Efficient Funding of Higher Education

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Abstract

I implement a basic tool of financial markets—namely, a portfolio—into student loans. In higher education funding, credit market loans (CMLs) lead to under-investment, while income-contingent loans (ICLs) produce over-investment. This research introduces a 'portfolio regime' (PR), which allows students to combine CMLs and ICLs. The model assumes that agents privately invest in higher education after receiving a noisy signal about their future incomes. The article compares a PR with a 'competition regime' (CR), which allows students to choose one type of loan but prohibits a portfolio. The key insight is that implementation of a PR may improve the efficiency of investment in higher education and social welfare. Nevertheless, the PR does not maximize social welfare because of adverse selection into ICL programs.

Keywords: Human capital accumulation; Education policy; Adverse selection; Higher education; Income contingent loans

JEL classification: I21; I22; I23; I24; I28; D31; H31

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1. Introduction

This article introduces a portfolio of higher education funding. The main advantages are that a portfolio of certain loans may improve the efficiency of the investment process and social welfare, though this portfolio does not maximize social welfare. I begin with a review of higher education funding. I then discuss the most common types of student loans. I conclude with a presentation of a portfolio of student loans.

The motivation of this research stems from the importance of higher education and the shift to private funding. Abundant empirical evidence highlights the important role of higher education in generating personal incomes and promoting the economic development of countries (Barro, 1998; Bassanini and Scarpenta, 2001; Restuccia and Urrutia, 2004). Correspondingly, investment in education has increased substantially in OECD countries during the second half of the twentieth century (Checchi, 2006; Greenaway and Haynes, 2003). As a result, budgetary pressures have recently led some European countries to shift from public funding of higher education (through various forms of income support transfers) to private funding (based on student loans). A structural transformation toward private higher education funding requires cautious treatment of inherent market failures.

The two existing types of student loans suffer from market failures. The first, credit market loans (CMLs), cause under-investment in higher education because of credit constraints, inadequate risk sharing and insufficient risk diversification. First, financial institutions avoid providing loans to disadvantaged students or provide loans with less favorable terms. The reason is that students' uncertain future incomes are insufficient collateral against the loans (see Galor and Zeira, 1993); even students themselves can partly grasp their future ability. Thus, without government intervention, disadvantaged but talented students may not obtain higher education. Even if the government guarantees access to CMLs, other factors may still lead to under-investment in higher education. CMLs provide inadequate risk sharing: while future incomes are uncertain, repayment is fixed. However, *ex ante*, risk averse students prefer a lower repayment in case of bad luck and a higher repayment in case of prosperity. Income-contingent repayment provides insurance against the uncertainty in future incomes. Furthermore, the fixed repayment damages not only students but also investors. That is, investors who buy shares of student loans have insufficient risk diversification. These market failures have led the literature to conclude that with CMLs, students under-invest in their higher education.

To overcome the inherent under-investment in higher education, several countries provide a second type of student loan: income-contingent loans (ICLs). The unique feature of ICLs—that is, income-contingent repayment—has both advantages and disadvantages. In addition to their positive implications on inequality measures, ICLs operate as an *ex ante* risk-sharing and diversification mechanism. The reason is cross-subsidization: agents with a higher income realization (discovered after the completion of higher education) provide a larger payback. As a result, people with high incomes subsidize those with low incomes. The economist Milton Friedman (1962) was the first to mention the potential advantages of ICLs to students and investors. He recommended ICLs because they allow students to forgo a fraction of their future income flow to finance their higher education investment. Students gain risk sharing and insurance against uncertainty in future incomes. Investors can buy ICLs and diversify their risks over students with different income prospects. Such diversification reduces the interest rate on ICLs and thus increases their attractiveness. Eckwert and Zilcha (2012) analyze the implications of alternative ICL programs, differing mainly in the degree of risk-pooling. Another advantage of ICLs is the ability to break even. The government can design the ICL program to break even without governmental funds. The advantages of ICL programs have spurred increasing interest in the literature and implementation by several countries, including Australia, New Zealand, Chile, Sweden and the United Kingdom. Chapman (2006) describes the experience in Australia, the first country to implement ICLs in 1989 (see also Barr and Crawford, 1998; Lleras, 2004; Nerlove, 1975; Woodhall, 1988).

Eckwert and Zilcha (2011) reveal the disadvantages of ICLs. Because of cross-subsidization, even people with negative net returns acquire higher education. Therefore, while funding higher education merely with CMLs leads to under-investment, pure ICLs cause the opposite situation—over-investment in higher education. This inefficiency in the two types of loans calls for improvement of the funding schemes for higher education.

To improve higher education funding, I introduce a 'portfolio regime' (PR), which allows students to combine CMLs and ICLs. A PR is a general case of the competition regime (CR) proposed by Eckwert and Zilcha (2011). Their CR allows students to choose one type of loan, a CML or an ICL, but *prohibits* a combination of them. In contrast, a PR does not constrain students to one market only. Students can choose any loan combination with non-negative shares of ICLs and CMLs. Therefore, a PR provides more financial flexibility than a CR. It is also more realistic because no country forces students to choose one type of loan. To investigate the implications of a PR, I develop a theory of educational production with uncertainty in future incomes and full access to private higher education funding. Risk averse agents invest in their higher education after receiving a noisy ability signal. The source of heterogeneity is agents' abilities, which are imperfectly correlated with their signals[1]. In this framework, I characterize the optimal loan decisions of students and derive key insights into social welfare.

I characterize which students choose CMLs only, ICLs only, and a portfolio of loans. The results suggest that intermediate-ability agents (according to their signal) choose a portfolio, while the rest of the students finance their higher education through one market only. Furthermore, agents with better signals or greater wealth prefer larger shares of CMLs than others under constant relative risk aversion (CRRA) or constant absolute risk aversion (CARA) utility functions. This pattern reverses under a quadratic utility function.

The main insight into social welfare is that a PR may lead to higher social welfare than a CR, though it does not maximize social welfare. By implementing a PR, some students enjoy the financial flexibility and choose a portfolio of loans, which makes them better off than under a CR. However, their shift to a portfolio may deteriorate the borrowing terms of ICL participants. Nevertheless, I prove that a PR Pareto dominates a CR in the common case when CMLs co-exist with ICLs in both regimes[2].

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[1] To simplify the analysis, I focus on one source of heterogeneity and ignore other influences (e.g., family background).

[2] An exception is the case when all students choose ICLs only under a CR. Here, a CR leads to higher social welfare than a PR.

To summarize, although introducing a PR into higher education funding improves social welfare overall, it is not 'the first best'.

The PR does not maximize social welfare because of externalities in students' behavior, which arises from students' belief that their entrance into ICL programs has a negligible effect on all ICL participants. However, augmented ICL share of high-signal students benefits all ICL participants. The reason is cross-subsidization; that is, students with high incomes subsidize those with low incomes, and as such, their participation improves the ICL borrowing terms. The externality causes adverse selection into the ICL program. High-signal students under-invest in ICLs. The adverse selection further worsens the borrowing terms of ICL participants and reduces the attractiveness of the program. As a result, high-signal students further depart to the credit market, which pushes the financing costs of the disadvantaged even higher, increases income inequality and reduces social welfare. Thus, a PR achieves greater social welfare than a CR, but it does not maximize social welfare.

Section 2 introduces the PR model. Section 3 characterizes the optimal loan decision in each signal group and conducts a comparative static analysis. In addition, I derive a closed-form solution for a quadratic utility function. Section 04 provides a social welfare analysis, and section 5 compares a PR with Eckwert and Zilcha's (2011) CR. Section 6 concludes. All proofs are relegated to the Appendix.

2. The Model

This section depicts the PR model. I begin by describing the timeline and the human capital formation. Then, I introduce the unique higher education funding scheme, a PR. Next, I present the individual behavior and production and define a welfare index. After describing the ingredients of the model, I define PR equilibrium.

2.1. Timeline

In an overlapping-generations model, the lifetime of agents consists of two periods: the youth period and the working period. In the youth period, agents acquire public education and higher education. All children obtain public education (K-12), which is compulsory, equal and free. Then, agents decide whether to take out a loan and acquire higher education, after which they enter the working period. In the working period, they earn a labor income based on their human capital, repay the student loan and consume the rest of their income. The two periods complete the lifetime of an agent[3].

2.2. Human capital formation

In this section, I describe the information on abilities and how abilities transform into human capital and incomes. The information on abilities changes at three points in life: birth, the completion of public education, and the completion of higher education. At birth, a continuum of agents $\left[0,1\right]$ is randomly endowed with innate ability, $\tilde{a}^i \in \left[a^1, a^2\right] \subset \mathbb{R}_+$. The realization of agent i's ability, a^i, is unknown. After the completion of compulsory public education, agents receive an ability signal, $y \in \left[y^1, y^2\right] \subset \mathbb{R}_+$, which has a simple interpretation—a result of matriculation tests or high school achievements *partly* correlated with actual ability. That is, larger signals are 'good news' because they forecast higher expected realization of ability and income. Therefore, I assume that signals and abilities satisfy 'the monotone likelihood ratio property' (MLRP) (see Milgrom, 1981)[4]. In addition, the ability signals are publicly observable, and I denote their distribution by $\nu\left(y\right)$. At this point, the information on each agent is the ability signal. Therefore, I denote all agents with signal y as signal group y. Thus, from the agent's perspective, ability is a realization of a random variable $\tilde{a}_y$ with an expectation of $\bar{a}_y = E\left[\tilde{a}_y\right]$. Yet there is no *aggregate* uncertainty in the economy, because the ability distribution of each signal group is known. To simplify the analysis and without loss of generality, I assume the following:

Assumption 1: $\tilde{a}_y = y + \tilde{\varepsilon}$, *and* $\tilde{\varepsilon} \sim \left(0, \sigma^2\right)$. ∘

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[3] The results remain in an extended model with an additional retirement period.

[4] The assumption of MLRP suggests that the ability distribution in a higher signal group 'first degree stochastically dominates' the ability distribution in a lower signal group; that is, $y' > y \Rightarrow \tilde{a}_{y'} \succ_1 \tilde{a}_y$.

Therefore, the signal reflects the mean ability in signal group y (i.e., $\bar{a}_y = E\left[\tilde{a}_y\right] = y$), and the variance, σ^2, measures the signal quality. If the variance is zero, there is no uncertainty. However, I assume that actual ability is *partly* correlated with the signal; that is, the variance is positive. After the completion of higher education, the realization of ability is fully revealed and determines labor income. Thus, the information on abilities changes at three points in life.

After describing the information on abilities, I now explain how they convert to human capital and income. After receiving the signal, agents choose their investment in higher education, $I \in \{0,1\}$. For simplicity, the investment choice is binary; that is, agents choose whether to invest in higher education ($I = 1$) or not ($I = 0$). Accordingly, the random human capital in signal group y is

$$\tilde{h}_y = \left\{ \begin{matrix} A & , if\ I = 0 \\ A + \tilde{a}_y, & if\ I = 1 \end{matrix} \right\}, \tag{1}$$

where A is the basic level of human capital after the completion of compulsory public education and $A + \tilde{a}_y$ is the random level of human capital of agents in signal group y in case they invest in higher education (*ex ante*). I assume that each person inelastically supplies l units of labor. Without loss of generality, let l=1. Incomes are based on the human capital. The *random* labor income, $\omega \tilde{h}_y$, equals the human capital in Eq. (1), multiplied by the wage rate for an effective unit of human capital, ω. Recall that the human capital and incomes are random until the completion of higher education. Then, in the working period, there is no uncertainty. Labor incomes are based on the *realization* of ability, a^i, which is already known. After describing the human capital and income formation, I now introduce the PR.

2.3. Higher education funding

The key novelty of a PR is allowing students to use the basic tool of capital markets—a portfolio. I begin with standard assumptions on student loans. Then, I depict the two types of loans that compose the portfolio. Afterwards, I introduce the PR. I assume that the costs of higher education are normalized to '1'. Accordingly, each student receives a loan of 1 unit and repays the loan in the working period. The government monitors the repayment through the tax system, and therefore there is no tax evasion. Now, the portfolio of loans consists of the two most common student loans, CMLs and ICLs, which differ in their repayment obligation. The payback of CMLs is the interest rate $R = 1 + r$. The interest rate is exogenously given by the gross international interest rate (see section 2.5). In contrast, the payback of ICLs, $R\frac{a^i}{\bar{a}}$, depends on the realization of ability, a^i, and on $\bar{a}$, a plug number that breaks even with student loans. Accordingly, the payback increases as ability rises. High-income ICL participants, with $a^i > \bar{a}$, cross-subsidize the remaining participants. That is, their repayment is larger than the interest rate, while all other participants' repayment is lower than the interest rate. Note that while $R\frac{a^i}{\bar{a}}$ is the *actual* ICL repayment in the working period, the repayment after the completion of public education, $R\frac{\tilde{a}_y}{\bar{a}}$, is the realization of a random variable with an expectation of $R\frac{\bar{a}_y}{\bar{a}}$ for each signal group y. After presenting the two loans that compose the portfolio, I now introduce the PR.

In a PR, students simply mix the two loans. That is, after the completion of public education, each agent in signal group y decides whether to finance his or her higher education entirely through an ICL or a CML or to combine them. I denote by $\theta_y \in [0,1]$ the CML share and by $1 - \theta_y$ the ICL share in his or her portfolio. Thus, the random payback of each agent in signal group y is a weighted average of the two, and the weights are the shares of each loan in the portfolio

$$(2) \qquad \theta_y R + (1 - \theta_y) R\frac{\tilde{a}_y}{\bar{a}}.$$

Recall that the government determines $\bar{a}$ to break even with the ICL program. Thus, the government equates the *ex post* mean repayment *across all signal groups* and the interest rate; that is,

(3) $$E\left[\theta_y R + \left(1-\theta_y\right) R \frac{\bar{a}_y}{\bar{a}}\right] = R.$$

Recall that the interest rate is exogenously given. Using $E\left[\theta_y R\right] = RE\left[\theta_y\right]$ and equality (3), $\bar{a}$ satisfies

(4) $$\bar{a} = \frac{E\left[\left(1-\theta_y\right)\bar{a}_y\right]}{E\left[1-\theta_y\right]} = E\left[\bar{a}_y \middle| \theta_y < 1\right] - \frac{cov\left[\bar{a}_y, \theta_y\right]}{E\left[1-\theta_y\right]}.$$

The first equality suggests that $\bar{a}$ is a *weighted* mean ability of ICL participants ($\theta_y < 1$). The second equality implies that the calculation of $\bar{a}$ takes into account the selection of students into the ICL program[5]. After presenting the borrowing terms in a PR and how it breaks even, I now turn to the optimal choice of students.

2.4. Individual behavior

In this section, I describe the optimal loan decision of students, that is, how they choose their optimal mixture of ICLs and CMLs. I then verify that a set of students exist who select a portfolio of the two loans. Agents gain utility from consumption in the working period. The random consumption of agents with signal y equals their labor income net of their repayment obligation (2) given their human capital (1); that is,

(5) $$\tilde{c}_y = \left\{ \begin{array}{lr} A\omega & , \quad if\ I = 0 \\ \underbrace{\left(A + \tilde{a}_y\right)\omega}_{income} - \left[\underbrace{\overbrace{\theta_y R}^{CML} + \overbrace{\left(1-\theta_y\right) R \frac{\tilde{a}_y}{\bar{a}}}^{ICL}}_{repayment\ obligation}\right] & , \quad if\ I = 1 \end{array} \right\}.$$

[5] Suppose that $cov\left[\bar{a}_y, \theta_y\right]$ is positive; that is, signal groups with higher expected ability choose larger shares of CMLs than lower signal groups. In this case, $\bar{a}$ is lower than the mean ability of ICL participants (i.e., $\bar{a} < E\left[\bar{a}_y \middle| \theta_y < 1\right]$). The intuition is clear: when high-signal groups (the subsidiaries) leave the ICL program, $\bar{a}$ declines, and therefore the expected payback, $R\frac{\bar{a}_y}{\bar{a}}$, increases for all ICL participants. In contrast, if $cov\left[\bar{a}_y, \theta_y\right]$ is negative, high-signal groups prefer larger shares of ICLs. Their augmented participation in the ICL program reduces the financing costs for all ICLs participants.

I assume that investment in higher education is profitable.

Assumption 2: $\bar{a}\omega > R$. ◦

With assumption 2, at least one student has positive net expected returns from financing higher education with ICLs only (i.e., $\bar{a}_y\left(\omega - \frac{R}{\bar{a}}\right) > 0$). However, as a result, investment in higher education is beneficial for *all* agents, though they may differ in the mixture of ICLs and CMLs they prefer[6]. The von Neumann–Morgenstern utility function $u\left(\tilde{c}_y\right)$: $\mathbb{R}_+ \rightarrow \mathbb{R}$ is twice differentiable, strictly increasing and concave. Each agent with signal y chooses the optimal share of CML, θ_y, by maximizing expected utility from consumption (5):

$$(6) \qquad \underset{\theta_y}{Max}\left\{E\left[u\left(\tilde{c}_y\right)\right]\right\}.$$

Definition 1: *The Appendix denotes standard CRRA, CARA and quadratic utility functions that I use in the rest of the article.*

After characterizing the optimization problem, I now verify that the portfolio set exists. That is, there is a set of students who prefer a portfolio $\theta_y \in (0,1)$ of the two loans rather than CMLs only or ICLs only. The first step defines three signal groups. Let

$$(7) \qquad \tilde{c}_{y,0} = \tilde{c}_y\left(\theta_y = 0\right) = A\omega + \tilde{a}_y\left(\omega - \frac{R}{\bar{a}}\right)$$

and

$$(8) \qquad \tilde{c}_{y,1} = \tilde{c}_y\left(\theta_y = 1\right) = A\omega + \tilde{a}_y\omega - R$$

[6] This undesirable feature of ICLs that all agents invest in higher education is usually prevented by a government policy that restricts access to higher education. Note that if I restrict access to agents with positive returns, the results are qualitatively similar.

denote the random consumption of an agent in signal group y if he or she finances higher education through ICLs only or CMLs only, respectively. The third signal group, $\hat{y}$, denotes agents who are indifferent between ICLs only and CMLs only.

Definition 2: *The cutoff signal* $\hat{y}$ *between ICLs only and CMLs only satisfies*[7]

$$(9) \qquad E\left[u\left(\tilde{c}_{\hat{y},0}\right)\right] = E\left[u\left(\tilde{c}_{\hat{y},1}\right)\right] > u\left(A\omega\right). \circ$$

The second step asserts that agents in the cutoff signal group belong to the portfolio set. Thus, the portfolio set exists if, as a sufficient condition, the cutoff signal $\hat{y}$ lies within the signals' distribution[8]. Furthermore, continuity implies that signal groups in a sufficiently small neighborhood around $\hat{y}$ also prefer to combine the two loans.

Proposition 1: *Assume that* $\hat{y} \in \left(y^1, y^2\right)$. *Then, the portfolio set is not empty.* $\circ$

The portfolio set lies between the following cutoffs:

Definition 3: *The portfolio cutoff signals* y' *and* y'' *are indifferent between a portfolio and ICLs (CMLs) only, respectively*[9]:

$$(10) \qquad E\left[u\left(\tilde{c}_{y',\theta}\right)\right] = E\left[u\left(\tilde{c}_{y',0}\right)\right] \textit{ and}$$

$$E\left[u\left(\tilde{c}_{y'',\theta}\right)\right] = E\left[u\left(\tilde{c}_{y'',1}\right)\right]. \circ$$

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[7] Assumption 2 ensures that the ICL program does not break down. Thus, the inequality $E\left[u\left(\tilde{c}_{\hat{y},1}\right)\right] > u\left(A\omega\right)$ holds. Note that there can be more than one cutoff signal, as I demonstrate in the sequel.

[8] Note that in CR equilibrium, the assumption that $\hat{y} \in \left(y^1, y^2\right)$ ensures that the sets of CMLs only and ICLs only are not empty.

[9] Note that agents in signal groups y' and y'' strictly differ from $\hat{y}$ because they prefer ICLs (CMLs) only to CMLs (ICLs) only, respectively. Combining the concavity of the utility function and (10) obtains

$E\left[u\left(\tilde{c}_{y',\theta}\right)\right] = E\left[u\left(\tilde{c}_{y',0}\right)\right] > \theta E\left[u\left(\tilde{c}_{y',0}\right)\right] + (1-\theta) E\left[u\left(\tilde{c}_{y',1}\right)\right]$. Thus, $E\left[u\left(\tilde{c}_{y',0}\right)\right] > E\left[u\left(\tilde{c}_{y',1}\right)\right]$,

$E\left[u\left(\tilde{c}_{y'',\theta}\right)\right] = E\left[u\left(\tilde{c}_{y'',1}\right)\right] > \theta E\left[u\left(\tilde{c}_{y'',0}\right)\right] + (1-\theta) E\left[u\left(\tilde{c}_{y'',1}\right)\right]$. Thus, $E\left[u\left(\tilde{c}_{y'',1}\right)\right] > E\left[u\left(\tilde{c}_{y'',0}\right)\right]$.

After presenting how the PR works, I now describe the production sector and the welfare index. I then define the equilibrium under the PR.

2.5 Production

I use standard simplifying assumptions of the production sector. Recall that with assumption 2, all agents invest in higher education. Accordingly, given the distribution of abilities, the stock of human capital (1) equals

$$(11) \qquad H = \int_{y^1}^{y^2} \overline{h}_y v(y)\,dy = A + E[\tilde{a}].$$

Using the stock of human and physical capital, competitive firms produce one consumption good. The production function $F(K,H)$ exhibits constant returns to scale, with positive and decreasing marginal returns. I assume a small country and international mobility of physical capital. This assumption is consistent with the empirical observation that globalization has promoted international mobility of physical capital more than that of labor. Therefore, the interest rate, $R = 1 + r$, is exogenously given by the gross international interest rate. Physical capital fully depreciates in the production process. Maximization of profits yields the stocks of physical capital and human capital:

$$(12) \qquad \begin{aligned} F_K\left(\frac{K}{H},1\right) &= R \\ F_L\left(\frac{K}{H},1\right) &= \omega \end{aligned}$$

That is, firms hire physical capital and human capital until their marginal product equals the interest rate and the wage rate, respectively.

2.6. Welfare index

In this section, I define the social planner's welfare index and its main features. The social welfare index equals

(13) $$W = \int_{y^1}^{y^2} \upsilon\left(\bar{c}_y\right) v\left(y\right) dy,$$

where $\bar{c}_y = E\left[\tilde{c}_y\right]$.

Assumption 3: *$\upsilon : \mathbb{R}_+ \rightarrow \mathbb{R}$ is twice differentiable, strictly increasing and concave.* ◦

The welfare function aggregates observable data—the mean consumption in each signal group given the distribution of signals. The concavity of the social planner's welfare index reflects inequality aversion. That is, social welfare rises if either the mean consumption increases in a particular signal group or income inequality declines across signal groups. I use the index in the subsequent welfare analysis. After introducing the ingredients of the model, I now define the PR equilibrium.

2.7. PR equilibrium

I first define the equilibrium and then explain how to solve it.

Definition 4: *Given the international gross interest rate, $R = 1 + r$, the distribution of abilities and the distribution of signals, PR equilibrium consists of a vector $\left(\omega, K, H\right) \in R_+^3$, a share of CMLs, θ_y, for each signal group , such that*

i. *The stock of human capital, H, satisfies (11);*
ii. *The wage rate and physical capital satisfy (12);*
iii. *The share of CMLs, θ_y, maximizes the expected utility in signal group y (6); and*
iv. *The ICLs program breaks even by $\bar{a}$ according* to *(4).* ◦

Given the distribution of abilities, the stock of human capital is determined by Eq. (11). The first equality in Eq. (12) implies that the exogenous interest rate, R, uniquely determines the ratio $\frac{K}{H}$ and, thus, K. Then, the wage rate, ω, is uniquely determined by substituting $\frac{K}{H}$ in the second equality. Given the interest rate, the wage rate, the signal and $\bar{a}$, each agent chooses the share of CMLs, θ_y, which completes the equilibrium. The following section analyzes the optimal loan decision of each signal group; the rest of the article conducts a welfare analysis and compares a PR with a CR.

3. Optimal loan decisions

This section first characterizes the tradeoff between the two types of loans. I then analyze low- and high-signal groups separately, after which I analyze a specific case—a quadratic utility function.

3.1. The tradeoff between ICLs and CMLs

The optimal loan decision involves a tradeoff, which is common in financial markets—that is, a tradeoff between risk and expected returns (loan repayment, in this case). Regarding the comparison between ICLs and CMLs, ICLs reduce uncertainty in future incomes but increase expected repayment of high-signal groups, both because of cross-subsidization. Deriving the expected utility (Eq. 6) by θ_y yields[10]

$$\frac{\partial E\left[u\left(\tilde{c}_y\right)\right]}{\partial \theta_y} = \frac{R}{\bar{a}} E\left[\left(\tilde{a}_y - \bar{a}\right)u'\left(\tilde{c}_y\right)\right] = \frac{R}{\bar{a}}\left(\left(\bar{a}_y - \bar{a}\right)E\left[u'\left(\tilde{c}_y\right)\right] + cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)\right) \tag{14}$$

Careful inspection of the first-order condition recognizes the tradeoff between risk and expected returns. The first component reflects the desire to reduce expected repayment. Its sign depends on the signal's magnitude, $\bar{a}_y - \bar{a}$. Recall that high-signal groups, with $\bar{a}_y > \bar{a}$, cross-subsidize low-signal groups in the ICL program. Therefore, their ICL expected repayment is larger than the interest rate, while the remaining participants enjoy a lower expected repayment. Thus, if students' only concerns were to reduce their expected payback, high-signal students would prefer CMLs only, while low-signal students would prefer ICLs only[11]. However, students are risk averse.

The second component of the first-order condition (14) reveals the desire to mitigate uncertainty in future incomes. Large absolute values of $cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)$ suggest that

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[10] Note that $\theta_y = ArgMax\left[E\left(u\left(\tilde{c}_y\right)\right)\right]$ is unique because the expected utility is strictly concave and the random consumption (5) is linear with respect to θ_y.

[11] As a result, if agents are risk neutral, then $\bar{a} \to 0$ and the equilibrium collapses to CMLs only for all y. Thus, risk aversion is necessary to avoid this trivial solution in both the PR and the CR.

the expected consumption is strongly related to the signal (see Lemma 1 in the Appendix)[12]. In this case, ICLs are a risk-sharing tool because they reduce the dispersion of *ex post* incomes. Thus, the optimal mixture of loans for each student balances the tradeoff between risk and expected returns.

Note that the first-order condition (14) reveals *externalities* in students' behavior. Students take $\bar{a}$ as given; thus, they assume that their decision to enter the ICL program has a negligible effect on the ICL expected repayment, $R\frac{\bar{a}_y}{\bar{a}}$. However, ICL participation of high- (low-) signal groups improves (worsens) the borrowing terms. The externalities imply that high- (low-) signal students under- (over-) invest in ICLs. I discuss the welfare implications of these externalities in sections 4 and 5. After explicating the tradeoff between ICLs and CMLs, the following sections analyze low- and high-signal groups separately.

3.2. Low-signal groups

The optimal loan decision of agents with low-income prospects (i.e., $\bar{a}_y \le \bar{a}$) is quite trivial. They prefer to fund their higher education entirely through the ICL program.

Proposition 2: *Assume that y satisfies $\bar{a}_y \le \bar{a}$. Then, $\theta_y = 0$.* ◦

Agents with low-income prospects choose ICLs for two reasons. First, the cross-subsidization provides relatively improved borrowing terms. That is, their expected repayment obligation, $\frac{R\bar{a}_y}{\bar{a}}$, is lower than that of CMLs (the interest rate). Second, they gain insurance against the uncertainty in future incomes. These reasons lead them to prefer ICLs only[13]. In contrast, for high-signal groups the optimal loan decision is more complicated.

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[12] The $cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)$ is negative because the signal is positively correlated to the expected consumption according to MLRP and agents are risk averse. Note that an ICL has a riskier payback. Thus, it is the 'safer' loan with regards to uncertainty in future income.

[13] According to the first-order condition (14), low-signal groups actually prefer a negative share of CMLs but are restricted to $\theta_y \in [0,1]$. Even signal group $y = \bar{a}$, with identical expected repayment

3.3. High-signal groups

In this section, I focus on the optimal loan decision of high-signal groups, with $\bar{a}_y > \bar{a}$, and discuss two arguments. First, high-signal groups may choose a portfolio of the two loans. Second, with CRRA or CARA utility functions (see definition 1), as the signal rises, they use increasingly larger shares of CMLs.

Corollary 1: *Assume that* $\hat{y} \in \left[\underline{y}, \bar{y}\right]$. *Then, the portfolio set* $\theta_y \in (0,1)$ *exhibits* $\bar{a}_y > \bar{a}$. ◦

Recall that a portfolio set exists and that low-signal groups prefer ICLs only (see propositions 1 and 2). Therefore, the portfolio set consists of high-signal groups[14]. This is not surprising, because high-signal groups face a tradeoff between ICLs and CMLs. ICLs provide them insurance against the uncertainty in future incomes and, at the same time, less favorable borrowing terms than CMLs. The balance of these contradicting incentives may produce a portfolio of CMLs only or ICLs only, depending on these groups' signal, risk aversion and wealth.

Now, using a comparative static analysis, I examine the effect of the signal on the optimal loan decision of high-signal groups. An increase in the signal augments the expected income, which has two effects. First, a higher expected income implies worse ICL borrowing terms. That is, the expected ICL repayment, $\frac{\bar{a}_y}{\bar{a}} R$, rises. As a result, ICLs become less attractive[15]. Second, the absolute risk aversion (ARA) alters. Assuming a CRRA utility function, wealthier agents are *less* risk averse (see definition 1). As a result, they are willing to tolerate a more risky income to raise the expected income. Combining the two effects, as the signal rises, CRRA students use

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under CMLs and ICLs, prefers ICLs only because of risk aversion. Its random consumption with a CML is a mean-preserving spread of its random consumption with an ICL.

[14] It is easy to verify that $\bar{a}_y > \bar{a}$ is a necessary condition for an interior solution in Eq. (14).

[15] Note that the price effect of a larger signal consists of a substitution effect (ICLs become more expensive) and a negative income effect. As income declines, agents may increase the share of ICLs. However, according to proposition 2 in Landsberger and Meilijson (1989, pp. 208–209), the substitution effect overcomes the income effect.

increasingly larger shares of CMLs to finance their higher education. The result further holds for CARA utility function, in which ARA does not depend on wealth.

Proposition 3: U*nder CRRA or CARA utility functions,* θ_y *is an increasing function of* y. ◦

3.4. Illustration of the optimal loan decisions

This section summarizes the optimal loan decisions of all signal groups with CRRA or CARA utility functions. Fig. 1 demonstrates the CML share under a PR (the solid line) and a CR according to Eckwert and Zilcha (2011) (the dashed line). Note that a CR is a private case of a PR, in which agents are not allowed to choose a portfolio.

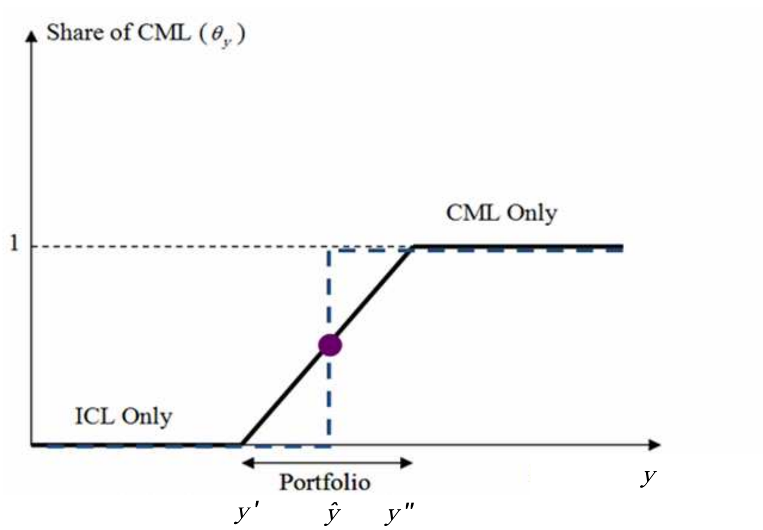

Fig. 1. CML share with a CRRA or a CARA utility function. The x-axis denotes the signal y, and the y-axis denotes the CMLs share $\theta_y \in [0,1]$. $\theta_y = 0$ if the funding scheme consists of ICLs only and $\theta_y = 1$ if the agent prefers funding entirely through CML. The completed line represents PR, and the dashed line denotes CR. For simplicity, θ_y is a straight line. y' denotes the cutoff signal between a portfolio and ICLs only; y'' denotes the cutoff signal between a portfolio and CMLs only (see definition 3).

Low-signal groups, with $\bar{a}_y \le \bar{a}$, finance their higher education entirely through the ICL program (propositions 2 and 5). In contrast, for high-signal groups, with $\bar{a}_y > \bar{a}$, selecting ICLs only has a cost in terms of expected consumption. As a result, a set of these students prefers a portfolio (corollary 1), including indifferent agents between

ICLs only and CMLs only (proposition 1). Therefore, the portfolio set has intermediate signals. However, agents with extreme signals select one market only. Although students with CRRA or CARA utility functions use increasingly larger shares of CMLs as the signal rises (proposition 3), the following section suggests that this pattern does not hold for quadratic utility functions.

3.5. A quadratic utility function: A closed-form solution

In this section, I assume that the utility function is quadratic with the range $u'(\tilde{c}_y) = \alpha - \beta \tilde{c}_y \geq 0$ (see definition 1). In this case, I obtain a closed-form solution, in which I reveal the concavity of the CML share and analyze the determinants of the optimal loan decision. Afterwards, I discuss existence. Rearranging the random consumption $\tilde{c}_y$ (5) and substituting $u'(\tilde{c}_y) = \alpha - \beta \tilde{c}_y$ in the first-order condition (14), an interior solution satisfies:

$$\begin{aligned} \frac{\partial E\left[u\left(\tilde{c}_y\right)\right]}{\partial \theta_y} &= \frac{R}{\bar{a}} E\left[\left(\tilde{a}_y - \bar{a}\right)\left(\alpha - \beta\left(\left(A + \tilde{a}_y\right)\omega - R\frac{\tilde{a}_y}{\bar{a}} + \frac{R}{\bar{a}}\theta_y\left(\tilde{a}_y - \bar{a}\right)\right)\right)\right] \\ (15) \qquad &= kE\left[\tilde{a}_y - \bar{a}\right] - \left(\omega - \frac{R}{\bar{a}}\left(1 - \theta_y\right)\right) E\left[\left(\tilde{a}_y - \bar{a}\right)^2\right] \\ &= 0 \end{aligned}$$

where $k = \frac{\alpha}{\beta} - \left(\left(A + \bar{a}\right)\omega - R\right)$ is positive[16].

Substituting $E\left[\left(\tilde{a}_y - \bar{a}\right)^2\right] = \left[E\left(\tilde{a}_y - \bar{a}\right)\right]^2 + \sigma^2$ in equality (15), I solve the CML share:

$$(16) \qquad \theta_y = \frac{k\bar{a}\left(y - \bar{a}\right)}{R\left(\left(y - \bar{a}\right)^2 + \sigma^2\right)} - \left(\frac{\bar{a}\omega - R}{R}\right)^{17}$$

under the constraint that $\theta_y \in [0,1]$.

[16] See Lemma 2 on the Appendix; I obtain the second equality in (15) using $E\left[\tilde{a}_y\left(\tilde{a}_y - \bar{a}\right)\right] = E\left[\left(\tilde{a}_y - \bar{a}\right)^2\right] + \bar{a}E\left[\tilde{a}_y - \bar{a}\right]$.

[17] $E\left[\left(\tilde{a}_y - \bar{a}\right)^2\right] = E\left[\tilde{a}_y^{\,2}\right] - 2\bar{a}E\left[\tilde{a}_y\right] + \bar{a}^2$. Using assumption 1, $E\left[\tilde{a}_y^{\,2}\right] = E\left[\left(\bar{a}_y + \varepsilon\right)^2\right] = \bar{a}_y^{\,2} + \sigma^2$. Note that it is easy to confirm from Eq. (16) that the results in proposition 2 and corollary 1 hold. That is, low-signal groups, with $y \leq \bar{a}$, participate solely in the ICL program, while high-signal students may choose a portfolio of the two loans.

Eq. (16) reveals that the CML share is *concave* because of two contradicting effects. First, a larger signal forecasts a higher expected income, and thus a larger ICL expected repayment. As a result, agents prefer to increase the share of CML. Second, in a quadratic utility function, wealthier agents are more risk averse. Thus, they are willing to forgo more consumption to acquire ICLs as a risk-sharing tool. Therefore, these two contradicting effects generate a *concave* CML share.

Proposition 4: θ_y *is a concave function of* y. ◦

Fig. 2 summarizes the optimal loan decisions of all signal groups.

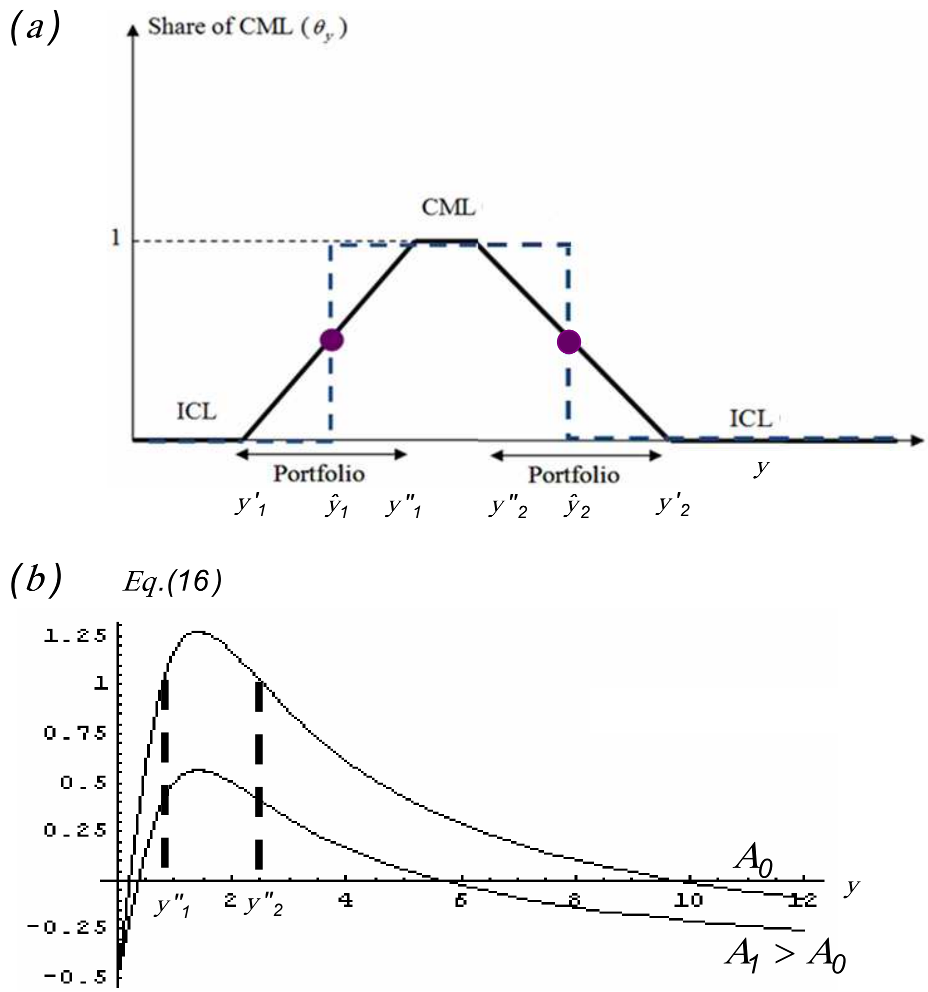

Fig. 2. (a) CML share under a quadratic utility function. The x-axis denotes the signal y, and the y-axis denotes the CMLs share $\theta_y \in [0,1]$. $y'_{1,2}$ are the cutoff signals between a portfolio and ICLs only; $y''_{1,2}$ are the cutoff signals between a portfolio and CMLs only (definition 3). The solid line represents PR, and the dashed line denotes CR. For simplicity, θ_y is a straight line. (b) Eq. (16). The lower illustration demonstrates a higher basic level of human capital; that is, $A_1 > A_0$ (Eq. (1)).

According to Fig. 2 and proposition 5, agents with sufficiently low signals (lower than $\bar{a}+\sqrt{\sigma^2}$) behave qualitatively similar to the cases of the CRRA and CARA utility functions (proposition 3). That is, they increase their CML share as the signal rises. Intermediate-signal agents, with $y \in [y''_1, y''_2]$, prefer CMLs only. However, for sufficiently large signals, the opposite case occurs. The increasing ARA becomes the dominant factor; that is, as the signal rises, agents reduce the CML share because they become more risk averse. The high-signal groups with $y \geq y'_2$, participate solely in the ICL program. Thus, the CML share is concave.

In addition to the concavity, Eq. (16) reveals the main determinants of the optimal loan decision: ARA, wealth and signal quality. First, several parameters affect the ARA. Recall that, assuming a quadratic utility function, agents are more risk averse if β/α rises or if they are more affluent (i.e., A or ω grow; definition 1). Higher ARA results in a larger ICL share. The second reason to increase insurance through ICLs is income uncertainty. Recall that the actual ability is *partly* correlated with the signal (assumption 1). The variance of the ability distribution of each signal group, σ^2, measures the signal quality. If the variance increases, the signal becomes less informative, and therefore the uncertainty in future incomes grows. To diminish the increasing uncertainty, students increase their ICL share.

Corollary 2: *θ_y is decreasing in β/α, A, ω and σ^2.* ◦

For example, Fig. 2(b) demonstrates that as agents become more risk averse, they increase their participation in the ICL program. Ceteris paribus, I increase the basic level of human capital (i.e., $A_1 > A_0$), which increases incomes and, thus, the ARA. As a result, the ICLs-only set expands at the expense of the portfolio set, which in turn increases at the expense of the CMLs-only set. Recall that a qualitatively similar results occur if β/α, ω, or σ^2 rise. That is, higher ARA, wealth or signal quality reduces CML shares.

Moreover, corollary 2 has implications for the existence of the three sets: portfolio, CMLs only and ICLs only. Corollary 2 implies that a sufficiently high ARA, wealth and σ^2 generate an equilibrium with all students choosing ICLs only.

Proposition 5: *If* $\sigma^2 \geq \left(\frac{k\bar{a}}{2(\bar{a}\omega - R)}\right)^2$*, then* $\theta_y = 0$ *for all* y*.* ◦

To avoid this trivial solution and have a portfolio set, I add a necessary assumption that the ARA, wealth and σ^2 are sufficiently low.

Assumption 4: $\sigma^2 < \left(\frac{k\bar{a}}{2(\bar{a}\omega - R)}\right)^2$. ◦

A more binding assumption is necessary to ensure existence of the CMLs-only set.

Assumption 5: $\sigma^2 \leq \left(\frac{k}{2\omega}\right)^2$. ◦

Corollary 3 summarizes the conditions for existence of the three sets: portfolio, CMLs only and ICLs only.

Corollary 3:

(a) The ICLs-only set is not empty.

(b) If assumption 4 holds and $y^2 > y'_1$*, then the portfolio set is not empty.*

(c) If assumption 5 holds and $y^2 > y''_1$*, then the CMLs-only set is not empty.* ◦

For example, in Fig. 2(b), when the basic level of human capital equals A_0, the three sets exist. In this case, assumption 5 holds and the upper bound of the ability distribution, y^2, is sufficiently large. However, an increase in the basic level of human capital, $A_1 > A_0$, eliminates the CMLs-only set completely. In this case, assumption 5 does not hold. The rest of the article examines the social welfare implications of the PR and compares it with the CR.

4. Social welfare

In this section, I demonstrate that the PR does not maximize social welfare (13). I describe the externalities in students' behavior and their two implications: over-investment in higher education and adverse selection into the ICL program. The source for inefficiencies in higher education funding is externalities in students' behavior. Students believe that their decision to join the ICL program or not has a negligible effect on all ICL participants. However, an increase in the ICL share of high-signal groups reduces the expected repayment obligation of *all* ICLs participants, $R\frac{\bar{a}_y}{\bar{a}}$ (because $\bar{a}$ rises; Eq. (4)). In contrast, augmented ICL participation of low-signal groups, which are subsidized by the program, damages the borrowing terms of *all* ICL participants. Because students do not internalize their influence on the borrowing terms, high-signal students under-invest in ICLs and low-signal students over-invest in ICLs relative to the social optimum. Furthermore, low-signal students, in spending high-signal groups' money, invest in higher education even if the expected net returns are negative. As a result, the ICL program is adversely selected toward low-signal groups. Therefore, the terms of loan repayment become worse for all ICLs participants, which further reduces the attractiveness of the program. Consequently, high-signal students further depart to the credit market to lower their repayment obligation. This effect pushes the financing costs of low-signal groups even higher, which further increases the income inequality and reduces social welfare.

Proposition 6:

(a) Aggregate investment in education is sub-optimally high.

(b) Investment of high-signal groups in ICLs is sub-optimally low. ◦

Thus, PR does not maximize social welfare. Fig. 3 demonstrates that the portfolio cutoff signals do not coincide with their socially optimal levels. The arrows indicate the direction for improving social welfare. The social planner prefers to increase the ICL participation of high-signal groups.

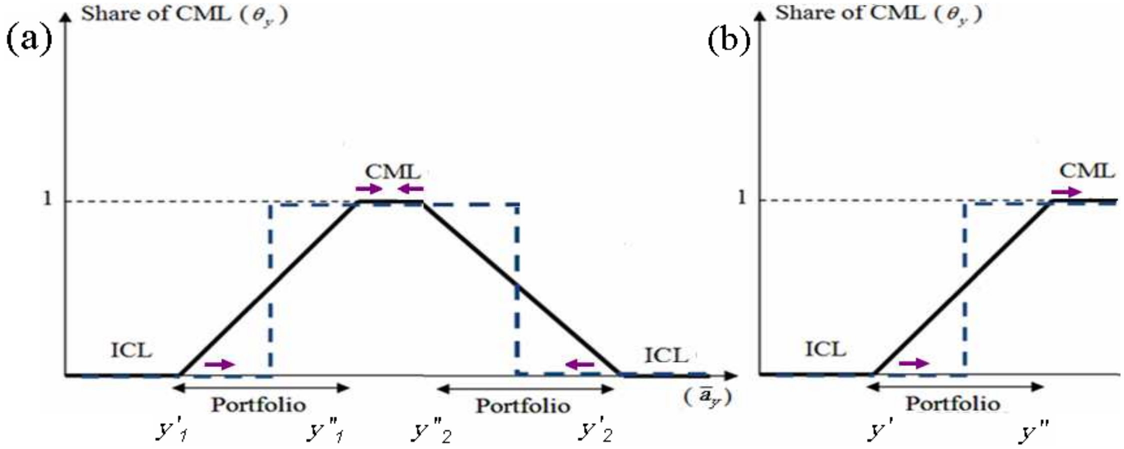

Fig. 3. CML share under (a) a quadratic utility function and (b) a CRRA or CARA utility function. The arrows indicate the direction for improving social welfare.

While a PR does not maximize social welfare, the following section demonstrates that in most cases, a PR Pareto dominates a CR.

5. PR versus CR

This section compares a PR and a CR. First, I discuss the effect of implementing a PR. Second, I prove that in most cases, a PR Pareto dominates a CR. Suppose that a CR exists and now the government establishes a PR. That is, agents are no longer restricted to one market only. What are the implications on each signal group? Some students enjoy this financial flexibility and decide to combine the two loans. They are clearly better off because they *voluntarily* shift to a portfolio. That is, their preferences reveal that they prefer a portfolio rather than borrow in one market only. The rest of the students do not change their loan decisions and still finance their higher education through one market only[18]. Signal groups that remain entirely in the credit market enjoy the same borrowing terms. Thus, they are indifferent between the two regimes. Therefore, a shift to a PR does not damage the CMLs-only set or the portfolio set.

Now, I analyze the effect on signal groups that remain entirely in the ICL program. Although they continue choosing ICLs only, they may be worse off. The shift of other students to a portfolio may deteriorate their borrowing terms. That is, their expected

[18] They must choose the same market because of continuity.

payback, $R\frac{\bar{a}_y}{\bar{a}}$, may rise (if $\bar{a}$ declines). Fig. 4 demonstrates the effect of the portfolio set on ICL participants under CRRA or CARA utility functions (A similar illustration is derived for a quadratic utility function). For this purpose, I divide the portfolio set into two sub-sets. The right set consists of high-signal groups that partially enter the ICL program (i.e., their θ_y declines). The left set includes high-signal groups that partially exit the ICL program (i.e., their θ_y increases; corollary 1). Consequently, while the right triangle improves the borrowing terms of ICLs participants, the left triangle worsens them.

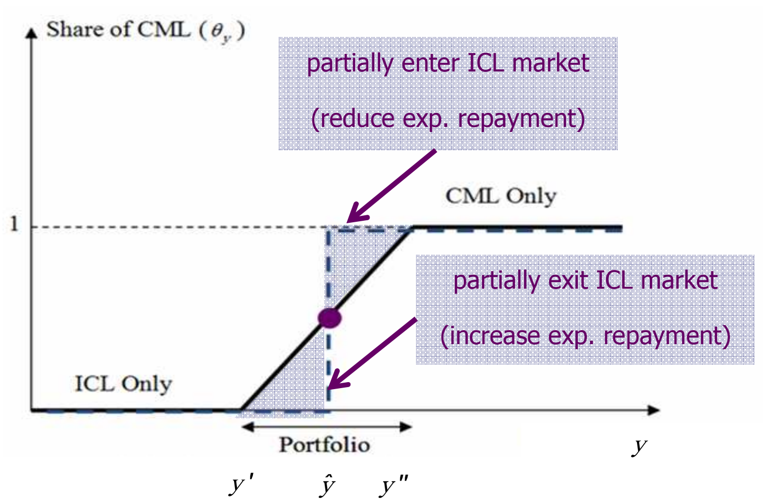

Fig. 4. CML share under CRRA or CARA functions. The solid line represents PR, and the dashed line denotes CR. Allowing agents to combine ICLs and CMLs, the right (left) triangle denotes students who partially enter (exit) the ICL program. For simplicity, θ_y is a straight line.

Because of the offsetting effects, detecting whether ICLs participants are better off requires a formal proof.

Proposition 7: *Assume that the utility function is CRRA or CARA or quadratic. Then,*

(a) if $\hat{y} \in (y^1, y^2)$, *then a PR Pareto dominates a CR, and*

(b) if $y^2 \in (y', \hat{y})$, *then a CR leads to higher social welfare than a PR.* ∘

Proposition 7 compares the PR and CR in two cases. The common case, demonstrated in Fig. 4, assumes that CMLs co-exist with ICLs; that is, $\hat{y} \in (y^1, y^2)$ (proposition 1). In this case, the PR Pareto dominates the CR because both ICL participants and the portfolio set are better off, while signal groups with CMLs only are not damaged.

The only exception to this result, demonstrated in Fig. 5, occurs if under a CR, all students choose ICLs only; that is, $y^2 \in (y', \hat{y})$. In this case, the CR leads to higher social welfare than the PR.

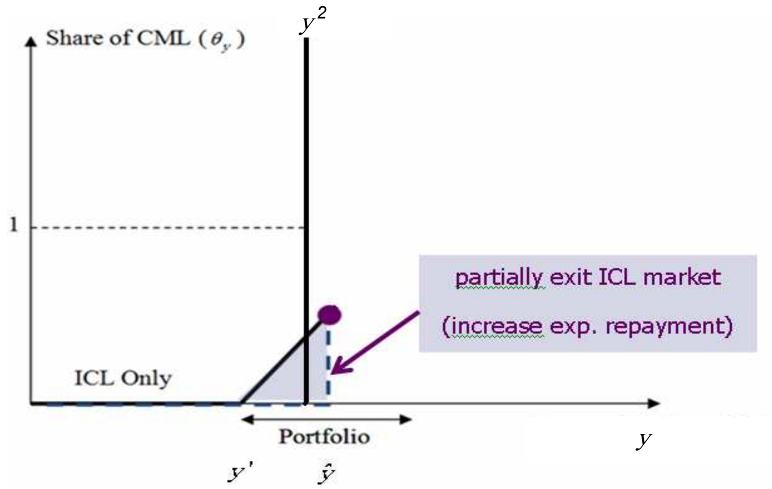

Fig. 5. CML share for $y^2 \in (y', \hat{y})$. The solid line represents PR, and the dashed line denotes CR. Allowing agents to combine ICLs and CMLs, the triangle denotes students who partially exit the ICL program. For simplicity, θ_y is a straight line.

By shifting to a PR, high-signal agents partially exit the ICL program. Consequently, ICL participants are damaged because their borrowing terms deteriorate. Then, adverse selection occurs, and the income distribution becomes less equal. This is the only exception to the Pareto dominance of the PR over the CR.

6. Conclusion

This article introduces the advantages of the basic tool of capital markets—a portfolio to develop efficient higher education funding schemes. First, characterizing the optimal loan decisions, I find that the portfolio set consists of intermediate-ability agents. Agents with better signals or greater wealth prefer larger shares of CMLs than others under CRRA or CARA utility functions. This pattern reverses under a quadratic utility function. Note that similar results can be generated using additional assumptions instead of specific utility functions. Second, I provide key insights into social welfare. In most cases, a PR leads to higher social welfare than a CR; nevertheless, the PR is still inefficient. The selection of students between the two types of loans involves externalities. Agents do not consider their effect on the borrowing terms of all ICLs participants; as a result, aggregate investment in education is sub-optimally high. Moreover, there is adverse selection of students into the ICL program, which damages the borrowing terms in the ICL program. Therefore, it is socially beneficial to increase the ICL share of high-signal groups at the expense of low-signal groups.

The next step is to analyze policies and circumstances that may alleviate the externalities. Suggestions include subsidizing ICLs in various ways. Another key factor is the quality of signals. According to the results, greater income uncertainty may encourage high-signal groups to increase their ICL share. Therefore, a surprising implication of this article is that greater income uncertainty may lead to higher social welfare.

7. Appendix

Definition 1:

a. *The utility function* $u(\tilde{c}) = \frac{\tilde{c}^{1-\gamma}}{1-\gamma}$ *exhibits CRRA.*

The ARA, $-\frac{u''}{u'} = \frac{\gamma}{\tilde{c}}$, is decreasing as income rises.

b. *The utility function* $u(\tilde{c}) = 1 - e^{-\lambda\tilde{c}}$ *exhibits CARA.*

The ARA equals λ.

c. *The utility function* $u(\tilde{c}) = \alpha\tilde{c} - \frac{1}{2}\beta\tilde{c}^2$ *is quadratic.*

The ARA, $\frac{\beta}{\alpha - \beta\tilde{c}}$, is increasing as income rises. Because the utility function is strictly increasing and concave, $\alpha > 0, \quad \beta > 0$, and $\frac{\alpha}{\beta} > \tilde{c}_y, \quad \forall \tilde{c}_y$. ∘

Proof of proposition 1:

Take

(17) $\tilde{c}_{y,\lambda} = \lambda\tilde{c}_{y,1} + (1-\lambda)\tilde{c}_{y,0}$, for all $\lambda \in (0,1)$.

Then, substituting $\tilde{c}_{y,0}$, $\tilde{c}_{y,1}$ in Eq. (17), I obtain

$\tilde{c}_{y,\lambda} = \tilde{c}_y(\theta_y = \lambda) = A\omega + \tilde{a}_y\omega - \lambda R - (1-\lambda)\frac{\tilde{a}_y}{\bar{a}}R$, for all $\lambda \in (0,1)$.

Now, because the utility function is concave, it satisfies the following:

$E\left[u\left(\tilde{c}_{y,\lambda}\right)\right] > \lambda E\left[u\left(\tilde{c}_{y,0}\right)\right] + (1-\lambda) E\left[u\left(\tilde{c}_{y,1}\right)\right].$

This inequality can be rewritten as

$$(18) \qquad E\left[u\left(\tilde{c}_{y,\lambda}\right)\right] > \lambda\left(E\left[u\left(\tilde{c}_{y,0}\right)\right] - E\left[u\left(\tilde{c}_{y,1}\right)\right]\right) + E\left[u\left(\tilde{c}_{y,1}\right)\right] \text{ or}$$
$$E\left[u\left(\tilde{c}_{y,\lambda}\right)\right] > (1-\lambda)\left(E\left[u\left(\tilde{c}_{y,1}\right)\right] - E\left[u\left(\tilde{c}_{y,0}\right)\right]\right) + E\left[u\left(\tilde{c}_{y,0}\right)\right].$$

Now, recall that the cutoff signal $\hat{y}$ satisfies Eq. (9). Substituting $E\left[u\left(\tilde{c}_{\hat{y},0}\right)\right] = E\left[u\left(\tilde{c}_{\hat{y},1}\right)\right]$ in Eq. (18) implies that $E\left[u\left(\tilde{c}_{\hat{y},\lambda}\right)\right] > E\left[u\left(\tilde{c}_{\hat{y},1}\right)\right]$ and $E\left[u\left(\tilde{c}_{\hat{y},\lambda}\right)\right] > E\left[u\left(\tilde{c}_{\hat{y},0}\right)\right]$ for all $\lambda \in (0,1)$. ∘

Lemma 1:
$cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)$ is negative and becomes more negative if θ_y rises.

<u>Proof</u>: The relationship between the expected consumption and the signal $\partial\bar{c}_y / \partial\bar{a}_y = \omega - \frac{\left(1-\theta_y\right)R}{\bar{a}}$ is positive (assumption 2). The $cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)$ is negative because agents are risk averse. An increase in θ_y amplifies the relationship between the signal and the expected consumption, $\partial^2\bar{c}_y / \partial\bar{a}_y\partial\theta_y = \frac{R}{\bar{a}} > 0$. ∘

Proof of proposition 2:

The first term in Eq. (14) is negative or zero because $\bar{a}_y \le \bar{a}$. The second term is negative according to Lemma 1 in the Appendix. ∘

Lemma 2: Assume that the utility function is quadratic and y satisfies $y > \bar{a}$. *Then,* $k > 0$.

<u>Proof</u>: The expected consumption (see Eq. (5)) is given by

$$\begin{aligned} & E\left(\tilde{c}_y\right) = A\omega + E\left(\tilde{a}_y\right)\left(\omega - \left(1-\theta_y\right)\frac{R}{\bar{a}}\right) - \theta_y R \\ (19) \qquad & > A\omega + \bar{a}\left(\omega - \left(1-\theta_y\right)\frac{R}{\bar{a}}\right) - \theta_y R \\ & = \left(A + \bar{a}\right)\omega - R \end{aligned}$$

The inequality derives from $y > \bar{a}$ and assumption 2. Recall that the utility function is strictly increasing in $\tilde{c}_y$; that is, $\frac{\alpha}{\beta} > \tilde{c}_y, \quad \forall \tilde{c}_y$. In particular,

$$(20) \qquad \frac{\alpha}{\beta} > E\left(\tilde{c}_y\right).$$

Combining Eqs. (19) and (20), $\frac{\alpha}{\beta} > \left(A + \bar{a}\right)\omega - R$. That is, $k > 0$ (recall that k is defined in Eq. (21)). ∘

Proof of proposition 4:

The cutoff signals between a portfolio and ICL (CML) only are $y'_{1,2}-\bar{a}=\frac{A'}{2C}\pm\sqrt{\left(\frac{A'}{2C}\right)^2-B}$ and $y''_{1,2}-\bar{a}=\frac{A'}{2(C+1)}\pm\sqrt{\left(\frac{A'}{2(C+1)}\right)^2-\sigma^2}$, respectively (definition 3). θ_y, given by Eq. (16), is positive if $-C(y-\bar{a})^2+A'(y-\bar{a})-C\sigma^2>0$. Assumption 4 ensures that there are two positive roots to this quadratic equation, given by $y'_1-\bar{a}<y'_2-\bar{a}$. θ_y is concave because $C>0$. Thus, θ_y assumes positive values between the roots. The maximum level of θ_y derives from $\frac{\partial\theta_y}{\partial(y-\bar{a})}=\frac{-A\left((y-\bar{a})^2-\sigma^2\right)}{\left((y-\bar{a})^2+\sigma^2\right)^2}=0$. That is, $y-\bar{a}=\pm\sqrt{\sigma^2}$. ◦

Proof of proposition 5:

To simplify the presentation, I define $A'=\frac{k\bar{a}}{R}$ and $C=\frac{\bar{a}\omega-R}{R}$. Substituting *A'* and *C* in Eq. (16) and rearranging, θ_y is non-positive for all *y* if $-C(y-\bar{a})^2+A'(y-\bar{a})-(\sigma^2)C\le 0$. There is a single root to this quadratic equation or no root if $A'^2\le 4\sigma^2C^2$. Its concavity follows from $C>0$. ◦

Proof of corollary 3:

(a) (b) see proof of proposition 5.

(c) θ_y, given by Eq. (16), equals '1' (recall that it cannot exceed 1) if $-(C+1)(y-\bar{a})^2+A'(y-\bar{a})-\sigma^2(C+1)\ge 0$. There is a single root or two positive roots to this quadratic equation, given by $y''_{1,2}-\bar{a}$, if $A'^2\ge 4\sigma^2(C+1)^2$. $\theta_y=1$ between the roots because θ_y is concave.

Proof of proposition 6:

(a) The returns of an agent in signal group y from acquiring higher education, $\bar{a}_y\omega$, is the expected excess income above his or her income as a high school graduate. The cost of higher education is normalized to '1', which in future values equals R. Thus, the expected net returns of signal group y is $\bar{a}_y\omega-R$. Accordingly, it is efficient that only students with signals higher than $y_e=\frac{R}{\omega}$ will acquire higher education. However, according to assumption 2, investment in higher education through ICL is beneficial for *all* students. As a result, all students acquire higher education. ◦

(b) Suppose that the utility function exhibits CRRA or CARA. First, I prove that $\frac{\partial W(y')}{\partial y'}>0$. Then, I prove that $\frac{\partial W(y'')}{\partial y''}>0$. Aggregation of the mean consumption (5) in each signal group yields

$$\int_{\underline{y}}^{\overline{y}} \overline{c}_y \nu(y)\,dy = \int_{\underline{y}}^{y'} \left(A\omega + \overline{a}_y \left(\omega - \frac{R}{\overline{a}} \right) \right) \nu(y)\,dy + \int_{y'}^{y''} \left(A\omega + \overline{a}_y \left(\omega - \frac{(1-\theta_y) R}{\overline{a}} \right) - \theta_y R \right) \nu(y)\,dy + \int_{y''}^{\overline{y}} \left(A\omega + \overline{a}_y \omega - R \right) \nu(y)\,dy$$

$$(21) \qquad = \int_{\underline{y}}^{\overline{y}} \left(A\omega + \overline{a}_y \omega - R \right) \nu(y)\,dy + R \left(\int_{\underline{y}}^{y'} \left(1 - \frac{\overline{a}_y}{\overline{a}} \right) \nu(y)\,dy + \int_{y'}^{y''} \left(1 - \frac{(1-\theta_y)\overline{a}_y}{\overline{a}} - \theta_y \right) \nu(y)\,dy \right)$$

$$= \int_{\underline{y}}^{\overline{y}} \left(A\omega + \overline{a}_y \omega - R \right) \nu(y)\,dy + \frac{R}{\overline{a}} \left(\underbrace{\overline{a} \int_{\underline{y}}^{y''} (1-\theta_y) \nu(y)\,dy - \int_{\underline{y}}^{y''} \overline{a}_y (1-\theta_y) \nu(y)\,dy}_{=0 \ (4)} \right)$$

I obtain the last two equalities because signal groups $\left[\underline{y}, y'\right]$ choose ICL only (i.e., $\theta_y = 0$) and because the ICL program breaks even according to equality (4). Note that equality (Eq. 4) equals $\overline{a} = \int_{\underline{y}}^{y''} (1-\theta_y)\overline{a}_y \nu(y)\,dy \Big/ \int_{\underline{y}}^{y''} (1-\theta_y) \nu(y)\,dy$ in the case of CRRA or CARA utility functions because signal groups with $(y'', \overline{y})$ choose CMLs only ($\theta_y = 1$). Because the right-hand side of equality (21) is independent of y', differentiation with respect to y' yields

$$(22) \qquad \frac{R(\partial \overline{a}/\partial y')}{\overline{a}^2} \left(\int_{\underline{y}}^{y'} \overline{a}_y \nu(y)\,dy + \int_{y'}^{y''} \left((1-\theta_y)\overline{a}_y \right) \nu(y)\,dy \right) = \nu(y') \left(\overline{c}_{y',\theta} - \overline{c}_{y',0} \right).$$

Now, increasing y' changes the size of the relevant sets and increases $\overline{a}$, which damages the borrowing terms for ICL participants. Differentiating the welfare function

$$W = \int_{\underline{y}}^{y'} \upsilon\left(\overline{c}_{y,0}\right) \nu(y)\,dy + \int_{y'}^{y''} \upsilon\left(\overline{c}_{y,\theta}\right) \nu(y)\,dy + \int_{y''}^{\overline{y}} \upsilon\left(\overline{c}_{y,1}\right) \nu(y)\,dy$$

using Eqs. (7) and (8) yields

$$\frac{\partial W}{\partial y'} = \nu(y') \left(\upsilon\left(\overline{c}_{y',0}\right) - \upsilon\left(\overline{c}_{y',\theta}\right) \right) +$$

$$\frac{R(\partial \overline{a}/\partial y')}{\overline{a}^2} \left(\int_{\underline{y}}^{y'} \left(\partial \upsilon\left(\overline{c}_{y,0}\right) / \partial \overline{c}_{y,0} \right) \overline{a}_y \nu(y)\,dy + \int_{y'}^{y''} \left(\partial \upsilon\left(\overline{c}_{y,\theta}\right) / \partial \overline{c}_{y,\theta} \right) \left((1-\theta_y)\overline{a}_y \right) \nu(y)\,dy \right)$$

$$> \nu(y') \left(\upsilon\left(\overline{c}_{y',0}\right) - \upsilon\left(\overline{c}_{y',\theta}\right) \right) +$$

$$\left(\partial \upsilon\left(\overline{c}_{y'',\theta}\right) / \partial \overline{c}_{y'',\theta} \right) \frac{R(\partial \overline{a}/\partial y')}{\overline{a}^2} \left(\int_{\underline{y}}^{y'} \overline{a}_y \nu(y)\,dy + \int_{y'}^{y''} \left((1-\theta_y)\overline{a}_y \right) \nu(y)\,dy \right)$$

$$\overset{(22)}{=} \nu(y') \left(\upsilon\left(\overline{c}_{y',0}\right) - \upsilon\left(\overline{c}_{y',\theta}\right) \right) + \left(\partial \upsilon\left(\overline{c}_{y'',\theta}\right) / \partial \overline{c}_{y'',\theta} \right) \nu(y') \left(\overline{c}_{y',\theta} - \overline{c}_{y',0} \right)$$

$$> 0$$

I assume that the frontier theorem holds. That is, I ignore feedback effects of $\overline{a}$ on the optimal choices of θ_y, because $\frac{\partial W(y')}{\partial \theta_y} \frac{\partial \theta_y}{\partial \overline{a}} \frac{\partial \overline{a}}{\partial y'} = 0$. The first and second inequalities follow from MLRP ($\overline{c}_y$ is strictly increasing in y; see assumption 1) and the concavity of $\upsilon(\cdot)$ (see assumption 3). Recall that the cutoff signal y' is indifferent between ICL only and a portfolio (definition 3). Thus, because of the risk aversion, $\overline{c}_{y',\theta} > \overline{c}_{y',0}$.

Now, I prove that $\frac{\partial W(y'')}{\partial y''} > 0$. Differentiation of equality (21) with respect to y'' yields

$$(23)\quad \frac{R(\partial\bar{a}/\partial y'')}{\bar{a}^2}\left(\int_{y'}^{y''}\left((1-\theta_y)\bar{a}_y\right)v(y)dy\right) = v(y'')\left(\bar{c}_{y'',1}-\bar{c}_{y'',\theta}\right).$$

Differentiating the welfare function (13)

$$W = \int_{\underline{y}}^{y'} \upsilon\left(\bar{c}_{y,0}\right)v(y)dy + \int_{y'}^{y''} \upsilon\left(\bar{c}_{y,\theta}\right)v(y)dy + \int_{y''}^{\bar{y}} \upsilon\left(\bar{c}_{y,1}\right)v(y)dy$$

yields

$$\begin{aligned}
&\frac{\partial W}{\partial y''} = v(y'')\left(\upsilon\left(\bar{c}_{y'',\theta}\right)-\upsilon\left(\bar{c}_{y'',1}\right)\right)+\\
&\frac{R(\partial\bar{a}/\partial y'')}{\bar{a}^2}\left(\int_{y'}^{y''}\left(\partial\upsilon\left(\bar{c}_{y,\theta}\right)/\partial\bar{c}_{y,\theta}\right)\left((1-\theta_y)\bar{a}_y\right)v(y)dy\right)\\
&> v(y'')\left(\upsilon\left(\bar{c}_{y'',\theta}\right)-\upsilon\left(\bar{c}_{y'',1}\right)\right)+\\
&\left(\partial\upsilon\left(\bar{c}_{y'',\theta}\right)/\partial\bar{c}_{y'',\theta}\right)\frac{R(\partial\bar{a}/\partial y'')}{\bar{a}^2}\left(\int_{y'}^{y''}\left((1-\theta_y)\bar{a}_y\right)v(y)dy\right)\\
&\overset{(23)}{=} v(y'')\left(\upsilon\left(\bar{c}_{y'',\theta}\right)-\upsilon\left(\bar{c}_{y'',1}\right)\right)+\left(\partial\upsilon\left(\bar{c}_{y'',\theta}\right)/\partial\bar{c}_{y'',\theta}\right)v(y'')\left(\bar{c}_{y'',1}-\bar{c}_{y'',\theta}\right)\\
&> 0
\end{aligned}$$

The explanation is similar to the previous proof of $\frac{\partial W}{\partial y'} > 0$. If the utility function is quadratic, then $\frac{\partial W(y'_1)}{\partial y'_1} > 0$, $\frac{\partial W(y''_1)}{\partial y''_1} > 0$, and $\frac{\partial W(y'_2)}{\partial y'_2} < 0$, $\frac{\partial W(y''_2)}{\partial y''_2} < 0$. The proof is similar with the necessary changes. (The proof is available from the author on request.)

Proof of proposition 7:

(a) To prove that the PR Pareto dominates the CR, I verify that $\bar{a}$ increases, which improves the borrowing terms of ICL (see discussion in section 5). Shifting from a CR to a PR, the change in $\bar{a}$ (4) is measured by

$$(24)\quad \begin{aligned}\Delta\bar{a} &= \frac{E^{PR}\left[(1-\theta_y)\bar{a}_y\right]}{E^{PR}\left[1-\theta_y\right]} - E^{CR}\left[\bar{a}_y\middle|\theta_y=0\right]\\ &> E^{PR}\left[(1-\theta_y)\bar{a}_y\right]-E^{CR}\left[\bar{a}_y\middle|\theta_y=0\right]\end{aligned}.$$

The equality derives because in a CR, $\bar{a} = E\left[\bar{a}_y\middle|\theta_y=0\right]$. The inequality derives because $E^{PR}\left[1-\theta_y\right] < 1$.

First, assume that the utility function exhibits CRRA or CARA with $\bar{y} > y''$. Then, θ_y is an increasing function of y (proposition 3). Therefore, shifting from a CR to a PR, agents $[y',\hat{y}]$ partially exit the ICL program, while agents $[\hat{y},y'']$ partially enter the ICL program (definition 3); that is,

$$
\begin{aligned}
&\Delta\bar{a} > E_{\hat{y}}^{y''}\left[\left(1-\theta_y\right)\bar{a}_y\right] - E_{y'}^{\hat{y}}\left[\theta_y\bar{a}_y\right] \\
&= E_{\hat{y}}^{y''}\left[\bar{a}_y\right] - E_{y'}^{y''}\left[\theta_y\bar{a}_y\right] \\
&> 0
\end{aligned}
$$

The inequality derives because MLRP implies that $E_{\hat{y}}^{y''}\left[\bar{a}_y\right] > E_{y'}^{y''}\left[\bar{a}_y\right]$ and $\theta_y < 1$. Now, if $\bar{y} < y''$, then the set $\left[\hat{y}, y''\right]$ becomes $\left[\hat{y}, \bar{y}\right]$ and the proof is identical.

Second, assume that the utility function is quadratic. If $\bar{y} \le \bar{a} + \sqrt{\sigma^2\tilde{\varepsilon}}$, then θ_y is an increasing function of y (see proposition 4). Therefore, the proof is similar to part (a), where y'_1, $\hat{y}_1$ and y''_1 correspond to y', $\hat{y}$ and y''. If $\sqrt{\sigma^2\tilde{\varepsilon}} + \bar{a} < \bar{y} \le \hat{y}_2$, then an additional set $\left[y''_2, \bar{y}\right]$ partially enters the ICL program and further increases $\bar{a}$. Thus, $\Delta\bar{a}$ is even larger. Now, assume that $\bar{y} > y'_2$. Then, shifting from a CR to a PR, agents $\left[y'_1, \hat{y}_1\right]$ and $\left[\hat{y}_2, y'_2\right]$ partially exit the ICL program and reduce $\bar{a}$, whereas agents $\left[\hat{y}_1, y''_1\right]$ and $\left[y''_2, \hat{y}_2\right]$ partially enter the ICL program and increase $\bar{a}$. Use of Eq. (24) yields

$$
\begin{aligned}
&\Delta\bar{a} > E_{\hat{y}_1}^{y''_1}\left[\left(1-\theta_y\right)\bar{a}_y\right] + E_{y''_2}^{\hat{y}_2}\left[\left(1-\theta_y\right)\bar{a}_y\right] - \left(E_{y'_1}^{\hat{y}_1}\left[\theta_y\bar{a}_y\right] + E_{\hat{y}_2}^{y'_2}\left[\theta_y\bar{a}_y\right]\right) \\
&= E_{\hat{y}_1}^{y''_1}\left[\bar{a}_y\right] + E_{y''_2}^{\hat{y}_2}\left[\bar{a}_y\right] - \left(E_{y'_1}^{y''_1}\left[\theta_y\bar{a}_y\right] + E_{y''_2}^{y'_2}\left[\theta_y\bar{a}_y\right]\right) \\
&= E_{\hat{y}_1}^{\hat{y}_2}\left[\bar{a}_y\right] - E_{y'_1}^{y'_2}\left[\theta_y\bar{a}_y\right] \\
&\rightarrow E_{\hat{y}_1}^{\hat{y}_2}\left[\bar{a}_y\right] \\
&> 0
\end{aligned}
$$

I obtain the second equality by adding and subtracting $E_{y''_1}^{y''_2}\left[\bar{a}_y\right]$ (recall that signal group $\left[y''_1, y''_2\right]$ chooses CMLs only; therefore, $E_{y''_1}^{y''_2}\left[\bar{a}_y\right] = E_{y''_1}^{y''_2}\left[\theta_y\bar{a}_y\right]$). To derive the last inequality, I use assumption 1 and integration by parts to obtain

$$
\begin{aligned}
&E_{y'_1}^{y'_2}\left[\theta_y\bar{a}_y\right] = \int_{y'_1}^{y'_2}\left(\theta_y y\right)v\left(y\right)dy \\
&= \left[y\left(\frac{\theta_y{}^2}{2\left(\partial\theta_y/\partial y\right)}\right)\right]_{y'_1}^{y'_2} - \int_{y'_1}^{y'_2}\left(\frac{\theta_y{}^2}{2\left(\partial\theta_y/\partial y\right)}\right)v\left(y\right)dy \\
&= \left[y\left(\frac{\theta_y{}^2}{2\left(\partial\theta_y/\partial y\right)}\right) - \frac{\theta_y{}^3}{6\left(\partial\theta_y/\partial y\right)^2}\right]_{y'_1}^{y'_2} \longrightarrow 0
\end{aligned}
$$

Recall from proposition 4 that $\partial\theta_{y'_1}/\partial y > 0, \quad \partial\theta_{y'_2}/\partial y < 0, \quad \theta_{y'_{1,2}} = 0$. Note that if $\hat{y}_2 < \bar{y} < y'_2$, then the set $\left[\hat{y}_2, y'_2\right]$ that partially exits the ICL program reduces to $\left[\hat{y}_2, \bar{y}\right]$. In this case, $\Delta\bar{a}$ is even larger.

(b) Recall that in both equilibria, all students invest in higher education. Therefore, aggregate consumption is identical in both equilibria; that is,

$$
(1) \qquad E\left[\bar{c}_y^{PR}\right] = E\left[\bar{c}_y^{CR}\right],
$$

where consumption in signal group y is defined in Eq. (5). Because $y^2 \in (y', \hat{y})$, under a CR all agents choose ICL only. Implementing a PR, agents $[y', y^2]$ partially exit the ICL program. Therefore, it can be easily verified that

$$(2) \qquad \bar{c}_y^{CR} \overset{(\leq)}{>} \bar{c}_y^{PR}, \quad if\ \bar{a}_y \overset{(\geq)}{<} \bar{a}^{CR}.$$

Eqs. (1) and (2) together imply that $\bar{c}_y^{PR}$ is a mean-preserving spread of $\bar{c}_y^{CR}$, from which I conclude

$$W^{CR} = \int_{\underline{y}}^{\bar{y}} \upsilon\left(\bar{c}_y^{CR}\right) v(y)\, dy > \int_{\underline{y}}^{\bar{y}} \upsilon\left(\bar{c}_y^{PR}\right) v(y)\, dy = W^{PR}.$$

Thus, CR dominates PR in welfare terms.

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