar{c}\sqrt{\mathbb{E}(c^2|nd)}+\frac{3}{2}\mathbb{E}(c^2|nd)-2\mathbb{E}(c|nd)+\mathbb{E}(c|nd)\sqrt{\mathbb{E}(c^2|nd)}$$

Recall that disclosure thresholds are increasing in c and observe that for the viable values of c, $c-c^2$ is an increasing function. Therefore,

$$2\mathbb{E}(c-c^2|d,q=1+\bar{c})\leq\mathbb{E}(c-c^2|nd)+\mathbb{E}(c^2|nd)$$

Moreover, $q_{nd}\leq ch+1/2$, and so this necessary condition becomes

$$\frac{1}{2}-\bar{c}\leq\frac{3}{2}(\bar{c})^2+(\bar{c}+\mathbb{E}(|nd))\sqrt{\mathbb{E}(c^2|nd)}-\frac{1}{2}\mathbb{E}(c^2|nd)$$

Note, that $\mathbb{E}(c|nd),\sqrt{\mathbb{E}(c^2|nd)}\in(0,(\bar{c}))$. Thus the necessary condition becomes

$$\frac{1}{2}-\bar{c}-(\bar{c})^2\leq 0$$

and this inequality is false since $\bar{c}\leq\frac{1}{4}$. □

Proof of Lemma 4

Assume there exists $c>c'$ such that $q_c<q_{c'}$. For firm c to find disclosure optimal at q_c we must have that

$$q_c+L_d(\mu_{q_c})+\delta(V(c)-c)\geq q_{nd}+L_{nd}(\mu_{nd})+\delta V(k_{nd},c)$$

Similarly, for a firm of type c' not to disclose at q_c, we must have that

$$q_c + L_d(\mu_{q_c}) + \delta(V(c') - c') \leq q_{nd} + L_{nd}(\mu_{nd}) + \delta V(k_{nd}, c')$$

Subtracting the two inequalities gives

$$c' - c + V(c) - V(c') \geq V(k_{nd}, c) - V(k_{nd}, c') \tag{7}$$

For any market beliefs the disclosure threshold in the second period is increasing in the cost. Therefore, the difference in the values of the two firms is less than the difference in their disclosure costs, $V(k, c) - V(k, c') \geq c' - c$. Moreover, the value of a firm is decreasing in its cost, $0 > V(c) - V(c')$. Adding these two inequalities shows that equation (7) is false.

To show the second part of this, assume to the contrary that firms c, c' use the same interior threshold q. For an interior threshold, we must have that

$$q + L_d(\mu_q) + \delta(V(c) - c) = q_{nd} + L_{nd}(\mu_{nd}) + \delta V(k_{nd}, c)$$
$$q + L_d(\mu_q) + \delta(V(c') - c') = q_{nd} + L_{nd}(\mu_{nd}) + \delta V(k_{nd}, c')$$

Subtracting the two indifference conditions and rearranging gives

$$0 = V(c') - V(c) + c - c' + V(k_{nd}, c) - V(k_{nd}, c'),$$

which, by the previous argument, is false.□

Proof of Lemma 5

The marginal effect of increasing δ on the firm's payoff after disclosure and nondisclosure is $V(c) - c$ and $V(k_{nd}, c)$, respectively.

$$V(k_{nd}, c) - V(c) + c = c - \frac{3}{2}c^2 + c\sqrt{k_{nd}} + \frac{k}{2} > 0$$

Therefore, increasing δ increases the inclination of each firm to avoid disclosure, which, in turn, implies that expected profits and liquidation conditional on nondisclosure increase, making nondisclosure even more profitable for all firms. Therefore, increasing δ increases interior disclosure thresholds. If a firm uses a full disclosure strategy, increasing δ cannot increase disclosure and thus the claim is vacuously true. The continuity of the disclosure thresholds follows from the continuity of all payoff functions. □

Proof of Proposition 2

To prove this proposition it is sufficient to show that disclosure of the minimal profit for a firm of type $\bar{c}$ is optimal. Moreover, conditional on nondisclosure the D.1 condition implies that the market believes that the firm has the maximal disclosure cost and minimal profits. Finally, if the market believes that all firms disclose all profits, the market does not update its belief's about the firm's disclosure cost following disclosure. By equation (5), the value of disclosing profits of $\bar{c}$ for a type-$\bar{c}$ firm is

$$\bar{c} + \mathbb{E}(L_d(G)) + \delta(V(\bar{c} - \bar{c})) = \frac{1}{2}(2\bar{c}(2\bar{c}\delta - \delta + 2) - 4\mathbb{E}(c) + 4\mathbb{E}(c^2) + \delta + 1),$$

while the value of nondisclosure is

$$\frac{1}{2}\left(4\bar{c}^2(\delta + 1) + 2\bar{c} + \delta + 1\right)$$

Therefore, disclosure is the optimal action if

$$(1 - 2\bar{c}) - \frac{2}{\bar{c}}\left(\mathbb{E}(c) - \mathbb{E}(c^2)\right) \geq \delta$$

□

Proof of Proposition 3

For ease of exposition, I present a proof of the case where a firm with $c = 0$ does not use a strategy of full disclosure. The proof where some low-cost firms use a strategy of full disclosure is analogous. Define the disclosure threshold of a type c in a static model by $q^1(c)$ and the disclosure threshold in the first period of the dynamic model by $q^2(c)$.

A firm's payoff after nondisclosure is

$$nd(c) \equiv q_{nd} + L_{nd} + V(k_{nd}, c).$$

A firm's payoff from disclosing profits at its disclosure threshold is

$$d(q^2(c)) \equiv q^2(c) + L_d(\tilde{c} < c) + V(c) - c.$$

As all firms use a partial disclosure strategy, it follows that $d(q^2(c)) = nd(c)$. Then, by subtracting $d(q^2(0)) = nd(0)$ from $d(q^2(c)) = nd(c)$ we get

$$\Delta(c) \equiv q^2(c) - q^2(0) = c(1 + \sqrt{k_{nd}}) - \frac{3}{2}c^2 + 2L(c)$$

Define the function

$$dif(c) = (q^2(c) - q^2(0)) - (q^1(c) - q^1(0)) = \Delta(c) - c = c\sqrt{k_{nd}} - \frac{3}{2}c^2 + 2L(c)$$

Lemma 6. $q^2(0) < q^1(0)$

Proof. To prove this lemma it is sufficient to show that if $q^2(0) = q^1(0) = \bar{c} + \sqrt{\mathbb{E}(c^2)}$, a firm with no disclosure cost strictly prefers to disclose a profit of $\bar{c} + \sqrt{\mathbb{E}(c^2)}$. That is,

$$q^1(0) + 2V(0) > q_{nd} + L_{nd} + V(k_{nd}, 0) \tag{8}$$

Since $q^2(c) \leq q^2(\bar{c}) = q^2(0) + \Delta(\bar{c})$

$$q_{nd} \leq \frac{\bar{c} + \bar{c} + \sqrt{\mathbb{E}(c^2)} + (1 + \sqrt{k_{nd}})\bar{c} - \frac{3}{2}\bar{c}^2 + \mathbb{E}(c) - \mathbb{E}(c^2)}{2}$$

By plugging this lower bound and the expressions for the various functions into equation (8) we get that it is sufficient to show that

$$\frac{\sqrt{\mathbb{E}(c^2)}}{2} - \mathbb{E}(c) + \mathbb{E}(c^2) - \frac{1}{2}\bar{c} + \frac{3}{4}\bar{c}^2 > (\sqrt{k_{nd}} - 1)\mathbb{E}(c|nd) + \frac{\sqrt{k_{nd}}}{2}\bar{c} + \frac{3}{2}k_{nd} \tag{9}$$

It is straightforward to show that for a given value of k_{nd}, $\mathbb{E}(c|nd) \in [\frac{k_{nd}}{\bar{c}}, \sqrt{k_{nd}}]$. Thus, as the RHS of equation (9) is decreasing in $\mathbb{E}(c|nd)$, it is sufficient to check that this condition holds for $\mathbb{E}(c|nd) = \frac{k_{nd}}{\bar{c}}$:

$$\frac{\sqrt{\mathbb{E}(c^2)}}{2} - \mathbb{E}(c) + \mathbb{E}(c^2) - \frac{1}{2}\bar{c} + \frac{3}{4}\bar{c}^2 > \frac{1}{\bar{c}}(k_{nd}^{\frac{3}{2}} + \sqrt{k_{nd}} - k_{nd}) + \frac{3}{2}k_{nd} \tag{10}$$

As $k_{nd} < \bar{c}^2 = \frac{1}{16}$, the RHS of equation (9) is increasing in k_{nd} and so we can check equation (10) at $k_{nd} = \bar{c}^2$:

$$(2 - 9\bar{c})\bar{c} - 4\mathbb{E}(c) + 2\sqrt{E(c^2)} + 4\mathbb{E}(c^2) > 0 \tag{11}$$

The inequality is satisfied by the second regularity condition. □

Lemma 7.

1. *$dif(0) = 0$.*
2. *$dif'(0) > 0$.*
3. *$dif(c)$ is a concave function.*

Proof. The first property is immediate. The second property follows from the fact that $L(c) = \int_0^c (x - x^2)\frac{dG(x)}{G(c)}$ is a strictly increasing function (recall that $c \leq \bar{c} \leq \frac{1}{4}$ and that $\sqrt{k_{nd}}c - \frac{3}{2}c^2$ is increasing for small values of c). The third property follows from the first regularity condition.

□

A firm of type c uses a lower disclosure threshold in the dynamic environment relative to the static environment if and only if $dif(c) < q^1(0) - q^2(0)$. The function $dif(c)$ is concave and satisfies $dif(0) = 0 < dif'(0)$; therefore, if $dif(\bar{c}) > (q^1(0) - q^2(0))$, there is exactly one firm for which $q^1(c) = q^2(c)$. Thus, it is sufficient to show that

$$dif(\bar{c}) = \bar{c}\sqrt{k_{nd}} + 2\mathbb{E}(c) - 2\mathbb{E}(c^2) - \frac{3}{2}\bar{c}^2 > \bar{c} + \sqrt{\mathbb{E}(c^2)} - q^2(0) \qquad (12)$$

As the probability of disclosure is decreasing in c it must be the case that $k_{nd} \geq \sqrt{\mathbb{E}(c^2)}$. Moreover, by definition $q^2(0) \geq \bar{c}$;, thus it is sufficient to show that

$$\bar{c}\sqrt{\mathbb{E}(c^2)} + 2\mathbb{E}(c) - 2\mathbb{E}(c^2) - \frac{3}{2}\bar{c}^2 > \sqrt{\mathbb{E}(c^2)} \qquad (13)$$

The inequality is satisfied due to the second regularity condition. □

Proof of Proposition 4
Rewriting the indifference conditions yields

$$q_c - c = q_{nd} + \delta(V(k_{nd}, c) - V(k_{q_c}, c))$$

As disclosure thresholds are increasing in cost we have that $k_{nd} > k_{q_c}$ for all c. Therefore, due to the monotonicity of the value function in its first argument, $V(k_{nd}, c) - V(k_{q_c}, c) < 0$. This implies that for each type of manager, the indifference equation in the dynamic model is the same as indifference condition in the static model $q_c - c = q_{nd}$, with a positive term added to the RHS. Therefore, all managers find disclosure less attractive in the dynamic model, and dynamic incentives decrease voluntary disclosure. □

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