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IMMIGRATION, EXPECTATIONS, AND NEIGHBORHOOD CHANGE

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Abstract

We study the dynamics of neighborhood change in a small country whose demographics are shocked by a stochastic flow of immigrants. We assume frictions in changing neighborhoods, such as leases. Since adverse demographic changes can occur while an agent is waiting to move, the appeal of a neighborhood depends on expectations. Without stochastic shocks, there can be multiple equilibria: the anticipation that a neighborhood will be stable makes its current residents willing to remain there, while predictions of a change can lead current residents to move out. Our main finding is that stochastic shocks *eliminate* the stable equilibrium when both are possible: if a transition can happen in a given neighborhood, it will. The resulting timing of neighborhood change is generally inefficient.

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1 Introduction

There are many groups in Israeli society that, to a greater or lesser extent, prefer to live in neighborhoods that contain only members of the same group. Religious and secular Jews are one example. Others include Ashkenazi vs. Sephardic vs. Ethiopian, Arab vs. Jew, rich and poor, etc.

There is considerable evidence for these preferences from the Israeli press. Many examples concern segregation between Arabs and Jews. One is from the town of Karmiel:

> Tempers are flaring in Karmiel after in one of the prestigious neighborhoods an apartment was sold to an Arab. The residents had a big demonstration and announced that they would prevent the buyers from entering their apartment.
>
> David Zeira, head of the neighborhood committee, explained that "Many of the neighborhood's residents spent a large chunk of their savings on these apartments because they wanted to move out of Akko's neighborhoods and other places where they lived close to Arabs. The residents are afraid that the sale of this apartment will lead to more sales to Arabs in the neighborhood, and that this will lead the neighborhood to become multiethnic. We have nothing against Arabs, but each of us has a different culture, and if enough families arrive we will eventually have to build a mosque. [Natti K. Zilberman, *Davar*, 6/9/95, p. 3]

It is not only residents who try to prevent Arabs from entering. Sometimes these efforts take on an official character. In Katzir, an Israeli Arab who applied to buy an apartment claims that he was rejected on the basis that "we don't sell apartments to Arabs here":

> Adel Kaadan, married and father of two, from Baka el-Gerbia, requested to buy an apartment in the new city of Katzir. Kaadan applied to the

municipal authority of Tal Eeron and asked how he could start the procedure to buy a piece of land, a house, or an apartment. In order to do so, he met, at the end of April, with an employee of the committee that deals with applications for housing in Katzir. According to Kaadan, it was made clear to him that a plot could not be sold to him, since the policy of the settlement was that apartments and plots were designated for the housing and settlement of Jews only, and would not be sold to Arabs. [Shlomo Tsezna, *Maariv*, 11/7/95, p. 14]

Religious authorities have also tried to influence the ethnic character of neighborhoods:

> In Tsfat, the head rabbis issued a 'psak halacha' [decision of religious law] forbidding residents to sell or rent apartments to Arabs. ...
>
> The Rabbi of Chabad in Tsfat, Levi Bistritski, said to Maariv that "even in Shulchan Aruch [an ancient religious document], it's written that one should not sell apartments to goyim [non-Jews]." However, he admitted that it was clear to him that this psak halacha would not survive an appeal to the Supreme Court of Justice, and that there is no legal way to prevent the sale of apartments to Arabs. [Yehuda Goren, *Maariv*, 5/1/95, p. 18]

Such attitudes are not directed only at Arabs. Yaron London interviewed residents of Ramat Eliahu, a neighborhood in Rishon le Zion, where a temporary settlement of caravans was planned to house new immigrants (Yaron London, *Yediot Achronot (Shiva Yamim)*, 16/8/91, pp. 16-18). One, Lilian Levy, said:

> From my window one can see the caravans, and if I want to sell my apartment this may discourage potential buyers. Not long ago, an apartment in this building was sold for $95,000. Now, it's certain that I won't be able to get as much.

Roi Makov, an activist in the struggle to prevent the building of a caravan site in another, wealthier neighborhood, said:

> A temporary neighborhood would not fit in with the landscape of our neighborhood. Caravans should be located at the outskirts of town, far from sight.

To London's question "Are you against the caravans, or against the inhabitants of the caravans?", responds Miki Kandler:

> This you can't distinguish. Cheap housing draws a certain type of people.

In summarizing, London quotes Yona Pitelson, an architect in charge of designing and locating new caravan sites:

> 80% of the plans were held up because of the objections of residents and/or the municipal authorities... Until now, objections of residents and municipal authorities have caused the complete erasal of 14 neighborhoods from the map. Thousands of new immigrants and those eligible for housing are suffering because of the selfishness of property owners.

These examples show two things. First, in many cases there is a clear and strong preference for segregation among neighborhood residents. Second, authorities often attempt to influence demographic composition. The main purpose of this paper is to present a descriptive model of how neighborhood demographics change in the absence of outside intervention. The welfare properties of our model also give some insights into motives that may explain why we see outside intervention.

One should be cautious, however, in interpreting our welfare results, since our social welfare function ignores social goals such as equality, homogeneity, absorption, or mutual understanding. For example, we assume that agents' preferences over neighborhood demographics are fixed and include a bias towards segregation. It could be that, after living in an integrated neighborhood for some time, people will start to appreciate diversity and to get along better with members of other groups. If, e.g., Arabs and Jews live together long enough, the violence between them might abate, permitting them to enjoy and take an interest in each other's culture.

There are other reasons to be cautious in deriving policy conclusions from our results. We assume that social welfare equals the sum of rents in a neighborhood. The idea is that rents capture the utility that agents get from living in the neighborhood.[1] This implicitly assumes that there are no interneighborhood externalities. For example, a person may prefer to live in a segregated neighborhood but prefer that other neighborhoods be mixed, since this gives rise to a cosmopolitan society in which people understand one another and share the same values.

Finally, the central planner may disagree with the tastes of agents. For example, even if all of the residents are racists, the president may still maintain a preference for mutual understanding and tolerance. He or she may thus prefer a policy that leads people to question or alter their existing preferences. Since our model precludes this possibility, it may fail to give the "right" prescriptions in such cases.

For all of these reasons, our model may underplay the advantages of integration and the need for an active policy that encourages it. Such considerations should also carry some weight when one transforms our results to a policy recommendation.

In the remainder of the paper, we study the dynamics of neighborhood change in a small country, such as Israel. We first consider the case of a fixed environment. In this case, there may be multiple rational expectations equilibria. In each, agents' expectations are self fulfilling. We then consider what happens if the environment changes according to a general stochastic process. We show that the multiplicity disappears in this case. A neighborhood must undergo a transition at the first time at which the prophecy of a change becomes self-fulfilling.

[1] This comes from our assumption that the number of agents exceeds the number of apartments in the neighborhood, so competition drives rents to the highest willingness to pay.

2 Fixed Environment

The Model

Time is continuous. There is a single neighborhood that contains a continuum of identical apartments, each of which can house one agent. There is a large pool of prospective tenants, who come in two types: 1 and 2. The number of agents of each type is large enough that some agents of each type must live outside the neighborhood.

All apartments are owned by absentee landlords. An agent who moves into an apartment signs a lease[2] committing her to pay rent at a fixed rate until the lease expires. Leases expire randomly, according to a Poisson process with arrival rate k. That is, in a small period of length dt, the lease expires with probability kdt.

The rental rate is determined competitively when the lease is signed. Since there are more prospective tenants of each type than apartments, the rental rate equals the highest willingness to pay of the two types. This willingness to pay reflects the agent's expected utility from living in the neighborhood as compared with her next best alternative, whose utility is normalized to zero. An agent's expected utility depends not only on the neighborhood's physical and geographical characteristics, but also on its demographic composition: while living in the neighborhood, the agent has a flow of one-on-one interactions with neighbors that give her a utility that depends on her neighbor's type. The following table gives an example.

	Type 1	**Type 2**
Type 1	3, 3	2, 0
Type 2	0, 2	4, 4

[2]Since there are no moving costs in the model, one could ask why agents sign leases at all. Why do they not instead pay the spot price for an apartment for as long as they remain in the neighborhood? First, we do observe leases in rental relationships. Second, there are other barriers to moving in the real world, such as thin housing markets, search costs, planning demands, and the costs of adjustment to a new environment. Leases also serve as a convenient proxy for these various impediments.

In this example, if a type 1 agent meets a type 2 agent, the type 1 agent gets a utility of two while the type 2 agent receives zero. At each point in time, the chance that the agent meets a neighbor of type 1 or type 2 equals the proportion of this type in the neighborhood. This is how the neighborhood's demographic composition has an effect on the agent's utility.

Let $u(i, j)$ be the utility of a type i agent from an interaction with a type j agent. Suppose an agent of type i pays rent at rate R during the period $[t, t+dt]$. Suppose the proportion of type 2 agents in the neighborhood is X_t. Then the agent's instantaneous utility from living in the neighborhood is $((1 - X_t)u(i, 1) + X_t u_i(i, 2) - R)dt$.

All players have the same rate of time preference, $r \geq 0$. Consider a player who signs a lease at time zero. At time t, the player is still locked into her lease with probability e^{-kt}. Since her pure discount factor is e^{-rt}, her effective discount factor is $e^{-(r+k)t}$. Therefore, an agent who signs a lease at time zero and predicts that the neighborhood's demographics will follow the path $(X_t)_{t=0}^{\infty}$ expects the utility

$$\int_{t=0}^{\infty} e^{-(r+k)t}((1 - X_t)u(i, 1) + X_t u_i(i, 2) - R)dt$$

In examining the case of a fixed environment, we assume that all agents expect the same path $(X_t)_{t=0}^{\infty}$. This assumption simply means that we are looking for a rational expectations equilibrium. One may argue that this assumption is not reasonable, particularly in the case of multiple equilibria. However, the assumption is not needed in the second case (of a changing environment), which we view as more realistic. The reader who does not like this assumption can view the case of a fixed environment as a starting point that makes it easier to understand the second case.

Since apartments are rented competitively, the agent with the higher expected utility will move in. The difference between the utility of a type 2 agent and that of a type 1 agent is:

$$\int_{t=0}^{\infty} e^{-(r+k)t}((1 - X_t)\Delta(1) + X_t\Delta(2))dt$$

where $\Delta(i) = u(2,i) - u(1,i)$, the difference in utility between the two types who encounter a neighbor of type i. If this integral is positive, type 2 agents will move in; if negative, type 1 agents will.

A preference for segregation is captured by the technical assumption that $\Delta(2) > \Delta(1)$. This means that a higher proportion of type 2 agents leads to an increase in their utility *relative* to that received by type 1 agents. This property is consistent with a number of different stories. One is that the two groups simply dislike each other, as in Miyao [28]; this implies the stronger condition that $\Delta(2)$ is positive while $\Delta(1)$ is negative. Or both types may prefer to live with type 2s, with type 2's having the stronger preference: $\Delta(2) > \Delta(1) > 0$. Alternatively, there may be educational externalities; our assumption is formally equivalent to Bénabou's assumption that the two groups have different skill levels and the cost of attaining high skills is decreasing in the proportion of neighborhood residents who do so [5]. The two groups may also differ in their tastes for public goods, as in Ellickson [15], Epple and Romer [17], Epple, Filimon, and Romer [18], and Fernandez and Rogerson [21]. The function Δ would then capture, in reduced form, the effects of neighborhood composition on the bundle of public goods that is selected.

Analysis

Let us consider a neighborhood that is initially inhabited entirely by type 2 agents: $X_0 = 1$. We will give conditions under which there are multiple equilibria. First, there may be an equilibrium in which the neighborhood remains stable: X_t equals 1 for all t. If agents expect this, type 2 agents will be willing to pay more than type 1 agents if:

$$\int_{t=0}^{\infty} e^{-(r+k)t}((1-1)\Delta(1) + 1\Delta(2))dt > 0$$

This holds if $\Delta(2) > 0$: if type 2 agents get higher utility from meeting type 2 agents than type 1 agents get. In other words, as long as it is not strictly dominant for type 1 agents to move in, this is a perfect foresight equilibrium.

There may also be an equilibrium in which the neighborhood undergoes a transition to type 1. In the fastest such transition, every apartment whose lease expires is rented by a type 1 agent. This gives rise to a path (X_t) that satisfies $dX_t/dt = -kX_t dt$: in every period of length dt, kdt leases expire, of which a proportion X_t were held by type 2 agents and are taken over type 1 agents. The solution to this differential equation is $X_t = e^{-kt}$.

This path can occur in equilibrium if, when it is expected, type 1 agents indeed are willing to pay more than type 2 agents for a lease. This holds if:

$$\int_{t=0}^{\infty} e^{-(r+k)t}((1-e^{-kt})\Delta(1) + e^{-kt}\Delta(2))dt < 0$$

This translates to:

$$\Delta(1)\left[\frac{1}{r+k} - \frac{1}{r+2k}\right] + \Delta(2)\left[\frac{1}{r+2k}\right] < 0 \tag{1}$$

If agents are perfectly patient ($r = 0$), this becomes

$$\Delta(1) + \Delta(2) < 0$$

That is, agents of type 1 get a higher utility than do agents of type 2 from living in a neighborhood that is split 50-50 between the two types. This condition is consistent with $\Delta(2) > 0$, so that there can be multiple equilibria. If $r > 0$, the formula (1) is equivalent to

$$(1-p)\Delta(1) + p\Delta(2) < 0 \tag{2}$$

where

$$p = \frac{1}{2} + \frac{1}{2}\left[\frac{r}{r+2k}\right] \tag{3}$$

That is, agents of type 1 get a higher utility than do agents of type 2 from living in a neighborhood in which $p > 1/2$ of the residents are of type 1. This is because impatience causes players to put more weight on the neighbors that they will meet early on in the lease period; since these neighbors are more likely to be type 2 agents, the conditions for a transition to be self-fulfilling become harder to satisfy.

Discussion

The case of a fixed environment is problematic because of the multiplicity of equilibria. Not only is the theory unable to give a unique prediction in many cases; the validity of its predictions are themselves doubtful. Without an explanation of how players coordinate their beliefs, it is hard to justify why equilibrium is a reasonable assumption. That is, if we do not know which equilibrium to expect, how do we know that the agents will coordinate their expectations on one of the two equilibria? They may instead be confused or hold conflicting views about the future of their neighborhood.

3 Changing Environment

The situation turns out not to be so bleak. In this section we introduce the reasonable assumption that the world undergoes exogenous stochastic changes. This leads to a unique prediction, in which a transition must occur if its expectation is self fulfilling. Not only do we get a unique prediction; we also can discard the assumption that players coordinate their beliefs and play according to rational expectations equilibrium. Rather, players come to expect a transition via an inductive process in which they use iterated dominance to determine what other agents will do.

We assume that the two types of players' utilities from living in the neighborhood depend also on an exogenous factor that changes stochastically over time. There are many examples. Relative wages of the two types in areas that are within commuting distance may change. The demographic composition of surrounding areas may change; for example, if many type 1's move into nearby areas, the neighborhood itself may become more attractive to type 1's. Because the two types may have different consumption patterns, relative prices in the neighborhood or nearby can affect the neighborhood's relative appeal to the two groups.

The assumption of exogenous shocks to relative payoffs is particularly suitable in the case of Israel. Waves of immigration and differences in birth rates among groups have affected both the demographic composition of cities, relative wages, and prices. The cancellation of the Arab boycott and the lowering of tariff and regulatory barriers with other countries (mainly the European Community and the United States) also changed the relative prices of consumption goods and relative wages. These changes have income effects that alter agents' willingness to pay to live in neighborhoods with different characteristics.

The Model

The model is the same, with one change. Payoffs now depend also on a random parameter B_t that changes over time, t: if a type i agent encounters a type j agent at time t, her payoff from this encounter is $u(i, j, B_t)$. We assume that higher values of B_t help type 2 agents more than type 1 agents, regardless of the demographic environment that each agent faces. That is, $u(2, i, B_t) - u(1, j, B_t)$ is continuous and strictly increasing in B_t at a bounded rate: there exists a positive constant $\overline{w}$ such that for all $b > \hat{b}$,

$$0 < [u(2, i, b) - u(1, j, b)] - [u(2, i, \hat{b}) - u(1, j, \hat{b})] < \overline{w}[b - \hat{b}] \tag{4}$$

Let $\Delta(j, B_t)$ be the relative payoff to type 2 when meeting an agent of type j: $\Delta(j, B_t) = u(2, j, B_t) - u(1, j, B_t)$. We continue to assume a preference for segregation: $\Delta(2, B_t) > \Delta(1, B_t)$. The following table gives an example of the time-t payoff matrix.

	Type 1	**Type 2**
Type 1	$3, 3$	$2, B_t$
Type 2	$B_t, 2$	$4 + B_t, 4 + B_t$

We assume that B_t follows a Brownian motion. This is essentially the continuous time version of a random walk and may also have a deterministic trend. The Brownian motion has two parameters. The variance σ^2 tells us how fast the Brownian motion spreads out. The trend μ gives the rate at which its mean changes over time. More precisely, a Brownian motion has the following properties (Billingsley [9, p. 522]):

1. It is continuous with probability one.
2. For any $t > \hat{t} > 0$, the random variable $B_t - B_{\hat{t}}$ (which takes values in $\Re$) is normally distributed with mean $\mu(t - \hat{t})$ and variance $\sigma^2(t - \hat{t})$.
3. Its increments are independent. For any $t > \hat{t} \geq v > \hat{v}$, the random variable $B_t - B_{\hat{t}}$ is independent of $B_v - B_{\hat{v}}$.

The difference between the utility of type 2 and that of type 1 is now:

$$E\left[\int_{t=0}^{\infty} e^{-(r+k)t}\left(X_t\Delta(2, B_t) + (1 - X_t)\Delta(1, B_t)\right) dt\right]$$

Type 2 players move in if this is positive; type 1 if it is negative.

Finally, to give our iterated dominance argument a place to start, we assume that for B_t large enough, type 2 agents will move into the neighborhood regardless of its current and expected composition: that $E\left[\int_{t=0}^{\infty} e^{-(r+k)t}\Delta(1, B_t)dt\right] > 0$. For sufficiently low values of B_t, an analogous condition guarantees that type 1 agents will enter. For example, surrounding areas may become inhabited entirely by agents of a given type. Or the mix of jobs within commuting distance may change such that only agents of a given type can feasibly live in the neighborhood.

Solving the Model

Rather than looking for equilibria, we analyze the game using a more primitive solution concept: the *iterative elimination of conditionally dominated strategies* (see Fudenberg & Tirole [23, pp. 128 ff.]). This is essentially the extension of backwards induction to infinite horizon games.

The iterative procedure works as follows. Suppose an agent's lease expires at time t. If B_t is large enough, a type 2 agent will move in regardless of her beliefs over what will happen to the neighborhood's demographics afterwards. Let f^0 be the boundary of the region where this is true. This is depicted in Figure 1. To the right of f^0, we know that every expiring lease must be take over by a type 2 player; to the left of f^0 we cannot yet say who moves in.

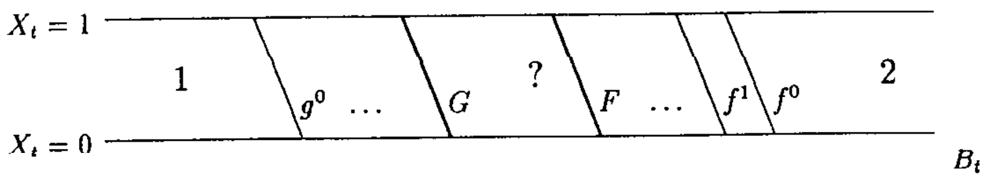

Figure 1: The iterative elimination procedure.

In the second step we assume that a player believes that type 2 agents will win every lease when the state is to the right of f^0. With this belief, there is a new boundary, f^1, such that type 2 agents win every lease when to the right of f^1. f^1 must lie to the left of f^0, since knowing that some leases will be taken over by type 2 agents makes the neighborhood more appealing to type 2 players. In the next step we find f^2 and so on. Let F be the limit of the sequence f^0, f^1,.... Whenever (B_t, X_t) is on the right side of F, any lease that expires must be taken over by a type 2 player. In a similar way, starting an iterative process from the left side of the environment space, we construct a bound G such that, to the left of G, type 1 players must win every lease.

The iterative elimination procedure divides the environment space into three regions. Type 2 agents win all leases to the right of F and type 1 agents always win to the left of G. We do not know what happens in the "?" region between the two lines.

Results

Theorem 1 states that when lease lengths are short relative to the rate at which the world is changing, the "?" region is small. The limit case is depicted in Figure 2. Here there is no "?" region, so that we can say in every state (X_t, B_t) which type will win the leases that expire.

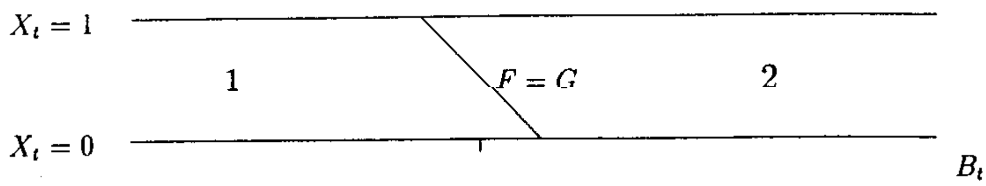

Figure 2: Illustration of Theorem 1

Let B^p be the value of B_t at which the two types of agents receive equal expected utilities from meeting a type 2 agent with probability p and type 1 with probability $1-p$:

$$p\Delta(2, B^p) + (1-p)\Delta(1, B^p) = 0 \tag{5}$$

Theorem 1

For any $k, \epsilon > 0$, there is a $\overline{\sigma} > 0$ and a $\overline{\mu} > 0$ such that if $\sigma < \overline{\sigma}$ and $\mu < \overline{\mu}$, type 2 agents win whenever $B_t > B^{h(X_t)} + \epsilon$ and type 1 whenever $B_t < B^{h(X_t)} - \epsilon$, where $h(x) = \frac{rx+k}{r+2k}$.

Proof: *The proof is analogous to that of Theorem 2 in Burdzy, Frankel, and Pauzner [11] and is therefore omitted.*

What does the theorem imply about how the neighborhood evolves? Whenever we are to the right of the division line, type 2 agents win every expiring lease. Thus, X_t rises. Note that this takes us *away* from the division line. Unless there is a strong movement of B_t to the left, type 2 agents will continue to enter until the neighborhood becomes completely segregated. Conversely, if we start to the left of the division line, type 1 agents win every lease, so X_t falls towards zero.

This shows that the neighborhood has two stable states: either all type 2 or all type 1. This means that almost all of the time we are on one of the two horizontal lines (X_t equal to zero or one). Changes occur when movements in B_t cause us to cross the division line. Theorem 2 shows that this is precisely the condition under which the prophecy of a transition becomes self fulfilling.

Theorem 2 *Assume that the neighborhood is initially segregated (X_0 equals zero or one). Then a transition to the other type begins at the first moment at which its expectation would be self fulfilling in a fixed environment.*

Proof

Consider the case $X_0 = 1$. By Theorem 1, if the world changes very slowly, type 1 agents begin to win all leases when B_t falls below $B^{h(1)}$. By definition of B^p, this means that type 1 agents have a higher payoff than type 2 agents from meeting a type 2 agent with probability $h(1)$ and a type 1 agent with probability $1 - h(1)$. But

$$h(1) = \frac{1}{2} + \frac{1}{2}\left[\frac{r}{r+2k}\right]$$

By (2) and (3), this is precisely the condition under which a transition to type 1 is self fulfilling in a fixed environment. The proof for $X_0 = 0$ is analogous.

4 Efficient Transitions

In this section we compare the actual timing of transitions to the timing that a social planner would choose. We assume that the social planner maximizes the expected present value of rents in the neighborhood. Since the rent equals an agent's expected utility from living in the neighborhood, and since agents get zero utility from living outside, this is equivalent to maximizing the sum of utilities of all agents (including, trivially, those who live outside).

This measure assumes that an agent's outside utility does not depend on the neighborhood's composition. If there is such an effect, the welfare analysis would change (but the prediction of when a transition will take place would not). The welfare analysis changes even if the population outside the neighborhood is 'very large'. Although in this case the neighborhood's demographics have a small effect on each person living outside, their aggregate effect might remain large due to the large population.

Patient Players

We first consider the case of complete patience ($r = 0$). Later we will discuss how impatience changes the welfare implications. From Theorem 1, we know that when $r = 0$, type 2 moves in when $B_t > B^{1/2}$ and type 1 moves in when $B_t < B^{1/2}$. Since the players (and the planner) are completely patient, the disutility of players *during* the transition can be ignored. Therefore, we can simplify by assuming that the neighborhood is inhabited entirely by type 2 when $B_t > B^{1/2}$ and entirely by type 1 otherwise.

When is it efficient for type 2 to inhabit the neighborhood? When $u(2,2,B_t) > u(1,1,B_t)$. In contrast, type 2's *do* inhabit the neighborhood whenever $B_t > B^{1/2}$. By (5), this holds whenever $\Delta(2, B_t) + \Delta(1, B_t) > 0$. These two conditions are generally not equivalent, so that sometimes the 'wrong' group inhabits the neighborhood. This is shown in Theorem 3.

Theorem 3 *Assume $r = 0$. The following three are equivalent:*

- *Type 2's inhabit the neighborhood inefficiently often.*
- $u(2, 1, B^{1/2}) > u(1, 2, B^{1/2})$.
- $u(2, 2, B^{1/2}) < u(1, 1, B^{1/2})$.

Conversely, these three are also equivalent:

- *Type 1's inhabit the neighborhood inefficiently often.*
- $u(2, 1, B^{1/2}) < u(1, 2, B^{1/2})$.
- $u(2, 2, B^{1/2}) > u(1, 1, B^{1/2})$.

Proof

By (4), $u(2,2,B_t) - u(1,1,B_t)$ *is increasing in* B_t. *This means that there is a threshold* b^{Eff} *such that it is efficient for 2's to inhabit the neighborhood if* $B_t > b^{Eff}$

and for 1's if $B_t < b^{Eff}$. *By Theorem 1, type 2's move in when* $B_t > B^{1/2}$ *and type 1's move in whenever* $B_t < B^{1/2}$. *Hence, type 2's inhabit the neighborhood inefficiently often if* $u(2,2,B^{1/2}) < u(1,1,B^{1/2})$. *This establishes that the first and third conditions are equivalent. The second condition is equivalent to the third since, by definition of* $B^{1/2}$,

$$u(1,1,B^{1/2}) + u(1,2,B^{1/2}) = u(2,1,B^{1/2}) + u(2,2,B^{1/2})$$

The proof of the second equivalence claim is analogous.

What is the intuition? Although the players and planner are perfectly patient, so that utilities during the transition are negligible, players decide where to live based only on the utility they will receive while their lease remains valid. This makes them effectively impatient: a player who decides where to live at the beginning of a transition will put considerable weight on the utility that she will receive during the transition. It turns out that during her lease, such a player expects to encounter equal numbers of neighbors of the two types. Therefore, *the lease is won by the type who gets more utility in a neighborhood that is split 50-50 between the two types.* That is, type i moves in if $u(i,i,B_t) + u(i,j,B_t) > u(j,j,B_t) + u(j,i,B_t)$. But from the point of view of social welfare, utility during the transition can essentially be ignored. Since the neighborhood is almost always segregated, what is important is that it be segregated in the 'right' way: the group with the higher payoff from living with *itself* should inhabit the neighborhood. This means that, to maximize social welfare, a transition from type i to type j should occur precisely when $u(i,i,B_t)$ becomes larger than $u(j,j,B_t)$. However, players do not internalize the effects of their locational decisions on the timing of transitions.

When do the two conditions coincide? When $u(2,1,B^{1/2}) = u(1,2,B^{1/2})$: when each group has the same effect on the other group's utility. If instead one group dislikes living with the other more, this group will tend to move out of the neighborhood 'too early' and move in 'too late'. This may give a guideline for policy intervention. A system of taxes and subsidies that equalizes these utilities will make actual play efficient. However, as explained in the introduction, this conclusion depends on two strong assumptions. One is our definition of social welfare as the sum of private willingness to pay. For example, in the case of poor vs. rich, it might be more appropriate to give the poor heavier weight than their willingness to pay would indicate, since willingness to pay is affected not only by utility but also by wealth. The second assumption is, as discussed above, that the neighborhood's composition has no effect on outside utility.

Impatient Players

We will not solve analytically for the social optimum. Instead, we will give qualitative conditions for maximizing social welfare and compare them to actual play. When players are impatient ($r > 0$), two things change. First, in terms of social welfare, utility during transitions cannot be ignored. Second, the actual dynamics of the neighborhood exhibit hysteresis (see Figure 2). That is, a transition to type 2 occurs after B_t rises a strictly positive distance beyond $B^{1/2}$. The reverse transition back to type 1 will not occur when B_t falls below the same threshold. Rather, B_t must fall some positive distance *below* $B^{1/2}$.

What is does an optimal policy look like for the timing of transitions? If we could ignore the utility during transitions, we would want to switch to type 2 when $u(2, 2, B_t) > u(1, 1, B_t)$ and to type 1 when the opposite holds. However, this policy is not optimal since transitions are costly: because players have a preference for segregation, utility is lower during transitions than afterwards. Taking this into account, the optimal policy would limit the frequency of transitions by requiring $u(i, i, B_t)$ to be sufficiently greater than $u(j, j, B_t)$ before a green light is given for type i to move in. To determine the optimal threshold, one should consider (a) the loss from retaining the wrong demographics, and (b) the chance that a reverse movement in B_t will make a transition back desirable in the near future. As $u(i, i, B_t)$ grows beyond $u(j, j, B_t)$, the loss (a) from not making a transition increases relative to the loss implied by (b) from making a transition. A transition becomes optimal when these two losses are equal.

Notice that both the social optimum and actual play exhibit hysteresis. However, the extent of hysteresis can differ. Moreover, the average of the upper and lower thresholds may also differ (as in the case of patient players). These two differences are the source of inefficiency when players are impatient.

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